Math 51 Second Exam — May 12, 2011

Name: ____ **SUID#:** ______________

Please circle your section:		
Jonathan Campbell 08 (11:00-11:50 AM) 14 (10:00-10:50 AM)	Minyu Peng 03 (11:00-11:50 AM) 11 (1:15-2:05 PM)	Joseph Man Chuen Cheng 06 (1:15-2:05 PM) 17 (2:15-3:05 PM)
Anca Vacarescu ACE (1:15-3:05 PM)	Henry Adams 02 (11:00-11:50 AM) 05 (1:15-2:05 PM)	Fernando Xuancheng Shao 12 (10:00-10:50 AM) 09 (11:00-11:50 AM)

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 90 minutes.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 9 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, May 26**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	**Total**
Points:	8	10	10	10	9	11	10	10	78
Score:									

1. (8 points) Compute, showing all steps, the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & -6 \\ 1 & -1 & 1 \end{bmatrix}$$

2. (10 points)

(a) Compute the following determinant (and show all work):

$$\begin{vmatrix} 3 & 1 & 0 & 2 & 1 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 4 & 0 & 1 & 3 & 2 \end{vmatrix} =$$

(b) Suppose $\mathbf{T}$ is the linear transformation with matrix

$$B = \begin{bmatrix} 2 & 7 \\ 3 & 5 \end{bmatrix}$$

and R is the triangular region in $\mathbb{R}^2$ with vertices $(0,0)$, $(3,-2)$, and $(2,0)$. Find the area of $\mathbf{T}(R)$, the image under $\mathbf{T}$ of R; show all steps in your reasoning.

3. (10 points) Let L be the line in $\mathbb{R}^2$ spanned by the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and let $\mathcal{B}$ be the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$$

Let $\mathbf{T} = 2\,\mathbf{Ref}_L$, that is, $\mathbf{T}$ is the map in $\mathbb{R}^2$ which reflects a vector across the line L and doubles its length.

(a) Find the matrix of $\mathbf{T}$ with respect to the basis $\mathcal{B}$. You may use any method you wish, but simplify your answer as much as possible.

(b) Find the matrix of $\mathbf{T}$ with respect to the standard basis for $\mathbb{R}^2$; simplify your answer as much as possible.

4. (10 points) Let A be the matrix

$$\begin{bmatrix} 6 & 14 \\ 2 & -6 \end{bmatrix}$$

(a) Find, showing all steps, a basis for $\mathbb{R}^2$ consisting of eigenvectors of A.

(b) Find a matrix B such that $B^3 = A^5$. (You may specify your answer for B as an explicit product of matrices and matrix inverses, without evaluating this product.)

5. (9 points) Suppose we know the following three facts about the matrix A:

- A has the form
$$A = \begin{bmatrix} 3 & -1 & -1 & 3 \\ -1 & 9 & -3 & -1 \\ -1 & -3 & 9 & -1 \\ a & b & c & d \end{bmatrix}$$
for some values a, b, c, d.

- A has four real eigenvalues, and $\mathbb{R}^4$ has a basis consisting of orthogonal eigenvectors of A.

- The vector $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A.

(a) Determine the eigenvalue associated to the eigenvector $\mathbf{u}$ given above. (Your answer should not depend on a, b, c, d.)

(b) Give values a, b, c, d of the fourth row of A for which *all* of the above conditions are satisfied; justify your answer.

(c) Show that A is not invertible.

6. (11 points) For this problem, let Q be the quadratic form

$$Q(x, y, z) = 6x^2 + 5y^2 + 4z^2 + 10xy + 4xz$$

(a) Write the matrix associated to the quadratic form Q.

(b) Note that Q can be written as $(x+2z)^2 + 5(x+y)^2$, a fact you do not have to verify. Is Q positive definite? If so, explain why; if not, find the definiteness of Q with justification.

For easy reference, here again is the function Q:

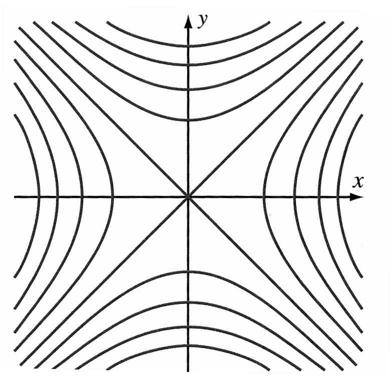
$$Q(x, y, z) = 6x^2 + 5y^2 + 4z^2 + 10xy + 4xz$$

Note: parts (c) and (d) do not depend on parts (a) and (b)!

