

# (Probabilistic) Context-Free Grammars

# A phrase structure grammar

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $NP \rightarrow N$   
 $VP \rightarrow V NP PP$   
 $NP \rightarrow NP NP$   
 $NP \rightarrow NP PP$   
 $NP \rightarrow e$   
 $PP \rightarrow P NP$

***people fish tanks***

*people fish with rods*

$N \rightarrow \text{people}$

$V \rightarrow \text{fish}$

$N \rightarrow \text{fish}$

$N \rightarrow \text{tanks}$

$N \rightarrow \text{rods}$

$V \rightarrow \text{people}$

$V \rightarrow \text{tanks}$

$P \rightarrow \text{with}$

Ambiguous: People people people, fish fish fish

# Phrase structure grammars = context-free grammars (CFGs)

- $G = (T, N, S, R)$ 
  - $T$  is a set of terminal symbols
  - $N$  is a set of nonterminal symbols
  - $S$  is the start symbol ( $S \in N$ )
  - $R$  is a set of rules/productions of the form  $X \rightarrow \gamma$ 
    - $X \in N$  and  $\gamma \in (N \cup T)^*$
- A grammar  $G$  generates a language  $L$ .

# Phrase structure grammars in NLP

- $G = (T, C, N, S, L, R)$ 
  - $T$  is a set of terminal symbols
  - $C$  is a set of preterminal symbols
  - $N$  is a set of nonterminal symbols
  - $S$  is the start symbol ( $S \in N$ )
  - $L$  is the lexicon, a set of items of the form  $X \rightarrow x$ 
    - $X \in C$  and  $x \in T$
  - $R$  is the grammar, a set of items of the form  $X \rightarrow \gamma$ 
    - $X \in N$  and  $\gamma \in (N \cup C)^*$
- By usual convention,  $S$  is the start symbol, but in statistical NLP, we usually have an extra node at the top (ROOT, TOP)
- We usually write  $e$  for an empty sequence, rather than nothing

# A phrase structure grammar (empty, unary, binary)

## Grammar Rules

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow V NP PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow N$

$NP \rightarrow e$

$PP \rightarrow P NP$

*EMPTY fish tanks*

*people fish EMPTY*

## Lexicon

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

# Probabilistic/stochastic – context-free grammars (PCFGs)

- $G = (T, N, S, R, P)$ 
  - $T$  is a set of terminal symbols
  - $N$  is a set of nonterminal symbols
  - $S$  is the start symbol ( $S \in N$ )
  - $R$  is a set of rules/productions of the form  $X \rightarrow \gamma$
  - $P$  is a probability function
    - $P: R \rightarrow [0, 1]$
    - $\forall X \in N, \sum_{X \rightarrow \gamma \in R} P(X \rightarrow \gamma) = 1$

