Regular Expression

Recap of the last lecture

NLP layers

- Understanding the semantics is a non-trivial task.
- Needs to performs a series of incremental tasks to achieve this.
- NLP happens in layers

Pragmatics & Discourse	*Study of semantics in context.*
Semantics	*Meaning of the sentence.*
Parsing	*Syntactic structure of the sentence.*
Chunking	*Grouping of meaningful phrases.*
Part of speech tagging	*Grammatical classes.*
Morphology	*Study of word structure.*

Increasing
Complexity Of
Processing

Regular Expression

Regular Expression (RE)

- A standard notation of characterizing a text sequence
- How can we search for any of the following:
  - woodchuck
  - woodchucks
  - Woodchuck
  - Woodchucks

- RE search requires a pattern and a corpus of texts to search through.

Regular Expression (RE)

RE	Example patterns matched
woodchunks	"interesting links to <u>woodchanks</u> and…."
a	"M<u>a</u>ry Ann stopped by Mona's"

- RE is case-sensitive
- Letters inside square brackets []

Pattern	Matches
[wW]oodchuck	Woodchuck, woodchuck
[1234567890]	Any digit

- Ranges [A-Z]

Pattern	Matches	
[A-Z]	An upper case letter	<u>D</u>renched Blossoms
[a-z]	A lower case letter	<u>m</u>y beans were impatient
[0-9]	A single digit	Chapter <u>1</u>: Down the Rabbit Hole

Negation

- Negations `[^Ss]`
  - Carat means negation only when it appears immediately after “[“

Pattern	Matches	Example patterns matched
`[^A-Z]`	Not an upper case letter	`Oyfn pripetchik`
`[^Ss]`	Neither ‘S’ nor ‘s’	`I have no exquisite reason”`
`[e^]`	Either ‘e’ or ‘^’	`Look here`
`a^b`	The pattern ‘a’ carat ‘b’	`Look up a^b now`

It solves the problem of woodchuck vs. Woodchuck

But not woodchuck vs woodchucks

Not woodchuck vs groundhog

Question mark ?

- ? Makes optimality of the pervious expression

Pattern	Matches
Woodchucks?	Woodchuck or Woodchucks
Colou?r	Color or Colour

It solves woodchuck vs woodchucks

Not woodchuck vs groundhog

pipe | for disjunction

Pattern	Matches
groundhog|woodchuck	Woodchucks is another name for groundhog
yours|mine	yours mine
??a|b|c	= [abc]
[gG]roundhog|[Ww]oodchuck	

It solves woodchuck vs groundhog

But not woodchuckssssssssss

Kleene *,   Kleene +

- Kleene * => zero or more occurrences of the immediately previous character or regular expression
- Kleene + => one or more of the previous character
- Period (.) matches any single character (except a carriage return)

Pattern	Matches	
`oo*h!`	0 or more of previous char	oh! ooh! oooh! ooooh!
[ab]*	Zero or more a's or b's	aaa ababab bbbb
`o+h!`	1 or more of previous char	oh! ooh! oooh! ooooh!
`baa+`		baa baaa baaaa baaaaa
`beg.n`		begin begun begun beg3n

Anchors ^ $

- Caret ^ matches the start of a line
  - Negations [^Ss] (careful!)
- \b matches a word boundary
- \B matches a non-boundary
- $ matches the end of a line

Pattern	Matches
`^[A-Z]`	
`^[^A-Za-z]`	
\bthe\b	
`\.$`	
`.$`	

Anchors ^ $

- Caret ^ matches the start of a line
  - Negations [^Ss] (careful!)
- \b matches a word boundary
- \B matches a non-boundary
- $ matches the end of a line

Pattern	Matches
`^[A-Z]`	`Palo Alto`
`^[^A-Za-z]`	`1 "Hello"`
\bthe\b	`the, not "other"`
`\.$`	`The end.`
`.$`	`The end?` `The end!`

Quiz

- Find all instances of the word "the" in a text.

`the`

You may miss capitalized examples

`[tT]he`

Incorrectly returns `other` or `theology`

`[^a-zA-Z][tT]he[^a-zA-Z]`

It won't find the word the when it begins or ends a line

`(^|[^a-zA-Z])[tT]he([^a-zA-Z]|$)` ⇒ Before the we require either the beginning-of-line or a non-alphabetic character, and the same at the end of the line.

