Signal Processing Project Report
Iterative estimation of sinusoidal signal parameters

Team 19

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Abstract

This report deals in the estimation methods for signal parameters. Specifically, it goes into a way to estimate signal parameters by continuous updation of initial estimates. This is an attempt at implementing the iterative algorithm to this end.

The need for a new algorithm

There already exist some successful methods for signal estimation like FFT-based estimation, subspace based estimation, spectrogram estimation, etc. Out of these we specifically talk about the FFT-Based algorithm. The major steps for the same are

1. Sampling the continuous time signal.
2. Finding N-point DFT of the discrete time signal obtained above.
3. Analysing the magnitude spectrum of the DFT to find peaks.
4. Mapping the amplitude and frequency of the peak to the amplitude and frequency of the actual signal.

The mapping described in step 4 above is of the following form, where f_0 and A are the frequency and amplitude of the original signal, k and A' are the frequency and amplitude of the peak in the DFT magnitude spectrum. α is a constant scaling factor. [(1)]

$$f_0 \leftrightarrow k$$
$$A \leftrightarrow \alpha A'$$

This method works pretty well, but there are some considerable limitations in this way of estimation. They are summarized below

- Works well only with monocomponent signals.
- While performing multicomponent estimation, it does not take into account the alteration of the peak positions and peak amplitudes due to presence of neighbouring components.
- Frequency estimation becomes harder and harder as the frequencies of

various components become closer and closer because the peaks become indistinguishible due to interference.

For the purpose of demonstration, consider the DFT magnitude spectra of a sum of 2 cosines with varying frequencies as mentioned in the respective figures below

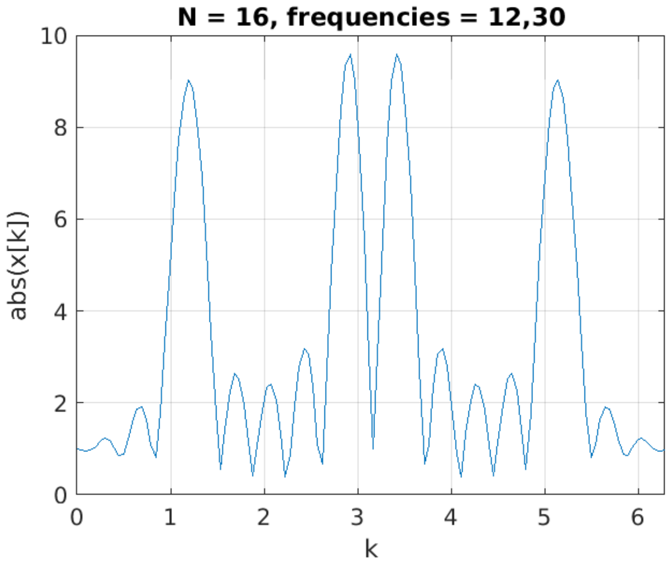

Figure 1: Here the peaks are clearly distinguishible

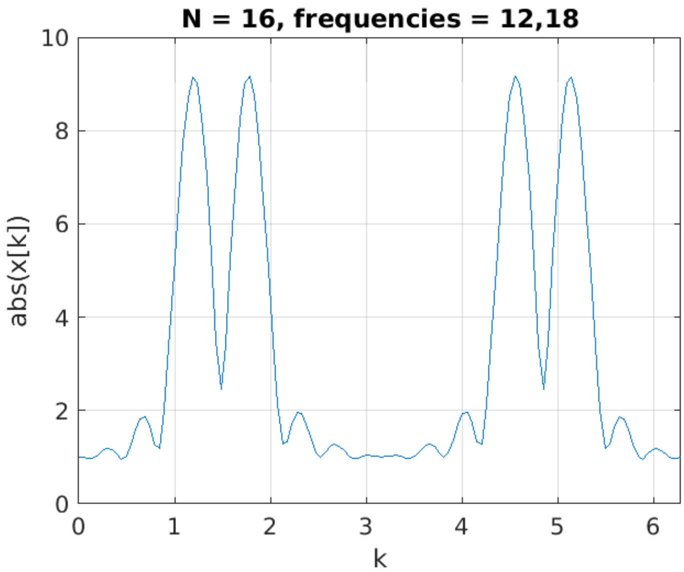

Figure 2: On decreasing the difference between the frequency components, the peaks come closer

