MATH 51 MIDTERM 2 (MARCH 1, 2012)

Nick Haber 11am 2:15pm	Fernando Shao 11am 1:15pm	Kaveh Fouladgar 11am 1:15pm	Ralph Furmaniak 11am 1:15pm
Sam Nariman 11am 1:15pm	Saran Ahuja 10am 1:15pm	Chris Henderson 11am 1:15pm	Amy Pang (ACE) 1:15pm

Your name (print):

Sign to indicate that you accept the honor code:

Instructions: Find your TA's name in the table above, and circle the time that your TTh section meets. During the test, you may not use notes, books, or calculators. Read each question carefully, show all your work, and CIRCLE YOUR FINAL ANSWER. Each of the 10 problems is worth 10 points. You have 90 minutes to do all the problems.

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10.	
Total	

1(a). Find the determinant of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{bmatrix}$.

1(b). Let D be the region inside the circle $x^2 + y^2 = 1$. Find the area of the image $T(D)$, where

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 5x + 3y \\ 2x - 7y \end{bmatrix}.$$

2. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by:

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2y \\ x - 3y \end{bmatrix}.$$

(a). Find the matrix A that represents the linear transformation T with respect to the standard basis $\mathcal{S} = \{\mathbf{e}_1, \mathbf{e}_2\}$.

(b). Consider the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ given by: $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find the change of basis matrix C for the basis $\mathcal{B}$. That is, find the matrix C such that $\mathbf{v} = C[\mathbf{v}]_{\mathcal{B}}$ for all vectors $\mathbf{v}$.

(c). Find the matrix B that represents the linear transformation T with respect to the basis $\mathcal{B}$.

3(a). Find all eigenvalues of the matrix $A = \begin{bmatrix} 1 & 5 & 2 \\ 0 & 13 & 0 \\ 1 & 3 & 1 \end{bmatrix}$.

3(b). Consider the matrix $B = \begin{bmatrix} 7 & 5 & -7 \\ -5 & -3 & 6 \\ 1 & 1 & 0 \end{bmatrix}$.

Find an eigenvector of B with eigenvalue $\lambda = 1$.

4(a). Find the eigenvalues of the matrix $M = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$.

4(b). Consider the quadratic form $\mathbf{x}^T A\mathbf{x}$, where A is a 3×3 symmetric matrix with eigenvalues -1, -2, and $t - 3$. (One of the entries of A contains the variable t.)

(i). For which values of t is the quadratic form positive definite?

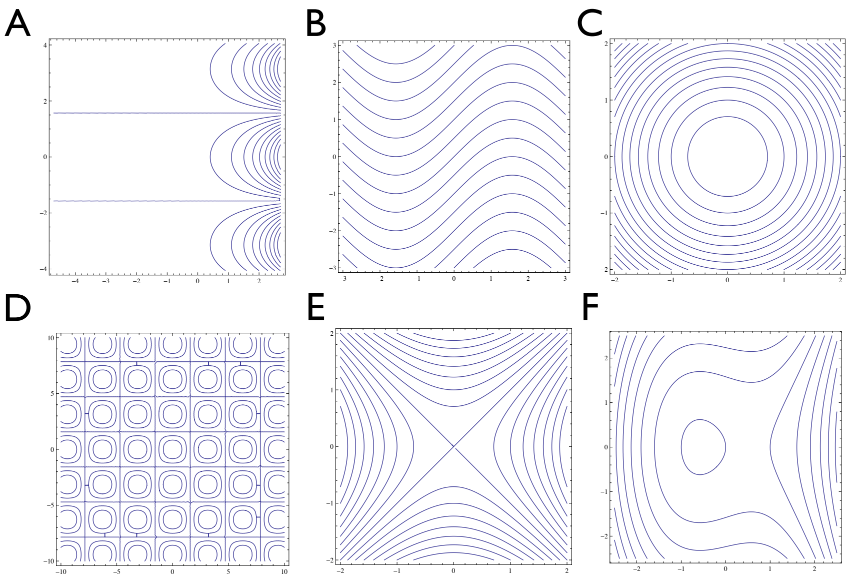
(ii). For which values of t is the quadratic form positive semidefinite but not positive definite?

(iii). For which values of t is the quadratic form negative definite?

(iv). For which values of t is the quadratic form indefinite?

(v). For which values of t is the quadratic form negative semidefinite but not negative definite?

5. Each figure below shows some of the level curves of a function.



For each of the five functions below, indicate which figure represents its level curves. (No explanations needed.)

$x^2 + y^2$

$x^2 - y^2$

$y^2 - x^3 + x$

$(\cos x)(\cos y)$

$y - \sin x$

6(a). The position of a particle at time t is $\mathbf{u}(t) = (3\sin(t-2), t, t+t^3)$. Find the velocity of the particle at time t.

6(b). Find the acceleration of the particle at time t.

6(c). Find the speed of the particle at time t.

6(d). Find the tangent line to the path of the particle at the point $(0, 2, 10)$.

7. Consider the function $f(x, y) = x^2y + 7\sin(x - 1)$.

(a). The position at time t of a particle moving in $\mathbf{R}^2$ is $(x(t), y(t))$. At time $t = 2$, the particle's position is $(1, 2)$ and its velocity is $(3, \sqrt{2})$. Find $\frac{d}{dt} f(x(t), y(t))$ at time $t = 2$.

(b). Find $\dfrac{\partial^2 f}{\partial y \partial x}(x, y)$.

(c). Find $\dfrac{\partial^2 f}{\partial x^2}(x, y)$.

8. Suppose $F : \mathbf{R}^3 \to \mathbf{R}^2$ is defined by

$$F(x,y) = \begin{bmatrix} 3x + 5y \\ x + 2\sin y \\ x^2 y \end{bmatrix}.$$

Find the matrix $DF(1,2)$.

9. Consider a hillside whose height (in feet above sea level) at the point (x, y) is given by

$$h(x, y) = 1000 + \frac{1}{100}(x^2 - 3xy + 2y^2)$$

with the positive x-axis pointing to the east and the positive y-axis pointing to the north. (Here x and y are also in feet.)

(a). Suppose you are walking due east along a path that passes through the point $(x, y) = (100, 50)$. What is the slope of your path at that point?

(b). Another hiker heads due south from the point $(x, y) = (100, 50)$. What is the slope of her path at that point? Is she initially ascending or descending?

10. Suppose $f : \mathbf{R}^2 \to \mathbf{R}^2$ is a map such that $f(1,1) = (2,3)$,

$$Df(1,1) = \begin{bmatrix} 5 & -2 \\ 3 & 0 \end{bmatrix} \quad \text{and} \quad Df(2,1) = \begin{bmatrix} 7 & 1 \\ 0 & 2 \end{bmatrix}.$$

Suppose $g : \mathbf{R}^2 \to \mathbf{R}^2$ is the map defined by

$$g(x,y) = \begin{bmatrix} x + y^2 \\ xy \end{bmatrix}.$$

Let $h = f \circ g$. Find $Dh(1,1)$.