§1

2 APRIL 2013

LINEAR ALG & MULTIVARIABLE CALC

"LINEAR ALGEBRA and multivariable calculus are two of the most widely used mathematical tools across all scientific disciplines."[1)] The course Math 51 aspires to teach both subjects in just one academic quarter. Students must learn, rapidly, material that is fairly different from prior math courses, which is why many students find this course challenging.

1) From math51.stanford.edu

Success in Math 51 requires a lot of work.

Here are some reasons why the effort is worthwhile:

- *More effort leads to a (much) better grade.* For Math 51, understanding and also *grade* correlate especially strongly with effort studying.[2)]
- *One hour of work now is worth two*[3)] *later.* Learn and internalize Math 51 and then down the road focus completely on, say, Chem 173 or Math 53. Otherwise you might end up forced to learn simultaneously both Chem 173 and the part of Math 51 required for Chem 173.[4)]
- *Effort rewarded generously with help.* I will provide as much assistance as desired (within reason). Past students have told me that this is not true of every teaching assistant.
- *Time is a sunk cost.* If you get test questions wrong, there might be mandatory meetings with me to discuss them, which effectively commits you to the time already. I recommend that you put in the effort beforehand[5)] when you can still get credit/points for your time. Also, if you score below the class average on a midterm, ACE will require you to double the number of problem session hours.

2) For some other courses, there doesn't seem to be as much of a correlation, if any. ^^;;

3) approximately …

4) It would be even harder for many reasons. Learning Math 51 would no longer be part of your work load, but would be in addition to your work load. Also you might not have course staff for assistance, and likely you'll need to do a fair amount of review/backtracking before you can learn any particular concept.

5) while meeting with me if you would like

1.1 Administrative Information

1.1.1 *TA/Section Information*

Daniel Kim Murphy

- dkmurphy@stanford.edu or dmurphy@math.stanford.edu[6)]
- http://www.dkmurphy.com[7)]
- Office 381 K (Math building, first floor)
- Office hours: Thursday at noon–13:15 and 15:15–17:00[8)]

6) One address has a 'k' while the other does not.

7) This is my personal webpage, which should contain a link to the section webpage, which is different from the course webpage: math51.stanford.edu

8) Office hours subject to change. Up to date information should be available at: www.stanford.edu/class/math51/oh.html

ACE[9)] Section

- Tuesdays and Thursdays, 13:15–15:05
- Hewlett 101

9) Accelerated Calculus for Engineers

1.1.2 *Homework*

Please write

> [your name]
> Section ACE
> TA: Daniel Murphy

with your name in place of [your name], at the top of the first page of each homework submission, and staple the pages together if there are multiple pages. Then turn in your homework submission to me either directly[10)] or to my office[11)] by the deadline currently set to be Tuesdays at 15:15 (3:15 PM).

10) during section, for example

11) slid under the door if necessarily

> "No late submissions will be accepted … [but] your lowest homework score will be dropped at the end of the quarter."[12)]

12) from www.stanford.edu/class/math51/homework.html

1.1.3 *Is Math 51 the Right Course? ACE the Right Section?*

This is the right course if you either want or need to know the material, and you have the proper background in single variable differential calculus. The question of correct section is more involved. This section is designated ACE[13)], and that means you will have more class time and

13) Accelerated Calculus for Engineers

contact with teaching staff. So far it sounds like it'd be better for everybody. It is probably better for *almost* everybody. One reason, however, that this section might not be for you is that many parts of it have mandatory attendance. If you are not going to take advantage of the extended contact with teaching staff, it might be better for your grade to enroll in a normal section. In such circumstances, a section change would also help the other students in the ACE section[14] as well as the students who were not so fortunate to get into the ACE section initially.

14) The less time I spend tracking down a student who does not want to put in the effort, the more time I can focus on those students who are willing to do so.

1.1.4 *Quiz on Some Course Policy*

To make sure everyone is on the same page, let's go over important points of course policy that might come up. We'll do this through a series of questions.

i) Students may go to the office hours of any TA or lecturer, even those other than their own. True or false?

ii) Students can go to office hours without making an appointment. True or false?

iii) Office hours is for failing students who need help to pass. True or false?

iv) Office hours is for A students who want an A$^+$. True or false?

v) Students may email their TAs concerning administrative questions, such as whether to withdraw or change to credit/no credit. True or false?

vi) Students may email their TAs concerning questions about concepts. True or false?

vii) Students may email their TAs concerning questions about homework. True or false?

Answers on next page.

The answers are all 'true'. I want to make sure everyone knows that they can always seek help. If, say, a TA is not allowed to give any more help on a particular homework question, the TA should say so.

Multiple choice. Choose the best response.