# A PCFG

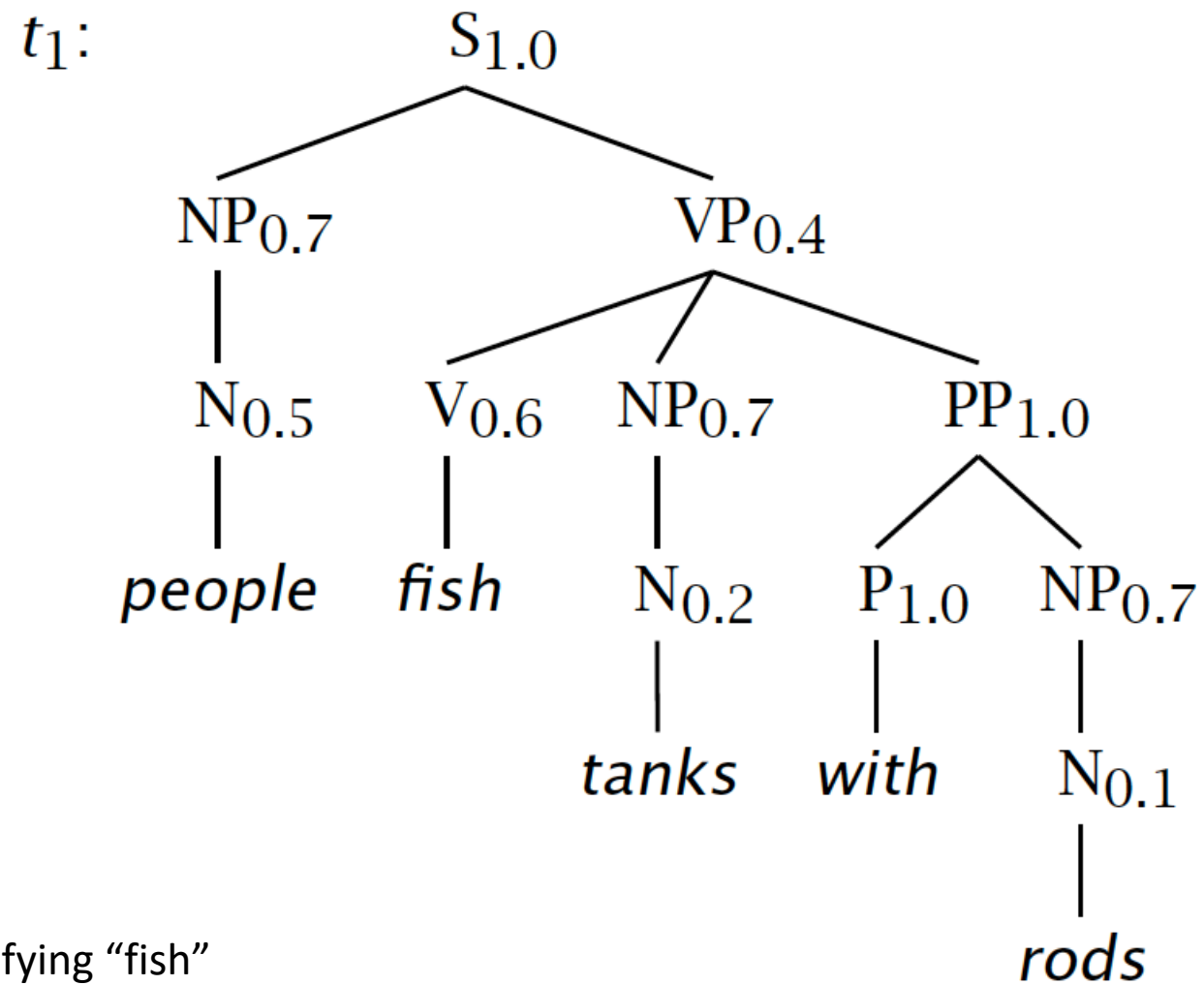
$S \rightarrow NP VP$	1.0	$N \rightarrow people$	0.5
$VP \rightarrow V NP$	0.6	$N \rightarrow fish$	0.2
$VP \rightarrow V NP PP$	0.4	$N \rightarrow tanks$	0.2
$NP \rightarrow NP NP$	0.1	$N \rightarrow rods$	0.1
$NP \rightarrow NP PP$	0.2	$V \rightarrow people$	0.1
$NP \rightarrow N$	0.7	$V \rightarrow fish$	0.6
$PP \rightarrow P NP$	1.0	$V \rightarrow tanks$	0.3
		$P \rightarrow with$	1.0

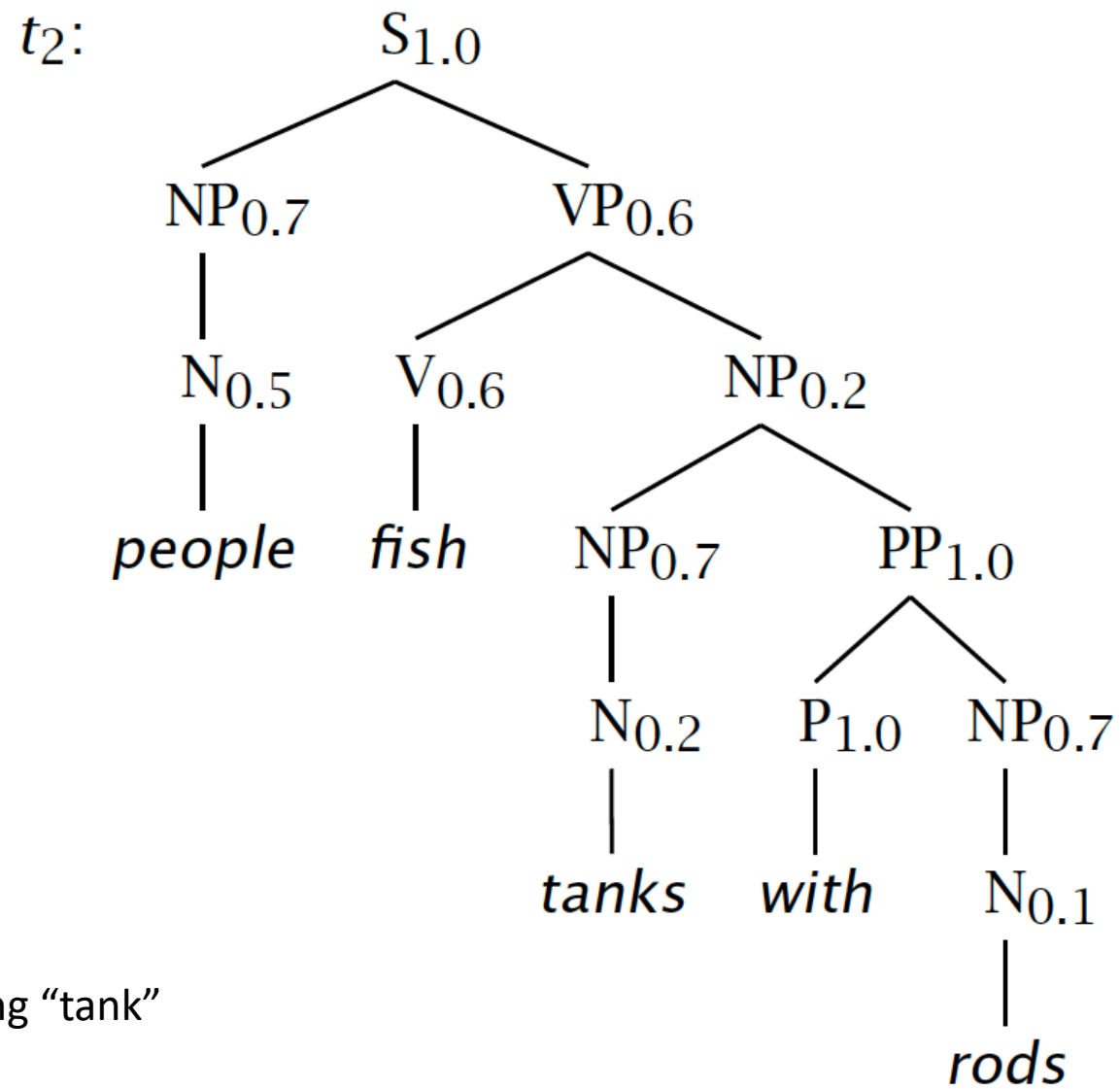
# The probability of trees and strings

- $P(t)$  – The probability of a tree  $t$  is the product of the probabilities of the rules used to generate it.
- $P(s)$  – The probability of the string  $s$  is the sum of the probabilities of the trees which have that string as their yield

$$\begin{aligned} P(s) &= \sum_t P(s, t) \text{ where } t \text{ is a parse of } s \\ &= \sum_t P(t) \end{aligned}$$







Preposition “with” modifying “tank”

# Tree and String Probabilities

- $s = \textit{people fish tanks with rods}$

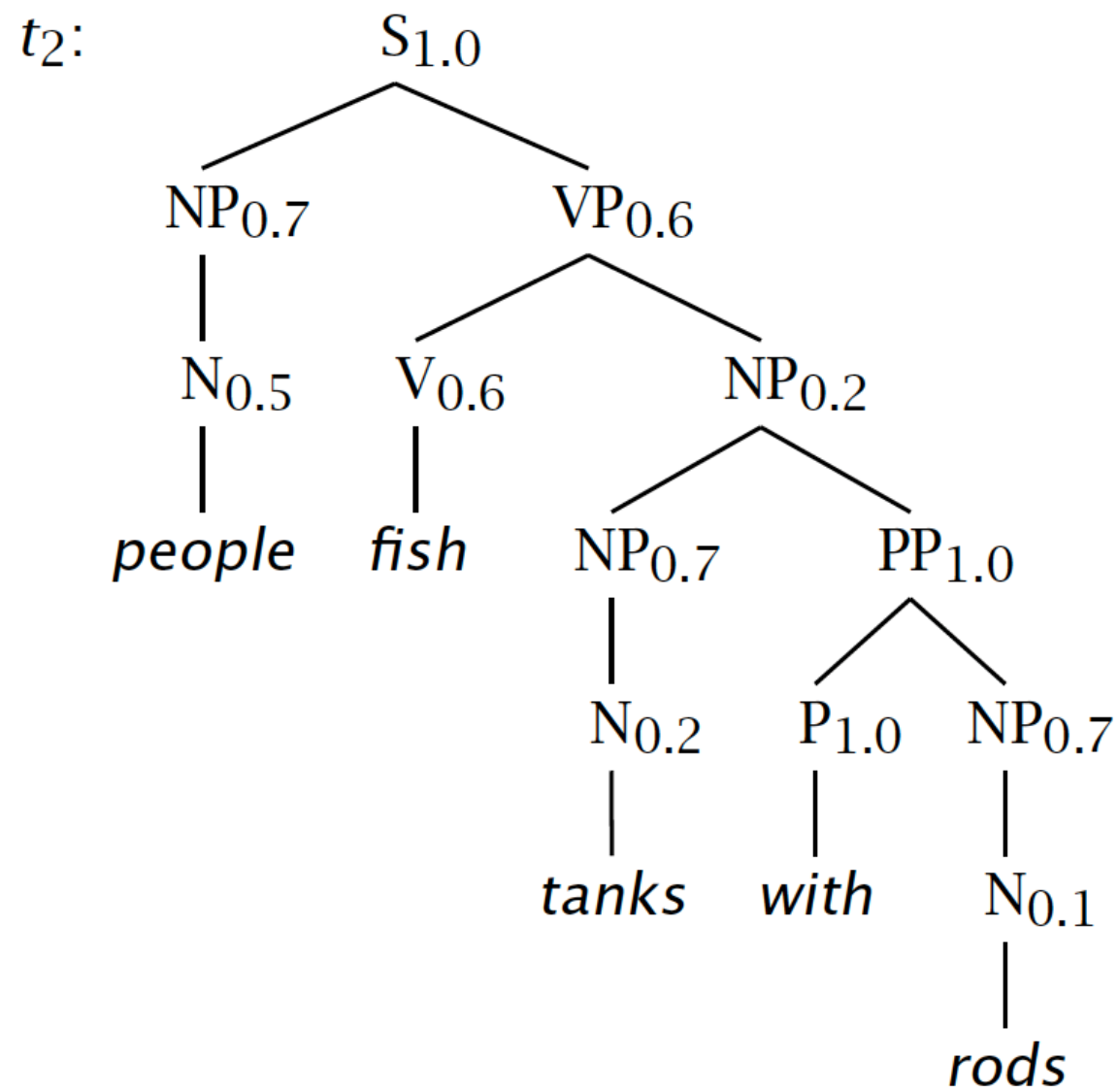
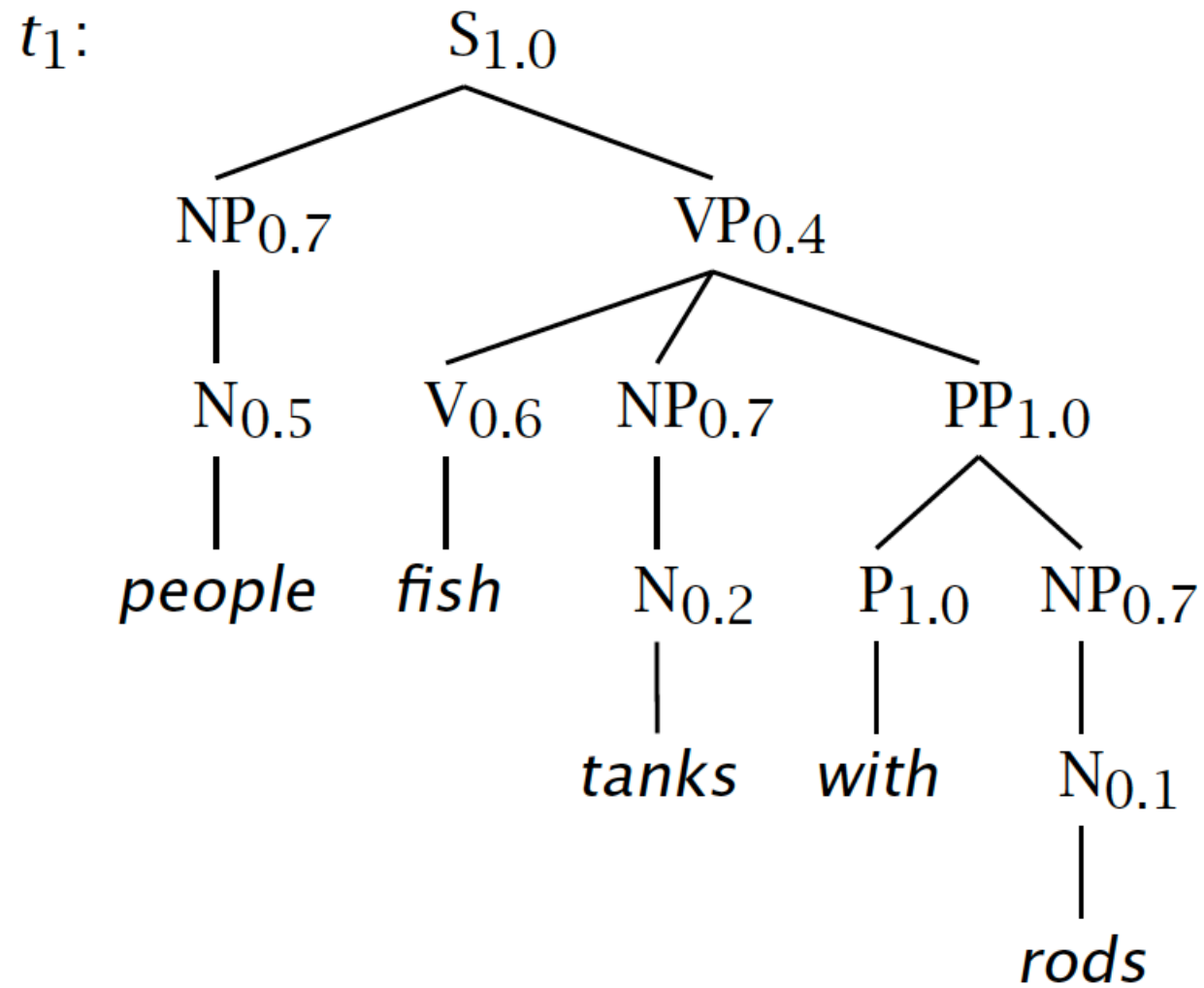
- $P(t_1) = 1.0 \times 0.7 \times 0.4 \times 0.5 \times 0.6 \times 0.7$   
 $\times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1$   
 $= 0.0008232$

