Assignment 1 Formation of a vanishing point

ChingShing, SJTU's junior

Three Steps of Forming a Vanishing Point

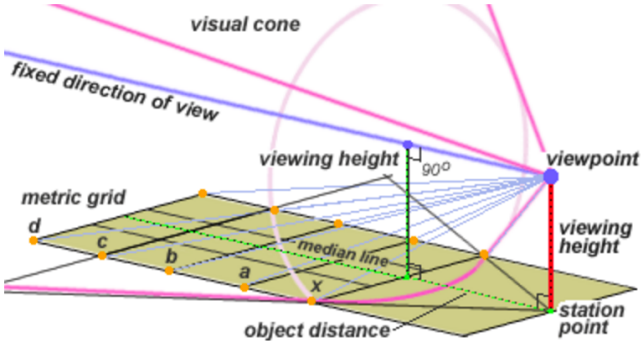
Outline:

① The basic perspective geometry ——the basic spatial condition

② An image plane——the perspective of projection

③ Perspective image on the image plane——forming a vanishing point

Main text:



①The basic perspective geometry ——the basic spatial condition

Figure 1. The basic perspective geometry

Features about Figure 1:

A. All lines in the metric grid are defined either **parallel or perpendicular to each other and to the direction of view**. This allows us to measure distances in any direction in relation to the viewpoint.

B. Five of **orange points**, and labeled **d, c, b, a** and **x** along one side of the direction of view; a matching row of unlabeled orange points is shown along the opposite side.

C. **The visual rays** (purple lines linked between **viewpoint** and **orange points**) from these points define the geometry of visual rays in physical space.

D. The human viewer stands on the **station point** and he sees at **viewpoint**. The **viewpoint** is at the tip or apex of the **visual cone**, and the origin of the **direction of view**.

E. A **median line** on the ground plane, extending from the station point and parallel to **the direction of view,** which divides the ground plane into symmetrical left and right halves.

F. An **object distance** is between the **viewpoint** (or **station point**) and any object within the **visual cone**.

② An image plane——the perspective of projection

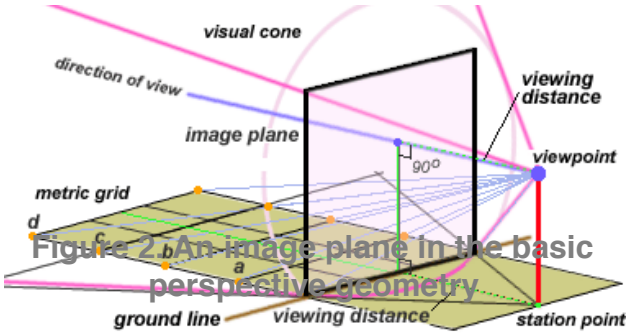


Figure 2.An image plane in the basic perspective geometry

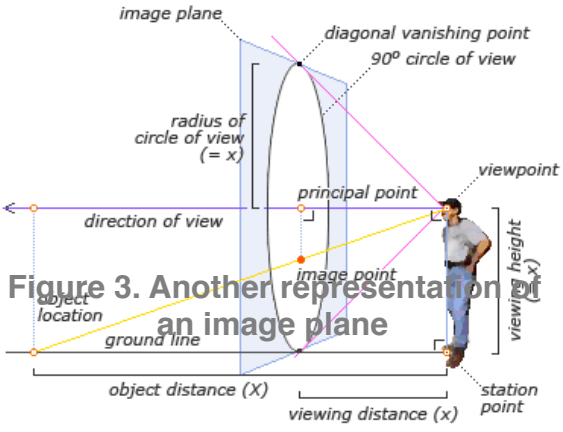


Figure 3. Another representation of an image plane

Features about Figure 2 & 3:

A. An **image plane** is conventionally a flat surface perpendicular to the **direction of view** and to the **ground plane**.

B. The **image plane** does not have fixed dimensions — its limits are only determined by the size of the **visual cone** or by the size of the support that we make the image on.

C. The **image plane** does have a fixed location: the **ground line** directly underneath it. This is equivalent to the base of a vertical wall on which the painting or fresco is displayed.

D. The **ground line** is at a fixed distance from the station point: this is the **viewing distance**.

③ Perspective image on the image plane——forming a vanishing point

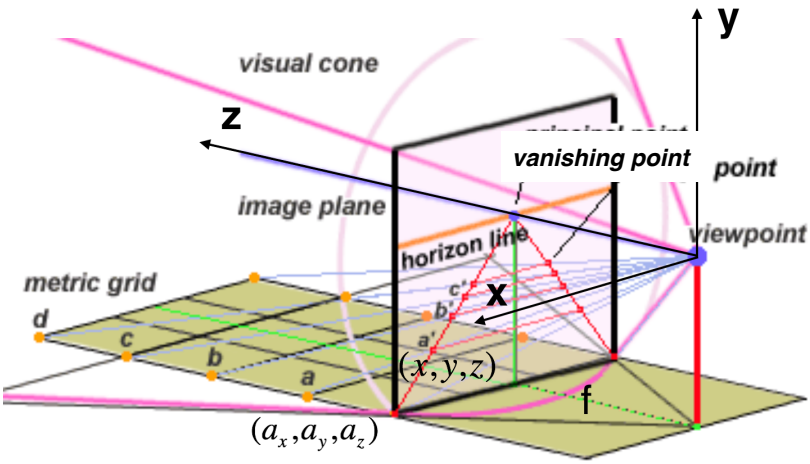


Figure 4. Perspective image on the image plane

Features about Figure 4:

A. Let x,y,z be an orthogonal system of coordinates. The x-axis is parallel to the **ground line**. The y-axis is parallel to the **image plane**. And the z-axis is coincident with the **direction of view**.

B. The **image plane** is defined by the equation $z=f$ where f is the **viewing distance.**

C. Choose any point on the straight line S that is parallel to z-axis on the **metric grid**. For instance, we take point $a(a_x,a_y,a_z)$, which is projected at point $a'(x,y,z)$ on the **image plane**. And we have:

$$\begin{cases} x = a_x + tn_x \\ y = a_y + tn_y \\ z = a_z + tn_z \end{cases}$$

where $\vec{n} = (n_x, n_y, n_z) = (0,0,1)$ is the unit vector giving the direction of S and *t* is a parameter.

D. The **vanishing point** of the straight line S is the point $v_s = (x_\infty, y_\infty, z_\infty)$ on the **image plane**, where

$$\begin{cases} x_\infty = \lim_{t\to\infty} f\dfrac{a_x + tn_x}{a_z + tn_z} = f\dfrac{n_x}{n_z} = 0 \\ y_\infty = \lim_{t\to\infty} f\dfrac{a_y + tn_y}{a_z + tn_z} = f\dfrac{n_y}{n_z} = 0 \\ z_\infty = f \end{cases}$$

Conclusion: the **vanishing point** locates the direction of view as it passes through the **image plane**.

Reference:

1.Figures: Bruce MacEvoy "Perspective in the world"(http://www.handprint.com/HP/WCL/perspect1.html)

2. Mathematical Properties: B. Caprile, V. Torre "Using Vanishing Points for Camera Calibration", International Journal of Computer Vision, Volume 4, Issue 2, pp. 127-139, March 1990 (http://link.springer.com/article/10.1007%2FBF00127813)