

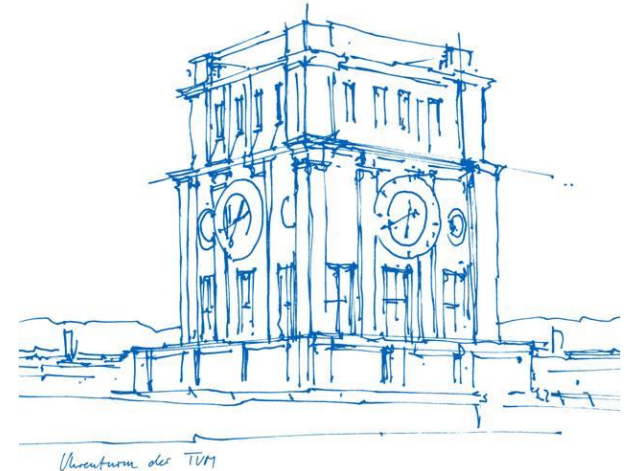
# Numerical optimization for robot design and control – Day 02

## Robot design

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Garching, 05th of September 2022



# Day-2: Robot design: Overview

## **Part-1: Robot design: Theory and introduction**

1. Introduction and background
2. Basic mathematics
3. Robot design

## **Part-2: Robot design as an optimization problem**

1. Problem formulation
2. Problem solving methods
3. Discussion

# Part-1: Robot design: Theory and introduction

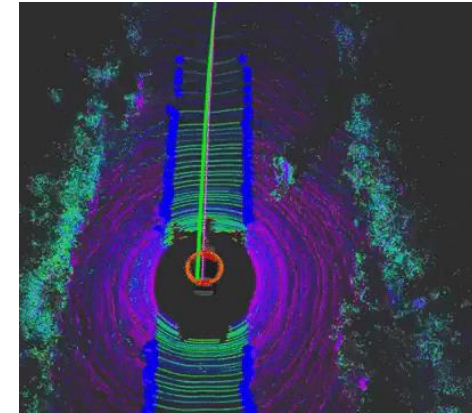
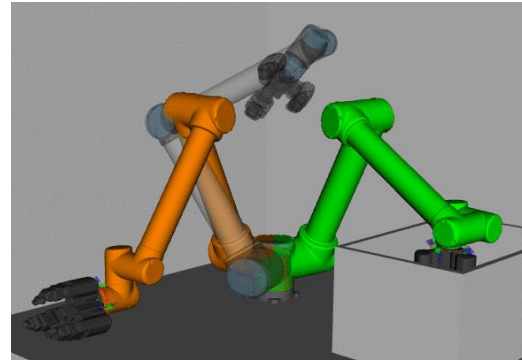
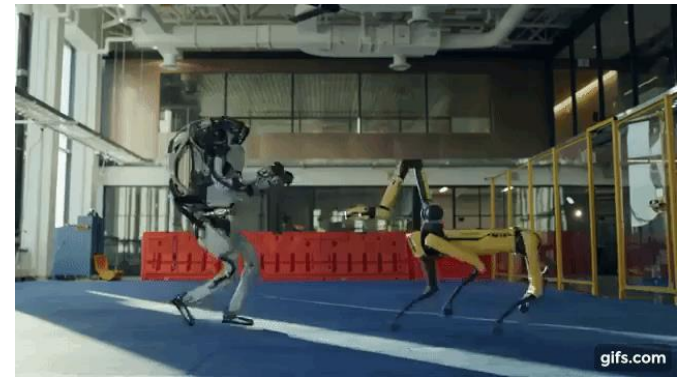
# Introduction and background

What is a robot?

*“A robot is an autonomous machine capable of **sensing** its environment, carrying out computations to make **decisions**, and performing **actions** in the real world.” [1]*

- **Body**       $\longrightarrow$     To effect the real world
- **Sensors**    $\longrightarrow$     To perceive the environment
- **Brain**       $\longrightarrow$     To take decisions

These elements are tightly coupled and are interdependent on each other



A repository of robots: <https://robots.ieee.org/robots/>

[1] <https://robots.ieee.org/learn/what-is-a-robot/#:~:text=A%20robot%20is%20an%20autonomous,actions%20in%20the%20real%20world.>

# Introduction and background

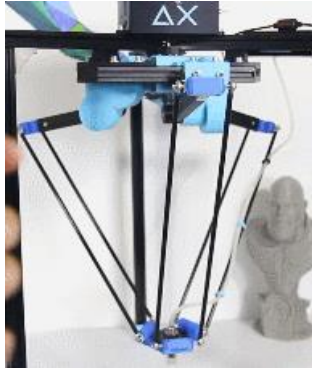
## Kinds of robots?

(A very loose classification)

Serial  
robots



Parallel  
robots



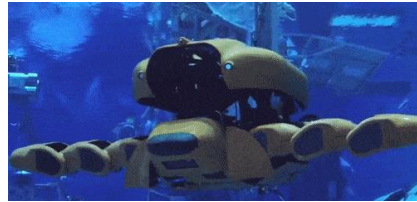
Mobile  
robots



Aerial  
robots

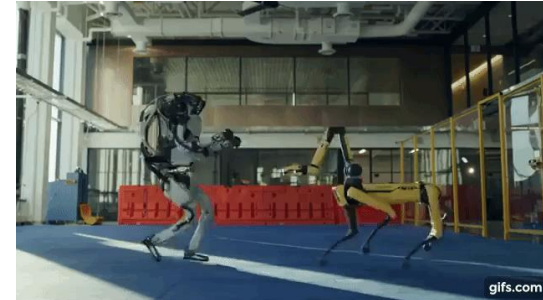


Underwater  
robots



And many  
many more..

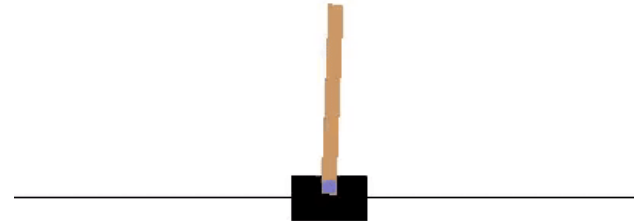
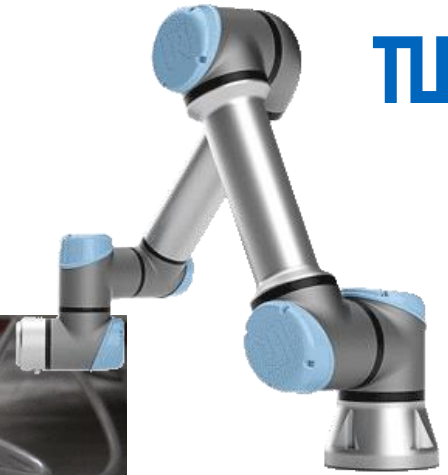
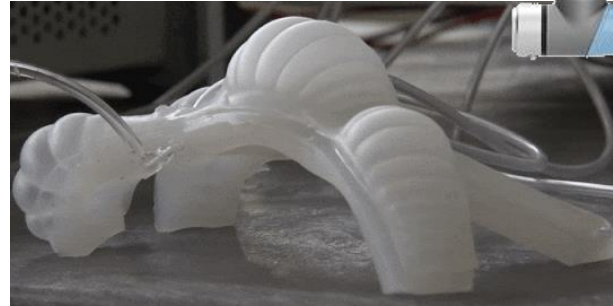
Can you  
name any  
more?



# Introduction and background

## Scope of the lecture

- Serial/parallel/mobile robots
- Fully actuated  
We can control all the **degrees of freedom** of the system
- Fully observable  
We can recover information regarding all the relevant **states** of the system



# Introduction and background

- We want to design a robot? But what does it mean?
  - What does a robot design consist of?
  - What kind of variables are required to describe the robot design?
  - What is a robot supposed to do?
  - How do we mathematically describe what its supposed to do?
  - What information is coupled with robot design here?

Workspace

Force  
applied

End-effector  
velocity

Power  
consumed

Cost

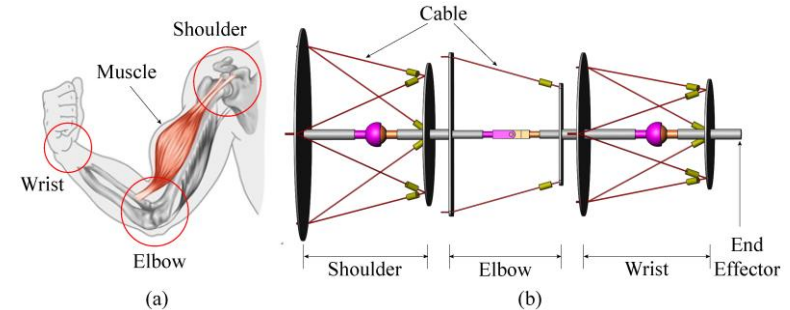
Aesthetics

Payload

# Introduction and background



- Lets consider a concrete example of a humanoid robot arm design
- What does a robot design consist of?
- What kind of variables are required to describe the robot design?
- What is a robot supposed to do?
- How do we mathematically describe what its supposed to do?
- What information is coupled with robot design here?



Source: Yang, K.; Yang, G.; Zhang, C.; Chen, C.; Zheng, T.; Cui, Y.; Chen, T. Cable Tension Analysis Oriented the Enhanced Stiffness of a 3-DOF Joint Module of a Modular Cable-Driven Human-Like Robotic Arm. *Appl. Sci.* **2020**, *10*, 8871. <https://doi.org/10.3390/app10248871>



# Introduction and background

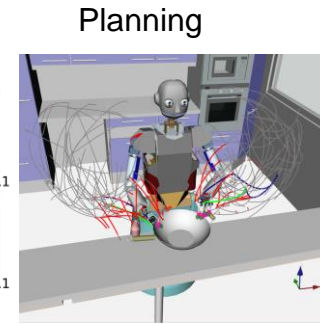
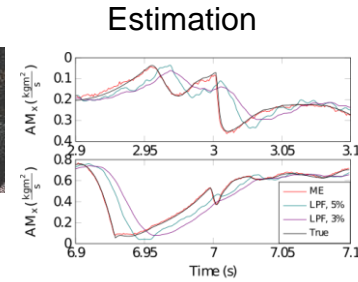
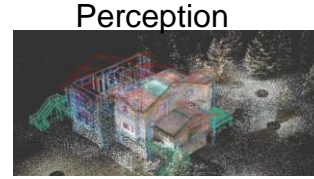
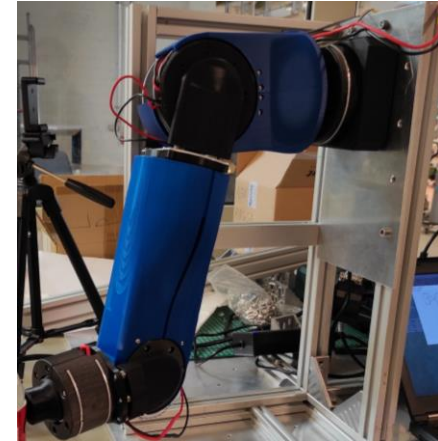
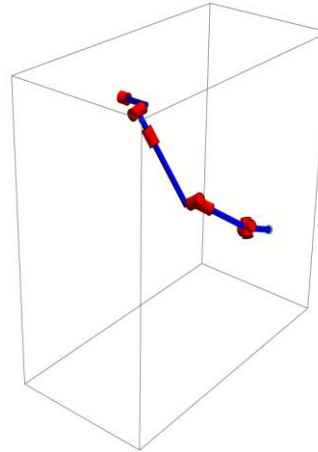
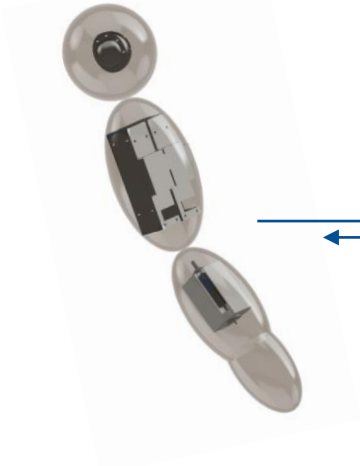
- Design of robotic systems has primarily been a sequential process

Concept Design

Detail Design

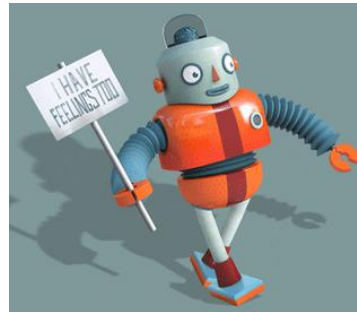
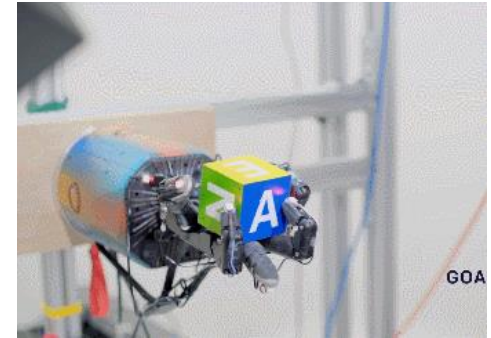
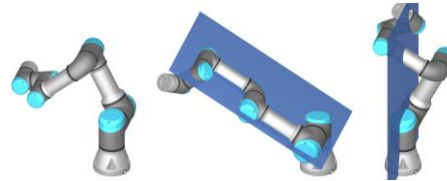
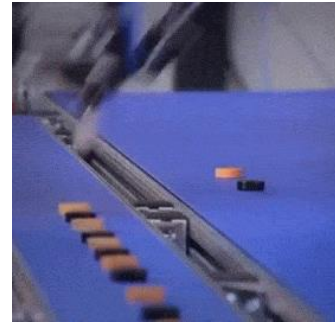
Simulation and Control

Prototype



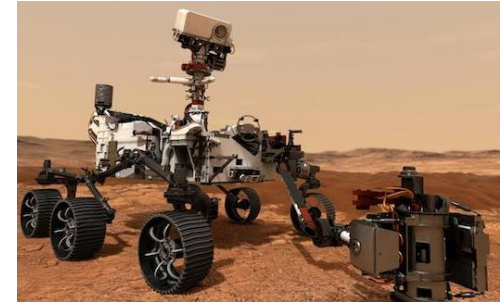
# Introduction and background

- Some aspects of interest for robot design:
  - Output properties
    - Speed
    - Manipulability
    - Dexterity
    - Workspace (Reachable, dexterous, wrench-closure, wrench-feasible...)
  - Behavior
    - Aggressive
    - Jerk free
    - Funny?



# Introduction and background

- Some aspects of interest for robot design:
  - Structural
    - Strong
    - Lightweight
    - Flexible ...
  - Packaging
    - Compact
    - Microscopic
    - Smaller base ...
  - Human-friendly
    - Friendly
    - Emotion reciprocation ...

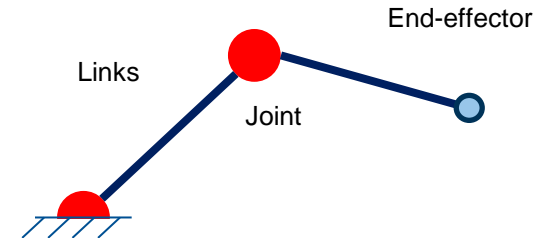
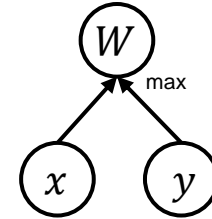


# Basic mathematics

- Say, we want to build a simple two link planar robot (planar 2R-robot)
- Workspace of this robot is the set of all the points in space the end-effector can reach. Now, what would that look like.
- We want the robot to have as large workspace as possible
- What would the ADG look like?
- Formulate the optimization problem?

$$\max_{x,y} W(x,y)$$

- What would be the trivial and only solution to this problem?



# Basic mathematics

- How do we set-up bottom-up mappings between workspace of the robot?
- The workspace definition,

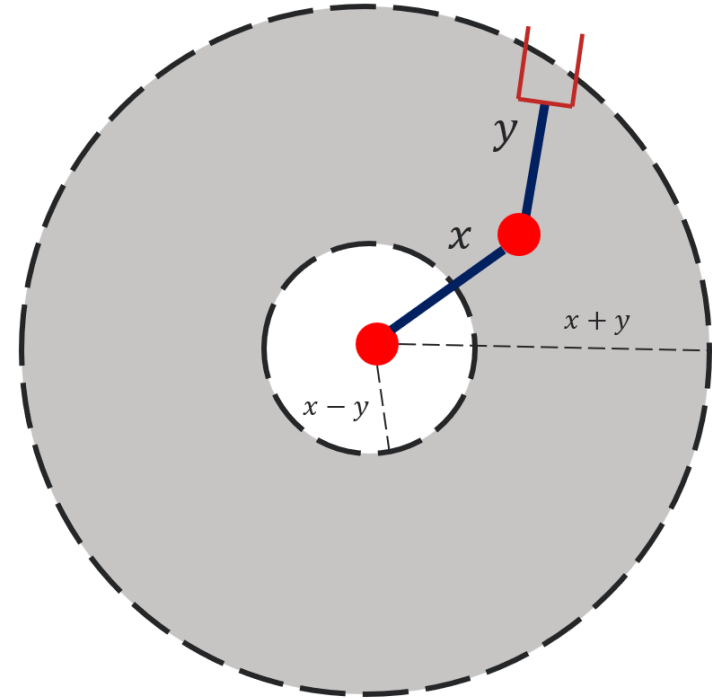
$$W(x, y) = \pi(x + y)^2 - \pi(x - y)^2$$

- The optimization problem is still trivial
- How do we add constraints to this problem,

$$\begin{aligned} h(x, y) &:= x + y = L \\ g(x, y) &:= x \geq y \end{aligned}$$

- Putting it all together,

$$\begin{aligned} \min_{x, y} \quad & -W(x, y) \\ \text{s.t.} \quad & x + y - L = 0 \\ & y - x \leq 0 \end{aligned}$$



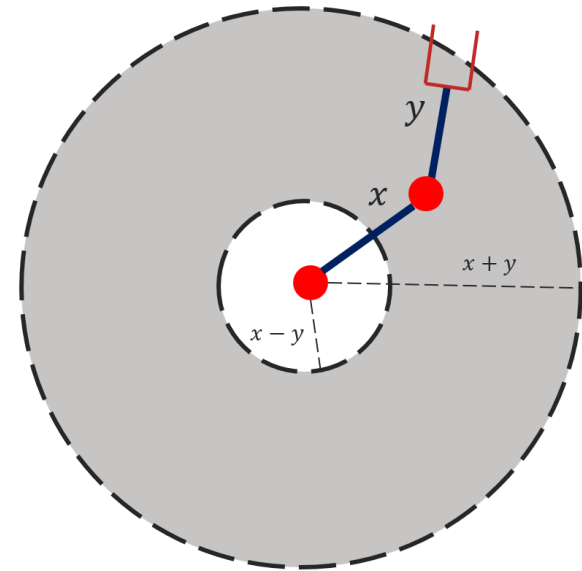
# Basic mathematics

- How do we solve this problem?
  - Reduce the number of variables by substituting the equality constraint (can be done because the problem is simple!)
  - Formulate a penalty function accommodating the equality and the inequality constraint
- Solve the problem!
- What are the optimal link lengths for this problem?
- How do we interpret the results?

$$\min_{x,y} -[\pi(x+y)^2 - \pi(x-y)^2]$$

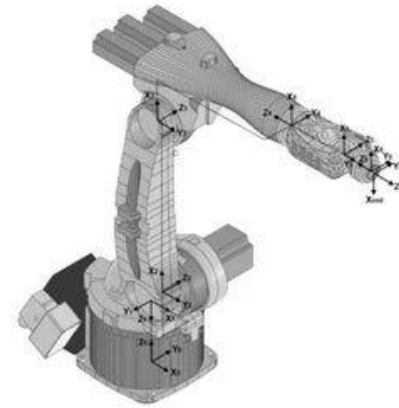
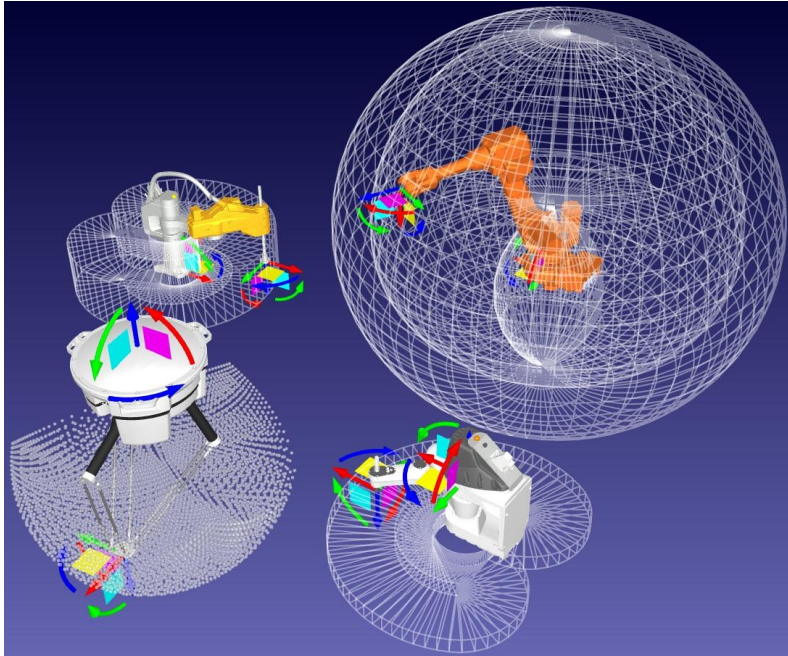
$$x + y - L = 0$$

$$y - x \leq 0$$

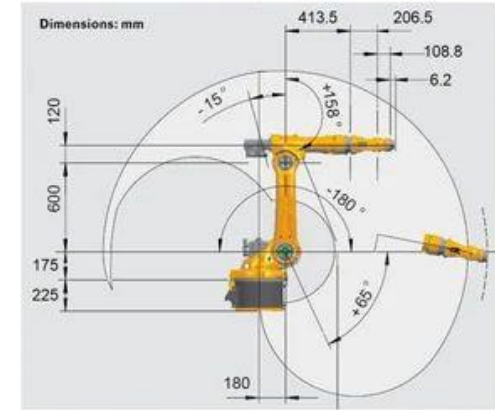


# Workspaces of industrial robots

- Some examples from industrial robots



(a)



(b)

How do we solve such problems for more complex robots?



# Robot design

- Consider a slightly complex scenario where we would like to optimise the link lengths of a design shown in the right
- Problem definition:

*“Design the tool holder in such a way that the person using it can move it, as much and as freely as possible.”*

- Formulate the above statement as an optimisation problem
  - Model the simplified system to identify design variables
  - Identify the quantities of interest
  - Draw the ADG
  - Mathematically model the bottom-up mapping from the design variables to the quantities of interest
  - Setup objective function and constraints



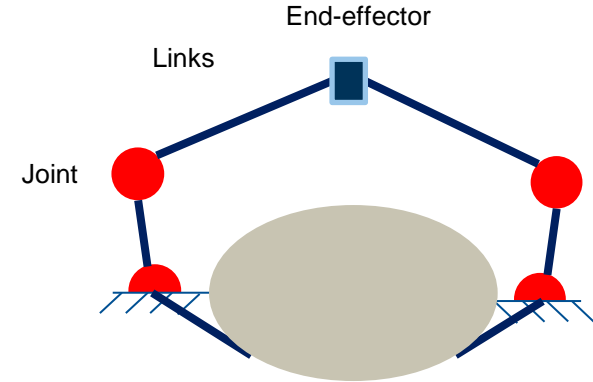


# Robot design

- Consider a slightly complex scenario where we would like to optimise the link lengths of a design shown in the right
- Problem definition:

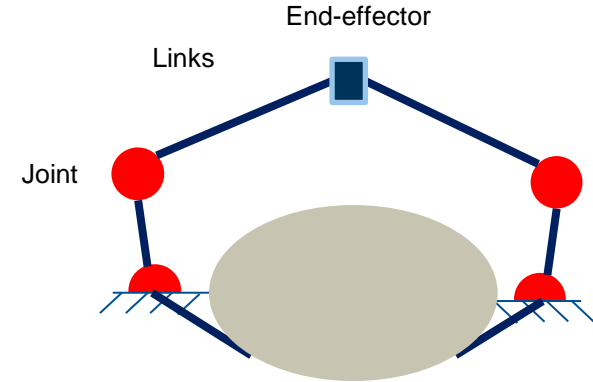
*“Design the tool holder in such a way that the person using it can move it, as much and as freely as possible.”*

- Formulate the above statement as an optimisation problem
  - Model the simplified system
  - Identify design variables
  - Identify the quantities of interest
  - Draw the ADG
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# Robot design

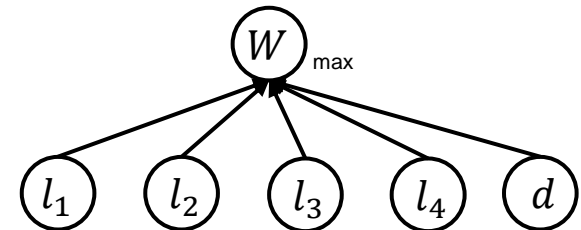
- Formulate the above statement as an optimisation problem
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  - Mathematically model the bottom-up mapping from the design variables to the quantities of interest
  - Setup objective function and constraints



Fixed links:  $l_1, l_2$

Distal links:  $l_3, l_4$

Distance between bases:  $d$

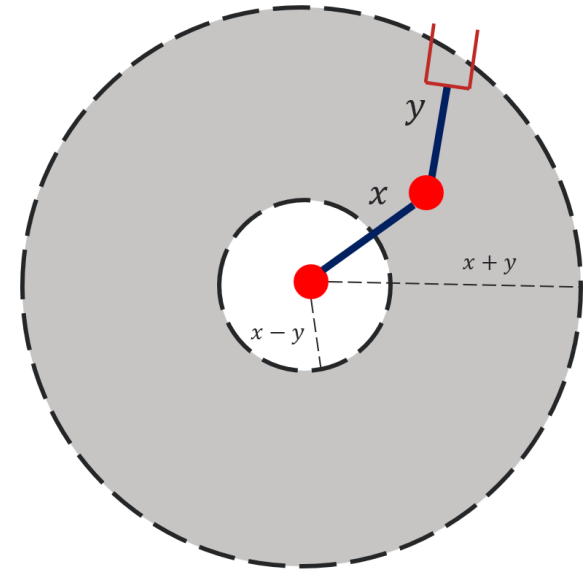


# Robot design

- Formulate the above statement as an optimisation problem
  - Model the simplified system
  - Identify design variables
  - Identify the quantities of interest
  - Draw the ADG
  - Mathematically model the bottom-up mapping from the design variables to the quantities of interest
- Use the same idea from one serial arm and model the workspace of the parallel robot
- How do we model the collision space between the robot and the human?
- Bonus: Setup collision constraints between the links and the human (ellipse)



$$\begin{aligned} \min_{x,y} & -[\pi(x+y)^2 - \pi(x-y)^2] \\ & x + y - L = 0 \\ & y - x \leq 0 \end{aligned}$$