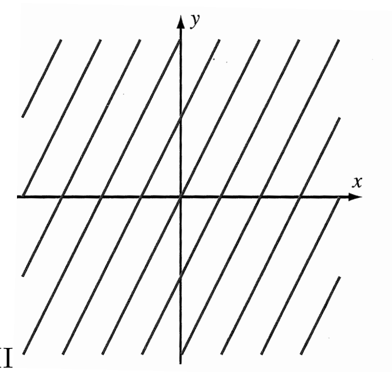
(c) Find $DQ|_{(-1,3,1)}$, the matrix of partial derivatives of Q evaluated at $(x, y, z) = (-1, 3, 1)$.

(d) Find $\dfrac{\partial^2 Q}{\partial y^2}$ and $\dfrac{\partial^2 Q}{\partial x\, \partial z}$.

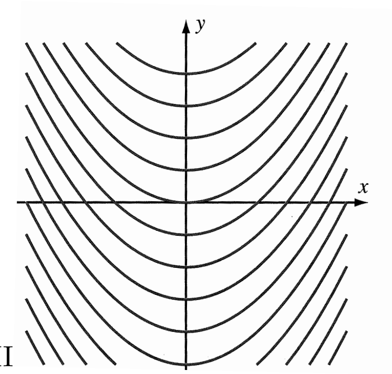
7. (10 points) Match each function below with a collection of its level curves, chosen from among the collections labeled I through VI below. No justification is necessary.



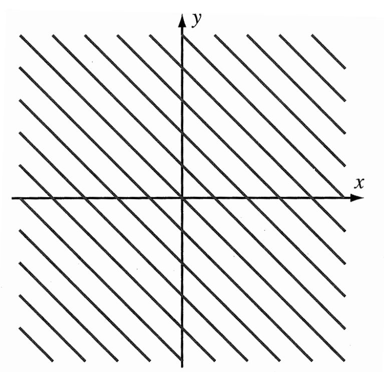
I



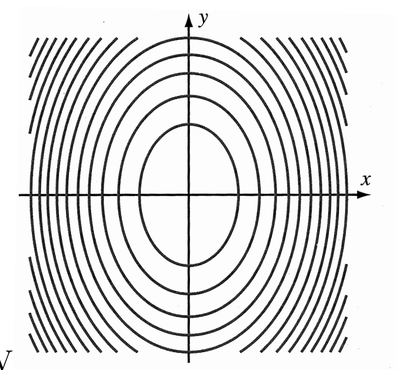
II



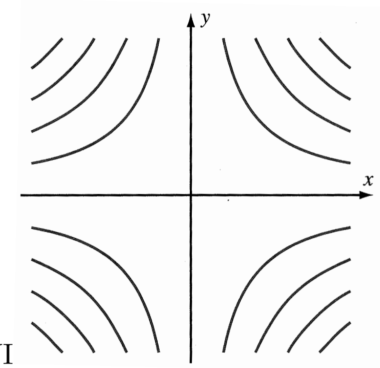
III



IV



V



VI

Function	I, II, III, IV, V, or VI	Function	I, II, III, IV, V, or VI
$f(x,y) = x^2 - y$		$f(x,y) = xy$	
$f(x,y) = \|x+y\|$		$f(x,y) = 2x^2 + y^2$	
$f(x,y) = x^2 - y^2$		$f(x,y) = (2x-y)^2$	

8. (10 points) If D is the set of positive real numbers, let $\mathbf{x} : D \to \mathbb{R}^2$ be the parametric curve given by

$$\mathbf{x}(t) = \left(\frac{1}{t^2} + 1,\ t^2 - 1\right)$$

(a) Find $\mathbf{x}'(t)$ and $\mathbf{x}''(t)$, also known as the velocity and acceleration vectors.

(b) Determine any values of t for which the velocity and acceleration are orthogonal to each other; show all your reasoning.

(c) Does the image of $\mathbf{x}$ lie on a level set of the function $f(x, y) = xy - y + x$? If so, specify which level set; if not, explain why not.

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