$\begin{array}{c} {\rm Math} \ 51 \\ {\rm First} \ {\rm Midterm} \ {\rm Exam} \end{array}$

Instructions.	Answer	the following	problems	carefully	and o	complete	ly. Ma	ke su	re
you show all you	ır work.	There are 10	0 points p	ossible. (Good 1	luck!			

3.7							
Name Section leader and time							
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6. (18)							
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1. (14 pts) (a). Solve the following system of equations using your method of choice. Write your answer in parametric form.

$$x + 2y - 3z - 2s + 4t = 1$$
$$2x + 5y - 8z - s + 6t = 4$$
$$x + 4y - 7z + 5s + 2t = 8$$

(b). For what vectors
$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathbb{R}^3$$
 does the matrix equation

$$\begin{pmatrix} 1 & 3 & 3 \\ 2 & 6 & 9 \\ -1 & -3 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

have a solution?

2. (14 pts) Consider the matrix

$$B = \begin{bmatrix} 1 & 1 & 0 & 5 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a). Find a set of vectors which spans the null space N(B).

(b). Find a set of vectors that span the column space C(B), where B is defined as in (a).

3. (18 pts) (a). Show that two vectors \mathbf{u}, \mathbf{v} in \mathbb{R}^3 , are linearly dependent if and only if

 $\mathbf{u} \times \mathbf{v} = 0.$

(b). Do the vectors
$$u=\begin{bmatrix}1\\1\\2\end{bmatrix},\ v=\begin{bmatrix}1\\2\\3\end{bmatrix},\ \mathrm{and}\ w=\begin{bmatrix}2\\-1\\1\end{bmatrix}$$
 form a basis in \mathbb{R}^3 ? Explain.

(c). Is the vector $\begin{bmatrix} 1\\1\\1 \end{bmatrix}$ in the span of the vectors u, v, and w in part (b)? Explain.

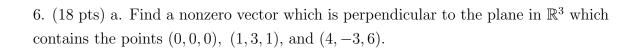
- 4. (24 pts) True or False.
- (a.) Every system of 2 equations in 3 variables has a solution.
- (b.) The set $\{(n,m) \mid n \text{ and } m \text{ are integers}\}$ is a subspace of \mathbb{R}^2 .
- (c.) Suppose that A is an $n \times n$ matrix and that its null space consists of a single point. Then every inhomogeneous equation Ax = b has a unique solution.
- (d.) If A is a 5×8 matrix, there is a vector $b \in \mathbb{R}^5$ so that the equation Ax = b has a unique solution.
- (e.) There exists a 4×3 matrix A, and a vector $b \in \mathbb{R}^4$ so that the equation Ax = b has exactly two solutions.
- (f.) If the null space of A is a line, and u_0 is a vector such that $Au_0 = b$, then the general solution to the nonhomogeneous equation Ax = b is also a line.
- (g.) The set $\{(x,y) \in \mathbb{R}^2 \mid xy = 0\}$ is a subspace of \mathbb{R}^2 .
- (h.) If A is a matrix and the equation Ax = b has at least two solutions, then the set of solutions contains a plane.

5. (12 pts) (a). State the Rank-Nullity Theorem

(b). Consider the matrix and vector

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & -3 & 1 \end{bmatrix} \quad v = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}.$$

Given that $Av = \mathbf{0}$ use the rank-nullity theorem to show that $dim(C(A)) \leq 2$.



b. Find the equation of the plane containing these three points. Your answer should be in the form ax+by+cz=d

c. Find the area of the triangle in \mathbb{R}^3 whose vertices are $(0,0,0),\ (1,3,1),$ and (4,-3,6).