MATH 51 FINAL, SUMMER 2014

Your name: ______________________________

Your Stanford University ID: ______________________

Please sign your name to the following: "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all aspects of the honor code with regards to this examination."

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- There are scratch pages at the end of the exam, and you may use the backs of pages. You may attach more white paper. However we will **not** give credit to work written in blue books.
- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers (unless told otherwise). You do not need to simplify your work unless instructed to do so. You may use any result proved in class or the text, unless asked to reprove it. Be sure to clearly state a result before using it, and to verify that all hypotheses are satisfied.
- Please fill out this cover sheet now. We will signal the time when you may open this booklet and after **180 minutes** we will signal the time when you must stop writing in it.
- Your exam and all scratch paper must remain inside the exam room, you may leave them on your desk if you wish to take breaks or use the restroom.
- Please check that your copy of this exam contains 17 *numbered* pages and is correctly stapled.
- We will keep this exam for our records, for a year after you are done with the course. You may contact me, `egoodman@math.stanford.edu`, to view it (if you are at Stanford) and we will make arrangements.

Problem	Points earned	Total possible
1		8
2		16
3		10
4		10
5		12
6		12
7		8
8		12
9		10
10		13
11		18
12		9
13		12
Total		150

(1) (8 points, 2 parts)

(a) Find an eigenvector with eigenvalue 1 for the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ -2 & 0 & 3 \end{bmatrix}$.

(b) Find an eigenvector with eigenvalue 3 for the matrix $A^{-1} + A + I$. (No additional computation should be needed, but you must justify your answer).

(2) (16 points, 4 parts) Find the limit (with justification) or show the limit doesn't exist.

(a)

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$$

(b)

$$\lim_{(x,y)\to(1,1)} \frac{y^2-1}{x^2-1}$$

(c)

$$\lim_{y\to 0} \frac{x\sin((\pi/4)+y) - x\sin(\pi/4)}{y}$$

For this problem assume $x \neq 0$ and express your answer in terms of x.

(d)

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{y}$$

(3) (10 points, 2 parts)

Consider the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2t & t & 2t \\ 2t+1 & 0 & 2t \end{bmatrix}$.

(a) For what values of t is A invertible?

(b) Find a value of t where A is not invertible. For this t, find the eigenvalues of A.

(4) (10 points, 2 parts)

Let A be a 2×3 matrix such that the set of solutions to $A\vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is the set of vectors of the form

$$\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1c \\ 0 \\ 2c \end{bmatrix}$$

where $c \in \mathbb{R}$.

(a) What is the nullspace of A?

(b) Find an example of a matrix A that has the given set of solutions for the given equation $A\vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

(5) (12 points)

Using the multivariable chain rule, find the total derivative matrix of $g \circ f$ at $(-1, 2, 2)$ where

$$g\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x^2 + y \\ ze^y - x \end{bmatrix}$$

and

$$f\left(\begin{bmatrix} r \\ s \\ t \end{bmatrix}\right) = \begin{bmatrix} r - t \\ s - t \\ t \end{bmatrix}.$$

(You may use matrices or tree diagrams to calculate the final answer.)

(6) (12 points, 2 parts)

(a) Let C be the surface $4x^2 - yz + z^2 = 2$. Find the tangent plane to C at $(0, 1, 2)$.

(b) Find the tangent plane to the graph of $f(x, y) = e^{xy} - x + y$ at the point $(\vec{a}, f(\vec{a}))$ where $\vec{a} = (2, 0)$.

(7) (8 points) Show that if v_1, v_2, v_3 are three non-zero orthogonal vectors, they are linearly independent.

(8) (12 points) Suppose you have a parallelogram with sides described by the vectors $\vec{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$. You are allowed to move the endpoint of $\vec{w}$ in some direction. In what direction should you move $\vec{w}$ so that the area of the parallelogram is increasing as fast as possible?

Please give a unit vector as your answer.

