MATH 51 MIDTERM (FEBRUARY 2, 2010)

Max Murphy 11am 1:15pm	Jonathan Campbell 11am 2:15pm	Jon Lee 10am 1:15pm	Eric Malm 11am 1:15pm
Xin Zhou 11am 1:15pm	Ken Chan (ACE) 1:15pm	Jose Perea 11am 1:15pm	Frederick Fong 11am 1:15pm

Your name (print):

Sign to indicate that you accept the honor code:

Instructions: Find your TA's name in the table above, and circle the time that your TTh section meets. During the test, you may not use notes, books, or calculators. Read each question carefully, and show all your work. Each of the 10 problems is worth 10 points. You have 90 minutes to do all the problems.

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10.	
Total	

1. Complete the following definitions.

(a). A set $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ of vectors in $\mathbf{R}^n$ is called *linearly dependent* provided

(b). A set V of vectors in $\mathbf{R}^n$ is called a *linear subspace* provided

(c). A map $T : \mathbf{R}^n \to \mathbf{R}^k$ is called a *linear* map provided

(d). A set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ of vectors in a linear subspace V is called a *basis* for V provided

(e). The *dimension* of a subspace V is

2. Find the row reduced echelon form rref(A) of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 2 & 5 \\ 2 & 4 & 7 & 10 & 8 \end{bmatrix}.$$

3(a). Consider the following matrix B and its row reduced echelon form $\mathrm{rref}(B)$:

$$B = \begin{bmatrix} 4 & 3 & 7 & 0 & 3 \\ 2 & 3 & 5 & 0 & 2 \\ 1 & 1 & 2 & 0 & 1 \\ 5 & 4 & 9 & 0 & 4 \end{bmatrix} \quad , \quad \mathrm{rref}(B) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(You do not need to check this.) Find a basis for the column space $C(B)$ of B.

3(b). Find a basis for the nullspace $N(B)$ of B (where B is as in part (a)).

4(a). Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \\ 1 & 2 \\ 3 & 10 \end{bmatrix}$. Find the condition(s) on a vector $\mathbf{b}$ for $\mathbf{b}$ to be in the column space of A. (Your answer should be one or more equations involving the components b_i of $\mathbf{b}$.)

4(b). Find a matrix B such that $N(B) = C(A)$. (Here A is the matrix in part (a).)

5. Let V be the set of all vectors $\mathbf{x}$ in $\mathbf{R}^4$ that are orthogonal to $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and to $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix}$. (To be in V, a vector must be orthogonal both to $\mathbf{u}$ and to $\mathbf{v}$.) Find a basis for V.

6(a). Suppose $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbf{R}^n$ such that $\|\mathbf{u}\| = \|\mathbf{v}\|$. Prove that the vectors $\mathbf{u} - \mathbf{v}$ and $\mathbf{u} + \mathbf{v}$ are orthogonal to each other.

6(b). Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly dependent vectors in $\mathbf{R}^n$. Suppose that A is an $m \times n$ matrix. Prove that the vectors $A\mathbf{v}_1$, $A\mathbf{v}_2$, and $A\mathbf{v}_3$ must also be linearly dependent.

7(a). Find a parametric equation for the line L through the points $A = (0, 1, 1)$ and $B = (1, 2, 3)$.

7(b). Find a point C on L such that the triangle ΔOAC has a right angle at C. (Here $O = (0, 0, 0)$ is the origin.)

8. Suppose $T : \mathbf{R}^3 \to \mathbf{R}^2$ is a linear transformation such that

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}3\\1\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}7\\13\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\1\\2\end{bmatrix}\right) = \begin{bmatrix}7\\20\end{bmatrix}.$$

Find the matrix for T.

9. Consider the points $A = (1, 1, 1, 1)$, $B = (1, 2, 0, 1)$ and $C = (1, 0, 1, 1)$ in $\mathbf{R}^4$.

9(a). Find the cosine of the angle at B of the triangle ABC.

9(b). Find a parametric equation for the plane through the points A, B, and C.

10. Short answer questions. (No explanations required.)

(a). Suppose that a linear subspace V is spanned by vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$. What, if anything, can you conclude about the dimension of V?

(b). Suppose that a linear subspace W contains a set $\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k\}$ of k linearly independent vectors. What, if anything, can you conclude about the dimension of W?

(c). Suppose $\mathbf{u} \cdot \mathbf{v} < 0$. What, if anything, can you conclude about the angle θ between $\mathbf{u}$ and $\mathbf{v}$? [Note: by definition, the angle θ between two nonzero vectors is in the interval $0 \leq \theta \leq \pi$.]

(d). Suppose $T : \mathbf{R}^k \to \mathbf{R}^n$ is a linear map and $\mathbf{b} \in \mathbf{R}^n$. If $k < n$, what, if anything, can you conclude about the number of solutions of $A\mathbf{x} = \mathbf{b}$?

(e). Suppose V is a 3 dimensional linear subspace of $\mathbf{R}^6$ and suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly independent vectors in V. What more, if anything, must you know in order to conclude that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for V?