Advanced Operators

RE	Expansion	Match	Example Patterns
\d	[0-9]	any digit	Party␣of␣5
\D	[^0-9]	any non-digit	Blue␣moon
\w	[a-zA-Z0-9␣]	any alphanumeric or space	Daiyu
\W	[^\w]	a non-alphanumeric	!!!!
\s	[␣\r\t\n\f]	whitespace (space, tab)	
\S	[^\s]	Non-whitespace	in␣Concord

Error

- We want to fix two kinds of errors
  - False positives (Type I)
    - Matching strings that we should not have matched (there, then, other)
  - False negatives (Type II)
    - Not matching things that we should have matched (The)
- Reducing error may require a trade-off between
  - Accuracy or Precision: minimizing false positives
  - Coverage or Recall: minimizing false negative

Morphology

Challenges…

- How do we know that
  - Both *woorchunk* and woodchuncks have same original/root word?
    - May be easy: the plural just tacks as s on to end
  - But what about goose vs geese or fox vs foxes?
- **Two kinds of knowledge:**
  - **Orthographic rules**: can solve *woorchunk* vs. woodchuncks
  - **Morphological rules:** can distinguish goose vs geese

Orthographic/Spelling Rules

- General rules used when breaking a word into its stem and modifiers.
- Example:
  - Singular English words ending with –y, when pluralized, end with –ies.
  - Peccary vs. Peccaries

Morphology: Definition

The study of words, how they are formed, and their relationship to other words in the same language.

Morphological Rules

- Morphological rules are exceptions to the orthographic rules used when breaking a word into its stem and modifiers.
- Example:
  - Goose vs Geese is due to vowel change

Morphological Parsing

- **Parsing:** Take an input and produce some sort of linguistic structure
- Morphological parsing the process of determining the morphemes from which a given word is constructed.

Text

A morpheme is the smallest meaningful constituent of a linguistic expression

Terminologies

	Tokens = N	Types = |V|
Switchboard phone conversations	2.4 million	20 thousand
Shakespeare	884,000	31 thousand
Google N-grams	1 trillion	13 million

Church and Gale (1990): $|V| > O(N^{½})$

- **Surface form:** Raw text present in a corpus
  - Example: going
- **Token:** a word present in running text (may be duplicated)
- **Type:** Unique word present in the running text
- **Vocabulary:** Set of types

they lay back on the San Francisco grass and looked at the stars and their

- 15 tokens (or 14)
- 13 types (or 12) (or 11?)

Infix

Tagalog: hingi (borrow) => h**<u>um</u>**ingi

Circumfix

German: Sagen (to say) =><u>ge</u>sag<u>t</u> (said)

- **Stem**
- **Affix : prefix/suffix/infix/circumfix**

Terminologies

- A word can have more than one affix
  - Example: rewrites *(re-, write,-s),* unbelievably *(un-, believe, -able, -ly)*

- English doesn’t tend to stack more than 4 or 5 affixes

- Languages that tend to string affixes together like Turkish does are called **agglutinative** languages
  - Turkish can have words with 9 or 10 affixes

Terminologies

- **Inflection:** A word stem with a grammatical morpheme, usually resulting in a word of the same class as the original stem.
  - **Example:** Plural (-s) or past (-ed)

- **Derivation:** the combination of a word stem with a grammatical morpheme, usually resulting in a word of a different class
  - **Example:** computerize (verb) vs. computerization (noun)

Compounding: combination of two stems: e.g., *doghouse*
Cliticization: Combination of a word stem with a clitic: e.g., *I've*

Inflectional Morphology

- **Two kinds of inflection**: an affix that marks **plural** and an affix that marks **possessive**

	Regular Nouns		Irregular Nouns	
Singular	cat	thrush	mouse	ox
Plural	cats	thrushes	mice	oxen

- Possessive suffix is realized by
  - apostrophe + -*s* for regular singular nouns (*llama's*)
  - plural nouns not ending in -s (children's)
  - often by a lone apostrophe after regular plural nouns (llamas')

Derivational Morphology

- **Recall:** derivation is the combination of a word stem with a grammatical morpheme, usually resulting in a word of a *different class*.
- **Nominalization**: Forming a new noun from verb/adjective

Suffix	Base Verb/Adjective	Derived Noun
-ation	computerize (V)	computerization
-ee	appoint (V)	appointee
-er	kill (V)	killer
-ness	fuzzy (A)	fuzziness

- Adjectives can also be derived from nouns and verbs

Suffix	Base Noun/Verb	Derived Adjective
-al	computation (N)	computational

Concatenative morphology is Easy!

- **Non-concatenative morphology**
  - Philipian language (Tagalog)
  - Um + hingi (request) = humingi (ask for)

- **Templatic morphology/root-and-pattern morphology**
  - In Hebrew, a verb is constructed from two components: a root (CCC) and a template (ordering of C and V to specify more semantic info)
  - *lmd* (learn/study) can be combined with active voice *CaCaC* template to produce *lamad* (he studied)
  - *CiCeC* => limed (he taught)
  - *CuCaC => lumad* (he was taught)

Finite-State Morphological Parsing

- Parsing English morphology

Input	Morphological parsed output
cats	`cat +N +PL`
cat	`cat +N +SG`
cities	`city +N +PL`
geese	`goose +N +PL`
goose	(`goose +N +SG`) or (`goose +V`)
gooses	`goose +V +3SG`
merging	`merge +V +PRES-PART`
caught	(`caught +V +PAST-PART`) or (`catch +V +PAST`)

Stems and morphological features

Finite-State Morphological Parsing

finite states can be used for parsing

1. **Lexicon:** the list of stems and affixes, together with basic information about them (Noun stem or Verb stem, etc.)