Verb attach

- $P(t_2) = 1.0 \times 0.7 \times 0.6 \times 0.5 \times 0.6 \times 0.2$   
 $\times 0.7 \times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1$   
 $= 0.00024696$

Noun attach

- $P(s) = P(t_1) + P(t_2)$   
 $= 0.0008232 + 0.00024696$   
 $= 0.00107016$



# Grammar Transforms

Restricting the grammar form for efficient parsing

# Chomsky Normal Form

normalizing the grammar may change the structure of the parse tree generated by the grammar

note that  $w$  has to be a terminal

- All rules are of the form  $X \rightarrow YZ$  or  $X \rightarrow w$ 
  - $X, Y, Z \in N$  and  $w \in T$
- A transformation to this form doesn't change the weak generative capacity of a CFG
  - That is, it recognizes the same language
    - But maybe with different trees
- Empties and unaries are removed recursively
- $n$ -ary rules are divided by introducing new nonterminals ( $n > 2$ )

# A phrase structure grammar

if NP can go to  $\epsilon$  then any where if NP appears on the right replace it with epsilon

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow V NP PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow N$

$NP \rightarrow \epsilon$

$PP \rightarrow P NP$

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

Start discussing epsilon removal

# Chomsky Normal Form steps

$S \rightarrow NP VP$

$S \rightarrow VP$

$VP \rightarrow V NP$

$VP \rightarrow V$

$VP \rightarrow V NP PP$

$VP \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP$

$NP \rightarrow NP PP$

$NP \rightarrow PP$

$NP \rightarrow N$

$PP \rightarrow P NP$

$PP \rightarrow P$

to remove this type of  $s \rightarrow vp$   
replace by  $s \rightarrow (something generated by BP)$

$N \rightarrow people$

$N \rightarrow fish$

$N \rightarrow tanks$

$N \rightarrow rods$

$V \rightarrow people$

$V \rightarrow fish$

$V \rightarrow tanks$

$P \rightarrow with$

Start discussing unary removal downwards: remove  $S \rightarrow VP$



# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V$

$S \rightarrow V$

$VP \rightarrow V NP PP$

$S \rightarrow V NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP$

$NP \rightarrow NP PP$

$NP \rightarrow PP$

$NP \rightarrow N$

$PP \rightarrow P NP$

$PP \rightarrow P$

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

Remove more unaries, next  $S \rightarrow V$

# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V$

$VP \rightarrow V NP PP$

$S \rightarrow V NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP$

$NP \rightarrow NP PP$

$NP \rightarrow PP$

$NP \rightarrow N$

$PP \rightarrow P NP$

$PP \rightarrow P$

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$S \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$S \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$S \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

After remove  $S \rightarrow V$  get this, and then do  $VP \rightarrow V$

# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V NP PP$

$S \rightarrow V NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP$

$NP \rightarrow NP PP$

$NP \rightarrow PP$

**$NP \rightarrow N$**

$PP \rightarrow P NP$

$PP \rightarrow P$

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$S \rightarrow \textit{people}$

$VP \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$S \rightarrow \textit{fish}$

$VP \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$S \rightarrow \textit{tanks}$

$VP \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

# Chomsky Normal Form steps

introduce new non terminals to capture rules which produce more than two non terminals

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V NP PP$

$S \rightarrow V NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow P NP$

$PP \rightarrow P NP$

$NP \rightarrow people$

$NP \rightarrow fish$

$NP \rightarrow tanks$

$NP \rightarrow rods$

$V \rightarrow people$

$S \rightarrow people$

$VP \rightarrow people$

$V \rightarrow fish$

$S \rightarrow fish$

$VP \rightarrow fish$

$V \rightarrow tanks$

$S \rightarrow tanks$

$VP \rightarrow tanks$

$P \rightarrow with$

$PP \rightarrow with$

And then binarize

# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V @VP\_V$

$@VP\_V \rightarrow NP PP$

$S \rightarrow V @S\_V$

$@S\_V \rightarrow NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow P NP$

$PP \rightarrow P NP$

$NP \rightarrow \textit{people}$

$NP \rightarrow \textit{fish}$

$NP \rightarrow \textit{tanks}$

$NP \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$S \rightarrow \textit{people}$

$VP \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$S \rightarrow \textit{fish}$

$VP \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$S \rightarrow \textit{tanks}$

$VP \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

$PP \rightarrow \textit{with}$

# A phrase structure grammar

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow V NP PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow N$

$NP \rightarrow e$

$PP \rightarrow P NP$

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V @VP\_V$

$@VP\_V \rightarrow NP PP$

$S \rightarrow V @S\_V$

$@S\_V \rightarrow NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow P NP$

$PP \rightarrow P NP$

$NP \rightarrow \textit{people}$

$NP \rightarrow \textit{fish}$

$NP \rightarrow \textit{tanks}$

$NP \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$S \rightarrow \textit{people}$

$VP \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$S \rightarrow \textit{fish}$

$VP \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$S \rightarrow \textit{tanks}$

