SPECIAL RELATIVITY IN ELECTROMAGNETICS

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A project report submitted to Department of Physics, Sardar Vallabhbhai National Institute of Technology, Surat for Mini project I 2022.

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C E R T I F I C A T E

This is to certify that the report entitled “Special Relativity in Electromagnetic ” is prepared and presented by Chinmay Burrewar and Aakash Akhouri in 3^{rd} *Year of Integrated MSc(Physics) on* 18^{th} *nov 2022, and their work is satisfactory.*

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Abstract

The present review was aimed to prove the correct derivation of Plane Waves in Empty Space and the concepts behind the Einstein theory of relativity with electromagnetism (i.e. electricity finding by Einstein was his first proposal of a long-suspected connection between electricity, and magnetism). This unintentional , and magnetism. However, according to the theory of relativity, the inertial frames are equivalent, with none preferred over the other. Also of the transformation of E and B, the force between moving charges, analysis of plane waves in empty space, and the , we presented a well detailed analysis of waves in empty space. In particular, we explained the concepts derivation of electromagnetic relations from relativity theory. Theory of relativity provides insight into electromagnetism, and the electrical force of repulsion is greater than the magnetic force of attraction in all inertial frames. Besides, the frames are related with one another by Lorentz's transformation.

Introduction

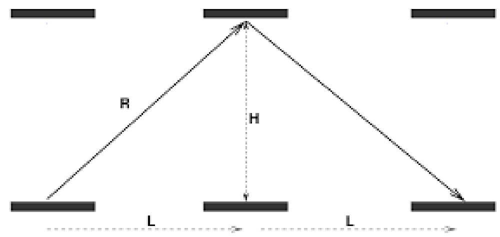
In 1819, the first accident that led to the establishment of the co-dependence of electricity and magnetism, credited to Hans Christian Oersted, presented more questions than answers. As the Maxwell's equations described, mathematically, the behaviour of electric and magnetic fields, they did not incorporate the concept of relativity. One of the most basic and important rule, the Lorentz force equation can be found to be inexplicably faulty because of this. The theory of relativity stated by Einstein is as follows:

I. The laws of physics apply, and remain unchanged in all inertial frames

II. The speed of light in vacuum is the same for all observers in their respective inertial frames, regardless of the motion of the force.

Time Dilation

Consider a observer A traveling in the train with velocity v where v is smaller than speed of light c that is ($v < c$) and a observer B at rest outside the train , observe the system in the figure . For observer A the light beam moves in a straight line with speed of light c and time t from source to the roof while for observer B, the light beam moves as shown in figure with speed c and time t' (as the distance travelled by the light is more than the distance observed by the observer A so the time taken will be different) [1].



$$(ct')^2 = (ct)^2 + (vt')^2 \tag{1}$$

$$(ct')^2 - (vt')^2 = (ct)^2 \tag{2}$$

$$t'\left(\frac{c^2 - v^2}{c^2}\right)^{\frac{1}{2}} = t \tag{3}$$

$$t' = \frac{t}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} \tag{4}$$

Here t' is known as time dilation. Let's consider,

$$\frac{t'}{t} = \gamma \tag{5}$$

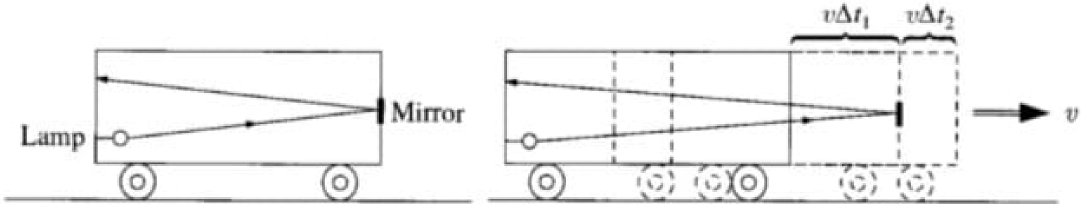
Then,

$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \tag{6}$$

This is known as the Lorentz factor.

