Lehrstuhl für Produktentwicklung und Leichtbau
TUM School of Engineering and Design
Technische Universität München

Numerical optimization for robot design and control – Day 02
Robot design

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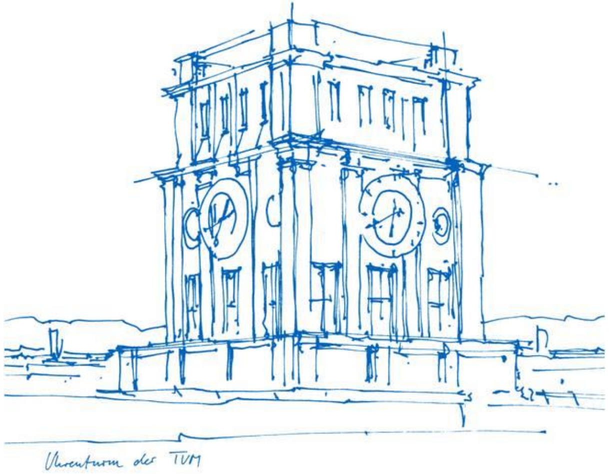
Garching, 05th of September 2022



Day-2: Robot design: Overview

Part-1: Robot design: Theory and introduction

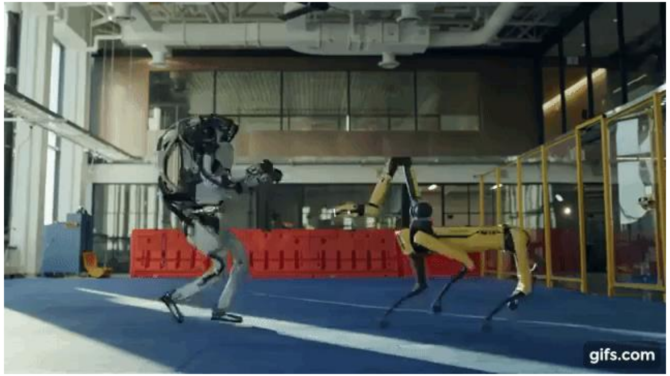
1. Introduction and background
2. Basic mathematics
3. Robot design

Part-2: Robot design as an optimization problem

1. Problem formulation
2. Problem solving methods
3. Discussion

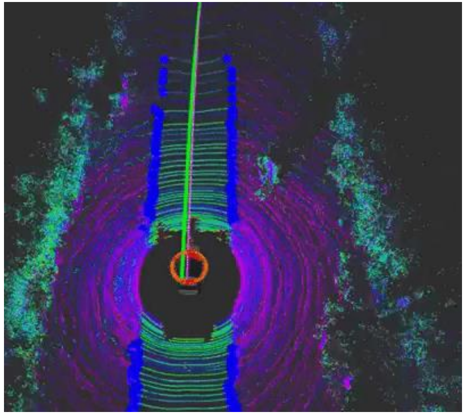
Part-1: Robot design: Theory and introduction

Introduction and background

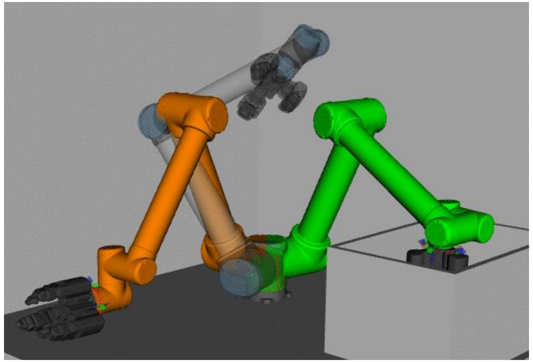


What is a robot?

“A robot is an autonomous machine capable of sensing its environment, carrying out computations to make decisions, and performing actions in the real world.” [1]



- Body → To effect the real world
- Sensors → To perceive the envoirnment
- Brain → To take decisions



These elements are tightly coupled and are interdependent on each other

A repository of robots: https://robots.ieee.org/robots/

[1] https://robots.ieee.org/learn/what-is-a-robot/#:~:text=A%20robot%20is%20an%20autonomous,actions%20in%20the%20real%20world.

Introduction and background

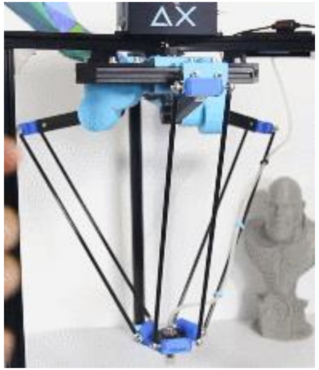
Kinds of robots?

