MATH 51 MIDTERM (FEBRUARY 9, 2012)

Nick Haber 11am 2:15pm	Fernando Shao 11am 1:15pm	Kaveh Fouladgar 11am 1:15pm	Ralph Furmaniak 11am 1:15pm
Sam Nariman 11am 1:15pm	Saran Ahuja 10am 1:15pm	Chris Henderson 11am 1:15pm	Amy Pang (ACE) 1:15pm

Your name (print):

Sign to indicate that you accept the honor code:

Instructions: Find your TA's name in the table above, and circle the time that your TTh section meets. During the test, you may not use notes, books, or calculators. Read each question carefully, show all your work, and CIRCLE YOUR FINAL ANSWER. Each of the 10 problems is worth 10 points. You have 90 minutes to do all the problems.

1.	
2.	
3.	
4.	
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7.	
8.	
9.	
10.	
Total	

1. Compute the following:

(a). $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

(b). $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 4 & 1 \\ 2 & -4 \end{bmatrix}$

(c). $3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

(d). $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

(e). $\begin{bmatrix} 2 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 3 \end{bmatrix}$

2. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 5 \\ 3 & 2 & 14 \end{bmatrix}$.

3(a). Consider the following matrix A and its row reduced echelon form $\mathrm{rref}(A)$:

$$A = \begin{bmatrix} 4 & 3 & 7 & 1 & 3 \\ 2 & 3 & 5 & -1 & 2 \\ 1 & 1 & 2 & 0 & 1 \\ 5 & 4 & 9 & 1 & 4 \end{bmatrix} \quad , \quad \mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(You do not need to check this.) Find a basis for the column space $C(B)$ of B.

3(b). As in part (a),

$$A = \begin{bmatrix} 4 & 3 & 7 & 1 & 3 \\ 2 & 3 & 5 & -1 & 2 \\ 1 & 1 & 2 & 0 & 1 \\ 5 & 4 & 9 & 1 & 4 \end{bmatrix}, \quad \mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find some vectors that span the nullspace $N(A)$ of A.

4(a). Consider the equation $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ 1 & 2 & 0 \end{bmatrix}$. Find the condition(s) on the vector $\mathbf{b}$ for this equation to have a solution. (Your answer should be one or more equations involving the components b_i of $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.)

4(b). Find a matrix B such that $N(B) = C(A)$, where

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ 1 & 2 & 0 \end{bmatrix}.$$

(This is the matrix from part (a).)

5(a). Find all solutions of the equation $A\mathbf{x} = \begin{bmatrix} 3 \\ 16 \end{bmatrix}$, where

$$A = \begin{bmatrix} 1 & -2 & 2 & 1 \\ 5 & -10 & 11 & 2 \end{bmatrix}.$$

5(b). Find a basis for the nullspace of A, where A is the matrix in part (a).

6(a). Suppose S is the sphere in $\mathbf{R}^3$ of radius 1 centered at the origin. Suppose that A, B, and C are three points on S and that side AC of the triangle ABC is a diameter of the sphere. Prove using vectors that the triangle has a right angle at B.

6(b). Suppose A is a matrix with 4 columns. Suppose $\mathbf{u}$ and $\mathbf{v}$ are linearly independent vectors in $\mathbf{R}^4$ such that $A\mathbf{u} = A\mathbf{v} = 0$. Prove that if $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ are any vectors in $\mathbf{R}^4$, then the vectors $A\mathbf{x}$, $A\mathbf{y}$, and $A\mathbf{z}$ must be linearly dependent.

7(a). Find a parametric equation for the line L through the points $A = (-1, 0, 3)$ and $B = (2, 1, 5)$.

7(b). Find a point C on L such the line OC and the line L are perpendicular to each other. (Here L is the line from part (a), and the line OC is the line through the origin $O = (0, 0, 0)$ and C.)

8. Find a matrix A such that:

$$A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix},\quad A\begin{bmatrix}3\\1\\1\end{bmatrix}=\begin{bmatrix}7\\13\end{bmatrix},\quad A\begin{bmatrix}3\\2\\1\end{bmatrix}=\begin{bmatrix}7\\18\end{bmatrix}.$$

9(a). Suppose Δ is an equilateral triangle in $\mathbf{R}^3$ each of whose sides has length 7. Let A, B, and C be the corners of Δ. Find

$$\left(2\,\overrightarrow{AB}\right) \cdot \left(3\,\overrightarrow{AC}\right).$$

[Notation: $\overrightarrow{AB}$ is the vector beginning at A and ending at B.]

9(b). Suppose $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are unit vectors in $\mathbf{R}^4$ such that each one is orthogonal to the other two. Find a number c so that $\mathbf{u}+2\mathbf{v}+3\mathbf{w}$ and $5\mathbf{u}+\mathbf{v}+c\mathbf{w}$ are orthogonal to each other. (Recall that a unit vector is a vector whose norm is 1.)

10. Short answer questions. (No explanations required.)

(a). Suppose that a linear subspace V is spanned by vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$. What, if anything, can you conclude about the dimension of V?

(b). Suppose that a linear subspace W contains a set $\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k\}$ of k linearly independent vectors. What, if anything, can you conclude about the dimension of W?

(c). Suppose $\mathbf{u} \cdot \mathbf{v} > 0$. What, if anything, can you conclude about the angle θ between $\mathbf{u}$ and $\mathbf{v}$? [Note: by definition, the angle θ between two nonzero vectors is in the interval $0 \leq \theta \leq \pi$.]

(d). Suppose that A is a $k \times m$ matrix (i.e., a matrix with k rows and m columns.) If $k < m$, what, if anything, can you conclude about the number of solutions of $A\mathbf{x} = \mathbf{b}$?

(e). Suppose A is a matrix with 5 rows and 4 columns. Suppose that the equation $A\mathbf{x} = 0$ has only one solution. What, if anything, can you conclude about the dimension of the column space of A?