2. **Morphotactics:** the model of morpheme ordering that explains which classes of morphemes can follow other classes of morphemes inside a word. E.g., the rule that English plural morpheme follows the noun rather than preceding it.

3. **Orthographic rules:** these **spelling rules** are used to model the changes that occur in a word, usually when two morphemes combine (e.g., the *y*→*ie* spelling rule changes *city* + *-s* to *cities*).

Finite-State Morphological Parsing

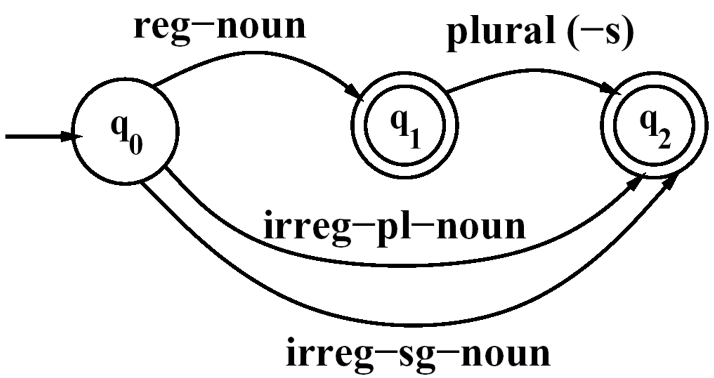
Morphological Parsing with FST finite state transducer

- A formal definition of FST (based on the **Mealy machine** extension to a simple FSA):
  - Q: a finite set of N states $q_0, q_1, \ldots, q_N$
  - Σ: a finite alphabet of complex symbols. Each complex symbol is composed of an input-output pair $i : o$; one symbol I from an input alphabet I, and one symbol o from an output alphabet O, thus $\Sigma \subseteq I \times O$. I and O may each also include the epsilon symbol ε.
  - q_0: the start state
  - F: the set of final states, $F \subseteq Q$
  - $\delta(q, i{:}o)$: the transition function or transition matrix between states. Given a state $q \in Q$ and complex symbol $i{:}o \in \Sigma$, $\delta(q, i{:}o)$ returns a new state $q' \in Q$. δ is thus a relation from $Q \times \Sigma$ to Q.

Finite-State Morphological Parsing

The Lexicon and Morphotactics

- A lexicon is a repository for words.
  - The simplest one would consist of an explicit list of every word of the language. ***Inconvenient or impossible!***
  - Computational lexicons are usually structured with
    - a list of each of the stems and
    - Affixes of the language together with a representation of morphotactics telling us how they can fit together.
  - The most common way of modeling morphotactics is the **finite-state automaton**.



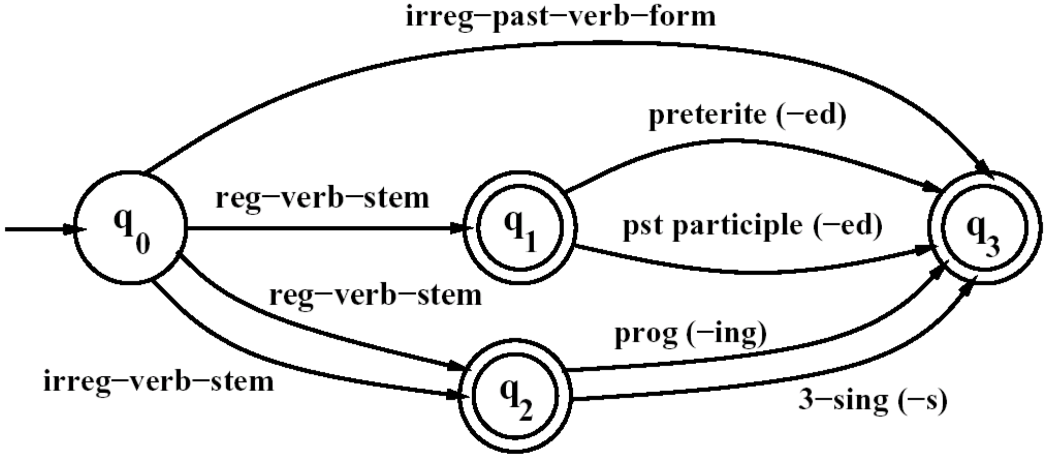
An FSA for English nominal inflection

Reg-noun	**Irreg-pl-noun**	**Irreg-sg-noun**	**plural**
fox	geese	goose	-s
fat	sheep	sheep	
fog	Mice	mouse	
fardvark			

