Math 51 - Autumn 2010 - Midterm Exam 2

Name: ______________________________

Student ID: ______________________________

Select your section:

Brandon Levin 05 (1:15-2:05) 15 (11:00- 11:50)	Amy Pang 14 (10:00-10:50) 17 (1:15-2:05)	Yuncheng Lin 06 (1:15-2:05) 21 (11:00-11:50 AM)	Rebecca Bellovin 09 (11:00-11:50) 23 (1:15-2:05)
Xin Zhou 02 (11:00-11:50) 08 (10:00- 10:50)	Simon Rubinstein-Salzedo 18 (2:15-3:05) 24 (1:15-2:05)	Frederick Fong 20 (10:00-10:50) 03 (11:00-11:50)	Jeff Danciger ACE (1:15-3:05)

Signature: ______________________________

Instructions:

- Print your name and student ID number, select your section number and TA's name, and **sign above to indicate that you accept the Honor Code**.
- There are nine problems on the pages numbered from 1 to 11, and each problem is worth 10 points. Please check that the version of the exam you have is complete and correctly stapled.
- Read each question carefully. **In order to receive full credit, please show all of your work and justify your answers unless specifically directed otherwise. If you use a result proved in class or in the text, you must clearly state the result before applying it to your problem.**
- Unless otherwise specified, you may assume all vectors are written in standard coordinates.
- You have 2 hours. This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of the teaching staff.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner.

Problem 1. Let $P : \mathbf{R}^2 \to \mathbf{R}^2$ be projection to the line $y = 2x$. (This problem continues on the next page.)

a) Find a basis for $\mathbf{R}^2$ consisting of eigenvectors of P.

b) Let $\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \end{bmatrix} \right\}$. Find the matrix A which represents the linear transformation T in terms of $\mathcal{B}$ coordinates.

(That is, find A such that $[T(\mathbf{x})]_{\mathcal{B}} = A[\mathbf{x}]_{\mathcal{B}}$ for all $\mathbf{x}$ in $\mathbf{R}^2$.)

Problem 2. Let A be the 3×3 matrix

$$A = \begin{bmatrix} a & 2 & b \\ 1 & 1 & 0 \\ c & -2 & d \end{bmatrix}$$

Assume that

$$\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

are eigenvectors of A. (This problem continues on the next page.)

a) Find the values of a, b, c, and d.

b) What are the eigenvalues of A?

c) For each eigenvalue you identified in Part (b), give a corresponding eigenvector of A.

Problem 3.

a) Define *positive definite quadratic form.*

b) Let $\{\mathbf{u},\ \mathbf{v}\}$ be a linearly independent set of vectors in $\mathbf{R}^n$. Show that the quadratic form generated by the symmetric matrix

$$A = \begin{bmatrix} \mathbf{u}\cdot\mathbf{u} & \mathbf{u}\cdot\mathbf{v} \\ \mathbf{v}\cdot\mathbf{u} & \mathbf{v}\cdot\mathbf{v} \end{bmatrix}$$

is positive definite.

(You may find the following fact helpful: If A and B are two matrices that can be multiplied, then $(AB)^T = B^T A^T$.

However, it is also possible to prove the statement without using this fact.)

Problem 4. Determine whether or not the following matrices are diagonalizable. Justify your answer.

a) $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 5 \end{pmatrix}$

b) $\begin{pmatrix} -1 & 2 \\ -3 & 5 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

Problem 5. Please clearly indicate your answer for each of the questions below. You do not need to justify your work.

a) Let A be a 3×3 matrix with eigenvalues $1, 2$, and -1. What are the eigenvalues of $5A$?

b) Let $A = \begin{pmatrix} 3 & -1 \\ 0 & 4 \end{pmatrix}$. Find the eigenvalues of A^{-10}.

c) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation which rotates each vector counterclockwise by an angle of $\frac{\pi}{3}$ and then projects it onto the line $y = 6x$. If A is a matrix which represents T, what is the determinant of A?

Problem 6.
Let P be the plane

$$\left\{ s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \;\middle|\; s, t \in \mathbf{R} \right\}.$$

Find a function G with the property that $G^{-1}(1) = P$.

Problem 7. For each of the limits below, decide whether the limit exists or not. Justify your answer.

a)

$$\lim_{(x,y)\to(0,0)} \frac{x\sqrt{|y|}}{\sqrt{x^2+y^2}}$$

b)

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}.$$

Problem 8. For each function listed below, choose one of the following statements:

Each non-empty level set is a line (**L**).
Each non-empty level set is a circle (**C**).
Not all the level sets are lines or circles (**N**).

Note that a single point is considered to be a circle of radius 0. You do not need to justify your answers.

a) $r(x, y) = e^{2x+y}$

b) $g(x, y) = \sin^2(xy) + \cos^2(xy)$

c) $h(x, y) = \sin(xy)$

d) $s(x, y) = (x + y)^2 + (x - y)^2$

e) $f(x, y) = x + (\ln \pi)y$

Problem 9. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be defined by $f(\mathbf{x}) = ||\mathbf{x}||$.

a) Compute $D_f(1, 0)$.

b) Show that $\mathbf{f}$ is not differentiable at $(0, 0)$.

The following boxes are strictly for grading purposes. Please do not mark.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		90