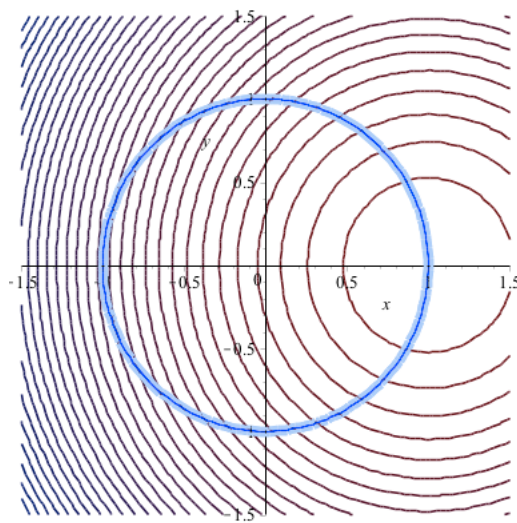


1. From DVC: 1.26, 12.2, 12.8, 12.11, 12.20, 13.1, 13.3, 13.4, 13.5, 13.6, 13.17, 13.21(a), 14.4, 14.5, 14.6, 14.8, 14.9, 14.10, 14.19.
2. Explain, in your own words and using illustrations, why the method of Lagrange multipliers works. Your explanation does not need to be a rigorous proof, but should be enough to help a classmate (who had not yet read Chapter 14) understand the reasoning behind the technique.
3. (a) Below is a contour plot of a function $f(x, y)$. Determine the points (x, y) where f , under its restriction to the blue curve, takes extreme values. Explain your reasoning.



- (b) Verify your solution to part (a) using the method of Lagrange multipliers. The function

$$f(x, y) = x^2 + y^2 - 2x + 1$$

and the blue constraint curve is the solution set to the equation $x^2 + y^2 = 1$.

(Hint: Don't divide by zero!) Identify the maximum and minimum values of $f(x, y)$ subject to the constraint $x^2 + y^2 = 1$.

- (c) Again verify the calculation by parameterizing the blue curve $x^2 + y^2 = 1$ and using the first derivative test (as in Proposition 13.1).

4. (a) Compute the inverse of the matrix $\begin{bmatrix} 3 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

- (b) Determine the eigenvalues of the following matrix, and compute the associated eigenspaces.

$$A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

- (c) Choose an eigenbasis for the matrix in part (b). Write the vector $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as a linear combination of these eigenvectors.

- (d) Compute $A^{100}\mathbf{e}_1$. Hint: Use part (c). You do not have to compute any matrix powers.