Finite-State Morphological Parsing

The Lexicon and Morphotactics

word => stem (stemming) (removing affixes) stem might not be part of dictionary
word => root (lemmitization) the root is a dictionary word



An FSA for English verbal inflection

Preterite: Past
Prog: present participle

Reg-verb-stem	Irreg-verb-stem	Irreg-past-verb	past	Past-part	Pres-part	3sg
walk fry talk impeach	cut speak sing sang spoken	caught ate eaten	-ed	-ed	-ing	-s

The Porter Stemmer (Porter, 1980)

- A simple rule-based algorithm for stemming
- An example of a HEURISTIC method
- Based on rules like:
  - ATIONAL -> ATE (e.g., *relational* -> *relate*)
- The algorithm consists of seven sets of rules, applied in order

The Porter Stemmer: definitions

- Definitions:
  - CONSONANT: a letter other than A, E, I, O, U, and Y preceded by consonant (e.g. SYZYGY)
  - VOWEL: any other letter
- With this definition, all words are of the form:

  $(C)(VC)^m(V)$

  C=string of one or more consonants (con+)

  V=string of one or more vowels

  m>=0
- E.g.,
  - Tr ou bl e s
  - C V C V C

The Porter Stemmer: rule format

- The rules are of the form:

  (condition) S1 -> S2

  Where S1 and S2 are suffixes

- Conditions:

m	The measure of the stem
*S	The stem ends with S
v	The stem contains a vowel
*d	The stem ends with a double consonant
*o	The stem ends in CVC (second C not W, X, or Y)

The Porter Stemmer: Step 1

- SSES -> SS
  - *caresses -> caress*
- IES -> I
  - *ponies -> poni*
  - *ties -> ti*
- SS -> SS
  - *caress -> caress*
- S -> ϵ
  - *cats -> cat*

The Porter Stemmer: Step 2a (past tense, progressive)

- (m>0) EED -> EE
  - Condition verified: *agreed -> agree*
  - Condition not verified: *feed -> feed*
- (*V*) ED -> ϵ
  - Condition verified: *plastered -> plaster*
  - Condition not verified: *bled -> bled*
- (*V*) ING -> ϵ
  - Condition verified: *motoring -> motor*
  - Condition not verified: *sing -> sing*

m	The measure of the stem
*S	The stem ends with S
v	The stem contains a vowel
*d	The stem ends with a double consonant
*o	The stem ends in CVC (second C not W, X, or Y)

The Porter Stemmer: Step 2b (cleanup)

- (These rules are ran if second or third rule in 2a apply)
- AT-> ATE
  - *conflat(ed) -> conflate*
- BL -> BLE
  - *Troubl(ing) -> trouble*
- (*d & ! (*L or *S or *Z)) -> single letter
  - <u>Condition verified</u>: *hopp(ing) -> hop, tann(ed) -> tan*
  - <u>Condition not verified</u>: *fall(ing) -> fall*
- (m=1 & *o) -> E
  - <u>Condition verified</u>: *fil(ing) -> file*
  - <u>Condition not verified</u>: *fail -> fail*

Why?

m	The measure of the stem
*S	The stem ends with S
v	The stem contains a vowel
*d	The stem ends with a double consonant
*o	The stem ends in CVC (second C not W, X, or Y)

(*V*) ED -> ϵ

<u>Condition verified</u>: *plastered -> plaster*

<u>Condition not verified</u>: *bled -> bled*

(*V*) ING -> ϵ

<u>Condition verified</u>: *motoring -> motor*

<u>Condition not verified</u>: *sing -> sing*

The Porter Stemmer: Steps 3 and 4

- Step 3: Y Elimination (*V*) Y -> I
  - <u>Condition verified</u>: *happy -> happi*
  - <u>Condition not verified</u>: *sky -> sky*
- Step 4: Derivational Morphology, I
  - (m>0) ATIONAL -> ATE
    - *Relational -> relate*
  - (m>0) IZATION -> IZE
    - *generalization-> generalize*
  - (m>0) BILITI -> BLE
    - *sensibiliti -> sensible*

m	The measure of the stem
*S	The stem ends with S
v	The stem contains a vowel
*d	The stem ends with a double consonant
*o	The stem ends in CVC (second C not W, X, or Y)

