first case

Table of Contents

parameters

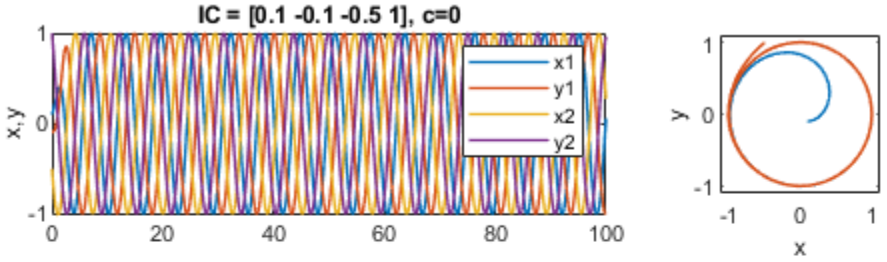
odeopts = odeset('RelTol',1e-11,'AbsTol',1e-13);

```
t_span = 100;
e=0.01;
varnames = {'x1','y1','x2','y2'};

% Coupling parameter and initial conditions
% v CHANGE ME v
c = 0.0;
init = [0.1, -0.1, -0.5, 1];
% ^ CHANGE ME ^
```

ODE solver

```
[t, y_eq] = ode45(@(t,y) PS5_eq(t,y,c), [0,t_span], init);%, odeopts);

% function plot over time
h = figure;
subplot(3,3,[1,2])
plot(t, y_eq(:,:),'LineWidth',1)
ylabel('x,y')
legend(varnames)
```

```
title(['IC = [',num2str(init(1)),' ',num2str(init(2)),'
 ',num2str(init(3)),...
        ' ',num2str(init(4)),'], c=',num2str(c)])
xlim([0, t_span])

% phase coupling plot
subplot(333)
plot(y_eq(:,1), y_eq(:,2),'LineWidth',1);hold on
plot(y_eq(:,3), y_eq(:,4),'LineWidth',1)
xlabel('x')
ylabel('y')
axis square
buff = .1;
axis([min([y_eq(:,1);y_eq(:,3)])-buff
 max([y_eq(:,1);y_eq(:,3)])+buff...
    min([y_eq(:,2);y_eq(:,4)])-buff max([y_eq(:,2);y_eq(:,4)])+buff])
```



DMD

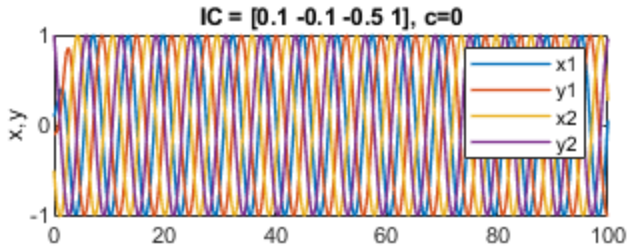
```
thresh = .9;
deltaT = mean(diff(t));
p = 20;

Output = DMD(y_eq',[],thresh);

% Dimension reduction selection
```

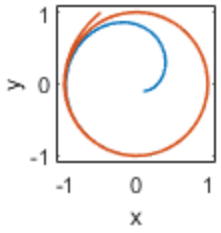
```
% figure,
% semilogy(diag(Output.DMD.Sig)/sum(diag(Output.DMD.Sig)),'.')
% hold on
% line([Output.DMD.r+.5 Output.DMD.r+.5],[min(diag(Output.DMD.Sig))
 max(diag(Output.DMD.Sig))])
% xlabel('Sig number')
% ylabel('Energy')

for i = 1:Output.DMD.r
    emp_omega_r_phi(i,1) = imag(log(Output.DMD.D(i))/deltaT)/(2*pi);
    emp_omega_r_phi(i,2) = abs(Output.DMD.D(i));
    emp_omega_r_phi(i,3) =
 atan2(imag(Output.DMD.D(i)),real(Output.DMD.D(i)))*(180/pi);
end
```