(A very loose classification)

Aerial robots

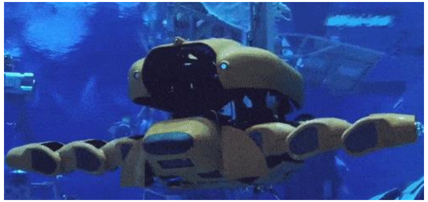
Serial robots

Parallel robots



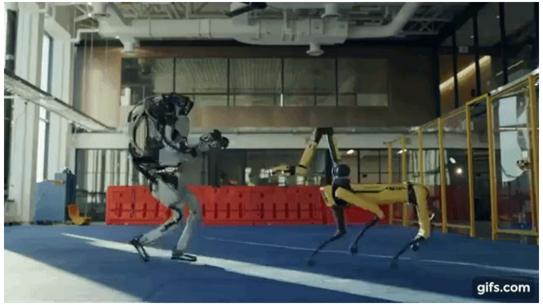
Mobile robots

Underwater robots

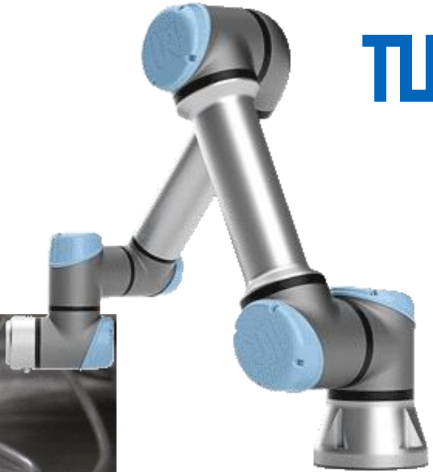


And many many more..

Can you name any more?

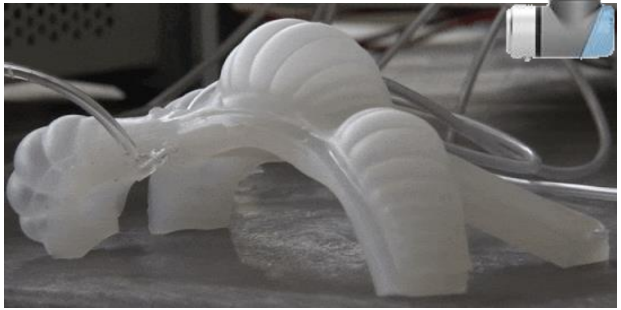


Introduction and background



Scope of the lecture

- Serial/parallel/mobile robots

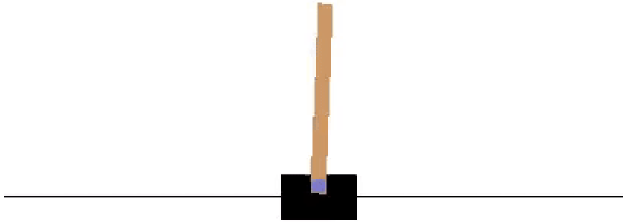


- Fully actuated

  We can control all the degrees of freedom of the system

- Fully observable

  We can recover information regarding all the relevant states of the system



Introduction and background

- We want to design a robot? But what does it mean?
  - What does a robot design consist of?
  - What kind of variables are required to describe the robot design?
  - What is a robot supposed to do?
  - How do we mathematically describe what its supposed to do?
  - What information is coupled with robot design here?

Workspace

Force applied

End-effector velocity

Power consumed

Cost

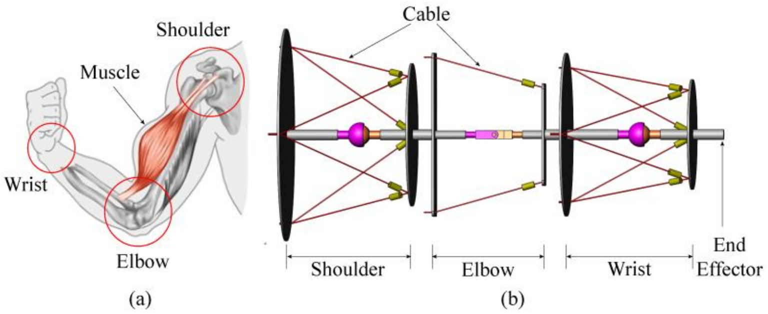
Aesthetics

Payload

Introduction and background

???

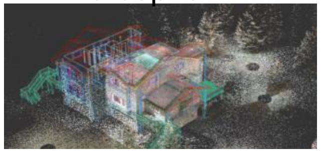
- Lets consider a concrete example of a humanoid robot arm design
  - What does a robot design consist of?
  - What kind of variables are required to describe the robot design?
  - What is a robot supposed to do?
  - How do we mathematically describe what its supposed to do?
  - What information is coupled with robot design here?



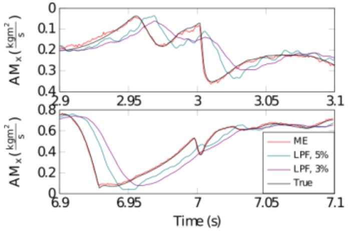
Source: Yang, K.; Yang, G.; Zhang, C.; Chen, C.; Zheng, T.; Cui, Y.; Chen, T. Cable Tension Analysis Oriented the Enhanced Stiffness of a 3-DOF Joint Module of a Modular Cable-Driven Human-Like Robotic Arm. *Appl. Sci.* **2020**, *10*, 8871. https://doi.org/10.3390/app10248871