The Porter Stemmer: Steps 5 and 6

- Step 5: Derivational Morphology, II
  - (m>0) ICATE -> IC
    - *triplicate -> triplic*
  - (m>0) FUL -> ϵ
    - *hopeful -> hope*
  - (m>0) NESS -> ϵ
    - *goodness -> good*
- Step 6: Derivational Morphology, III
  - (m>0) ANCE -> ϵ
    - *allowance-> allow*
  - (m>0) ENT -> ϵ
    - *dependent-> depend*
  - (m>0) ANT -> ϵ
    - *irritant -> irrit*
  - (m>0) IVE -> ϵ
    - *effective -> effect*

m	The measure of the stem
*S	The stem ends with S
v	The stem contains a vowel
*d	The stem ends with a double consonant
*o	The stem ends in CVC (second C not W, X, or Y)

The Porter Stemmer: Step 7 (cleanup)

- Step 7a
  - (m>1) E -> ϵ
    - *probate -> probat*
  - (m=1 & !*o) NESS -> ϵ
    - *goodness -> good*
- Step 7b
  - (m>1 & *d & *L) -> single letter
    - <u>Condition verified</u>: *controll -> control*
    - <u>Condition not verified</u>: *roll -> roll*

m	The measure of the stem
*S	The stem ends with S
v	The stem contains a vowel
*d	The stem ends with a double consonant
*o	The stem ends in CVC (second C not W, X, or Y)

Examples

- *computers*
  - Step 1, Rule 4: -> *computer*
  - Step 6, Rule 4: -> *compute*
- *singing*
  - Step 2a, Rule 3: -> *sing*
- *controlling*
  - Step 2a, Rule 3: -> *controll*
  - Step 7b : -> *control*
- *generalizations*
  - Step 1, Rule 4: -> *generalization*
  - Step 4, Rule 11: -> *generalize*
  - Step 6, last rule: -> *general*

Problems

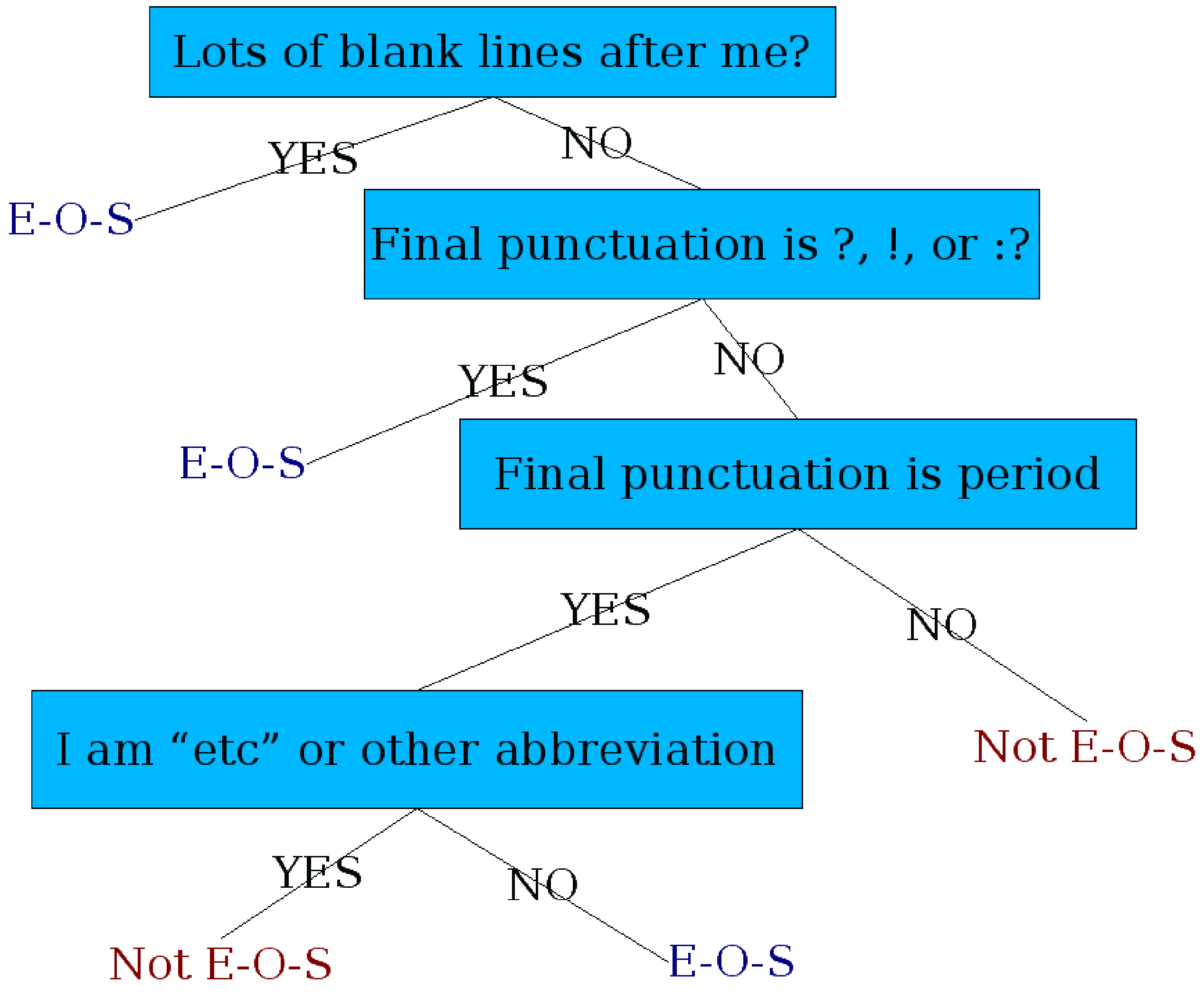
- *elephants -> eleph*
  - Step 1, Rule 4: -> *elephant*
  - Step 6, Rule 7: -> *eleph*
- *Etc……*