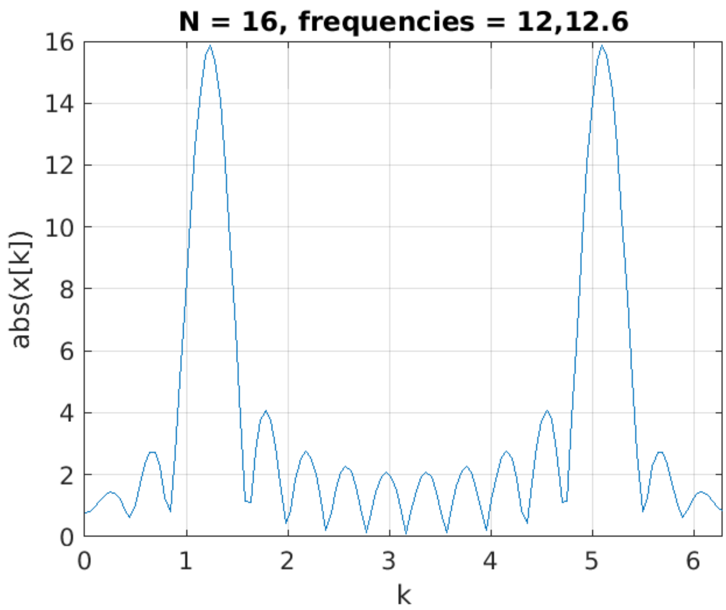

Figure 3: When the frequency components are very close, we see interference and consequently 2 separate peaks are not visible, meaning the freqencies cannot be estimated accurately

This is where the proposed algorithm steps in. It allows us to estimate signal parameters for multicomponent signals while taking into account their interference with neighbouring components. Simulations for the same set of frequency components shows that the new algorithm outperforms the FFT-based estimation and provides accurate estimations even for closely spaced sinusoids.

Signal Representation

For this algorithm, the signal is represented using a Quasi-Harmonic Model (QHM), which is given as

$$s_q(t) = \left(\sum_{k=-L}^{L} (a_k + tb_k) e^{2\pi jk\hat{f}_k t} \right) w(t) \quad (2)$$

- Where L is the number of sinusoids used to reconstruct the signal
- $\hat{f}_k$ is the analysis frequency of the kth sinusoid, i.e. the frequency at which the kth sinusoid is being generated (this need not be equal to its true frequency)
- a_k and b_k are the complex amplitude and complex slope, respectively. $^{(c)}$
- $w(t)$ is a time-domain window function and is typically a Hamming

window. (For our project, we have chosen a rectangular window)

As seen in the formula, a QHM has an added time dependency, in the form of an additional coefficient of the complex exponential. Specifically, it refers to tb_k. One benefit of the QHM is that this time dependency allows the user to choose from a wider range of signals, some of which may not be processable using a conventional signal processing method. Another added perk is that this model allows us to capture the instantaneous complex amplitude, phase and frequency during estimation, given by these formulae (3):

$$M_k(t) = |a_k + tb_k| = \sqrt{\left(Re\{a_k\} + tRe\{b_k\}\right)^2 + \left(Im\{a_k\} + tIm\{b_k\}\right)^2}$$

$$\phi_k(t) = 2\pi\widehat{f}_k t + tan^{-1}\left(\frac{Im\{a_k\} + t \cdot Im\{b_k\}}{Re\{a_k\} + t \cdot Im\{b_k\}}\right)$$

$$F_k(t) = \widehat{f}_k + \frac{1}{2\pi}\frac{\left(Re\{a_k\}*Re\{b_k\}\right) + \left(Im\{a_k\}*Im\{b_k\}\right)}{M_k(t)^2}$$

Additionally, it can handle small errors in frequency to give accurate results in almost any case. For the sake of this algorithm, will be looking at one specific property of this signal model. Knowing that both a_k and b_k can be complex-valued, then we are able to represent them as vectors and express one in terms of its projections on the other.

$$b_k = \rho_{1,k}a_k + j\rho_{2,k}a_k \quad (4)$$

Using this, we are able to calculate a_k and b_k, which upon further computation gives us these formulae(5):

$$\rho_{1,k} = \frac{\frac{dM_k(t)}{dt} \; at\ t = 0}{M_k(0)}$$

$$\rho_{2,k} = 2\pi(F_K(0) - \widehat{f}_k$$

(In our case $\rho_{1,k}$ will be 0 at all times)

Again, we use more computations to obtain the required formulae for error function $\widehat{\eta}_k$

$$\widehat{\eta}_k = \frac{\rho_{2,k}}{2\pi}$$

We can express this error function in terms of our signal parameters a and b. For a signal $h(t) = (a_1 + tb_1)e^{2\pi j\widehat{f_1}t}$, having amplitude A_1, we have[6]

$$a_1 = A_1 \frac{sin(2\pi\eta_1 T)}{2\pi\eta_1 T}$$

$$b_1 = 3jA_1\left(\frac{sin(2\pi\eta_1 T)}{(2\pi\eta_1)^2 T^3} - \frac{cos(2\pi\eta_1 T)}{2\pi\eta_1 T^2}\right)$$

$$\widehat{\eta_1} = \frac{b_1}{2j\pi a_1}$$

Where η_1 is our initial error, calculated at the start of each iteration, given by the formula $\eta_k = F_K(0) - \widehat{f_k}$