How many hours per week should a student expect to dedicate to Math 51 in order to succeed?

a) fewer than 5 hours per week

b) 5–9 hours per week

c) 10–14 hours per week

d) 15–19 hours per week

e) more than 19 hours per week

The best responses are probably d) and e). That d) and e) are better than a)–c) is based on the observation of professors who have taught Math 51. The main point is that the expected number of hours of work per week required for success[15)] might be unexpectedly high. The course webpage mentions 5 hours of class and 10–15 hours outside of class. That would indicate that the answer is d), but ACE has more class hours so perhaps e) is more appropriate.

15) Of course some students, for example those who already took a similar course, could succeed with fewer hours.

Here are some more true/false questions.

i) As long as a student finishes the homework within one week after the due date, the student can receive credit. True or false?

ii) As long as a student finishes the homework within the day after the due date, the student can receive partial credit. True or false?

iii) A student misses the homework deadline by 10 minutes. That student is entitled to credit. True or false?

iv) Points will not be deducted for poorly/improperly[16)] formatted assignment submissions. True or false?

16) multiple pages unstapled or messy write-ups

v) A student is very ill for a week. With a doctor's note, that student can turn in the assignment one week later. True or false?

Answers on next page.

The answers to these questions are all 'false' by policy. The purpose of dropping the lowest homework score is to accommodate exceptional circumstances in which a student does not submit a homework assignment – for any reason, even if, say, a student completed it on time, but forgot to submit it.[17] What about missing multiple assignments? In such situations, which usually come along with missing multiple weeks of class, the best course of action might be to withdraw.

17) The purpose is not to encourage students to argue for scores that, in fairness, they shouldn't receive, so they can drop one of their lower scores. If you miss an assignment, that is your lowest score.

❧ *You might lose a point if you submit a multiple page assignment and do not staple it.* Submission of multiple unattached pages risks losing credit for entire pages of work and inconveniences course staff. Please staple multiple page submissions.

1.1.5 *Miscellaneous*

For issues not covered by the above[18], please contact me. In particular, feel free to contact me concerning lecture registration (on Axess[19]) and section registration (on Coursework[20]).

18) as well as clarifications of the above

19) `axess.stanford.edu`

20) `coursework.stanford.edu`

1.1.6 *Survey*

Please fill out the short survey in §1.5 on page 11, the last page, and give it to me today.

1.2 Vectors in $\mathbf{R}^n$

Try not to let the later parts discourage you – they are designed to be extra challenging.

Example 1.

(a) Compute $2\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} + 3\begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$.

(b) Parts (i) and (ii) ask the 'inverse' of part (a).

(i) Determine c_1, c_2, c_3 satisfying:

$$c_1\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} + c_3\begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

or show that no solution exists.

(ii) Determine c_1, c_2, c_3 satisfying:

$$c_1\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} + c_3\begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ 2 \end{bmatrix}$$

or show that no solution exists.

(c) For the part(s) of (b) admitting a solution c_1, c_2, c_3, how many solutions are there?

(d) The above parts illustrate the following fact: For fixed vectors $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_k, \boldsymbol{w}$ in $\mathbf{R}^n$, $\boldsymbol{w}$ may or may not be a linear combination of $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_k$, and $\boldsymbol{w}$ may be a linear combination of $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_k$ in more than one way.[21)] For simplicity, take $k = 2$. Show that if $\boldsymbol{w}$ is a linear combination of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ in more than one way[22)], then $\boldsymbol{w}$ is a linear combination of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ in infinitely many ways. ❧

21) In other words, the equation $c_1\boldsymbol{v}_1 + \cdots + c_k\boldsymbol{v}_k = \boldsymbol{w}$ might have no solutions or might have more than one solution in the unknowns $c_1, \ldots, c_k$.

22) This means $c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 = \boldsymbol{w}$ and $d_1\boldsymbol{v}_1 + d_2\boldsymbol{v}_2 = \boldsymbol{w}$ where either $c_1 \neq d_1$ or $c_2 \neq d_2$.

Example 2.

(a) Represent the vectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ as well as the scalar multiple $-2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and the sum $\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ and the difference $\begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ graphically.

(b) Use the graphical representation to parametrize[23)] the line segment determine by the points[24)] $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(c) Use the graphical representation to explain why every vector in $\mathbf{R}^2$ may be written as $a\begin{bmatrix} 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ for some real numbers a and b.

23) In this case, the parametrization may be written in the form $\{\text{expr in } t \mid t \in [a, b]\}$.

24) Here, as is customary, the point corresponding to a vector is the coordinate of the head when in standard position with tail at the origin.

1.3 Linear Combinations & Spans

Example 3.