Introduction and background

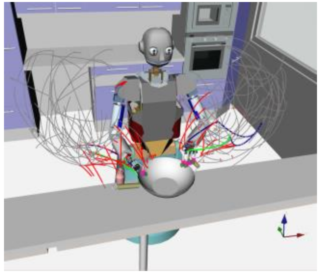
Perception



Estimation



Planning

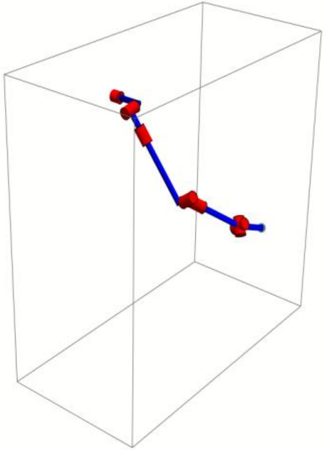


- Design of robotic systems has primarily been a sequential process

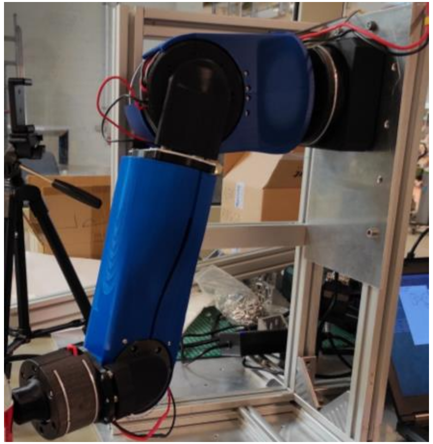
Concept Design

Detail Design

Simulation and Control

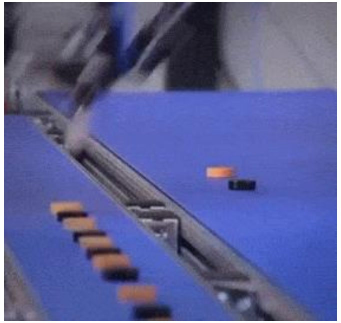


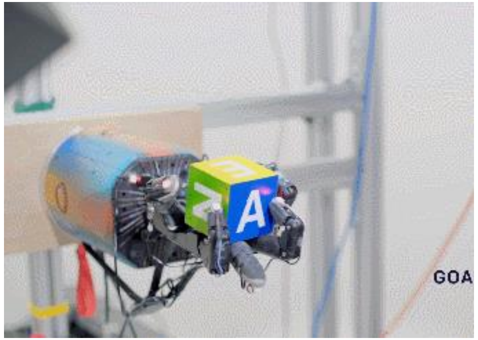
Prototype

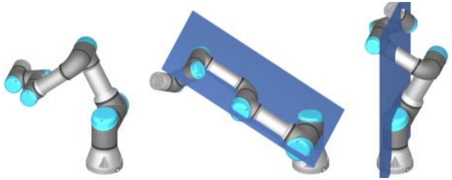


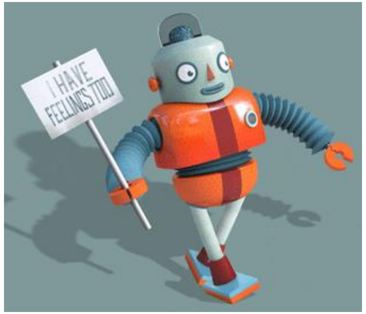
Introduction and background

- Some aspects of interest for robot design:
  - Output properties
    - Speed
    - Manipulability
    - Dexterity
    - Workspace (Reachable, dexterous, wrench-closure, wrench-feasible…)
  - Behavior
    - Aggressive
    - Jerk free
    - Funny?



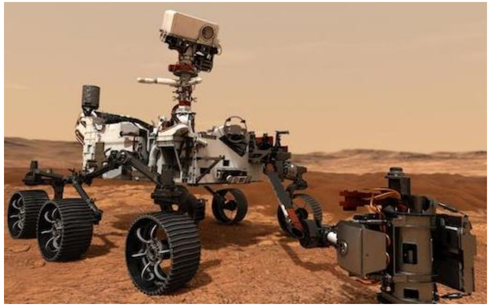






Introduction and background

- Some aspects of interest for robot design:
  - Structural
    - Strong
    - Lightweight
    - Flexible …
  - Packaging
    - Compact
    - Microscopic
    - Smaller base …
  - Human-friendly
    - Friendly
    - Emotion reciprocation …

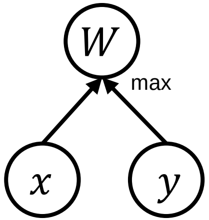


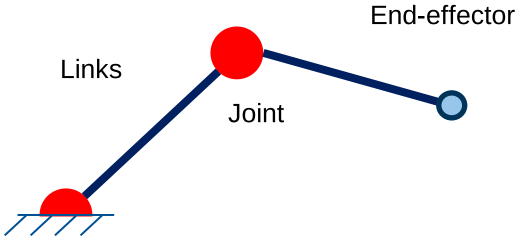
Basic mathematics

- Say, we want to build a simple two link planar robot (planar 2R-robot)
- Workspace of this robot is the set of all the points in space the end-effector can reach. Now, what would that look like.
- We want the robot to have as large workspace as possible
- What would the ADG look like?
- Formulate the optimization problem?

$$\max_{x,y} \; W(x, y)$$

- What would be the trivial and only solution to this problem?





Basic mathematics

- How do we set-up bottom-up mappings between workspace of the robot?
- The workspace definition,

$$W(x,y) = \pi(x+y)^2 - \pi(x-y)^2$$

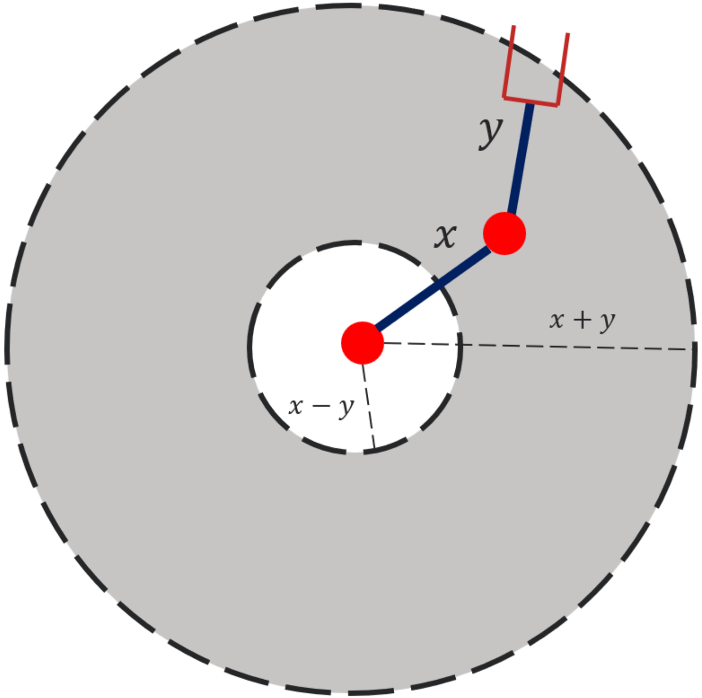
- The optimization problem is still trivial

- How do we add constraints to this problem,

$$h(x,y) \coloneqq x + y = L$$
$$g(x,y) \coloneqq x \geq y$$

- Putting it all together,

$$\min_{x,y} -W\,(x,y)$$
$$x + y - L = 0$$
$$y - x \leq 0$$



Basic mathematics

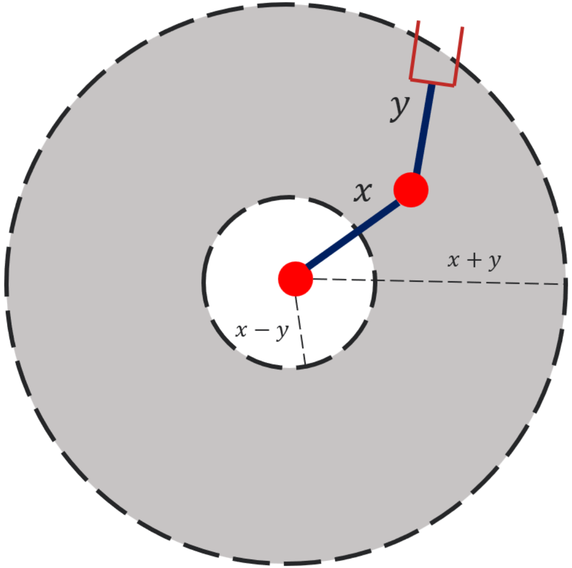
- How do we solve this problem?
  - Reduce the number of variables by substituting the equality constraint (can be done because the problem is simple!)
  - Formulate a penalty function accommodating the equality and the inequality constraint

- Solve the problem!

- What are the optimal link lengths for this problem?

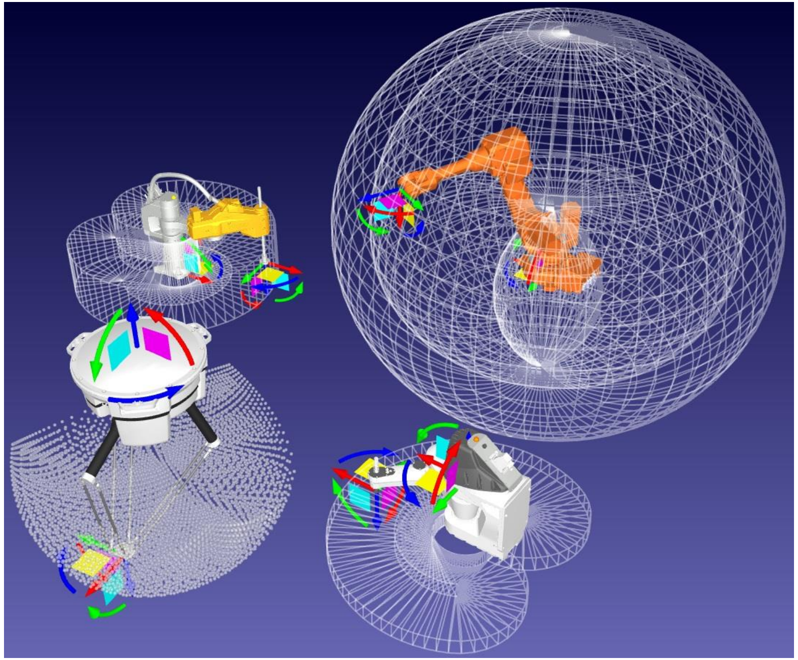
- How do we interpret the results?

