1. From the textbook: 23.2, 23.4, 23.8, 25.7, 25.1 (Hint: Use Proposition 18.1), 25.10, 26.2, 26.4, 26.12, 26.19.

2. Let A be an $n \times n$ matrix, and suppose that $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are all eigenvectors of A with the *same* eigenvalue λ. Show that any vector $\mathbf{v}$ in the linear subspace $V = \text{span}(\mathbf{v}_1, \ldots, \mathbf{v}_k)$ is also an eigenvector of A with eigenvalue λ.

3. Let A be an $n \times n$ matrix.

   (a) Suppose $\mathbf{v}_1$ is an eigenvector of A with eigenvalue λ_1. Show that $\mathbf{v}_1$ is also an eigenvector of the matrix $A^2 = AA$, and find its eigenvalue.

   (b) For p a positive integer, let A^p denote A multiplied by itself p times. Show that $\mathbf{v}_1$ is an eigenvector of A^p, and find its associated eigenvalue.

   (c) Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ be eigenvectors of A corresponding to eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_k$, respectively. Suppose that

   $$\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k.$$

   Find $A\mathbf{v}$ (as a linear combination of the eigenvectors $\mathbf{v}_i$).

   (d) Find $A^p\mathbf{v}$.

   Side remark: The mathematical field of *dynamics* is concerned with describing how points in a space X behave as we repeatedly apply some function $f : X \to X$. This question shows that if X is a vector space and f is a linear map, then when f is diagonalizable, knowing the eigenvectors and eigenvalues of f allows us to understand its dynamics very well (and without having to compute any large matrix powers).

4. Suppose A is a matrix that has eigenvector $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ with eigenvalue $\lambda_1 = 2$, and eigenvector $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ with eigenvalue $\lambda_2 = 1$.

   (a) If $\mathbf{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, find the coordinates $[\mathbf{x}]_\beta$ of $\mathbf{x}$ with respect to the basis $\beta = \{\mathbf{v}_1, \mathbf{v}_2\}$.

   (b) Find $A^8\mathbf{x}$. (Hint: Use part (a), and Question 3.)

5. Let $Q(\mathbf{x}) = \mathbf{x}^T I_n \mathbf{x}$ be the quadratic form associated to the identity matrix I_n.

   (a) Describe the definiteness of Q.

   (b) We have another name for the number $Q(\mathbf{x})$. What is it? (Hint: Page 22).