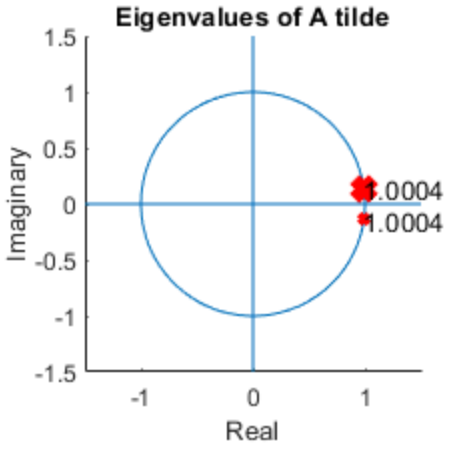
Performing DMD

eigen value spectrum of A tilde

```
figure(h);
subplot('position',[.05 .1 .4 .4])
D = Output.DMD.D;
D_plot = zeros(length(D),2);
for i = 1:length(D)
    D_plot(i,1) = real(D(i));
    D_plot(i,2) = imag(D(i));
end

M_size = sort(1:Output.DMD.r,'descend');
M_size_norm = normalize_var(M_size,5,10);

circle(0,0,1);hold on
line([0 0],[-2 2])
line([-2 2], [0 0])
for i = 1:length(D)
    plot(D_plot(i,1), D_plot(i,2), 'rx','MarkerSize',
 M_size_norm(i),'LineWidth', M_size_norm(i)/2)
    text(D_plot(i,1),D_plot(i,2),num2str(norm(D_plot(i,:))))
end
xlim([-1.5 1.5])
ylim([-1.5 1.5])
axis square
xlabel('Real')
ylabel('Imaginary')
title('Eigenvalues of A tilde')

% Mode selection
for i = 1:Output.DMD.r
    x(i) = abs(emp_omega_r_phi(i,1));
    y(i) =
 (abs(Output.DMD.D(i))^p)*norm(Output.DMD.DynamicModes(:,i));
end
```

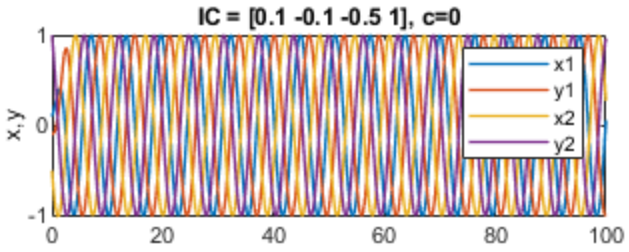


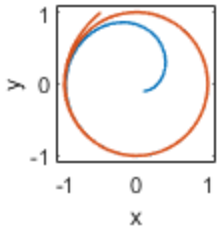


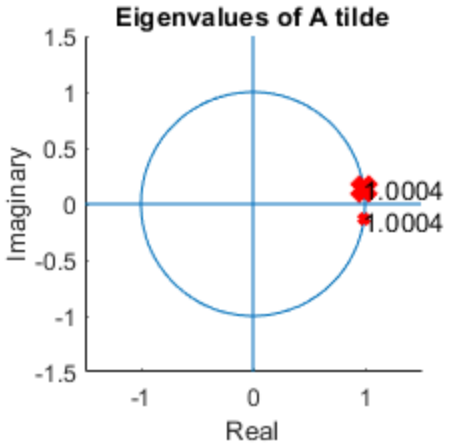


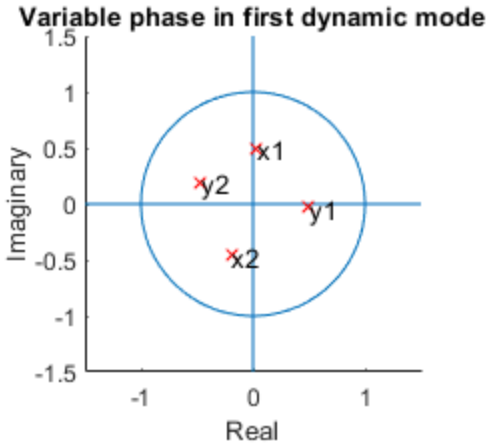
phase of variables to first dynamic mode

```
figure(h);

subplot('position',[.55 .1 .4 .4])
circle(0,0,1);hold on
line([0 0],[-2 2])
line([-2 2], [0 0])
plot(Output.DMD.DynamicModes(:,1),'rx')
for i = 1:length(Output.DMD.DynamicModes(:,1))

 text(real(Output.DMD.DynamicModes(i,1)),imag(Output.DMD.DynamicModes(i,1)),varnam
end
xlim([-1.5 1.5])
ylim([-1.5 1.5])
axis square
xlabel('Real')
ylabel('Imaginary')
title('Variable phase in first dynamic mode')
```









