Math 51 - Winter 2009 - Midterm Exam I

Name: ____________________________________

Student ID: ________________________________

Select your section:

Penka Georgieva 02 (11:00-11:50 AM) 06 (1:15-2:05 PM)	Anssi Lahtinen 03 (11:00-11:50 AM) 11 (1:15-2:05 PM)	Man Chun Li 12 (1:15-2:05 PM) 08 (11:00-11:50 AM)	Simon Rubinstein-Salzedo 17 (1:15-2:05 PM) 21 (11:00-11:50 AM)
Aaron Smith 09 (11:00-11:50 AM) 20 (10:00-10:50 AM)	Nikola Penev 14 (1:15-2:05 PM) 24 (2:15-3:05 PM)	Eric Malm 15 (11:00-11:50 AM) 23 (1:15-2:05 PM)	Yu-jong Tzeng 51A

Signature: ________________________________

Instructions: Print your name and student ID number, print your section number and TA's name, write your signature to indicate that you accept the honor code. During the test, you may not use notes, books, calculators. Read each question carefully, and show all your work.
There are .....ten.... problems on the pages numbered from 1 to ....14...., with the total of 100 points. Point values are given in parentheses. You have 2 hours (until 9PM) to answer all the questions.

In the exam all vectors are columns, but sometimes we use transpose to write them horizontally.

Thus $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix} = [v_1, v_2, \ldots, v_k]^T$.

Similarly $\mathbf{v}^T$ is a row $[v_1, v_2, \ldots, v_k]$.

The dot product of two vectors is denoted as $\mathbf{v} \cdot \mathbf{w}$.

Problem 1. (10 pts.) Mark as TRUE/FALSE the following statements. If a statement is false, give a simple example. If a statement is true, give a justification.

a) The null space of the matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is $\mathbb{R}^2$ TRUE FALSE

b) The cross product of two vectors belong to the plane spanned by them. TRUE FALSE

c) Let A be a 2×4 matrix. Then $\dim N(A) \geq 2$. TRUE FALSE

d) Null space $N(A)$ of an $n \times k$ matrix A is a subspace of $\mathbb{R}^n$. TRUE FALSE

e) Span $\{\mathbf{v_1}, \mathbf{v_2}\}$ is always two dimensional linear subspace. TRUE FALSE

Problem 2. (12 pts.) Consider the following system of equations:

$$\begin{cases} x + 3y = 1 \\ 2x + a \cdot y = 2 \end{cases}$$

where x and y are unknowns, and a is some real number.

a) For what values of a the above system of equations has exactly one solution?

b) For what values of a the above system of equations has exactly two solutions?

c) For what values of a the above system of equations has more than two solutions?

Problem 3. (10 pts.) a) For what values of a is the set

$$\operatorname{Span}\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\a\end{bmatrix}\right)$$

a linear subspace?

b) For given number a let V_a be the translate of $\operatorname{Span}\left(\begin{bmatrix}2\\5\end{bmatrix}\right)$ by the vector $\begin{bmatrix}1\\a\end{bmatrix}$, i.e.

$$V_a = \begin{bmatrix}1\\a\end{bmatrix} + \operatorname{Span}\left(\begin{bmatrix}2\\5\end{bmatrix}\right)$$

For what number(s) a is the V_a a linear subspace?

Problem 4. (12 pts.) Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

where a, b, c are real numbers.

1. Give a condition on a, b, c to ensure that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly dependent.

2. Give a condition on a, b, c to ensure that $\mathbf{v}_3$ is perpendicular to $\mathbf{v}_1$ and $\mathbf{v}_2$.

3. Use the preceeding question to give an equation of the plane passing through the origin with directions $\mathbf{v}_1, \mathbf{v}_2$.

Problem 5. (12 pts.) Let $\mathbf{e_1}$ and $\mathbf{e_2}$ be the standard basis of $\mathbb{R}^2$. Show that

$$\{2\mathbf{e_1} + \mathbf{e_2}, \mathbf{e_1} - 3\mathbf{e_2}\}$$

is also a basis of $\mathbb{R}^2$.

Problem 6. (10 pts.) Let $\mathbf{u} = [1, -1, 1, -1]^T$ and $\mathbf{w} = [0, 3, 3, 1]^T$.
a) Find the cosine of the angle between the vectors $\mathbf{u}$ and $\mathbf{w}$.
(It is OK to leave the answer in the form like "$\frac{\sqrt{12+345^6}}{789}$".)

b) Find the numbers a and b such that the vector $[2, 4, a, b]^T$ is in the $\mathrm{Span}\,(\mathbf{u}, \mathbf{w})$.

Problem 7. (12 pts.) Let $\mathbf{v} = [1, 0, -1]^T$.
a) Show that the set $V = \{\mathbf{x} \in \mathbb{R}^3 | \mathbf{x} \cdot \mathbf{v} = 0\}$ is a linear subspace.

b) Find a matrix A such that $N(A) = V$.

c) Find a matrix A such that $C(A) = V$.

Problem 8. (12 pts.) For a given matrix

$$A = \begin{bmatrix} 2 & 4 & -1 & 3 & 1 & -1 \\ -1 & -2 & 3 & -3 & 2 & 3 \\ 1 & 2 & 0 & -2 & 1 & 0 \\ 2 & 4 & 2 & -2 & 4 & 2 \end{bmatrix} \qquad \text{with} \qquad \text{rref}\,(A) = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(you don't have to verify that rref (A) is equal to the above matrix.)
a) find a basis of $N(A)$.

b)Find **all** solutions to

$$A\mathbf{x} = [-1, 3, 0, 2]^T$$

Notice that the right hand side of this equation is equal to one of the columns of A.

c) find a basis of $C(A)$.

Problem 9. (10 pts.) Let $\mathbf{v_1} = [1, 1, -1]^T$ and $\mathbf{v_2} = [3, 2, 1]^T$.
a)Check if $[1, 0, 0]^T$ is in the $\text{Span}\,\{\mathbf{v_1}, \mathbf{v_2}\}$.

b) Using the fact that the vector $\mathbf{w} = [0, 1, 0]^T$ is not in the $\text{Span}\,\{\mathbf{v_1}, \mathbf{v_2}\}$, write all solutions to the system of equations:

$$\begin{cases} x + 3y = 0 \\ x + 2y = 1 \\ -x + y = 0 \end{cases}$$

Question	Score	Maximum
1		10
2		10
3		12
4		12
5		10
6		12
7		12
8		12
9		10
Total		100