Sentence Segmentation

Sentence Segmentation

- !, ? are relatively unambiguous
- Period "." is quite ambiguous
  - Sentence boundary
  - Abbreviations like Inc. or Dr.
  - Numbers like .02% or 4.3
- Build a binary classifier
  - Looks at a "."
  - Decides EndOfSentence/NotEndOfSentence
  - Classifiers: hand-written rules, regular expressions, or machine-learning

Determining if a word is end-of-sentence: a Decision Tree



More sophisticated decision tree features

- Case of word with ".": Upper, Lower, Cap, Number
- Case of word after ".": Upper, Lower, Cap, Number

- Numeric features
  - Length of word with "."
  - Probability(word with "." occurs at end-of-s)
  - Probability(word after "." occurs at beginning-of-s)

Decision Trees and other classifiers

- We can think of the questions in a decision tree
- As features that could be exploited by any kind of classifier
  - Logistic regression
  - SVM
  - Neural Nets
  - etc.

- Course Instructor: Tanmoy Chakraborty (**tanmoychak.com**)
  (NLP, Social Media, Graph Neural Networks)
  tanchak@iitd.ac.in
- Guest Lecture: Graham Neubig (CMU)
- Course page: https://sites.google.com/view/ell881-iitd/home
- Piazza: https://piazza.com/iitd.ac.in/spring2023/ell881
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- Group Email: TBD

Spelling Error:
Minimum Edit Distance

How similar are two strings?

- Spell correction
  - The user typed "Appl"

    Which is closest?
    - App
    - Appeal
    - Apple

- Computational Biology
  - Align two sequences of nucleotides

    AGGCTATCACCTGACCTCCAGGCCGATGCCC
    TAGCTATCACGACCGCGGTCGATTTGCCCGAC

  - Resulting alignment:

    -AGGCTATCACCTGACCTCCAGGCCGA--TGCCC---
    TAG-CTATCAC--GACCGC--GGTCGATTTGCCCGAC

- Also for Machine Translation, Information Extraction, Speech Recognition

Edit Distance

- The minimum edit distance between two strings
- Is the minimum number of editing operations
  - Insertion **(I)**
  - Deletion **(D)**
  - Substitution **(S)**
- Need to transform one into the other

Minimum Edit Distance

- Two strings and their **alignment**: TRIAL vs ZEIL

```
T R I A L
| | | | |
Z E I * L
s s   d
```

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

- If each operation has cost of 1
  - Distance between these is 3

edit distance considers all the costs 1
levenshtein cost only differs in considering the substitution cost as 2 instead of 1

- If substitutions cost 2 (Levenshtein)
  - Distance between them is 5

Other uses of Edit Distance in NLP

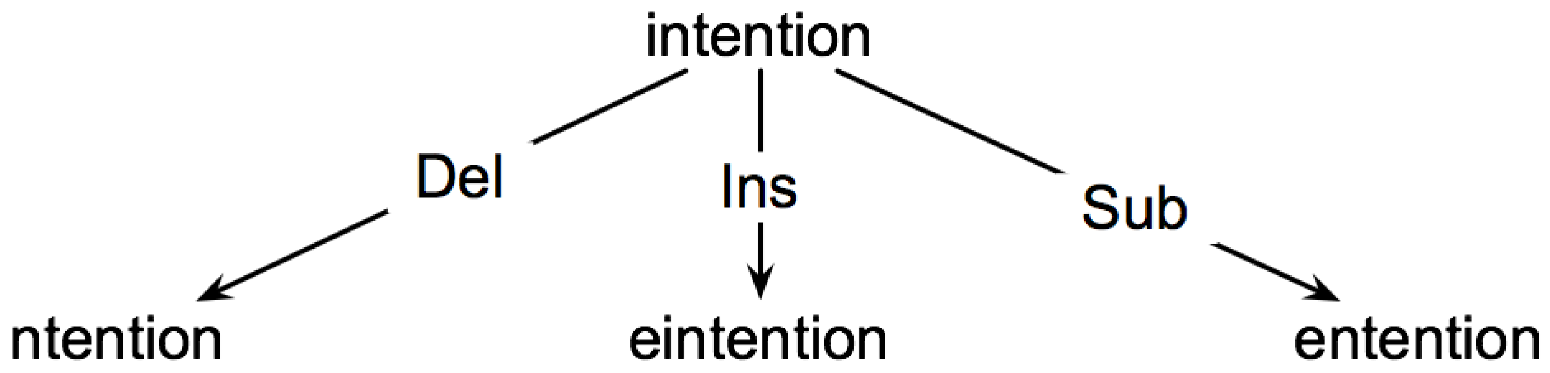
- Evaluating Machine Translation and speech recognition