second case

parameters

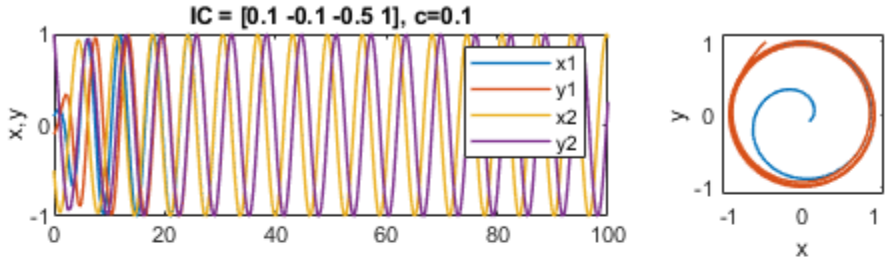
odeopts = odeset('RelTol',1e-11,'AbsTol',1e-13);

```
t_span = 100;
e=0.01;
varnames = {'x1','y1','x2','y2'};

% Coupling parameter and initial conditions
% v CHANGE ME v
c = 0.1;
init = [0.1, -0.1, -0.5, 1];

% ^ CHANGE ME ^
```

ODE solver

```
[t, y_eq] = ode45(@(t,y) PS5_eq(t,y,c), [0,t_span], init);%, odeopts);

% function plot over time
h_2 = figure;
subplot(3,3,[1,2])
plot(t, y_eq(:,:),'LineWidth',1)
```

```
ylabel('x,y')
legend(varnames)
title(['IC = [',num2str(init(1)),' ',num2str(init(2)),'
 ',num2str(init(3)),...
       ' ',num2str(init(4)),'], c=',num2str(c)])
xlim([0, t_span])

% phase coupling plot
subplot(333)
plot(y_eq(:,1), y_eq(:,2),'LineWidth',1);hold on
plot(y_eq(:,3), y_eq(:,4),'LineWidth',1)
xlabel('x')
ylabel('y')
axis square
buff = .1;
axis([min([y_eq(:,1);y_eq(:,3)])-buff
 max([y_eq(:,1);y_eq(:,3)])+buff...
    min([y_eq(:,2);y_eq(:,4)])-buff max([y_eq(:,2);y_eq(:,4)])+buff])
```



DMD

```
thresh = .9;
deltaT = mean(diff(t));
p = 20;

Output = DMD(y_eq',[],thresh);
```

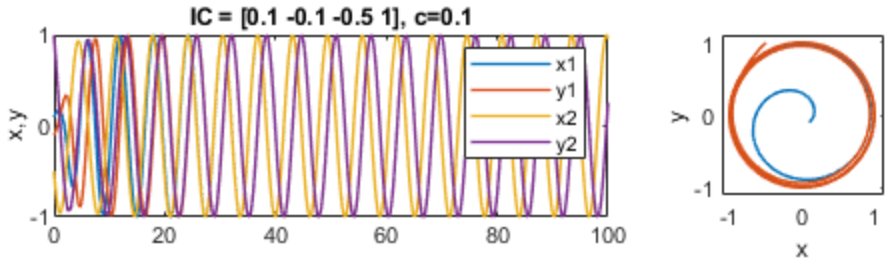
```
% Dimension reduction selection
% figure,
% semilogy(diag(Output.DMD.Sig)/sum(diag(Output.DMD.Sig)),'.')
% hold on
% line([Output.DMD.r+.5 Output.DMD.r+.5],[min(diag(Output.DMD.Sig))
 max(diag(Output.DMD.Sig))])
% xlabel('Sig number')
% ylabel('Energy')

for i = 1:Output.DMD.r
    emp_omega_r_phi(i,1) = imag(log(Output.DMD.D(i))/deltaT)/(2*pi);
    emp_omega_r_phi(i,2) = abs(Output.DMD.D(i));
    emp_omega_r_phi(i,3) =
 atan2(imag(Output.DMD.D(i)),real(Output.DMD.D(i)))*(180/pi);
end
```

