Practice 5: The Power of Approximation
Type: Group Assignment
Due: Feb 17, Friday

`Motivation:`

1. The value of $\log_{10} 2 = 0.3010$. In practice, it is written as 0.3. The value of π is 3.141592653589793238462643... In most of the calculations, we write it as 3.14. Why? `Reason:` Approximations are considered good in practice as we do not lose much between a strict and an approximation analysis.

2. In algorithm analysis, we often assume the input size $n = 2^k$. Why ? In asymptotic sense, increasing the input size by adding dummy numbers does not affect the analysis. Thus, approximate analysis is considered good in practice.

3. We witness approximation in all walks of life. (i) We work hard to get grade 'S', however, we end up in 'A'. (ii) I enter the class with a good preparation so that I get to see 100% reachability, however, by the end of the lecture, I see only 80%. In both (i) and (ii), we wish to **maximize** the parameter of interest, however, due to approximation, we achieve below the optimum. (iii) The shortest path between Ashwatha and Akshaya is through banyan hostel (paver blocks), however, many students prefer to use service road (bitumen) which is longer than the shortest path (iv) The duration of Test 1 practice is 4 hrs, however, all students took at least 4 hrs. The last two items are examples of **minimization** problems. The solution is little more than the optimum due to the presence of approximation.

4. You must understand that we do NOT end up in near optimum always. For example, in some courses, we get 'S' (thus, hard work pays). In some lectures, I get 100% reachability. Some students use paver block road.

5. Can we explore approximation in algorithm design ? We shall look at three case studies for which we propose strategies that give approximate solutions (need not give optimum always).

6. Case study 1: (Policemen Problem) Consider a road network with roads and junctions. Policemen are placed in junctions to monitor the road traffic and to take care of any mishap. A policeman can monitor more than one road. The objective is to minimize the number of policemen so that

every road is monitored by some policeman. Finding a minimum may be a difficult task. Instead, you are allowed to present an approximate solution; your solution may take little more than the optimum number. Most importantly, the underlying strategy (algorithm) must run in polynomial time. **Tasks expected from you:** Present two strategies to find approximate number of policemen in any road network. Compare the performance of the strategies with the optimum solution. For example, for a given road network, the minimum is 5, however, the Strategy 1 outputs 7 and Strategy 2 outputs 9. Generate 100 different samples of various sizes to validate your strategies.

7. Case study 2: (Community size in Social Network) Consider any social network; the larger the community size, the better is the social network. The objective is to find the size of the maximum community size. This problem may be difficult to tackle, instead, we look for approximate solution. That is, for a given network, the maximum is 100, whereas, our strategy outputs 75. Like before, present two strategies and compare their performances. **Interesting Note:** Ramsey's magic gives a 3-size community or 3-size enemy network in any group of 6. This need not be maximum always. The generalization of Ramsey's magic is to identify k-size community in a n-size social network. What is the largest k for which this is possible.

   Tasks expected from you: Present two strategies to find approximate size of the community in any social network. Compare the performance of the strategies with the optimum solution. Generate 100 different samples of various sizes to validate your strategies.

8. Case study 3: Finding the size of the largest enemy set in a social network. This is again a generalization of Ramsey's magic for finding k-size enemy set in a n-size social network with k being maximum. Perform simulation to compare the performance of strategies proposed.

   Questions to ponder on:
   1. How do we find an optimum solution for a problem ?
   2. Can optimum solution be found out in polynomial time ?
   3. Why should approximation strategies run in polynomial time ?
   4. Are we compromising on feasibility ?