Math 51 - Winter 2011 - Midterm Exam I

Name: ______________________________

Student ID: ______________________________

Circle your section:

Nick Haber 11:00 AM 1:15 PM	James Zhao 10:00 AM 1:15 PM	Henry Adams 11:00 AM 1:15 PM
Ralph Furmaniak 11:00 AM 1:15 PM	Jeremy Miller 11:00 AM 2:15 PM	Ha Pham 11:00 AM 1:15 PM
Sukhada Fadnavis 10:00 AM 1:15 PM	Max Murphy 11:00 AM 1:15 PM	Jesse Gell-Redman 1:15 PM

Signature: ______________________________

Instructions: Print your name and student ID number, select the time at which your section meets, and **write your signature to indicate that you accept the Honor Code**. There are 10 problems on the pages numbered from 1 to 12. Each problem is worth 10 points. In problems with multiple parts, the parts are worth an equal number of points unless otherwise noted. Please check that the version of the exam you have is complete, and correctly stapled. In order to receive full credit, please show all of your work and justify your answers. You may use any result from class, but if you cite a theorem be sure to verify the hypotheses are satisfied. **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. GOOD LUCK!

Question	1	2	3	4	5	6	7	8	9	10	Total
Score											

1. Complete the following definitions.

(a). A set $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ of vectors in $\mathbf{R}^n$ is called *linearly independent* provided

Solution

(b). A function $T : \mathbf{R}^n \to \mathbf{R}^k$ is called a *linear transformation* provided

(c). A set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ of vectors in a subspace V is called a *basis* for V provided

(d). A set V of vectors in $\mathbf{R}^n$ is called a *subspace* of $\mathbf{R}^n$ provided

(e). The *dimension* of a subspace V is

2. Find the row reduced echelon form rref(A) of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 2 & 6 \\ 2 & 4 & 100 & 10 & 8 \end{bmatrix}.$$

3. Consider the following matrix A and its row reduced echelon form $\mathrm{rref}(A)$:

$$A = \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 & -3 \\ 1 & 0 & 1 & 2 & 3 \end{bmatrix} \quad , \quad \mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(You do not need to check that the row reduction is correct).

(a). Find a basis for the column space $C(A)$.

(b). Find a basis for the nullspace $N(A)$.

4. Consider the matrix $M = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & z \end{bmatrix}$. For which values of z will the system $M\mathbf{x} = \begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix}$ have:

(a). (2 points) A unique solution? (Show your work below.)

(b). (2 points) An infinite number of solutions?

(c). (2 points) No solutions?

Show your work here:

4(d). (4 points) For $z = 7$, find the complete solution to the system

$$M\mathbf{x} = \begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix}.$$

5. Let V be the set of all vectors $\mathbf{x}$ in $\mathbf{R}^5$ that are orthogonal to $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and to $\mathbf{v} = \begin{bmatrix} -1 \\ -2 \\ -3 \\ -4 \\ -5 \end{bmatrix}$. (To be in V, a vector must be orthogonal both to $\mathbf{u}$ and to $\mathbf{v}$.) Find a basis for V.

6(a). Suppose that A is an $m \times n$ matrix of rank n. Find all the solutions $\mathbf{v}$ of $A\mathbf{v} = \mathbf{0}$. Explain your answer.

6(b). Suppose that A is an $m \times n$ matrix of rank n as in part (a). Suppose $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are vectors such that $A\mathbf{v}_1$, $A\mathbf{v}_2$ and $A\mathbf{v}_3$ are linearly dependent. Prove that the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ must also be linearly dependent.

7(a). Find a parametric equation for the line L passing through the points $A = (0, 4, 1)$ and $B = (1, 3, 1)$.

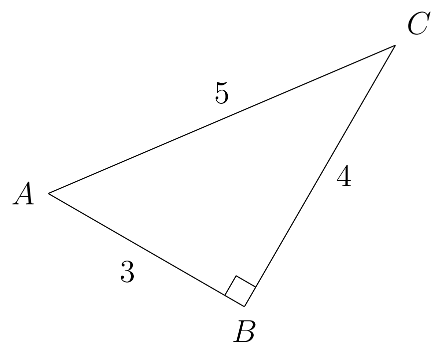
7(b). Find a point C on L such that the triangle ΔOAC has a right angle at C. (Here $O = (0, 0, 0)$ is the origin.)

8(a). Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation with $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}-1\\5\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}2\\3\end{bmatrix}$. Find a matrix A for T such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbf{R}^2$. [Hint: What is $\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}1\\1\end{bmatrix}$?]

8(b). Let ΔABC be a 3-4-5 right triangle in $\mathbf{R}^2$ as shown below. Let $S : \mathbf{R}^2 \to \mathbf{R}^2$ be the rotation about the origin such that

$$S\left(\overrightarrow{AB}\right) = \frac{3}{5}\left(\overrightarrow{AC}\right).$$

FInd the matrix M such that $S(\mathbf{x}) = M\mathbf{x}$ for all $\mathbf{x} \in \mathbf{R}^2$.



9(a). Consider the points $A = (2, 1, 3, 1)$, $B = (4, 1, 5, 1)$ and $C = (2, 3, 5, 1)$ in $\mathbf{R}^4$. Find a parametric equation for the plane through the points A, B, and C.

9(b). Consider the triangle ABC (where A, B and C are the points given in part (a)). Find the cosine of the angle between the two sides AB and AC.

10(a). (3 points) Consider the set $V = \{(x_1, x_2) \in \mathbf{R}^2 | x_1 \leq 0, x_2 \leq 0\}$. Is V a linear subspace of $\mathbf{R}^2$? Explain.

(b). (3 points) Suppose that $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear transformation with matrix $B = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$ and that $\mathbf{x}$ is a unit vector in $\mathbf{R}^2$. What, if anything, can you conclude about the length of the vector $T(\mathbf{x})$?

(c). (4 points) Suppose that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are three linearly independent vectors. Show that $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are linearly independent.