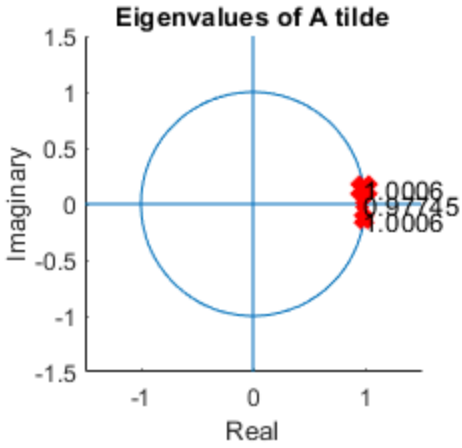
Performing DMD

eigen value spectrum of A tilde

```
figure(h_2);
subplot('position',[.05 .1 .4 .4])
D = Output.DMD.D;
D_plot = zeros(length(D),2);
for i = 1:length(D)
    D_plot(i,1) = real(D(i));
    D_plot(i,2) = imag(D(i));
end

M_size = sort(1:Output.DMD.r,'descend');
M_size_norm = normalize_var(M_size,5,10);

circle(0,0,1);hold on
line([0 0],[-2 2])
line([-2 2], [0 0])
for i = 1:length(D)
    plot(D_plot(i,1), D_plot(i,2), 'rx','MarkerSize',
 M_size_norm(i),'LineWidth', M_size_norm(i)/2)
    text(D_plot(i,1),D_plot(i,2),num2str(norm(D_plot(i,:))))
end
xlim([-1.5 1.5])
ylim([-1.5 1.5])
axis square
xlabel('Real')
ylabel('Imaginary')
title('Eigenvalues of A tilde')

% Mode selection
for i = 1:Output.DMD.r
    x(i) = abs(emp_omega_r_phi(i,1));
    y(i) =
 (abs(Output.DMD.D(i))^p)*norm(Output.DMD.DynamicModes(:,i));
```

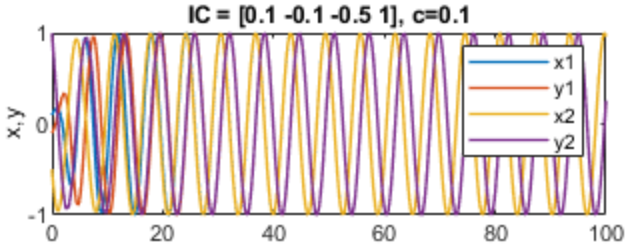
```
end
```

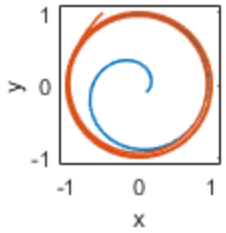


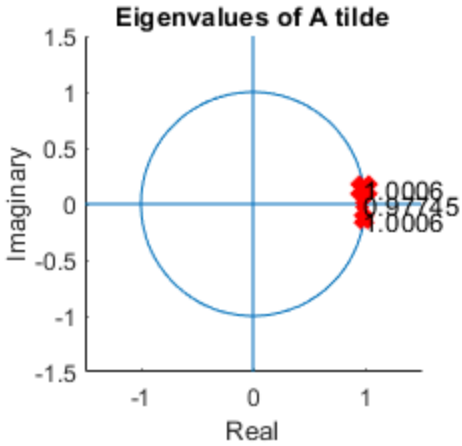


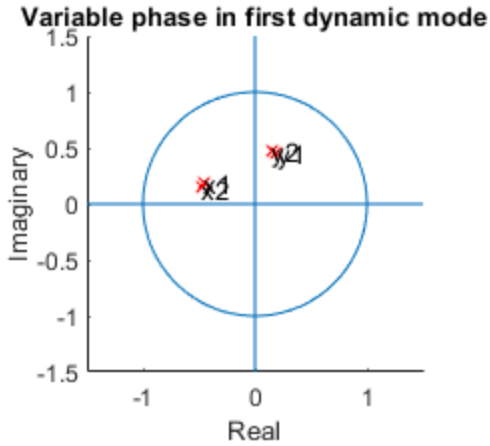
phase of variables to first dynamic mode

```
figure(h_2);

subplot('position',[.55 .1 .4 .4])
circle(0,0,1);hold on
line([0 0],[-2 2])
line([-2 2], [0 0])
plot(Output.DMD.DynamicModes(:,1),'rx')
for i = 1:length(Output.DMD.DynamicModes(:,1))

 text(real(Output.DMD.DynamicModes(i,1)),imag(Output.DMD.DynamicModes(i,1)),varnam
end
xlim([-1.5 1.5])
ylim([-1.5 1.5])
axis square
xlabel('Real')
ylabel('Imaginary')
title('Variable phase in first dynamic mode')
```