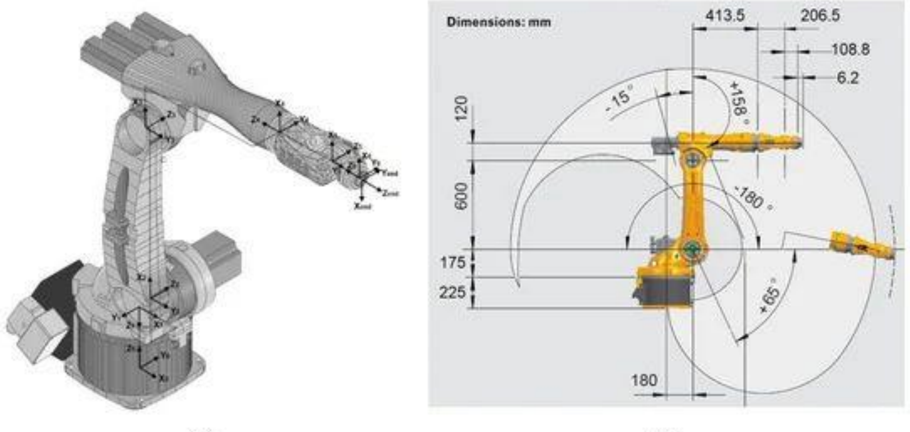
$$\min_{x,y} -[\pi(x+y)^2 - \pi(x-y)^2]$$
$$x + y - L = 0$$
$$y - x \leq 0$$



Workspaces of industrial robots

- Some examples from industrial robots





(a) (b)

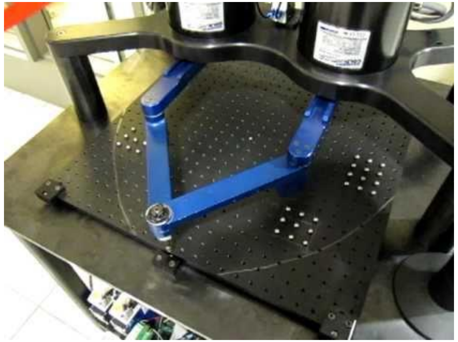
How do we solve such problems for more complex robots?

Robot design

- Consider a slightly complex scenario where we would like to optimise the link lengths of a design shown in the right
- Problem definition:

"Design the tool holder in such a way that the person using it can move it, as much and as freely as possible."

- Formulate the above statement as an optimisation problem
  - Model the simplified system to identify design variables
  - Identify the quantities of interest
  - Draw the ADG
  - Mathematically model the bottom-up mapping from the design variables to the quantities of interest
  - Setup objective function and constraints

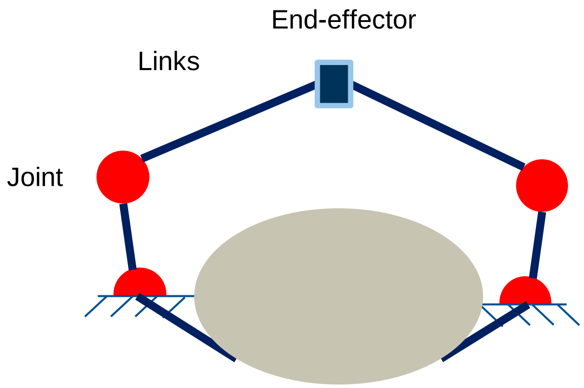


Robot design

- Consider a slightly complex scenario where we would like to optimise the link lengths of a design shown in the right
- Problem definition:

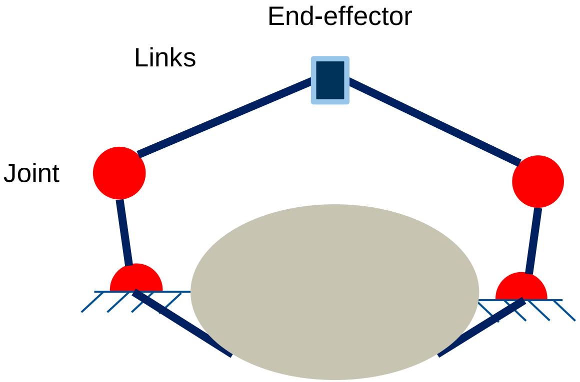
“Design the tool holder in such a way that the person using it can move it, as much and as freely as possible.”