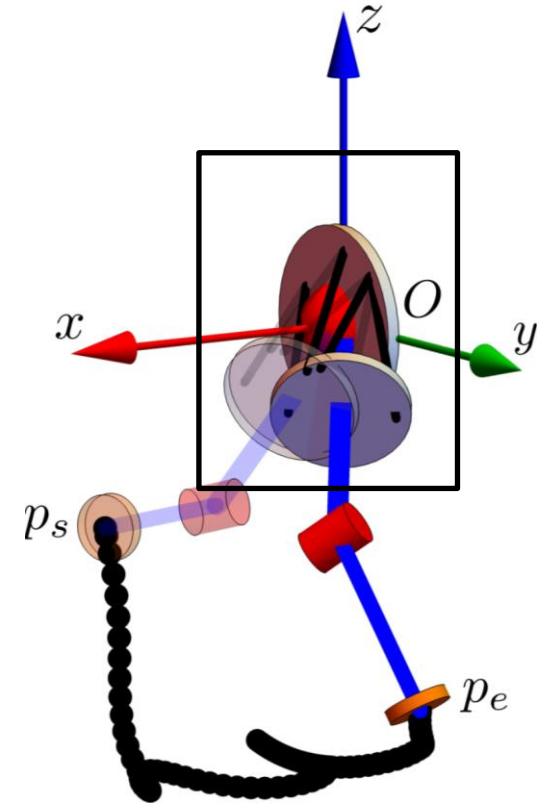
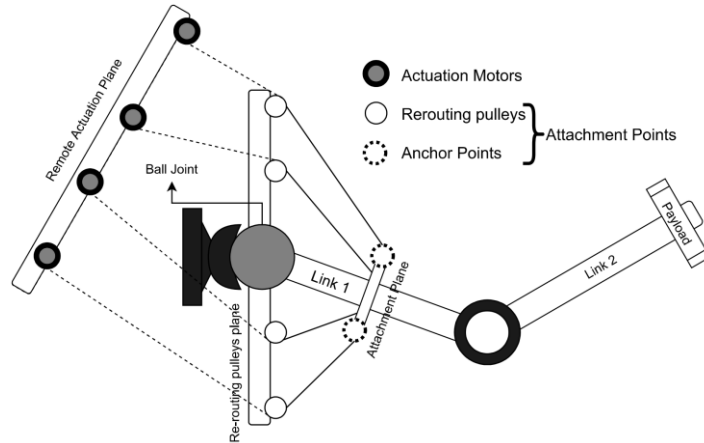




# Part-2: Robot design as an optimization problem

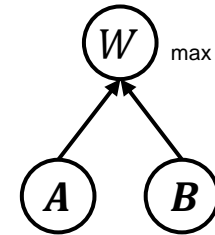
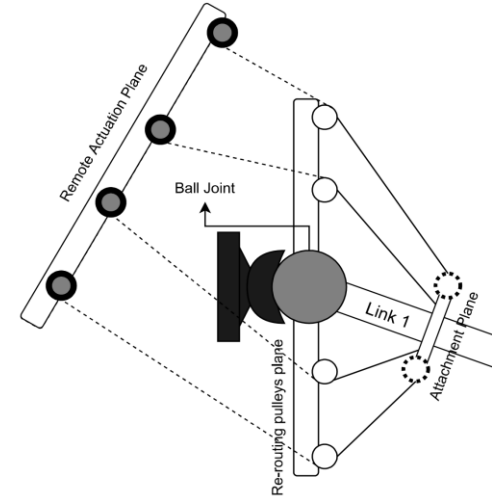
# Robot design

- Workspace of the CDPR: (there are several definitions)
  - Wrench-closure-workspace (WCW): A set of all configurations where the cable-driven-parallel-robot (CDPR) can admit positive tensions. If these tensions can be achieved via chosen actuators, then the workspace can also be called wrench-feasible-workspace
- What we deal with is called a multilink cable-driven-robot (MCDR)



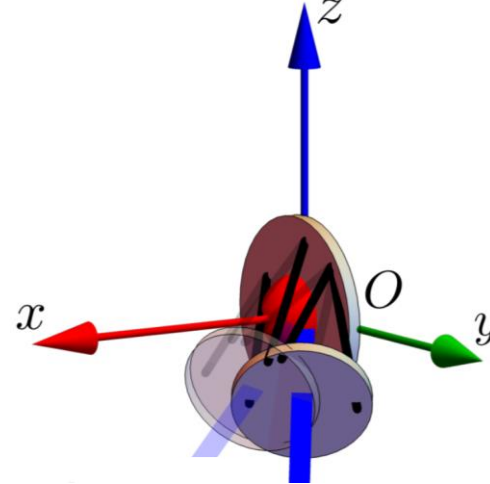
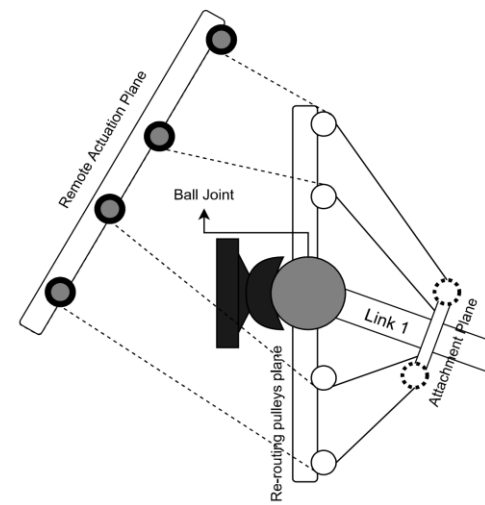
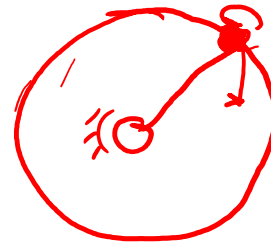
# Robot design

- Why do we want to build such a robot?
- How do we describe the robots pose?
- What are the variables that are available for us to modify the robots pose?
- What are the design variables whose values we would like to find?
- What is our objective in this robot design problem?
- Is the model complete or are we missing anything?
- ADG for the current problem?