Spokesman confirms		senior	government	adviser was shot	
Spokesman said	the	senior		adviser was shot	dead
S	I		D		I

- **Named Entity Extraction** and **Entity Coreference**
  - IBM Inc. announced today
  - IBM profits
  - US President Donald Trump announced yesterday
  - for United States President Donald Trump

How to find the Min Edit Distance?

- Searching for a path (sequence of edits) from the start string to the final string:
  - **Initial state**: the word we're transforming
  - **Operators**: insert, delete, substitute
  - **Goal state**: the word we're trying to get to
  - **Path cost**: what we want to minimize: the number of edits



Minimum Edit as Search

- But the space of all edit sequences is huge!
  - We can’t afford to navigate naïvely
  - Lots of distinct paths wind up at the same state.
    - We don’t have to keep track of all of them

Defining Min Edit Distance

- For two strings
  - X of length n
  - Y of length m
- We define $D(i,j)$
  - the edit distance between $X[1..i]$ and $Y[1..j]$
    - i.e., the first i characters of X and the first j characters of Y
- The edit distance between X and Y is thus $D(n,m)$

Dynamic Programming for Minimum Edit Distance

- **Dynamic programming**: A tabular computation of D(n,m)
- Solving problems by combining solutions to sub-problems.
- Bottom-up
  - We compute D(i,j) for small i,j
  - And compute larger D(i,j) based on previously computed smaller values
  - i.e., compute D(i,j) for all i ($0 < i < n$) and j ($0 < j < m$)

Defining Min Edit Distance (Levenshtein)

- Initialization

```
D(i,0) = i
D(0,j) = j
```

- Recurrence Relation:

```
For each  i = 1…M
      For each  j = 1…N
```

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} \end{cases}$$

- Termination:

```
D(N,M) is distance
```

The Edit Distance Table

L	5				
A	4				
I	3				
R	2				
T	1				
#	0	1	2	3	4
	#	Z	E	I	L

i

j

The Edit Distance Table

i						
5	L	5	6	7	6	**5**
4	A	4	5	6	5	6
3	I	3	4	5	4	5
2	R	2	3	4	5	6
1	T	1	2	3	4	5
0	#	0	1	2	3	4
		#	Z	E	I	L
j		0	1	2	3	4

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

The Edit Distance Table

(INTENSION vs EXECUTION)

N	9	8	9	10	11	12	11	10	9	**8**
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

Computing Alignments

- Edit distance isn't sufficient
  - We often need to **align** each character of the two strings to each other
- We do this by keeping a "backtrace"
- Every time we enter a cell, remember where we came from
- When we reach the end,
  - Trace back the path from the upper right corner to read off the alignment

Edit Distance with Backtrace

L	5	6	7	6	**5**
A	4	5	6	5	6
I	3	4	5	4	5
R	2	3	4	5	6
T	1	2	3	4	5
#	0	1	2	3	4
	#	Z	E	I	L

← Substitute

← No Change

← Delete

```
T R I A L
| | | | |
Z E I * L
s s   d
```

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

Edit Distance with Backtrace (Another Path)

L	5	6	7	6	**5**
A	4	5	6	5	6
I	3	4	5	4	5
R	2	3	4	5	6
T	1	2	3	4	5
#	0	1	2	3	4
	#	Z	E	I	L

- Substitute
- No Change
- Delete
- Insert

Cost is same, i.e., 5

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

MinEdit with Backtrace

n	9	↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙←↓ 12	↓ 11	↓ 10	↓ 9	↙ **8**
o	8	↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↓ 10	↓ 9	↙ **8**	← 9
i	7	↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	↙ **8**	← 9	← 10
t	6	↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙ **8**	← 9	← 10	←↓ 11
n	5	↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ **8**	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙↓ 10
e	4	↙ 3	← 4	↙← **5**	← **6**	← 7	←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9
t	3	↙←↓ 4	↙←↓ **5**	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙ 7	←↓ 8	↙←↓ 9	↓ 8
n	2	↙←↓ **3**	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↓ 7	↙←↓ 8	↙ 7
i	**1**	↙←↓ 2	↙←↓ 3	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙ 6	← 7	← 8
#	**0**	1	2	3	4	5	6	7	8	9
	#	**e**	**x**	**e**	**c**	**u**	**t**	**i**	**o**	**n**