- Formulate the above statement as an optimisation problem
  - Model the simplified system ✓
  - Identify design variables
  - Identify the quantities of interest
  - Draw the ADG
  - Mathematically model the bottom-up mapping from the design variables to the quantities of interest
  - Setup objective function and constraints

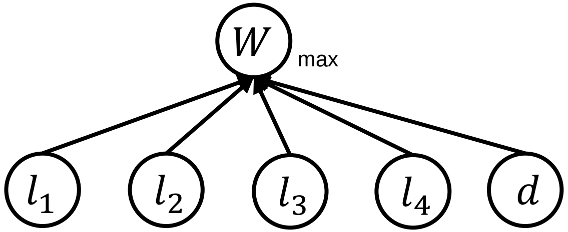


Robot design

- Formulate the above statement as an optimisation problem
  - Model the simplified system
  - Identify design variables
  - Identify the quantities of interest
  - Draw the ADG
  - Mathematically model the bottom-up mapping from the design variables to the quantities of interest
  - Setup objective function and constraints



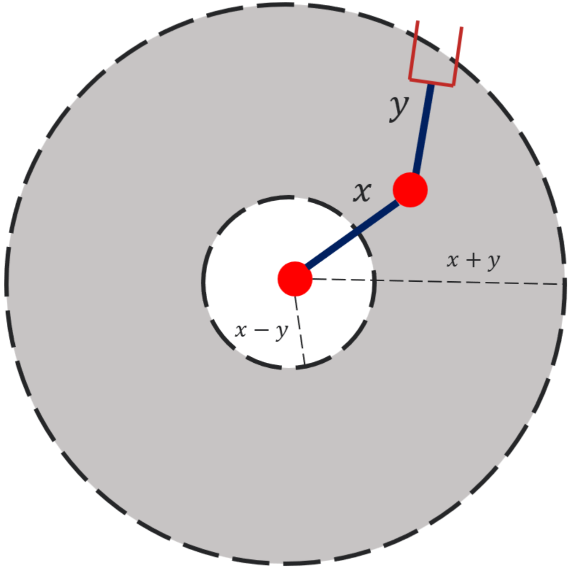
Fixed links: l_1, l_2
Distal links: l_3, l_4
Distance between bases: d



Robot design

$$\min_{x,y} -[\pi(x+y)^2 - \pi(x-y)^2]$$
$$x + y - L = 0$$
$$y - x \leq 0$$

- Formulate the above statement as an optimisation problem
  - Model the simplified system ✓
  - Identify design variables ✓
  - Identify the quantities of interest ✓
  - Draw the ADG ✓
  - Mathematically model the bottom-up mapping from the design variables to the quantities of interest
    - Use the same idea from one serial arm and model the workspace of the parallel robot
    - How do we model the collision space between the robot and the human?
  - Bonus: Setup collision constraints between the links and the human (ellipse)

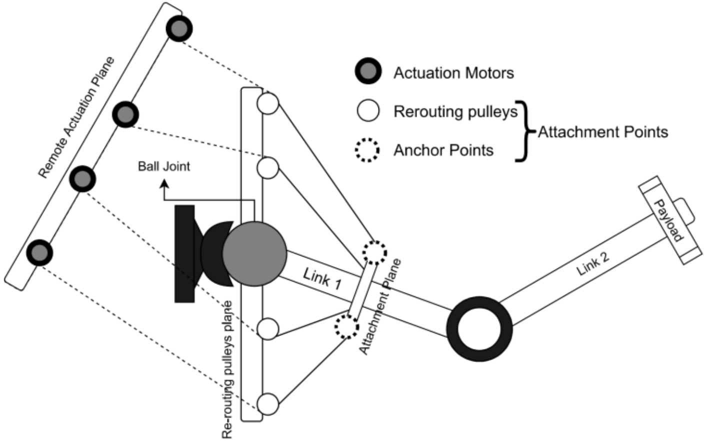


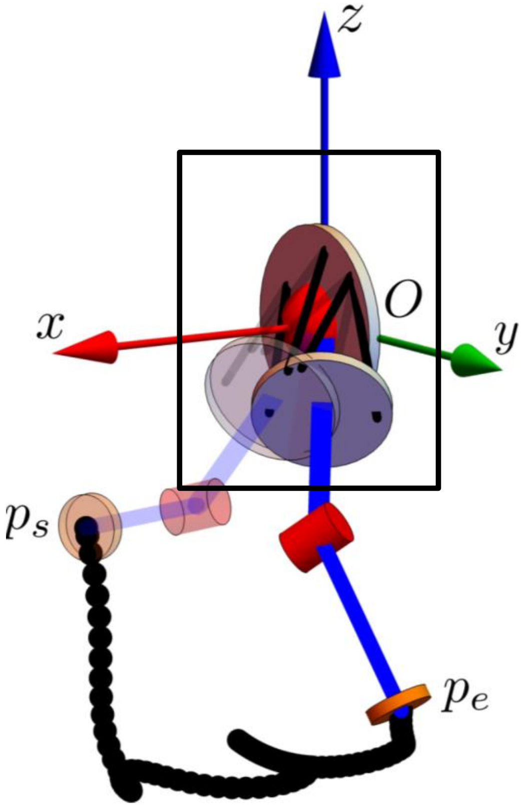
Part-2: Robot design as an optimization problem

Robot design

- Workspace of the CDPR: (there are several definitions)
  - Wrench-closure-workspace (WCW): A set of all configurations where the cable-driven-parallel-robot (CDPR) can admits positive tensions. If these tensions can be achieved via chosen actuators, then the workspace can also be called wrench-feasible-workspace