$VP \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

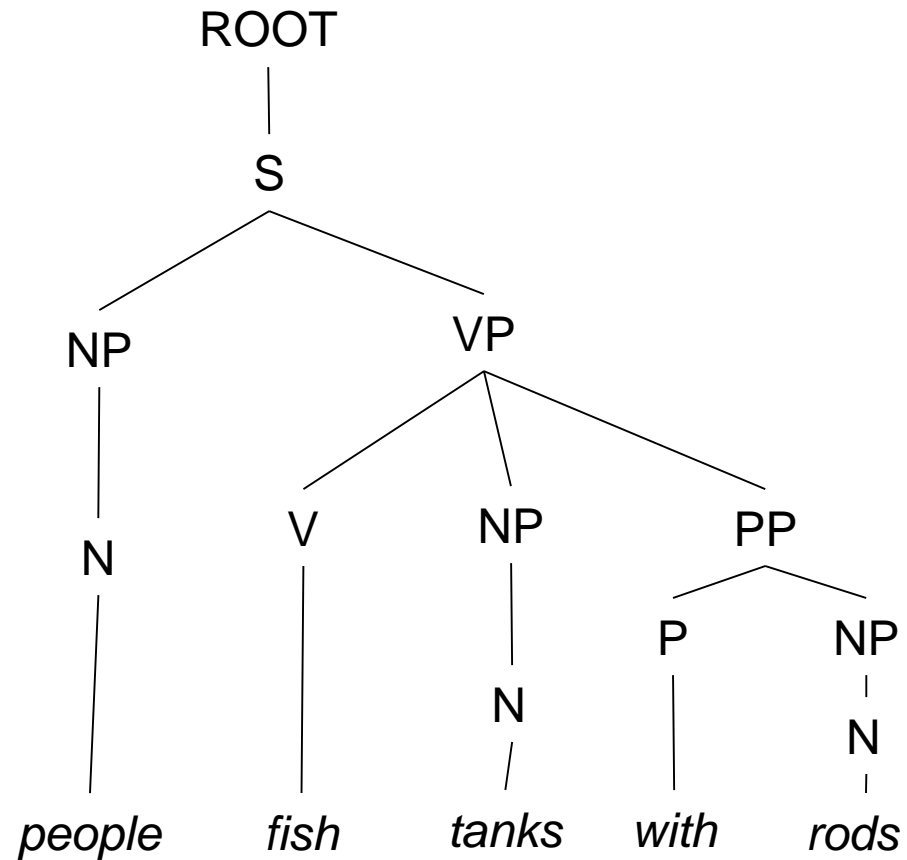
$PP \rightarrow \textit{with}$

# Chomsky Normal Form

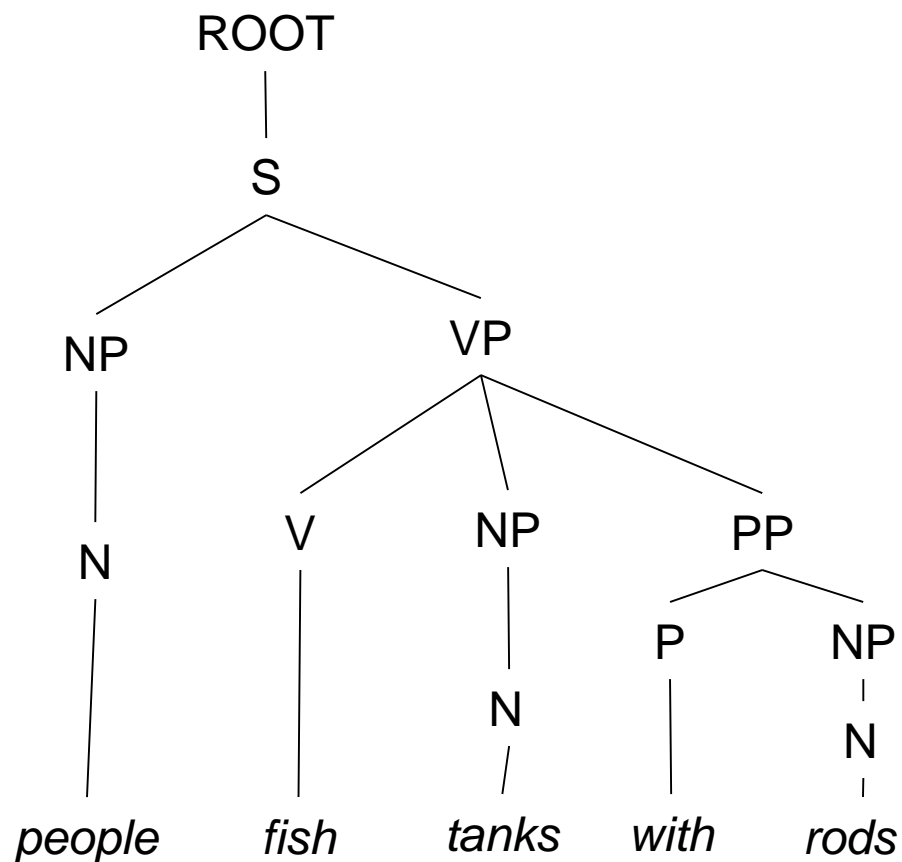
- You should think of this as a transformation for efficient parsing
- With some extra book-keeping in symbol names, you can even reconstruct the same trees with a detransform
- In practice full Chomsky Normal Form is a pain
  - Reconstructing n-aries is easy
  - Reconstructing unaries/empties is trickier
- **Binarization is crucial for cubic time CFG parsing**
- The rest isn't necessary; it just makes the algorithms cleaner and a bit quicker



