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Math 51

First Midterm Exam

Instructions. Answer the following problems carefully and completely. Make sure you show all your work. There are 100 points possible. Good luck!

Name ______________________________

Section leader and time ______________________________

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1. (14)	____________
2. (14)	____________
3. (18)	____________
4. (24)	____________
5. (12)	____________
6. (18)	____________
Total (100)	____________

1. (14 pts) (a). Solve the following system of equations using your method of choice. Write your answer in parametric form.

$$
\begin{aligned}
x + 2y - 3z - 2s + 4t &= 1 \\
2x + 5y - 8z - s + 6t &= 4 \\
x + 4y - 7z + 5s + 2t &= 8
\end{aligned}
$$

(b). For what vectors $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathbb{R}^3$ does the matrix equation

$$\begin{pmatrix} 1 & 3 & 3 \\ 2 & 6 & 9 \\ -1 & -3 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

have a solution?

2. (14 pts) Consider the matrix

$$B = \begin{bmatrix} 1 & 1 & 0 & 5 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a). Find a set of vectors which spans the null space $N(B)$.

(b). Find a set of vectors that span the column space $C(B)$, where B is defined as in (a).

3. (18 pts) (a). Show that two vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^3$, are *linearly dependent* if and only if

$$\mathbf{u} \times \mathbf{v} = 0.$$

(b). Do the vectors $u = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $w = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ form a basis in $\mathbb{R}^3$? Explain.

(c). Is the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in the span of the vectors u, v, and w in part (b)? Explain.

4. (24 pts) True or False.

(a.) Every system of 2 equations in 3 variables has a solution.

(b.) The set $\{(n, m) \mid \ n \text{ and } m \text{ are integers}\}$ is a subspace of $\mathbb{R}^2$.

(c.) Suppose that A is an $n \times n$ matrix and that its null space consists of a single point. Then every inhomogeneous equation $Ax = b$ has a unique solution.

(d.) If A is a 5×8 matrix, there is a vector $b \in \mathbb{R}^5$ so that the equation $Ax = b$ has a unique solution.

(e.) There exists a 4×3 matrix A, and a vector $b \in \mathbb{R}^4$ so that the equation $Ax = b$ has exactly two solutions.

(f.) If the null space of A is a line, and u_0 is a vector such that $Au_0 = b$, then the general solution to the nonhomogeneous equation $Ax = b$ is also a line.

(g.) The set $\{(x, y) \in \mathbb{R}^2 \mid xy = 0\}$ is a subspace of $\mathbb{R}^2$.

(h.) If A is a matrix and the equation $Ax = b$ has at least two solutions, then the set of solutions contains a plane.

5. (12 pts) (a). State the Rank-Nullity Theorem

(b). Consider the matrix and vector

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & -3 & 1 \end{bmatrix} \quad v = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}.$$

Given that $Av = \mathbf{0}$ use the rank-nullity theorem to show that $dim(C(A)) \leq 2$.

6. (18 pts) a. Find a nonzero vector which is perpendicular to the plane in $\mathbb{R}^3$ which contains the points $(0, 0, 0)$, $(1, 3, 1)$, and $(4, -3, 6)$.

b. Find the equation of the plane containing these three points. Your answer should be in the form $ax + by + cz = d$

c. Find the area of the triangle in $\mathbb{R}^3$ whose vertices are $(0, 0, 0)$, $(1, 3, 1)$, and $(4, -3, 6)$.