Math 51 First Exam — April 25, 2013

Name: ____ **SUID#:** ______________

Circle your section:		
Amy Pang 15 (11:00-11:50 AM) 17 (2:15-3:05 PM)	Daniel Murphy ACE (1:15-3:05 PM)	Xin Zhou 02 (11:00-11:50 AM) 12 (10:00-10:50 AM)
Michael Lipnowski 06 (1:15-2:05 PM) 14 (10:00-10:50 AM)	Yuncheng Lin 08 (10:00-10:50 AM) 18 (2:15-3:05 PM)	Evita Nestoridi 09 (11:00-11:50 AM) 11 (1:15-2:05 PM)

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result proved in class or the text, but be sure to clearly state the result before using it, and to verify that all hypotheses are satisfied.
- Please check that your copy of this exam contains 8 numbered pages and is correctly stapled.
- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.
- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by teaching staff is prohibited. If you need extra room for your answers, use the back side of a page or one of the extra sheets provided in this packet, and clearly indicate that your answer continues there. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, May 9**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	**Total**
Points:	10	10	10	10	10	10	10	10	80
Score:									

1. (10 points)

   (a) Complete the following sentence: a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ of vectors in $\mathbb{R}^n$ is defined to be *linearly dependent* if

   (b) Let $\mathbf{v}_1, \ldots, \mathbf{v}_k \in \mathbb{R}^n$ and A be an $m \times n$ matrix. Show that if $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent, then so is $\{A\mathbf{v}_1, \ldots, A\mathbf{v}_k\}$.

   (c) Give specific numerical examples of vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ *and* a 3×3 matrix A so that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly *independent*, but $\{A\mathbf{u}, A\mathbf{v}, A\mathbf{w}\}$ is linearly *dependent*.

2. (10 points) Let P be the plane in $\mathbb{R}^3$ containing the three points $(0, 0, 1)$, $(0, -3, 0)$, and $(2, 0, 0)$.

   (a) Find a parametric representation of the plane P.

   (b) Let Q_1 be the point $(1, 1, 1)$. Find a point Q_2 in the plane P so that the vector from Q_1 to Q_2 is perpendicular (normal) to P.

3. (10 points) Be careful to answer *both* parts of the following:

(a) Compute, showing all steps, the reduced row echelon form of the matrix

$$A = \begin{bmatrix} 2 & 4 & -2 & 2 & 8 & -2 \\ 3 & 6 & 1 & 2 & 13 & 1 \\ 0 & 0 & 3 & -2 & -3 & -2 \\ 3 & 6 & -2 & 3 & 13 & -1 \end{bmatrix}$$

(b) Fill in the blanks (no reasoning needed): Rank of A:__________ Nullity of A:__________

4. (10 points) Suppose all we know about the 4×9 matrix A is the following information:

$$A = \begin{bmatrix} | & | & | & | & | & | & | & | & | \\ \mathbf{a_1} & \mathbf{a_2} & \mathbf{a_3} & \mathbf{a_4} & \mathbf{a_5} & \mathbf{a_6} & \mathbf{a_7} & \mathbf{a_8} & \mathbf{a_9} \\ | & | & | & | & | & | & | & | & | \end{bmatrix} \quad \text{and} \quad \text{rref}(A) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using this information, specify each of the following as completely as you can (expressing in terms of the vectors $\mathbf{a_1}, \ldots, \mathbf{a_9} \in \mathbb{R}^4$ if necessary), showing all your reasoning:

(a) a basis for $N(A)$, the null space of A

(b) a basis for $C(A)$, the column space of A

5. (10 points) Suppose b is an unspecified real number, and consider the following system of equations involving variables x, y, z:

$$(*)\begin{cases} x + 4y + 3z = 2 \\ 3x + 5y + bz = 9 \end{cases}$$

(a) *For this part only*, suppose $b = 2$; express the solution to the above system in parametric form.

(b) Find, with complete reasoning, all values of b so that the system $(*)$ has no solution (x, y, z); if no such value of b exists, explain why.

(c) Find, with complete reasoning, all values of b so that the system $(*)$ has infinitely many solutions; if no such value of b exists, explain why.

6. (10 points) Let

$$V = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

(a) Let $\mathbf{v} \in \mathbb{R}^5$. Find one or more conditions that determine precisely whether $\mathbf{v}$ lies in V. (Your answer should be given in the form of one or more equations involving the components of $\mathbf{v}$.)

(b) It's a fact that there exist matrices A that satisfy $V = N(A)$. For this question, you don't have to find any such matrices, but consider what can be said about the possible *size* of such an A. Among the choices below, circle all sizes "$m \times n$" for which it's *possible* to find some matrix A, consisting of m rows and n columns, whose null space equals V. (No justification is necessary.)

2×5 3×5 7×5

2×2 3×3 5×5 7×7

5×2 5×3 5×7

7. (10 points) Let W be the set of vectors $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$ in $\mathbb{R}^4$ for which $2w_2 + 3w_3 + 4w_4 = 0$.

(a) Show that W is a subspace of $\mathbb{R}^4$.

(b) Find, with reasoning, a 4×4 matrix A such that $C(A) = W$.

8. (10 points)

(a) Suppose $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation satisfying:

$$\mathbf{T}\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix} \qquad \mathbf{T}\left(\begin{bmatrix}0\\0\\3\end{bmatrix}\right) = \begin{bmatrix}0\\-3\end{bmatrix} \qquad \mathbf{T}\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix}$$

Find the matrix of $\mathbf{T}$; show all your steps.

(b) Let $\mathbf{S} : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects vectors across the line $y = x$. Find the matrix of $\mathbf{S}$; show all your steps.