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Amath482
HW1

Abstract

By using data from 24-hour broad spectrum recoding of acoustics with half an hour increment, this paper will get the center frequency as well as the path of a submarine technology. We filter out the data around center frequency by eliminate the noise to get the path of that submarine in plot.

Introduction and Overview

Noise exists in our life and negatively influence us ability to analyze sound. However, by adapting Fourier transform, we are able to decrease or eliminate the influence of sound. This is accomplished by filtering out the noise around center frequency. This will make us clearly see the path of that submarine. At first, I will average the spectrum to get the center frequency. Then, due to the noise included in data around center frequency, I have to eliminate it to know the path clearly.

Theoretical Background

In some situation, we need to use frequency to look at our plot. Then, we introduce Fourier transform which convert x-axis from time or something else to frequency as we need by formula:

$$ft(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx \qquad (1)$$

Which can be recovered by a similar formula to change the position of k and x and replace -I by I. This can only solve function in infinity domain. However, in real world, we need an finite domain equation which is Fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_n cos(kx) + b_n sin(kx)) \quad (2)$$

To eliminate some shortcoming in maximum frequency, we will use DFT:

$$\hat{x}_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{\frac{2\pi ikn}{N}}$$

(3)

But, in this assignment, we are dealing with signal which has a large N, and FFT will same a lot of time compared with DFT, so we do fftshift for most of time. However, in most of time, noise negatively influence our judgement of signal. So, we introduce a filter function to filter out such noise by:

$$F(k) = e^{-\tau(k-k_0)^2} \quad (4)$$

Algorithm Implementation and Development

The data is 262144 * 49, so we need to reshape it into 64*64*64 * 49 at half an hour period, an then average the frequency after Fourier transform to transform it from time domain to frequency domain. To get the center frequency, we need to do the average of all data in subdata after fast Fourier transform by using the sum in all column to divide the 49. And by applying the maximum index value into the meshgrid after fast Fourier transform, we will get a center point which is our center frequency. Also, For the wavenumber k, it has to be rescale by 2π/L since FFT is 2π periodic signal. Then next step is to filter it. By applying the point with gaussian filter formula, we get a function which can eliminate the noise for the data around center frequency which can make us better locate where the submarine is. In addition, the τ in the function usually result in different path. This is because some τ cannot completely remove some obvious noise. I range the value from 5 to 1 and try some decimal less than 1 and some negative numbers (it cannot be negative in this question, and it should be positive always), and I found 1 is a best value for it, so the filter function becomes:

$$F(k) = e^{(-1\times((Kx-center(1))^2+(Ky-center(2))^2+(Kz-center(3))^2))} \quad (5)$$

Where center is the array contains the xyz position of center frequency. Then we filter it and then do the reverse of Fourier transform by ifftn to get the xyz position of submarine. Then there will be 49 number for each of x, y, z, which is the path of the submarine. And the x,y position is where we need to send our P-8 aircraft to because we don't need to consider the z(how much deep under water).

<u>Computational Result</u>

Center Frequency: 5.3407 -6.9115 2.1991

The path we should send our aircraft to is the xy points as below:

For x position: 3.125 3.125 3.125 3.125 3.125 3.125 3.125 3.125 3.125 2.8125 2.8125 2.5 2.1875 1.875 1.875 1.5625 1.25 0.9375 0.3125 0 -0.3125 -0.9375 -1.25 -1.875 -2.1875 -2.8125 -3.125 -3.75 -4.0625 -4.375 -4.6875 -5.3125 -5.625 -5.9375 -5.9375 -6.25 -6.5625 -6.875 -6.875 -6.875 -6.875 -6.875 -6.875 -6.5625 -6.5625 -6.25 -5.625 -5.625 -5

For y position:

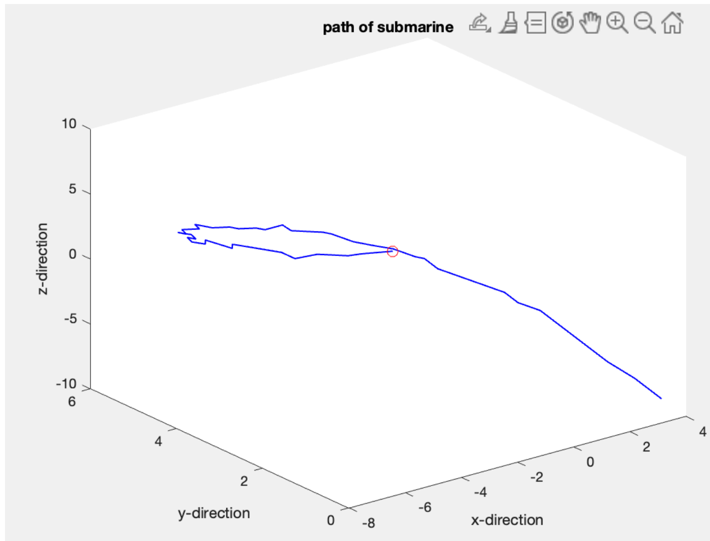
0 0.3125 0.6250 1.2500 1.5625 1.8750 2.1875 2.5000 2.8125 3.1250 3.4375 3.7500 4.0625 4.3750 4.6875 4.6875 5.0000 5.3125 5.3125 5.6250 5.6250 5.9375 5.9375 5.9375 5.9375 5.9375 5.9375 5.9375 5.9375 5.9375 5.6250 5.6250 5.3125 5.3125 5.0000 4.6875 4.6875

4.3750   4.0625   4.0625 3.4375   3.4375   3.1250   2.5000   2.1875   1.8750

1.5625   1.2500   0.9375

The path of

submarine：



Summary and Conclusions

To conclusion, by using Fast Fourier transform, we can successfully find out the center frequency of the submarine. However, because the origin data mix some noise in it, we have to adapt filter function to eliminate such negative influence. Then, we can successfully find out the exact path of submarine, and make it’s possible for aircraft to follow it. Fourier transform is useful in finding frequency of some signal, and by eliminate the noise from the frequency plot, we can get the original path of the object.

Appendix A - function in code

linspace(a,b,n): create equally space of n number between a and b
zeros: create a vector or matrix with all 0 in given dimension
reshape: reshape a matrix or vector to given dimension
fftshift: shift the array, and shift zero frequency to the center of array

meshgrid: create a grid containing all points in same dimension
max: picks the maximum value in given input
fftn: gives the Fourier transform of input
iffth: reverse operation of fftn.
plot3: plot in 3d

Appendix B - Matlab code

```
% Clean workspace
clear all; close all; clc

load subdata.mat
L = 10; % spatial domain
n = 64; % Fourier modes

x2 = linspace(-L,L,n+1);
x = x2(1:n);
y =x;
z = x;
k = (2*pi/(2*L))*[0:(n/2 - 1) -n/2:-1];
ks = fftshift(k);

[X,Y,Z]=meshgrid(x,y,z);
[Kx,Ky,Kz]=meshgrid(ks,ks,ks);

ave = zeros(n,n,n);
for j=1:49
   Un(:,:,:)=reshape(subdata(:,j),n,n,n);
   Unt = fftn(Un);
   ave = ave + Unt;
end
ave = abs(fftshift(ave))/49;
[M,I] = max(ave(:));
center = [Kx(I),Ky(I),Kz(I)];

%filter function
tau = 1;
filter = exp(-tau*((Kx-center(1)).^2+(Ky-center(2)).^2+(Kz-center(3)).^2));
xp = zeros(1,49);
```

```
yp = zeros(1,49);
zp = zeros(1,49);
for k = 1:49
    data = reshape(subdata(:,k),n,n,n);
    ftnd = fftn(data);
    filt = ftnd.* fftshift(filter);
    unfilt = ifftn(filt);
    unfilt = unfilt / max(unfilt(:));

    [M,I] = max(unfilt(:));
    center2 = [X(I),Y(I),Z(I)];
    xp(1,k) = center2(1);
    yp(1,k) = center2(2);
    zp(1,k) = center2(3);
end

plot3(xp, yp, zp, 'b', 'LineWidth', 1)
hold on
plot3(xp(49), yp(49), zp(49), 'ro', 'Markersize', 8)
xlabel('x-direction');
ylabel('y-direction');
zlabel('z-direction');
title("path of submarine");
%q3
position = [xp; yp];
```