Math 51- Autumn 2013- Midterm Exam II

Please circle the name of your TA:

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Circle the time your TTh **section** meets: 9:00 10:00 11:00 1:15 2:15

Your name (print):

Student ID:

Please sign the following: ”On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

Signature:____

Instructions: Circle your TA’s name and the time that you attend the TTh section. Read each question carefully, and show all your work. You have 90 minutes to do all the problems. During the test, **you may NOT use any notes, books, calculators or electronic devices**

Question	1	2	3	4	5	6	7	Total
Maximum	8	20	18	10	18	16	10	100
Score								

Problem 1. (8pts) For which values of a is the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 3 & a & 5 \end{bmatrix}$ NOT invertible?

Problem 2. (20 pts total) Consider the matrix $A = \begin{pmatrix} 3 & 2 \\ -2 & -2 \end{pmatrix}$

(a)(10 pts) find the eigenvalues and the corresponding eigenvectors of A.

(b) (3pts) what are the eigenvalues of A^{99}? (A is the matrix given above)

(c) (3pts) is A^{99} diagonalizable? Please explain!

(d) (4pts) if R is a region in $\mathbb{R}^2$ of area 4, what is the area of its image under the linear transformation whose associated matrix is A?

Problem 3. (18 pts total) Consider the linear transformation T that reflects vectors in $\mathbb{R}^2$ across the line $x = -2y$.

(a) (8 pts) Find the eigenvalues of T and a basis for each eigenspace;

(a)(8 pts) Find the matrix A associated to this linear transformation in the standard basis

(c) (2pts) is T surjective? Please explain!

Problem 4. (10pts total) The position of a particle at time t is given by $\mathbf{x}(t) = \begin{bmatrix} t^2 \\ 5 \\ \sin(7t) \end{bmatrix}$.

(a) (6pts) calculate its velocity and acceleration

(b) (4pts) find the equation of the tangent line to the curve traced by the particle at time $t = 0$.

Problem 5. (18pts total) Calculate:

(a) (4pts) $\dfrac{\partial}{\partial y}(x^2 \sin y + e^z)$ at the point $(x, y, z) = (3, 0, 7)$.

(b) (6pts) the matrix of total derivatives of the function $f(x, y, z) = (x^4 + zy,\ x^2 \sin y + e^z)$.

(c) (8pts) Assume $h(x, y, z) = g(f(x, y, z))$ where f is the function in part (b) and $g : \mathbb{R}^2 \to \mathbb{R}$ is a function $g(u, v)$ such that $\dfrac{\partial g}{\partial u} = 2$ and $\dfrac{\partial g}{\partial v} = 3$ at the point $(0, 1)$. Calculate $\dfrac{\partial h}{\partial z}$ at the point (0,0,0).

Problem 6. (16pts total) Consider an anthill whose height in mm above sea level is given by

$$h(x, y) = 500 - x^2 + 2xy + 3y^2,$$

where x points E (east), and y points N (north).

(a)(6pts) If an ant is crawling on this hill in such a way that its x-coordinate is *increasing* at 2mm/sec and its y-coordinate is *decreasing* at 1mm/sec, at what rate is its height changing when the ant is at the point P whose coordinates are $x = 20$mm, $y = 10$mm?

(b) (6pts) Suppose another ant is now moving at the same point P above but in the SW direction. Does it ascend or descend? Please explain.

(c) (6pts) Find the equation of the tangent line to the level sets of the height function $h(x, y)$ at the point $x = 20$, $y = 10$.

Problem 7. (10 points) Assume $\mathbf{x}(t)$ is the position vector at time t of a particle moving smoothly on a sphere of radius 5 centered at the origin. Prove that at any moment the velocity vector $\dfrac{d\mathbf{x}}{dt}$ of the particle is perpendicular to its position vector.