Math 51 - Autumn 2010 - Final Exam

Name: ______________________________

Student ID: ___________________________

Select your section:

Brandon Levin 05 (1:15-2:05) 15 (11:00- 11:50)	Amy Pang 14 (10:00-10:50) 17 (1:15-2:05)	Yuncheng Lin 06 (1:15-2:05) 21 (11:00-11:50 AM)	Rebecca Bellovin 09 (11:00-11:50) 23 (1:15-2:05)
Xin Zhou 02 (11:00-11:50) 08 (10:00- 10:50)	Simon Rubinstein-Salzedo 18 (2:15-3:05) 24 (1:15-2:05)	Frederick Fong 20 (10:00-10:50) 03 (11:00-11:50)	Jeff Danciger ACE (1:15-3:05)

Signature: ____________________________

Instructions:

- Print your name and student ID number, select your section number and TA's name, and **sign above to indicate that you accept the Honor Code**.
- There are 11 problems on the pages numbered from 1 to 12, and each problem is worth 10 points. Please check that the version of the exam you have is complete and correctly stapled.
- Read each question carefully. **In order to receive full credit, please show all of your work and justify your answers unless specifically directed otherwise. If you use a result proved in class or in the text, you must clearly state the result before applying it to your problem.**
- Unless otherwise specified, you may assume all vectors are written in standard coordinates.
- You have 3 hours. This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of the teaching staff.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

Problem 1. Let $V = \text{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}\right)$, and let S be the set of all the vectors in $\mathbf{R}^4$ which are orthogonal to V.

a) Show that S is a subspace of $\mathbf{R}^4$.

b) Find a matrix A with $C(A) = V$.

Problem 2. Suppose that $\{\mathbf{u}, \mathbf{v}\}$ is a linearly independent set of vectors in $\mathbf{R}^n$. For what values of $t \in \mathbf{R}$ is the set $\{\mathbf{v} + t\mathbf{u}, \mathbf{u} - \mathbf{v}\}$ linearly independent?

Problem 3.

a) Complete the following definition:

A function $f : X \to Y$ is *one-to-one* if

b) Let L be a line through the origin in $\mathbf{R}^2$, and suppose that $T : \mathbf{R}^2 \to \mathbf{R}^2$ is projection to L. Show that T is not one-to-one.

Problem 4. Let

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}.$$

a) Is A invertible?

b) Find the eigenvalues of A and compute the dimension of each eigenspace.

Problem 5. Let $\mathcal{D} = \{(x, y) \in \mathbf{R}^2 \mid y \geq 0\}$, and let $f : \mathcal{D} \to \mathbf{R}$ be defined by

$$f(x, y) = e^x \sqrt{y}.$$

a) Find the linearization of f at $(0, 1)$.

b) Find the second order Taylor polynomial of f at $(0, 1)$.

c) Use the Taylor polynomial from the previous part to approximate $e\sqrt{2}$.

Problem 6.
Define

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix},$$

and let $Q : \mathbf{R}^3 \to \mathbf{R}$ be the quadratic form associated to A.

a) Classify Q as positive definite, positive semidefinite, indefinite, negative semidefinite, or negative definite.

b) Compute $\nabla Q(2, 1, 0)$.

Problem 7.
Suppose that $z(x,y) = x^2 + y^2$, $x(u,v) = uv$, and $y(u,v) = u^2 + v$.

a) Compute $\frac{\partial z}{\partial u}(1,0)$.

b) Now suppose that u and v are functions of r, s, and t, with

$$u(1,2,3) = 1, \qquad v(1,2,3) = 0, \qquad \frac{\partial u}{\partial r}(1,2,3) = 2, \text{ and } \quad \frac{\partial v}{\partial r}(1,2,3) = -1.$$

Compute $\frac{\partial z}{\partial r}(1,2,3)$.

Problem 8. For each limit below, evaluate the limit or show it does not exist.

a)

$$\lim_{(x,y)\to(0,0)} \frac{x^3}{y^2}$$

b)

$$\lim_{(x,y,z)\to(0,0,0)} \frac{yz^2}{x^2+y^2+z^2}$$

Problem 9. Let S be the surface defined by

$$S = \{(x, y, z) \mid x^2 + y^2 = 4z^2 + 16.\}.$$

(This problem continues on the next page.)

a) Define a function $g : \mathbf{R}^3 \to \mathbf{R}$ with the property that S is a level set of g.

b) Find the tangent plane to S at the point $(4, 2, 1)$.

c) Let $\mathbf{r}(t) = (\sqrt{20}\cos t^3, \sqrt{20}\ \sin t^3, 1)$, and let $t_0 \in \mathbf{R}$ satisfy $\mathbf{r}(t_0) = (4, 2, 1)$. With g as in Part (a), find the directional derivative of g at $(4, 2, 1)$ in the direction $\mathbf{r}'(t_0)$.

Problem 10. Let $g : \mathbf{R}^2 \to \mathbf{R}$ be defined by $g(x, y) = ax^2 - 2ax - y^2 + by^2$.

a) Show that g has a critical point at $(1, 0)$.

b) Under what conditions on the constants a and b does the Second Derivative Test guarantee that g has a local minimum at $(1, 0)$?

Problem 11. Let D be the disc

$$D = \{(x, y) \mid x^2 + y^2 \leq 18\}$$

and let $f : D \to \mathbf{R}$ be defined by $f(x, y) = x^2 + y^2 + 4x + 4y + 7$.

a) Explain why f must attain an absolute maximum on D.

b) Find the point on D where f attains its absolute maximum.

The following boxes are strictly for grading purposes. Please do not mark.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
Total		110