Math 51 First Exam — April 26, 2012

Name: ______________________________ **SUID#:** ____________

Circle your section:			
Xiaodong Li 03 (11:00-11:50 am) 14 (10:00-10:50 am)	Frederick Tsz-Ho Fong 02 (11:00-11:50 am) 11 (1:15-2:05 pm)	Daniel Kim Murphy 09 (11:00-11:50 am) 18 (2:15-3:05 pm)	Tracy Nance ACE
Charles Minyu Peng 06 (1:15-2:05 pm) 08 (10:00-10:50 am)	James Zhao 05 (1:15-2:05 pm) 17 (2:15-3:05 pm)	Sukhada Fadnavis 12 (10:00-10:50 am) 15 (11:00-11:50 am)	

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result proved in class or the text, but be sure to clearly state the result before using it, and to verify that all hypotheses are satisfied.
- Please check that your copy of this exam contains 9 numbered pages and is correctly stapled.
- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.
- **You have 90 minutes.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- If you need extra room for your answers, use the back sides of each page. If you must use extra paper, use only that provided by teaching staff; make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, May 10**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ______________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	**Total**
Points:	12	10	12	16	8	8	10	76
Score:								

1. (12 points) Complete the following sentences.

   (a) Vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^7$ are defined to be *orthogonal* if

   (b) A *basis* for a subspace V of $\mathbb{R}^n$ is defined to be

   (c) A function $\mathbf{T} : \mathbb{R}^n \to \mathbb{R}^m$ is called a *linear transformation* if

2. (10 points) Be careful to answer *both* parts of the following:

   (a) Compute, showing all steps, the reduced row echelon form of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 & -2 \\ 3 & 0 & 6 & 2 & 5 \\ 1 & 2 & 0 & -1 & -2 \\ 2 & 3 & 1 & 1 & 0 \end{bmatrix}$$

   (b) Fill in the blanks (no reasoning needed): Rank of A:__________ Nullity of A:__________

3. (12 points) Consider the following three points A, B, C in $\mathbb{R}^3$:

$$A = (1, -1, 3), \quad B = (4, 1, -2), \quad C = (-1, -1, 1)$$

(a) In the triangle ΔABC, determine the cosine of the angle at vertex B.

(b) Let P be the plane in $\mathbb{R}^3$ that passes through the points A, B, C. Find a parametric representation for P.

For quick reference, here again are the points A, B, C:

$$A = (1, -1, 3), \quad B = (4, 1, -2), \quad C = (-1, -1, 1)$$

(c) Find an equation for the plane P of part (b), in the form $ax + by + cz = d$. (Here a,b,c,d are scalars, and x,y,z are the usual variables for coordinates of points in $\mathbb{R}^3$.)

4. (16 points) Let

$$A = \begin{bmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

As usual, we'll write $N(A)$ and $C(A)$, respectively, for the null space and column space of A.

(a) Find, with reasoning, a basis for $N(A)$.

(b) Find all solutions to the equation $A\mathbf{x} = \begin{bmatrix} 3 \\ 5 \\ -7 \end{bmatrix}$.

For quick reference, here again is the matrix: $A = \begin{bmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(c) Find a basis for $N(A)$ that contains the vector $\begin{bmatrix} 11 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}$, or state why no such basis exists.

(d) Find a basis for $C(A)$ that contains $\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$, or state why no such basis exists.

5. (8 points) Let L be the line in $\mathbb{R}^3$ that is the intersection of the planes whose equations are

$$x + y + z = 1 \qquad \text{and} \qquad x - y + z = 1$$

(a) Find L in parametric form.

(b) Find, with reasoning, a matrix A such that L is the set of solutions to the system $A\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$, or state why no such A exists.

6. (8 points) Suppose $\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^4$ is a linear transformation, and that

$$\mathbf{T}\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}4\\-1\\1\\0\end{bmatrix} \quad \text{and} \quad \mathbf{T}\left(\begin{bmatrix}0\\2\end{bmatrix}\right) = \begin{bmatrix}8\\4\\0\\-2\end{bmatrix}$$

(a) Find $\mathbf{T}\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and $\mathbf{T}\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$.

