Solutions to review notes

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Autumn 2014

For most of the questions appeared in review notes, we just put its reference to previous midterms.

Practice question 1 2009 Spring, question 4.

Practice question 2 2009 Spring, question 7.

Practice question 3 2008 Autumn, question 4-(c).

Practice question 4 2011 Autumn, question 9.

Practice question 5 2013 Spring, question 7.

Practice question 6

Solution. a) Use squeeze theorem. Now $\frac{x^3y-xy^3}{x^2+y^2} = \frac{xy(x^2-y^2)}{x^2+y^2}$. And we know $-1 \leq \frac{x^2-y^2}{x^2+y^2} \leq 1$. So

$$-|xy| \leq \frac{xy(x^2-y^2)}{x^2+y^2} \leq |xy|.$$

By squeeze theorem the desired limit is 0.

b) Again use squeeze theorem. We know $1 \leq \sin(\frac{1}{x}) \leq 1$, so

$$|re^{\sin(\frac{1}{x})}| \leq |r|e^1 = e|r|.$$

By squeeze theorem the original limit is 0.

c) Despite its complicated form the function $r \tan\theta$ is continuous at $(r,\theta) = (0,0)$. So the limit is just 0. $\square$