Third case

parameters

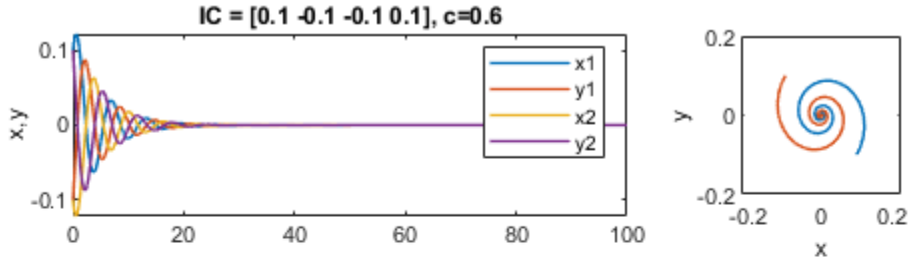
odeopts = odeset('RelTol',1e-11,'AbsTol',1e-13);

```
t_span = 100;
e=0.01;
varnames = {'x1','y1','x2','y2'};

% Coupling parameter and initial conditions
% v CHANGE ME v
c = 0.6;
init = [0.1, -0.1, -0.1, 0.1];

% ^ CHANGE ME ^
```

ODE solver

```
[t, y_eq] = ode45(@(t,y) PS5_eq(t,y,c), [0,t_span], init);%, odeopts);

% function plot over time
h_3 = figure;
subplot(3,3,[1,2])
plot(t, y_eq(:,:),'LineWidth',1)
```

```
ylabel('x,y')
legend(varnames)
title(['IC = [',num2str(init(1)),' ',num2str(init(2)),'
 ',num2str(init(3)),...
        ' ',num2str(init(4)),'], c=',num2str(c)])
xlim([0, t_span])

% phase coupling plot
subplot(333)
plot(y_eq(:,1), y_eq(:,2),'LineWidth',1);hold on
plot(y_eq(:,3), y_eq(:,4),'LineWidth',1)
xlabel('x')
ylabel('y')
axis square
buff = .1;
axis([min([y_eq(:,1);y_eq(:,3)])-buff
 max([y_eq(:,1);y_eq(:,3)])+buff...
    min([y_eq(:,2);y_eq(:,4)])-buff max([y_eq(:,2);y_eq(:,4)])+buff])
```



DMD

```
thresh = .9;
deltaT = mean(diff(t));
p = 20;

Output = DMD(y_eq',[],thresh);
```

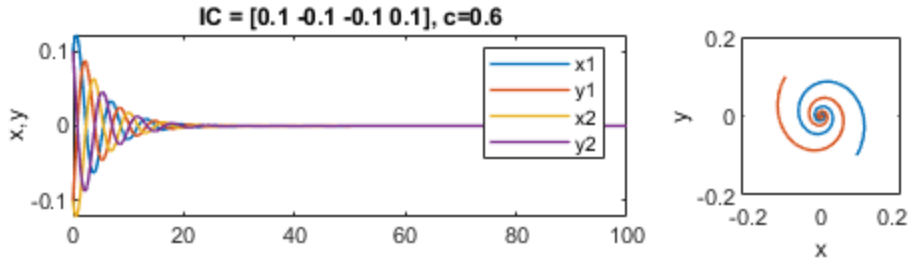
```
% Dimension reduction selection
% figure,
% semilogy(diag(Output.DMD.Sig)/sum(diag(Output.DMD.Sig)),'.')
% hold on
% line([Output.DMD.r+.5 Output.DMD.r+.5],[min(diag(Output.DMD.Sig))
 max(diag(Output.DMD.Sig))])
% xlabel('Sig number')
% ylabel('Energy')

for i = 1:Output.DMD.r
    emp_omega_r_phi(i,1) = imag(log(Output.DMD.D(i))/deltaT)/(2*pi);
    emp_omega_r_phi(i,2) = abs(Output.DMD.D(i));
    emp_omega_r_phi(i,3) =
 atan2(imag(Output.DMD.D(i)),real(Output.DMD.D(i)))*(180/pi);
end
```