(b) Let $\mathbf{b} \in \mathbb{R}^4$. Find one or more conditions on $\mathbf{b}$ that determine precisely whether $\mathbf{b}$ is equal to $\mathbf{T}(\mathbf{x})$ for some $\mathbf{x} \in \mathbb{R}^2$; that is, whether $\mathbf{b}$ belongs to $\mathrm{im}(\mathbf{T})$. (Your answer should be given in the form of one or more equations involving the components b_1, b_2, b_3, b_4 of $\mathbf{b}$.)

7. (10 points) Each of the statements below is either *always true* ("T"), or *always false* ("F"), or *sometimes true and sometimes false, depending on the situation* ("MAYBE"). For each part, decide which and circle the appropriate choice; you *do not* need to justify your answers.

   In all these statements, the vector $\mathbf{e_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (in $\mathbb{R}^2$), and similarly $\mathbf{e_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

   (a) Given a 2×5 matrix A, the equation $A\mathbf{y} = \mathbf{e_1}$ has no solutions $\mathbf{y}$ in $\mathbb{R}^5$. T F MAYBE

   (b) Given a 5×2 matrix B, the equation $B\mathbf{z} = B\mathbf{e_1}$ has infinitely many solutions $\mathbf{z}$ in $\mathbb{R}^2$. T F MAYBE

   (c) Given vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ in $\mathbb{R}^2$ with the property that each of the sets T F MAYBE

   $$\{\mathbf{v_1}, \mathbf{v_2}\}, \quad \{\mathbf{v_2}, \mathbf{v_3}\}, \quad \text{and} \quad \{\mathbf{v_1}, \mathbf{v_3}\}$$

   is linearly independent, the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is also linearly independent.

   (d) Given vectors $\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}$ in $\mathbb{R}^5$ with the property that each of the sets T F MAYBE

   $$\{\mathbf{w_1}, \mathbf{w_2}\}, \quad \{\mathbf{w_2}, \mathbf{w_3}\}, \quad \text{and} \quad \{\mathbf{w_1}, \mathbf{w_3}\}$$

   is linearly independent, the set $\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ is also linearly independent.

   (e) Given nonzero $\mathbf{v} \in \mathbb{R}^2$, the set T F MAYBE

   $$V = \{\mathbf{x} \in \mathbb{R}^2 \mid \|\mathbf{x} + \mathbf{v}\|^2 = \|\mathbf{x}\|^2 + \|\mathbf{v}\|^2\} \quad \text{is a subspace of } \mathbb{R}^2.$$

   (f) Given nonzero $\mathbf{w} \in \mathbb{R}^2$, the set T F MAYBE

   $$W = \{\mathbf{x} \in \mathbb{R}^2 \mid \|\mathbf{x} + \mathbf{w}\| = \|\mathbf{x}\| + \|\mathbf{w}\|\} \quad \text{is a subspace of } \mathbb{R}^2.$$

   (g) Given a counterclockwise rotation $\mathbf{Rot}_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ through an angle θ, the set $\{\mathbf{Rot}_\theta(\mathbf{e_1}), \mathbf{Rot}_\theta(\mathbf{e_2})\}$ is a basis for $\mathbb{R}^2$. T F MAYBE

   (h) Given a counterclockwise rotation $\mathbf{Rot}_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ through an angle θ, the set $\{\mathbf{e_1}, \mathbf{Rot}_\theta(\mathbf{e_1})\}$ is a basis for $\mathbb{R}^2$. T F MAYBE

   (i) Given a projection $\mathbf{Proj}_L : \mathbb{R}^2 \to \mathbb{R}^2$ onto a line L containing the origin, the set $\{\mathbf{Proj}_L(\mathbf{e_1}), \mathbf{Proj}_L(\mathbf{e_2})\}$ is a basis for $\mathbb{R}^2$. T F MAYBE

   (j) Given a reflection $\mathbf{Refl}_L : \mathbb{R}^2 \to \mathbb{R}^2$ through a line L containing the origin, the set $\{\mathbf{Refl}_L(\mathbf{e_1}), \mathbf{Refl}_L(\mathbf{e_2})\}$ is a basis for $\mathbb{R}^2$. T F MAYBE