# Robot design

- How do we formulate the workspace of the robot in this case?
- Are there any constraints in this problem?
- How do we model the constraints?
- How do we bound the design variables?






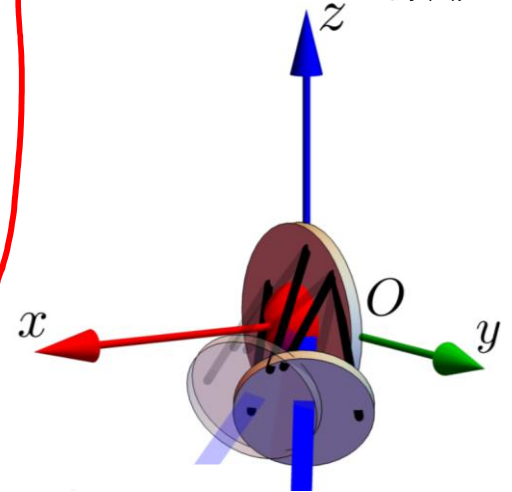
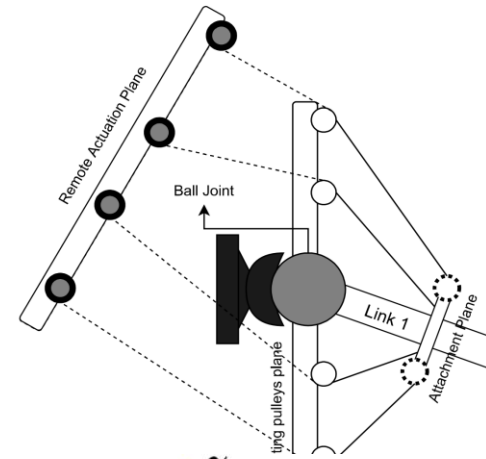
# Robot design

- Use a bi-level optimization scheme to maximize the workspace
- Problem solving strategy and MATLAB implementation

Optimiser - 1

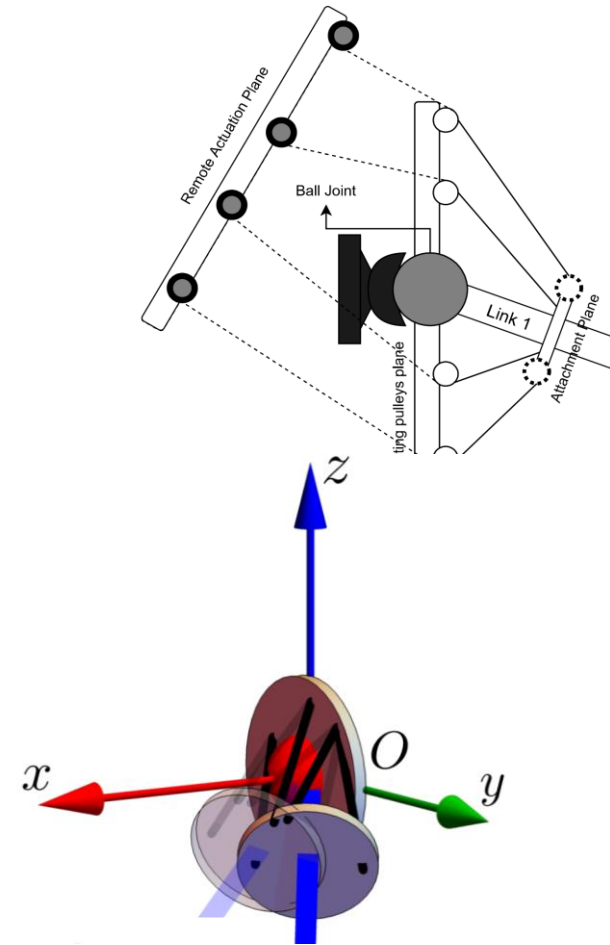
- ① → Sample a design  $[x_0]$
- ② → Check all pts on 
- ③ → Find cable forces for each pose
- ④ → Count feasible poses -  $N$

A red arrow originates from the word 'Optimiser - 1' and points to step ①. Another red arrow originates from step ④ and points back to step ②, indicating a feedback loop.



# Robot design

- Design parameter list:
  - Link length = 350
  - Ball radius = 20
  - Arm radius = 10
  - Ball center =  $[0,0,0]$
  - Number of cables = 4
  - Domain radius of the cables = 60
- Home position =  $[0,0]$
- Loads
  - 20Nm about Z-axis
  - -20Nm about Z-axis
  - 20Nm about Y-axis
  - -20Nm about Y-axis



Questions?

# End of Day-2

Thank you!