(9) (10 points, 2 parts)

(a) Find an example of a parametrized curve that lies on $y^2 + z^2 = 1 + x^2$ and goes through the point $(1, 1, 1)$.

(b) Find the tangent line to the image of your parametrized curve at $(1, 1, 1)$.

(10) (13 points)

Find the maximum and minimum values of y^2 on the domain $D^2 = \{(x, y) \in \mathbb{R}^2 : x^2 + xy + y^2 \leq 3.\}$

You may use any methods you wish to solve this problem, but show your reasoning clearly.

(11) (18 points, 3 parts)

Let $f(x, y) = x^2 + 3xy + y^2$.

(a) Find and classify any critical points of f.

(b) At the point $(2, 1)$ in what direction is f decreasing most rapidly? Also, find a direction in which f is not increasing or decreasing. (Specify which answer is which! Your answers do not have to be unit vectors.)

(c) Find the linearization and 2nd degree Taylor approximations of $f(x, y) = x^2 + 3xy + y^2$ at $\vec{a} = (2, 1)$. **Simplify** your answers. (Meaning for example, you should have one answer in the form $Lf(x, y) = c_1x + c_2y + c_3$, and T_2f also simplified.)

(12) (9 points, 3 parts)

In the following examples, is S a subspace? If so, find a basis; if not, give a justification.

(a) $S = \{x, y, z : 2x - z = -1\}$

(b) $S = \{x, y, z : \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ is orthogonal to } \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}\}.$

(c) $S = \{x, y : x^2 - y^2 = 0\}.$

(13) (12 points, 6 parts)

Match the following functions with their contour maps. No justification is needed.

(a) $f(x, y) = x^2 + x - y$

(b) $f(x, y) = x^2 + 4xy + y^2$

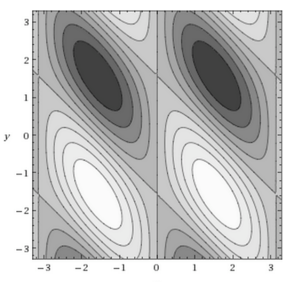
(c) $f(x, y) = x^3 - 3xy^2$

(d) $f(x, y) = \sin x \cos(x + y)$

(e) $f(x, y) = \cos x \cos(2y) - \sin x \sin(2y)$

(f) $f(x, y) = ye^x$

1.



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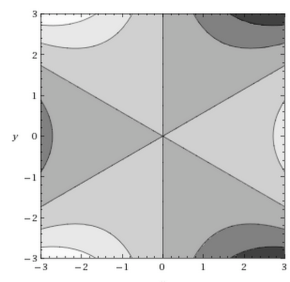

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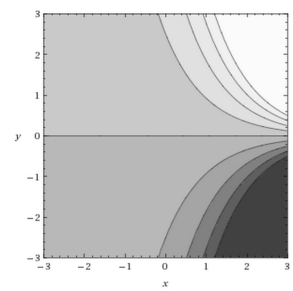

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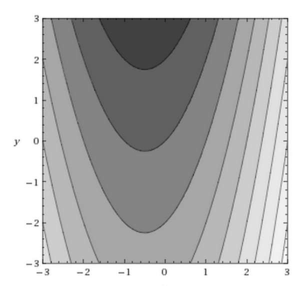

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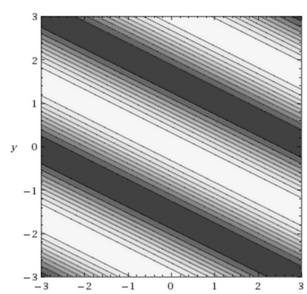

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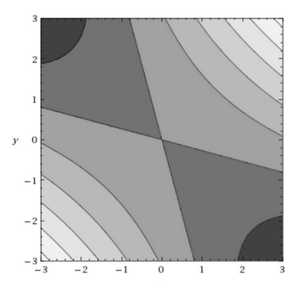

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