Lorentz Contraction

Consider a lamp at one end of the boxcar and a mirror at the other. To an observer in the car the time taken by light beam to come back is δt and $\delta x'$ is the length of the train then [1],



$$\Delta t' = 2\frac{\Delta x'}{c} \tag{7}$$

Now for the observer on ground or rest Let's Δt_1 is the time to reach the front end and Δt_2 is the return time, then

$$\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c} \tag{8}$$

$$\Delta t_2 = \frac{\Delta x - v\Delta t_2}{c} \tag{9}$$

Total time

$$\Delta t = \Delta t_1 + \Delta t_2 \tag{10}$$

$$\Delta t' = \sqrt{1 - \frac{v^2}{c^2}}\Delta t \tag{11}$$

$$\Delta x' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\Delta t \tag{12}$$

Lorentz transformation

Method 1:-

In classical non relativistic dynamics, one often needs to change the coordinates (x,y,z,t) of an event E in inertial frame S to (x',y',z',t') in another inertial frame S'. Pay heed to the following figure, where an inertial frame of reference S' moves with a velocity v along the positive x direction of another inertial frame S. Here, the transformation of coordinates, while disregarding relativity, would be,

$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

These rules of transformation are called Galilean transformation. But after studying time dilation and Lorentz length contraction, we know that this transformation is hardly true. This transformation has failed to account for the length contraction in the x direction, and the time dilation in general. Now, assuming that the coordinates of an event E are (x,y,z,t) in S,

$x' = \gamma d$

Where d is the distance between the origin O' and the apparent x coordinate of the event. The term apparent here implies that the apparent x coordinate is Lorentz contracted as measured form S.

Now,

$d = x - vt$

Hence,

$x' = \gamma(x - vt)$

Now, if one looks at the event from the perspective of S' and tries to calculate the contraction in x, they arrive at

$x = \gamma(x' + vt')$

On substituting x' from the above equations, we arrive at,

$t' = \gamma(t - v/c^2 x)$

Together, they constitute the Lorentz transformation,

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$t' = \gamma(t - v/c^2 x)$

Method 2:-

The wavefront of light emitted at t = 0 when reaches at P, the position and time observed by observers at O and O‘ are (x, y, z , t) and (t‘, x‘, y‘, z‘)respectively [3].

Time taken to reach O from P as observed in frame S is:

$$t = \frac{OP}{c} = \frac{\sqrt{x^2 + y^2 + z^2}}{c} \tag{13}$$

$$x^2 + y^2 + z^2 = (ct)^2 \tag{14}$$

$$x^2 + y^2 + z^2 - (ct)^2 = 0 \tag{15}$$

The same equation can be obtained for t‘ time taken by light to reach from O‘ to P,

$$x'^2 + y'^2 + z'^2 - (ct')^2 = 0 \tag{16}$$

Since both equations (15) and (16) represent the same spherical wavefront in S and S' frame, they can be equated as:

$$x^2 + y^2 + z^2 - (ct)^2 = x'^2 + y'^2 + z'^2 - (ct')^2 \tag{17}$$

Since frame S and S' are moving relative to S along the x-axis, length in direction is perpendicular to direction of motion are unaffected.
$i.e., y' = y and z' = z$

therefore,

$$x^2 - (ct)^2 = x'^2 - (ct')^2 \tag{18}$$

In the frame S

x = vt or, x – vt = 0

Whereas for frame S'

$$x' = k(x - vt) \tag{19}$$

Since both are relative, we can assume s is moving relatively along with s' having a velocity v along the negative x – axis.