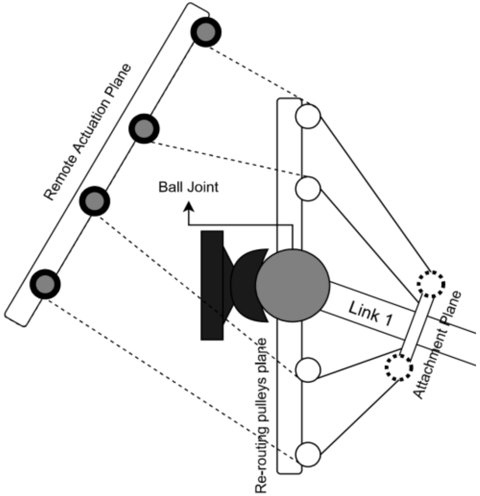
- What we deal with is called a multilink cable-driven-robot (MCDR)

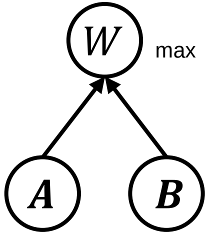




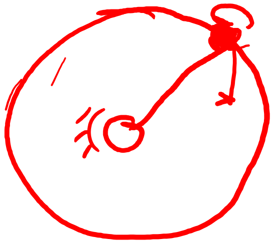
Robot design

- Why do we want to build such a robot?
- How do we describe the robots pose?
- What are the variables that are available for us to modify the robots pose?
- What are the design variables whose values we would like to find?
- What is our objective in this robot design problem?
- Is the model complete or are we missing anything?
- ADG for the current problem?

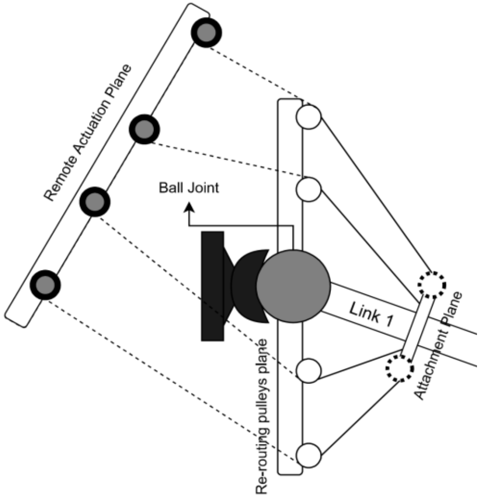


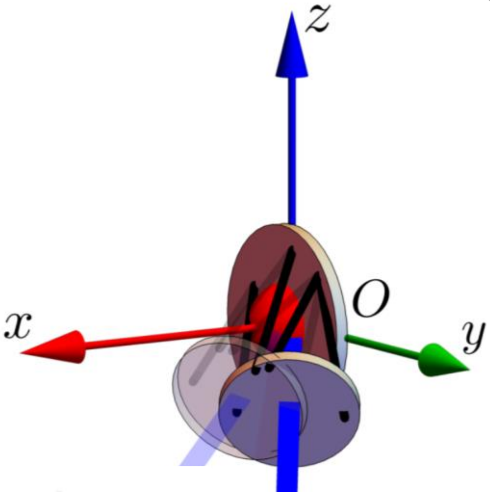


Robot design



- How do we formulate the workspace of the robot in this case?
- Are there any constraints in this problem?
- How do we model the constraints?
- How do we bound the design variables?



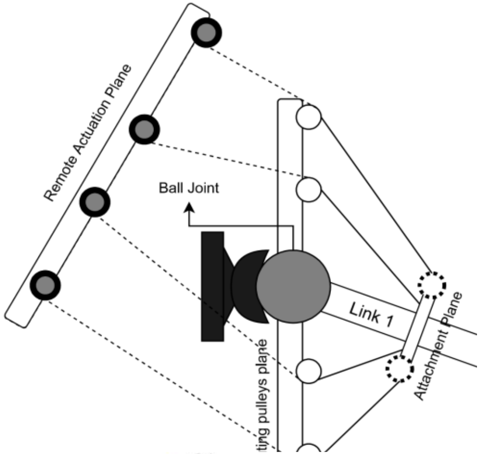


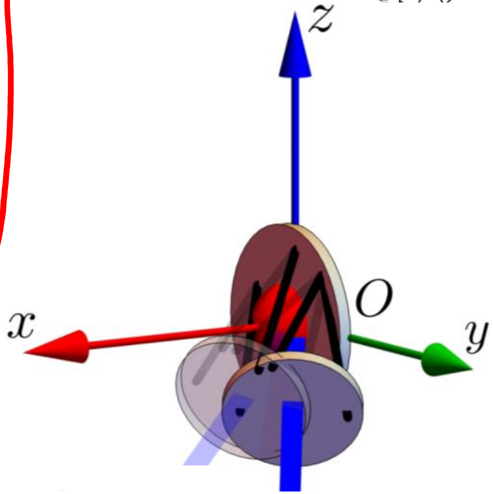
Robot design

- Use a bi-level optimization scheme to maximize the workspace
- Problem solving strategy and MATLAB implementation

