

Math 51 - Autumn 2011 - Midterm Exam II

Name: _____

Student ID: _____

Select your section:

Jonathan Campbell ACE (1:15–3:05pm)	Elizabeth Goodman 03 (11:00–11:50am) 17 (1:15–2:05pm)	Julio Gutierrez 15 (11:00–11:50am) 25 (1:15–2:05pm)
Seungki Kim 08 (10:00–10:50am) 09 (11:00–11:50am)	Kenji Kozai 14 (10:00–10:50am) 24 (1:15–2:05pm)	Yuncheng Lin 02 (11:00–11:50am) 05 (1:15–2:05pm)
Michael Lipnowski 21 (11:00–11:50am) 23 (1:15–3:05pm)	Jeremy Miller 11 (1:15–2:05pm) 18 (2:15–3:05pm)	Ho Chung Siu 20 (10:00–10:50am) 06 (1:15–2:05pm)

Signature: _____

Instructions:

- Print your name and student ID number, select your section number and TA's name, and write your signature to indicate that you accept the Honor Code.
- There are 9 problems on the pages numbered from 1 to 15, for a total of 100 points. Point values are given in parentheses. Please check that the version of the exam you have is complete, and correctly stapled.
- Read each question carefully. In order to receive full credit, please show all of your work and justify your answers unless you are *explicitly* instructed not to.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 90 minutes.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **December 2, 2011**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

Problem 1. (10 pts.) Let A be the matrix $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$. Find a basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ of \mathbb{R}^2 where \mathbf{v}_1 and \mathbf{v}_2 are eigenvectors of A .

Problem 2. (15 pts.) Let A be the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

a) Find the eigenvalues and eigenvectors of A .

b) Find a diagonal matrix D and a matrix C such that

$$A = CDC^{-1}.$$

c) Use part b) to compute A^{-2} , which is the inverse of A^2 .

Problem 3. (10 pts.)

The position of a particle at time t is given by

$$f(t) = \begin{bmatrix} t^2 \\ \sin t \\ e^t \end{bmatrix}.$$

a) Find $f'(t)$, also known as the velocity of the particle at time t .

b) Find $f''(t)$, also known as the acceleration of the particle at time t .

c) Find an equation for the tangent line to the path of the particle at the point $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Problem 4. (10 pts.)

Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be given by $f(x, y, z) = \begin{bmatrix} x^2 \sin yz \\ y^2 + e^{z-x} \end{bmatrix}$.

a) Compute the total derivative $Df(1, 2, 3)$ at the point $(1, 2, 3)$.

b) Compute the total derivative $Df(x, y, z)$ at a general point (x, y, z) .

Problem 5. (15 pts.) For each of the following questions, circle either “Always **TRUE**” or “Sometimes **FALSE**”. You do not need to supply reasons for your answer.

a) “Always **TRUE**” / “Sometimes **FALSE**”.

Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$. Then $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$ exists if both $\lim_{x \rightarrow 0} f(x,0)$ and $\lim_{y \rightarrow 0} f(0,y)$ exist.

b) “Always **TRUE**” / “Sometimes **FALSE**”.

Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and write $f(x,y) = (f_1(x,y), f_2(x,y))$. Then $\lim_{(x,y) \rightarrow (1,1)} f(x,y)$ exists if and only if both $\lim_{(x,y) \rightarrow (1,1)} f_1(x,y)$ and $\lim_{(x,y) \rightarrow (1,1)} f_2(x,y)$ exist.

c) “Always **TRUE**” / “Sometimes **FALSE**”.

If v and w are two different eigenvectors of a matrix A corresponding to two different eigenvalues, then v and w are linearly independent.

d) “Always **TRUE**” / “Sometimes **FALSE**”.

If A is a symmetric n -by- n matrix, then it has n distinct eigenvalues.

e) “Always **TRUE**” / “Sometimes **FALSE**”.

If a differentiable function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ satisfies $\frac{\partial f}{\partial x}(x,y) = 0$ and $\frac{\partial f}{\partial y}(x,y) = 0$ for all $(x,y) \in \mathbb{R}^2$, then f is a constant function.

Problem 6. (10 pts.)

Let A be the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & -2 & 1 \\ -1 & 3 & 1 \end{bmatrix}.$$

- a) Compute the determinant $\det(A)$.

b) Compute the inverse A^{-1} .

Problem 7. (10 pts.)

Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the function given by

$$f(x, y) = \begin{bmatrix} x^2 + xy \\ x - y \end{bmatrix}.$$

Assume that $g: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is a function satisfying

$$g(0, 0, 0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \text{and} \quad Dg(0, 0, 0) = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 0 \end{bmatrix}.$$

If $h: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is the composition $h = f \circ g$, compute $Dh(0, 0, 0)$.

Problem 8. (10 pts.) Let $Q: \mathbb{R}^3 \rightarrow \mathbb{R}$ be the quadratic form

$$Q(x, y, z) = 3x^2 + 3y^2 + 3z^2 + 2xy + 2xz + 2yz.$$

Determine whether the quadratic form Q is positive definite, negative definite, or indefinite. If none of these hold, determine whether Q is positive semidefinite or negative semidefinite.

Problem 9. (10 pts.)

Let A be a 2×3 matrix. Consider the function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ defined by $f(\mathbf{x}) = \|A\mathbf{x}\|^2$.

- a) Show that f is a quadratic form. What is the matrix B associated to f ? (The answer should be in terms of A .)

- b) Prove that all the eigenvalues of this matrix B are ≥ 0 .

c) Show that the matrix B always has nullity greater than 0.

The following boxes are strictly for grading purposes. Please do not mark.

Question	Score	Maximum
1		10
2		15
3		10
4		10
5		15
6		10
7		10
8		10
9		10
Total		100