So, position of O at any instant of t' relative to observer is

x'=-vt' or, x'+vt' = 0

Whereas position of O relative to observer O in frame S is x = 0 , so x and x‘ must be related as

x‘ = k‘ (x‘ - v t‘)

Where, k‘ is another constant

Substituting the value of x‘ from equation , we get;

$$x = k‘(x - vt) + vt‘ \tag{20}$$

$$t‘ = (kt - [\frac{x}{v}])(1 - [\frac{1}{k'}]) \tag{21}$$

Putting x’ and t’ , in equation (17) we get

$$x^2 - c^2t^2 = K^2(x - vt)^2 \tag{22}$$

$$-c^2k^2(t - [\frac{x}{v}])(1 - [\frac{1}{k'}])^2 \tag{23}$$

By simplifying and equating coefficient of t^2 we get

$$k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{24}$$

From equation (6),

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{25}$$

Substituting the value of k and k' in equation

$$x' = \frac{x - vt}{\gamma} \tag{26}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\gamma} \tag{27}$$

This is Lorentz transformations

Einstein Addition Rule

The non-relativistic velocity addition rule, called the Galilean velocity addition rule is quite faulty in the light of Lorentz contraction. The Galilean rule gives,

$$v' = v + u \tag{28}$$

Where v is the velocity of a particle in frame S and v' is the velocity of a particle in a frame S' that has a constant velocity u with respect to S, in the same direction as v. But here, the Lorentz contraction in S' along the direction of u is neglected. Invoking Lorentz transformation,

$$x' = \gamma(x - ut) \tag{29}$$

$$t' = \gamma\left(t - \frac{u}{c^2}x\right) \tag{30}$$

Writing in differential form

$$dx' = \gamma(dx - udt) \tag{31}$$

$$dt' = \gamma(dt - \frac{u}{c^2}dx) \tag{32}$$

Dividing (31) and (32)

$$\frac{dx'}{dt'} = \frac{\gamma(dx - u dt)}{\gamma(dt - \frac{u}{c^2}dx)} \tag{33}$$

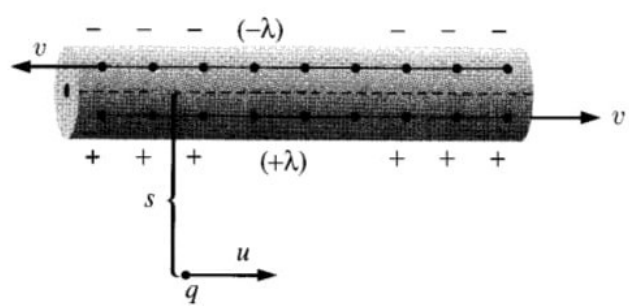
Dividing the numerator and denominator on RHS by dt,

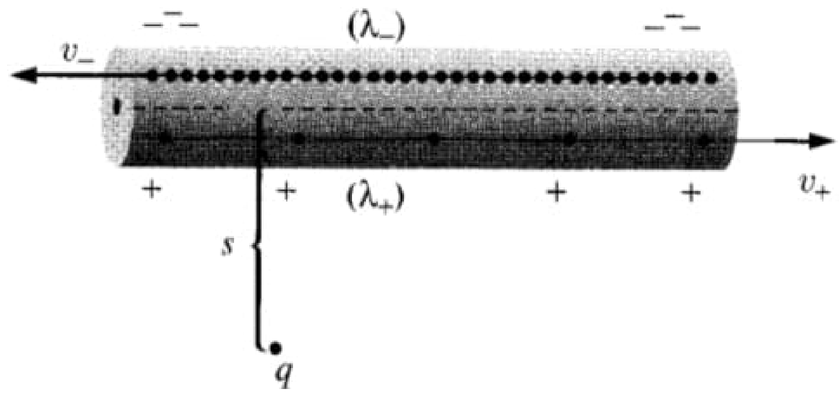
$$v' = \frac{v - u}{1 - \frac{uv}{c^2}} \tag{34}$$

This is known as Einstein's velocity addition rule.

Magnetism As A Relativistic Phenomenon

Consider a line charge α composed of positively charged beads going to the right along its length with a velocity v with respect to a frame S, superimposed with a similar line charge $-\alpha$ composed of negatively charged beads going to the left with the same velocity v. Another charge q is going to the right in the same direction at a distance s from the line charges and a velocity $u < v$ [1].