(Since only one iteration has been performed here, k has been set to 1)

The Iterative Algorithm (Taken from original research paper)

1. Initialization
   - 1.1. Get an initial estimate of frequencies $\left\{\widehat{f_k}\right\}_{k=1}^{L}$. This refers to the starting estimates for the frequencies of all the sinusoids that constitute the signal
   - 1.2. Estimate $\{a_k, b_k\}_{k=1}^{L}$ given the array obtained above.

2. Iteration
   - 2.1. For the k^{th} component
     - 2.1.1. Estimate and η_k (k^{th} mismatch error) from $\widehat{f_k}$ and compute $\widehat{\eta_k}$ (error function)
     - 2.1.2. Update the frequencies $\hat{f}_k \rightarrow \hat{f}_k + \widehat{\eta_k}$
     - 2.1.3. Recalculate $\{a_k, b_k\}$ for that k^{th} component
   - 2.2. Repeat for all k

Implementation in MATLAB (Simulations and Observations)

We immplemented and plotted the error in frequency estimation using MATLAB for a signal which consists of a sum of 3 complex sinusoids

$$x(t) = \sum_{k=1}^{K} c_k e^{j2\pi f_k t} \quad (7)$$

Additionally, we tried running this algorithm on a single-component signal and tried running the algorithm on a monocomponent signal with added white noise. We observed that our estimates ended up accurate for both multicomponent signals and single-component signals with added white noise. It is also accurate for the monocomponent signal.