Optimiser – 1

1. → Sample a design $[x_0]$
2. → Check all pts on
3. → Find cable forces for each pose
4. → Count feasible poses – N

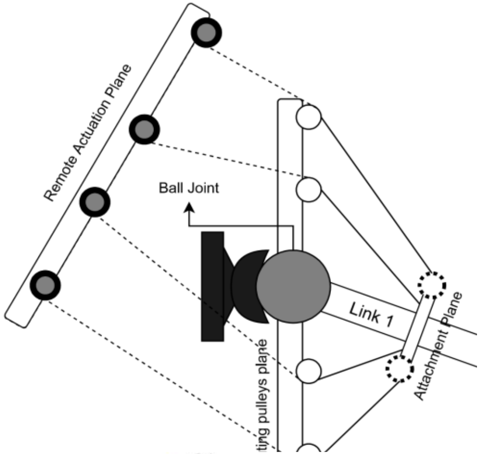


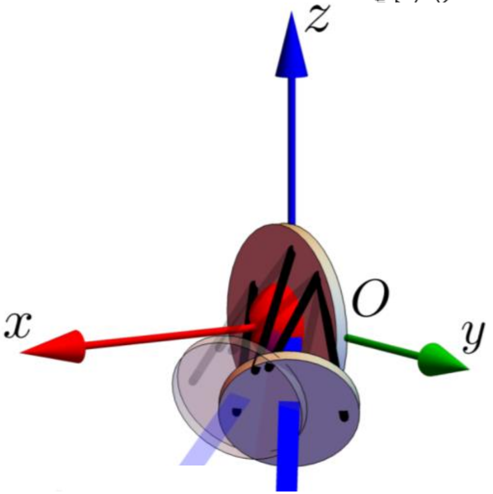


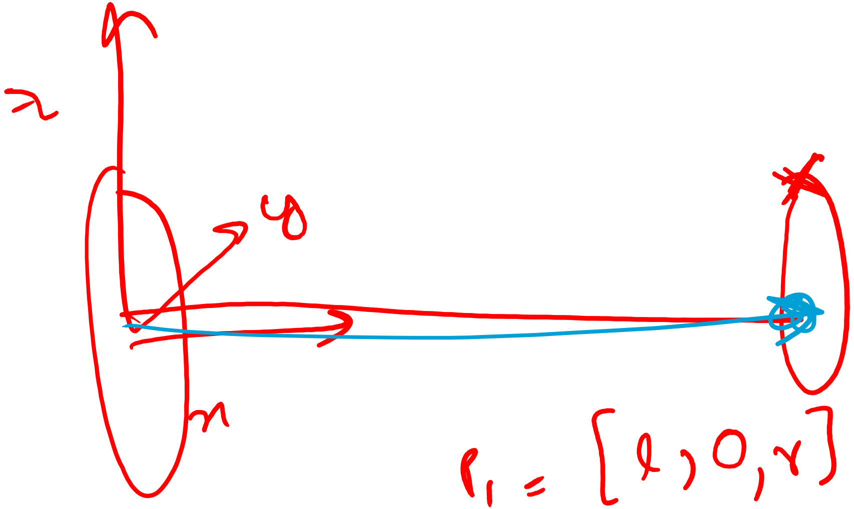
Robot design

- Design parameter list:
  - Link length = 350
  - Ball radius = 20
  - Arm radius = 10
  - Ball center = [0,0,0]
  - Number of cables = 4
  - Domain radius of the cables = 60

  - Home position = [0,0]
  - Loads
    - 20Nm about Z-axis
    - -20Nm about Z-axis
    - 20Nm about Y-axis
    - -20Nm about Y-axis

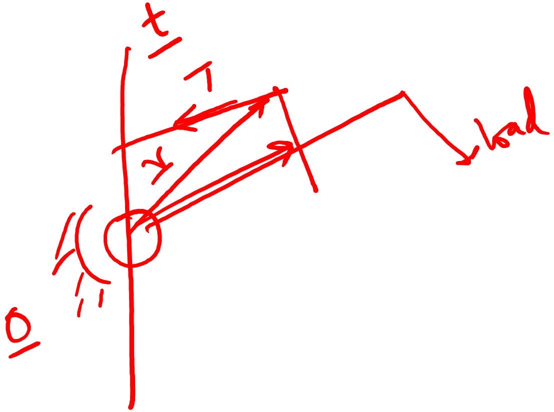






$P_1 = [l, 0, r]$

$BO = \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \end{pmatrix}$



$\underline{T} = \underline{t}\,T$ → magnitude

direction

Moment about $\underline{O}$

$$= \underline{r} \times \underline{T} = \underbrace{\left(\underline{r} \times \underline{t}\right)}_{jt} T$$

$jt\,[T] = f_1$ → for one load case

$jt\,(T) = f_2$ → 2nd load case

⋮

$$\Rightarrow JT \begin{bmatrix} T \\ \vdots \\ T \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \end{bmatrix}$$

✓ Solve using linear programming

Questions?

End of Day-2

Thank you!