Math 51 First Exam — April 21, 2011

Name: ____ **SUID#:** ______________

Please circle your section:		
Jonathan Campbell 08 (11:00-11:50 AM) 14 (10:00-10:50 AM)	Minyu Peng 03 (11:00-11:50 AM) 11 (1:15-2:05 PM)	Joseph Man Chuen Cheng 06 (1:15-2:05 PM) 17 (2:15-3:05 PM)
Anca Vacarescu ACE (1:15-3:05 PM)	Henry Adams 02 (11:00-11:50 AM) 05 (1:15-2:05 PM)	Fernando Xuancheng Shao 12 (10:00-10:50 AM) 09 (11:00-11:50 AM)

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 90 minutes.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 7 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, May 5**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	**Total**
Points:	12	12	8	12	10	12	12	78
Score:								

1. (12 points)

(a) Give the precise definition of a (linear) subspace V of $\mathbb{R}^n$.

(b) Complete the following sentence: A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is defined to be *linearly independent* if

(c) Suppose $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly independent set of vectors in $\mathbb{R}^7$. Is $\{\mathbf{u} + \mathbf{v} + \mathbf{w},\ 3\mathbf{v} - \mathbf{w},\ 2\mathbf{w}\}$ linearly independent? Justify your answer completely.

2. (12 points) Let Z be the plane in $\mathbb{R}^3$ containing the points $(1, 0, 1)$, $(0, 1, -1)$, and $(1, 2, 3)$.

(a) Find a parametric representation of Z.

(b) Give an equation for Z of the form $ax + by + cz = d$. (Here a,b,c,d are scalars, and x,y,z are the usual variables for coordinates of points in $\mathbb{R}^3$.)

(c) Find the coordinates of a point P in Z having the property that the vector from the origin to P is perpendicular (normal) to Z.

3. (8 points) Compute, showing all steps, the reduced row echelon form of the matrix

$$\begin{bmatrix} 0 & 0 & -1 & 4 & 1 \\ 1 & 2 & 3 & 4 & 3 \\ 2 & 4 & 6 & 2 & 6 \\ 3 & 6 & 10 & 8 & 8 \end{bmatrix}$$

4. (12 points) Suppose that all we know about the 3×4 matrix A is that its entries are all nonzero, and that its reduced row echelon form is

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(\textbf{Note:} this is \textit{not} the matrix A!)}$$

(a) Find a basis for the null space of A; show your reasoning.

(b) If the columns of A, in order, are $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4 \in \mathbb{R}^3$, circle *all* sets of vectors below that give a basis for the column space of A. You do not need to justify your answer(s).

$$\{\mathbf{a}_1\} \quad \{\mathbf{a}_2\} \quad \{\mathbf{a}_3\} \quad \{\mathbf{a}_4\}$$

$$\{\mathbf{a}_1, \mathbf{a}_2\} \quad \{\mathbf{a}_1, \mathbf{a}_3\} \quad \{\mathbf{a}_2, \mathbf{a}_3\} \quad \{\mathbf{a}_1, \mathbf{a}_4\} \quad \{\mathbf{a}_2, \mathbf{a}_4\} \quad \{\mathbf{a}_3, \mathbf{a}_4\}$$

$$\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\} \quad \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_4\} \quad \{\mathbf{a}_1, \mathbf{a}_3, \mathbf{a}_4\} \quad \{\mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$$

$$\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$$

(c) Suppose we also know that the second and third columns of A are, respectively,

$$\mathbf{a}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{a}_3 = \begin{bmatrix} 9 \\ 5 \\ 3 \end{bmatrix}$$

Use this information to find the first column $\mathbf{a}_1$ of A; give your reasoning.

5. (10 points) Each of the statements below is either *always true* ("T"), or *always false* ("F"), or *sometimes true and sometimes false, depending on the situation* ("MAYBE"). For each part, decide which and circle the appropriate choice; you *do not* need to justify your answers.

(a) A set of 4 vectors in $\mathbb{R}^5$ is linearly independent. T F MAYBE

(b) A set of 4 vectors in $\mathbb{R}^5$ spans $\mathbb{R}^5$. T F MAYBE

(c) A set of 5 vectors in $\mathbb{R}^4$ is linearly independent. T F MAYBE

(d) A set of 5 vectors in $\mathbb{R}^4$ spans $\mathbb{R}^4$. T F MAYBE

(e) A set of 5 vectors which spans $\mathbb{R}^5$ is linearly independent. T F MAYBE

(f) A set of 5 linearly independent vectors in $\mathbb{R}^5$ spans $\mathbb{R}^5$. T F MAYBE

(g) The span of 4 vectors in $\mathbb{R}^5$ is a 4-dimensional subspace. T F MAYBE

(h) The span of 5 vectors in $\mathbb{R}^4$ is a 5-dimensional subspace. T F MAYBE

(i) A set of 4 vectors in $\mathbb{R}^4$ forms a basis for $\mathbb{R}^4$. T F MAYBE

(j) A basis for $\mathbb{R}^4$ contains exactly 4 vectors. T F MAYBE

6. (12 points) For each part, provide with reasoning an example of a matrix (A, B, or C, respectively) that satisfies the given property, or briefly explain why no such matrix can exist.

   (a) The linear system $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ has infinitely many solutions.

   (b) The linear system $B \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ has exactly one solution.

   (c) The linear system $C \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ has no solutions.

7. (12 points)

(a) Suppose $\mathbf{S} : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that

$$\mathbf{S}\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\3\\1\end{bmatrix} \quad \text{and} \quad \mathbf{S}\left(\begin{bmatrix}2\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\2\\2\end{bmatrix}$$

Find the matrix of $\mathbf{S}$.

(b) Let $\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that projects vectors onto the line $y = x$. Find the matrix of $\mathbf{T}$.

(c) For $\mathbf{S}$ and $\mathbf{T}$ as above, find the matrix of $\mathbf{S} \circ \mathbf{T}$ or explain why it cannot be defined.

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