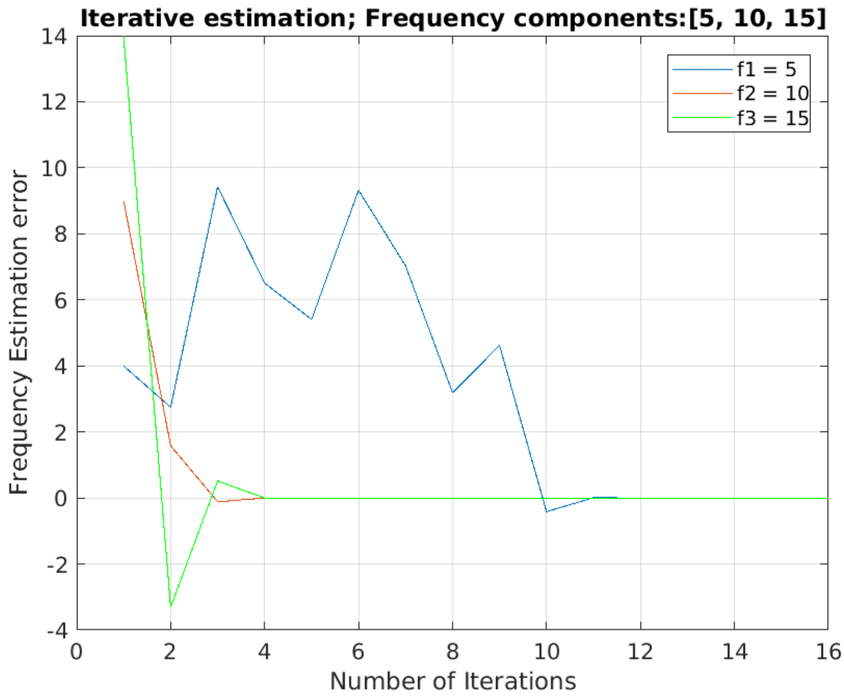

Figure 4: Multicomponent Frequency Estimation

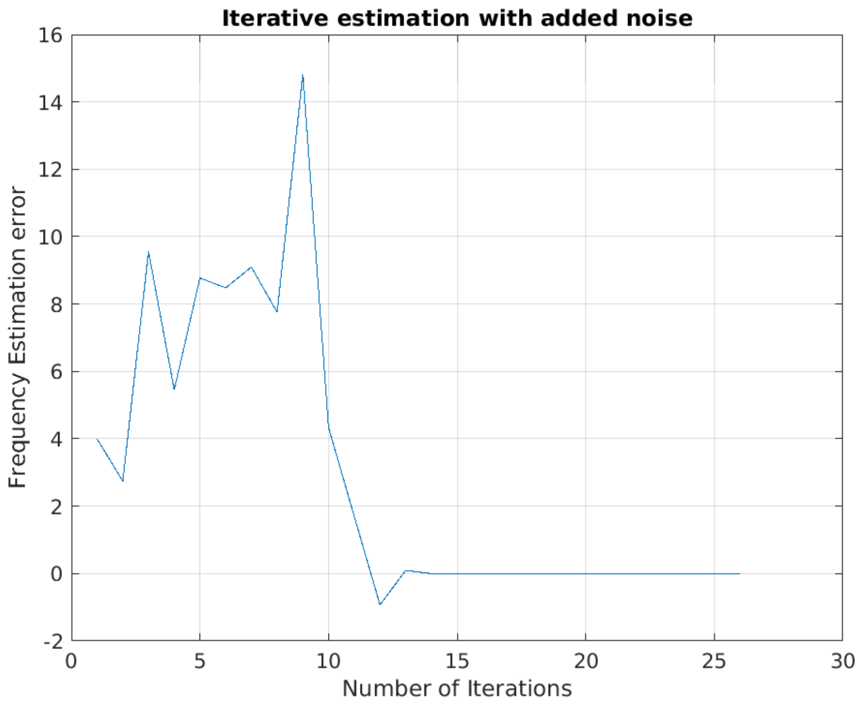

Figure 5: Monocomponent Frequency Estimation with additional white noise

As we can see, the frequency estimation error is seen to converge to zero after a certain number of iterations in both of these graphs. The number of iterations required for different signals depends on the difference between the target estimate and the initial estimate, as well as the magnitude and frequency of the signals. Even with added noise working on the signal with a frequency of 5 in the second graph, the number of iterations until convergence was attained had changed.

Comments and Comparisons of the proposed Algorithm with other similar techniques

This is similar in its application, to the Newton-Raphson iterative algorithm for the estimation of the roots of any given polynomial, where, at every iteration the initial estimated value of the root is added with a fixed error quantity $\frac{f'(x)}{f(x)}$ (for the input value 'x' at that iteration) and plugged into the polynomial again until the value of the polynomial, at some iteration goes below a small error factor tending to zero.

We start with our initial frequency estimate $\widehat{f_k}$, and can think of each η_k as a linear polynomial of sorts, taking our error at the k^{th} component to be η_k , then repeatedly incrementing the frequency estimate by the error function $\widehat{\eta_k}$ at the next iteration, and recalculating, until our initial error η_k for the k^{th} component falls within a certain acceptable value. This is done for all $\widehat{f_k}$, until we reach the expected value.

Caveats and Limitations

As useful as this method may be, it isn't exactly infallible. There is one very important caveat to this algorithm, which must be considered to ensure both accuracy of results and computational efficiency. The limitation is that in order to ensure that this algorithm is computationally efficient and accurate it is required to use some pre-existing method of estimation, before using this iterative algorithm. In other words, this algorithm is not exactly a one-shot kill method to estimate any signal. It is, rather, a means to refine the estimates taken until they meet the target value.

Another important thing to keep in mind is that for this algorithm to work, we need to have a good idea of the band in which the frequency components of the signal lie in order to calculate the intial error and further refine the estimates.

Conclusion

This iterative algorithm seems to be effective at estimating multicomponent signal parameters and is noise robust. However, a close estimate must already be provided using existing methods in order to ensure accuracy and computational efficiency.ferences

References

Original Research Paper - Iterative Estimation of Sinusoidal Signal Parameters: Yannis Pantazis, Olivier Rosec, and Yannis Stylianou --- https://ieeexplore.ieee.org/document/5411753

General References (Used to understand the methods) :

a) SINUSOID PARAMETER ESTIMATION USING THE FAST FOURIER TRANSFORM: Regis J. CRINON. https://ieeexplore.ieee.org/document/100528
b) Decomposition of AM-FM Signals with Applications in Speech Processing: Yannis Pantazis (PHD Thesis) https://www.csd.uoc.gr/~pantazis/source/phd_thesis.pdf
c) Iterative Estimation of Sinusoidal Signal Parameters: Yannis Pantazis, Olivier Rosec, and Yannis Stylianou --- https://ieeexplore.ieee.org/document/5411753
d) ON THE PROPERTIES OF A TIME-VARYING QUASI-HARMONIC MODEL OF SPEECH,
Yannis Pantazis, Olivier Rosec and Yannis Stylianou ---
https://www.csd.uoc.gr/~hannover/LabsiteDraft/MMILab_files/QuasiHarmonicModel_inter08.pdf

Specific References (From where the formulae and equations have been taken):

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Regis J. CRINON. https://ieeexplore.ieee.org/document/100528
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