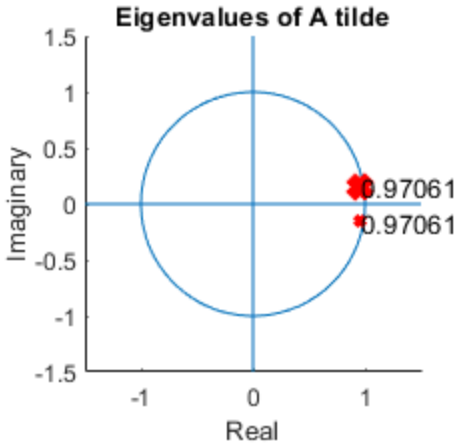
Performing DMD

eigen value spectrum of A tilde

```
figure(h_3);
subplot('position',[.05 .1 .4 .4])
D = Output.DMD.D;
D_plot = zeros(length(D),2);
for i = 1:length(D)
    D_plot(i,1) = real(D(i));
    D_plot(i,2) = imag(D(i));
end

M_size = sort(1:Output.DMD.r,'descend');
M_size_norm = normalize_var(M_size,5,10);

circle(0,0,1);hold on
line([0 0],[-2 2])
line([-2 2], [0 0])
for i = 1:length(D)
    plot(D_plot(i,1), D_plot(i,2), 'rx','MarkerSize',
 M_size_norm(i),'LineWidth', M_size_norm(i)/2)
    text(D_plot(i,1),D_plot(i,2),num2str(norm(D_plot(i,:))))
end
xlim([-1.5 1.5])
ylim([-1.5 1.5])
axis square
xlabel('Real')
ylabel('Imaginary')
title('Eigenvalues of A tilde')

% Mode selection
for i = 1:Output.DMD.r
    x(i) = abs(emp_omega_r_phi(i,1));
    y(i) =
 (abs(Output.DMD.D(i))^p)*norm(Output.DMD.DynamicModes(:,i));
```

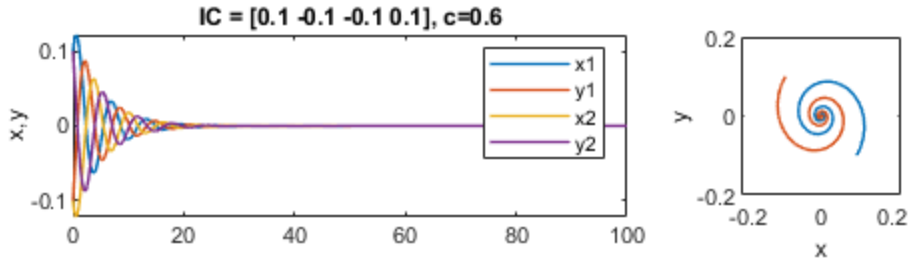
```
end
```

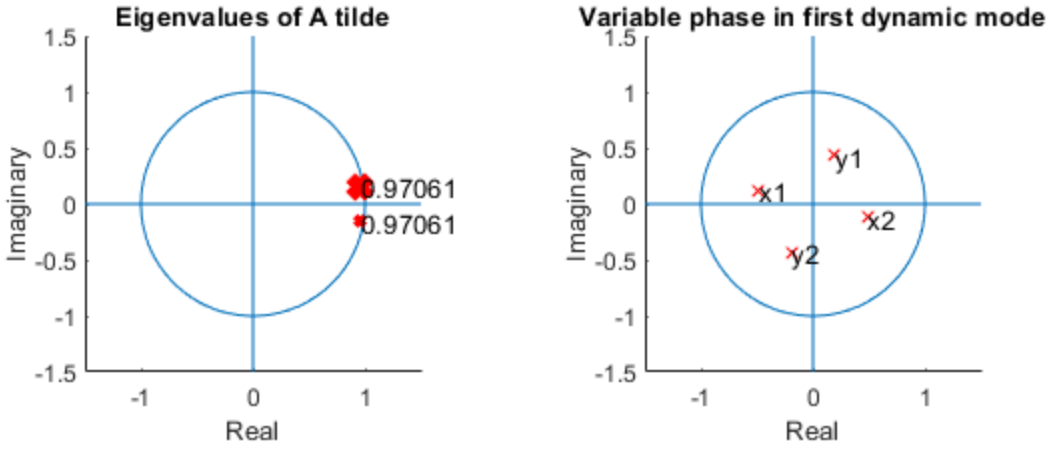




phase of variables to first dynamic mode

```
figure(h_3);

subplot('position',[.55 .1 .4 .4])
circle(0,0,1);hold on
line([0 0],[-2 2])
line([-2 2], [0 0])
plot(Output.DMD.DynamicModes(:,1),'rx')
for i = 1:length(Output.DMD.DynamicModes(:,1))

 text(real(Output.DMD.DynamicModes(i,1)),imag(Output.DMD.DynamicModes(i,1)),varnam
end
xlim([-1.5 1.5])
ylim([-1.5 1.5])
axis square
xlabel('Real')
ylabel('Imaginary')
title('Variable phase in first dynamic mode')
```





Explanation

for both of the cases which are in unison, the eigenvalue of A tilde is larger than one and for the one that is not beating in unison, the eigenvalue of a tilde is less than one. Suggesting the eigenvalue of A tilde indicate the amplitude and synchronicity of variables.

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