Notice the ingenuity of this setup. The total line charge in this frame is zero. Hence there can't be any electric force on the charge q but the net current to the right is,

$$I = 2\alpha v \tag{35}$$

Hence the only force this charge will experience is magnetic. Now consider another frame S' moving to the right with the same velocity u of the charge. In this frame, the magnetic force must be zero as the velocity of the charge gets nullified. Now, according to the STR, this is not allowable as in S, there is a force on q which vanishes in S' but the Lorentz contraction comes to the rescue, as on close observation, one can notice that in the frame S' there would be unequal Lorentz contraction in both the line charges. The velocity of the line charge is given by

$$v\pm = \frac{v \mp u}{1 \mp \frac{uv}{c^2}} \tag{36}$$

Where v+ is the velocity of the positive line charge in S' and v- is the velocity of the negative line charge in S'. due to these different velocities, the contraction of these line charges would be different and note that a contraction in line charge increases the line charge density α. With more contraction in the negative line charge, the net current would be towards the left. Now the Lorentz factor for both the charges would be different namely γ_+ and γ_-. Hence the current in S' is,

$$\alpha = \alpha\gamma_+ - \alpha\gamma_- \tag{37}$$

Where,

$$\gamma\pm = \frac{1}{\sqrt{1 - \frac{uv\pm}{c^2}}} \tag{38}$$

Now substituting v_-^+ in γ_-^+, one arrives at

$$\gamma\pm = \gamma\frac{1 \pm''' \frac{uv}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{39}$$

This leaves with

$$\alpha' = \frac{-2\alpha uv}{c^2\sqrt{1 - \frac{u^2}{c^2}}} \tag{40}$$

So, resultant electric field in S' is,

$$E = \frac{\alpha'}{2\pi\epsilon s} \tag{41}$$

The force is thus,

$$F' = qE \tag{42}$$

$$F' = \frac{-aquv}{\pi\epsilon c^2 s\sqrt{1 - \frac{u^2}{c^2}}} \tag{43}$$

Transformation of E and B

The electromagnetic force on a particle of a charge moving with velocity at a point, and a time at which the electric field E,[4] and magnetic field B is given by

$$f = q(E + u \times B) \tag{44}$$

The force transformation equations among a frame S , and another frame S' in which the particle experiencing the force is rapidly at rest are

$$f_x = f'_x \tag{45}$$

$$f_y = f_y\sqrt{1-\frac{v^2}{c^2}} = \frac{f'_y}{\gamma} \tag{46}$$

$$f_x = f_z\sqrt{1-\frac{v^2}{c^2}} = \frac{f'_z}{\gamma} \tag{47}$$

Suppose a particle carrying a charge is instantly at rest in S', where there is a magnetic field B' and electric field E' .The Lorentz or electromagnetic force on the particle is given by

$$f' = qE' \tag{48}$$

Since there is no magnetic force on the particle at rest. Note that we have assumed the invariance of electric charge. In the S frame, the corresponding force. Since the charge has velocity which is the velocity v of S' relative to S. If v is along with the common x'-x axes, then

$$v_x = v \tag{49}$$

$$v_y = v_z = 0 \tag{50}$$

Consider the x components. From equation (45) ,

$$f'_x = qE'_x \tag{51}$$

From equation (48),

$$f_x = q(E_x + v \times B_x) \tag{52}$$

Therefore,

$$f_x = qE_x \tag{53}$$

Applying equation (44), we have,

$$qE'_x = qE_x \tag{54}$$

Hence,

$$E'_x = E_x \tag{55}$$

Let consider the y components: From equation (46) ,

$$f'_y = qE'_y \tag{56}$$

From equation (48) ,

$$f_y = q(E_y + v \times b_y) \tag{57}$$

Applying equation (44) ,

$$qE'_y = q(E_y - vB_z) \tag{58}$$

We have,

$$E_y = \gamma(E_y - vB_z) \tag{59}$$

This the transformation equation for E_y

Conclusion

For now, we conclude that magnetism as a phenomenon can be attributed to relativity. We have understood the relative nature of time and length, which both arise from the fact that speed of light is constant in all inertial frames. We understood the relativity of velocities in the new light of STR. Using the basic results of time dilation and length contraction, we derived the Lorentz transformation of coordinates in all inertial frames.

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