(a) Describe

$$\operatorname{span}\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) := \left\{ r\begin{bmatrix}1\\0\\0\end{bmatrix} \,\middle|\, r, s \in \mathbf{R} \right\}$$

geometrically. Describe

$$\begin{bmatrix}1\\1\\1\end{bmatrix} + \operatorname{span}\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \left\{ \begin{bmatrix}1\\1\\1\end{bmatrix} + r\begin{bmatrix}1\\0\\0\end{bmatrix} \,\middle|\, r \in \mathbf{R} \right\}$$

geometrically. Describe

$$\operatorname{span}\left(\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right) := \left\{ r\begin{bmatrix}1\\0\\0\end{bmatrix} + s\begin{bmatrix}0\\1\\0\end{bmatrix} \,\middle|\, r, s \in \mathbf{R} \right\}$$

geometrically. Describe

$$\begin{bmatrix}1\\1\\1\end{bmatrix} + \operatorname{span}\left(\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \left\{ \begin{bmatrix}1\\1\\1\end{bmatrix} + r\begin{bmatrix}1\\0\\0\end{bmatrix} + s\begin{bmatrix}0\\1\\0\end{bmatrix} \,\middle|\, r, s \in \mathbf{R} \right\}$$

geometrically.

(b) Find a parametric representation[25)] for the plane that goes through the points $(1, 0, -1)$, $(0, 1, -1)$ and $(0, 0, 0)$. Find a parametric representation for the plane that goes through the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

25) something like part (a)

1.4 Additional Problems

Problem 1. Describe

$$\operatorname{span}\left\{\begin{bmatrix}-1\\4\\3\end{bmatrix},\begin{bmatrix}0\\-2\\1\end{bmatrix},\begin{bmatrix}-1\\0\\5\end{bmatrix}\right\}$$

geometrically.

Problem 2. Describe

$$\operatorname{span}\left\{\begin{bmatrix}0\\4\\3\end{bmatrix},\begin{bmatrix}0\\-2\\1\end{bmatrix},\begin{bmatrix}-1\\0\\5\end{bmatrix}\right\}$$

geometrically.

Problem 3. Show that if $\boldsymbol{v}_1$ is in $\operatorname{span}\{\boldsymbol{v}_2, \boldsymbol{v}_3\}$, then it must be that $\operatorname{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\} = \operatorname{span}\{\boldsymbol{v}_2, \boldsymbol{v}_3\}$.

Problem 4. Find three vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ in $\mathbf{R}^n$ for some n so that the span of any two of the vectors is a proper subset of the span of all three of the vectors.

Problem 5. Is it true that $\boldsymbol{v}_1 \in \operatorname{span}\{\boldsymbol{v}_2, \boldsymbol{v}_3\}$ necessarily implies $\boldsymbol{v}_2 \in \operatorname{span}\{\boldsymbol{v}_1, \boldsymbol{v}_3\}$? Prove or provide a counterexample.

Problem 6. Is it true that $\boldsymbol{v}_1 \in \operatorname{span}\{\boldsymbol{v}_2\}$ necessarily implies $\boldsymbol{v}_2 \in \operatorname{span}\{\boldsymbol{v}_1\}$? Prove or provide a counterexample.

Problem 7. Find a parametric representation for the line described implicitly by $x + 2y = 4$.

Problem 8. Find a parametric representation for the plane described implicitly by $x + 2y + 3z = 4$.

Problem 9. Show that if $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ are in $\operatorname{span}\{\boldsymbol{v}_1, \dots, \boldsymbol{v}_k\}$, then $\boldsymbol{w}_1 + \boldsymbol{w}_2$ is in $\operatorname{span}\{\boldsymbol{v}_1, \dots, \boldsymbol{v}_k\}$.

Problem 10 (2000 Autumn Midterm 1, Q10(c)). True or false? Any three vectors in $\mathbf{R}^3$ span $\mathbf{R}^3$?

Problem 11 (2004 Spring Midterm 1, Q5). Show that if three distinct[26] vectors $\boldsymbol{p}$, $\boldsymbol{q}$, and $\boldsymbol{r}$ are collinear (in other words, there exists a line – not necessarily through the origin, of course – containing all three of those vectors), then one of the vectors must be in the span of the other two[27]. *Hint:* Represent the line parametrically, and start by showing that the direction vector of that line must be a linear combination of $\boldsymbol{p}$ and $\boldsymbol{q}$; then use that to write $\boldsymbol{r}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$.

26) the qualification 'distinct' is unnecessary since the question is trivial when two of the vectors coincide

27) that is, they must be linearly dependent, but this term might not be introduced until next time

This page intentionally contains only this sentence.

1.5 Information about You

My full name:

My preferred name:

The letter grade I predict I will get in this class:[28)]

28) As a baseline, assume that ⅓ of the class receives A and ⅓ receives B, which might not be the case this quarter, but has been the case in some past quarters.

Something unique about me that will assist in remembering me:

(Optional) Here is a drawing: