Date:

# * Geometric Properties of Binary Images:

- Assume:

  $b(x,y)$ is continuous & only one object.

- Area (zeroth moment):

$$\boxed{A = \iint_I b(x,y)\,dx\,dy}$$

- Position: Centre of Area (first moment):

$$\boxed{\bar{x} = \frac{1}{A}\iint_I x\,b(x,y)\,dx\,dy} \;,\; \boxed{\bar{y} = \frac{1}{A}\iint y\,b(x,y)\,dx\,dy}$$

- How to define orientation of object & use of Second moment

$$\boxed{E = \iint_I r^2\, b(x,y)\,dx\,dy}$$

- Which eqⁿ to use to axis ?..

  $y = mx + b$ ?.  $-\infty \le m \le \infty$

  use: $\boxed{x\sin\theta - y\cos\theta + \rho = 0}$  $\rho, \theta$ are finite

- find $\rho$ & $\theta$ that minimize $E$ for given $b(x,y)$

  given line $ax + by + c = 0$.

  distance of point $(x,y)$ from line:

$$\boxed{r = \left|\frac{ax + by + c}{\sqrt{a^2+b^2}}\right|}$$

Similarly, $x \sin\theta - y \sin\theta + \rho = 0$

$$\therefore r = \left| \frac{x \sin\theta - y \sin\theta + \rho}{\sqrt{\sin^2\theta + \cos^2\theta}} \right|$$

$$\boxed{r = |x \sin\theta - y \sin\theta + \rho|} \quad \text{--- (1)}$$

using eq$^n$ (1) in ~~Axis~~ of least second moment:

$$E = \iint_I r^2 \, b(x,y) \, dx \, dy$$

So,

$$\boxed{E = \iint_I (x \sin\theta - y \cos\theta + \rho)^2 \, b(x,y) \, dx \, dy}$$

using $\frac{\partial E}{\partial \rho} = 0$ we get, $\boxed{A(\bar{x} \sin\theta - \bar{y} \sin\theta + \rho) = 0}$

Axis passes through centre $(\bar{x}, \bar{y})$

Chang co-ordinates:

$x' = x - \bar{x}$, $y' = y - \bar{y}$

$x \sin\theta - y \cos\theta + \rho$

$= x' \sin\theta - y' \cos\theta$

$\therefore$ write $E$ as,

$$\boxed{E = a \sin^2\theta - b \sin\theta \cos\theta + c \cos^2\theta}$$

where:

$$a = \iint_{I'} (x')^2 \, b(x,y) \, dx' dy'$$

$$b = 2 \iint_{I'} (x' y') \, b(x,y) \, dx' dy'$$

$$c = \iint_{I} (y')^2 \, b(x,y) \, dx' dy'$$

— [a, b, c are constants & are easy to compute].

- Minimize E

using $dE/d\theta = (a-c)\sin 2\theta - b\cos 2\theta = 0$

we get: $\boxed{\tan 2\theta = \frac{b}{a-c}}$

we know, $\tan 2\theta = \tan(2\theta + \pi) = \frac{-b}{c-a}$

$\theta$ have 2 solutions:

(i) to maximize E → $\theta = \theta_2 + \theta_1 + \pi/2$

(ii) to minimize E → $\theta = \theta_1$

using second derivative test:

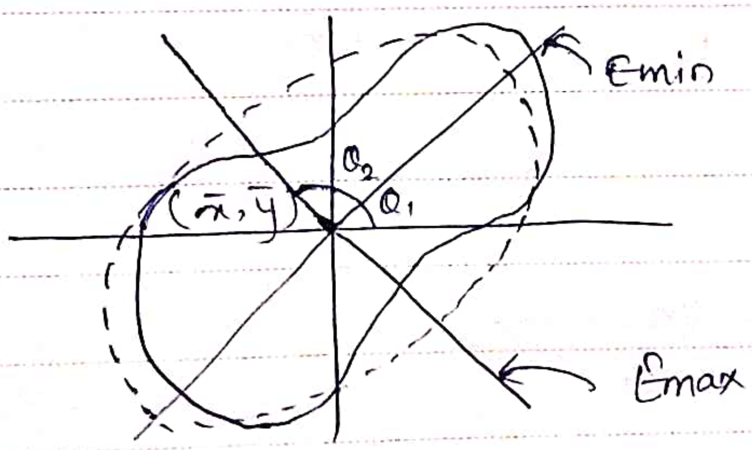
If $\frac{d^2E}{d\theta^2} = (a-c)\cos 2\theta + b\sin 2\theta \begin{cases} >0 \text{ then min} \\ <0 \text{ then max} \end{cases}$

Substitute $\cos 2\theta_1$, $\sin 2\theta_1$, $\cos 2\theta_2$, $\sin 2\theta_2$ :

$\frac{d^2E}{d\theta^2}(\theta_1) > 0$ & $\frac{d^2E}{d\theta^2}(\theta_2) < 0$.

- Orientation: $\boxed{\theta = \theta_1 = \frac{\operatorname{atan2}(b, a-c)}{2}}$

- Roundedness:




Roundedness $= \boxed{E_{min}/E_{max}}$

where $E_{min} = E(\theta_1)$ & $E_{max} = E(\theta_2)$

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- Discrete Binary image:

$b_{ij}$ = value at cell in row $i$ & column $j$.

area: $\boxed{A = \sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij}}$

- position: centre of area (first moment):

$\boxed{\bar{x} = \frac{1}{A} \sum_{i=1}^{n} \sum_{j=1}^{m} i b_{ij}}$ $\boxed{\bar{y} = \frac{1}{A} \sum_{i=1}^{n} \sum_{j=1}^{m} j b_{ij}}$

- second moment:

$\boxed{a' = \sum_{i=1}^{n} \sum_{j=1}^{m} i^2 b_{ij}}$

$\boxed{b' = 2 \sum_{i=1}^{n} \sum_{j=1}^{m} ij\, b_{ij}}$

$\boxed{c' = \sum_{i=1}^{n} \sum_{j=1}^{m} j^2 b_{ij}}$

- [$a', b', c'$ are second moments w.r.to origin]
[$a, b, c$ (w.r.to centre) can be found $a', b', c'$, $\bar{x}, \bar{y}, A$]