Adding Backtrace to Minimum Edit Distance

- Base conditions:

  $D(i,0) = i \qquad D(0,j) = j$

Termination:

$D(N,M)$ is distance

- Recurrence Relation:

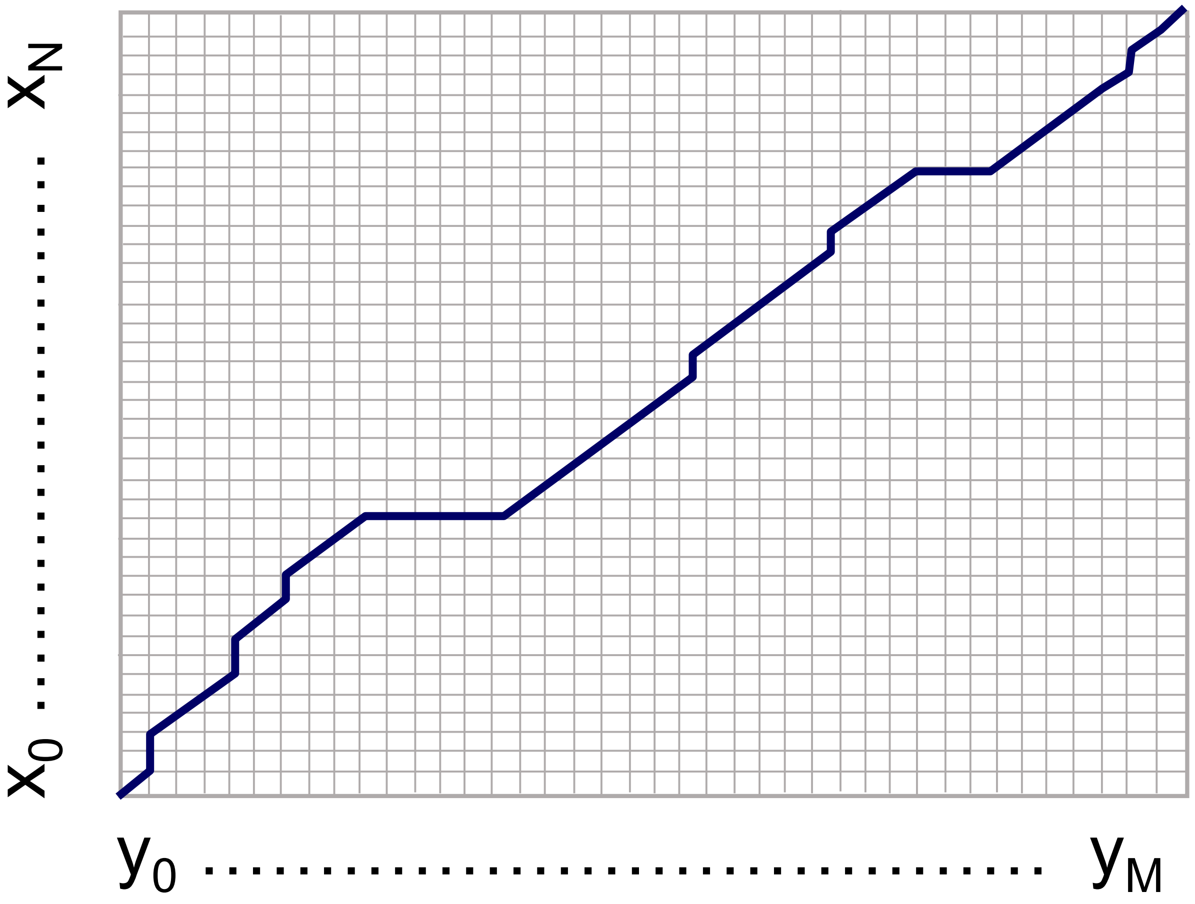
  For each $i = 1\ldots M$

  For each $j = 1\ldots N$

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 & \text{deletion} \\ D(i,j-1) + 1 & \text{insertion} \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} & \text{substitution} \end{cases}$$

$$ptr(i,j) = \begin{cases} \text{LEFT} & \text{insertion} \\ \text{DOWN} & \text{deletion} \\ \text{DIAG} & \text{substitution} \end{cases}$$

The Distance Matrix



Every non-decreasing path

from (0,0) to (M, N)

corresponds to
an alignment
of the two sequences

An optimal alignment is composed of optimal subalignments

Performance

- Time: $O(nm)$
- Space: $O(nm)$
- Backtrace: $O(n+m)$