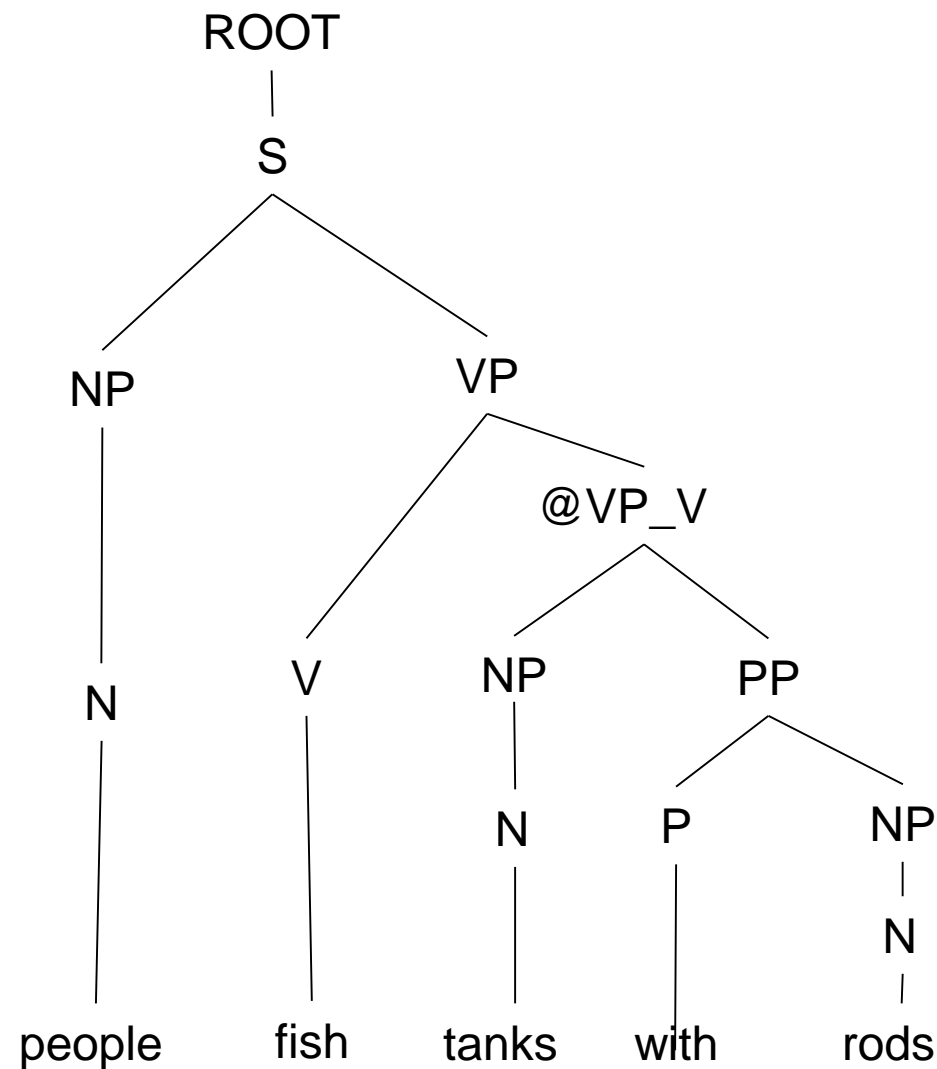
## An example: before binarization...



## An example: before binarization...



## After binarization...



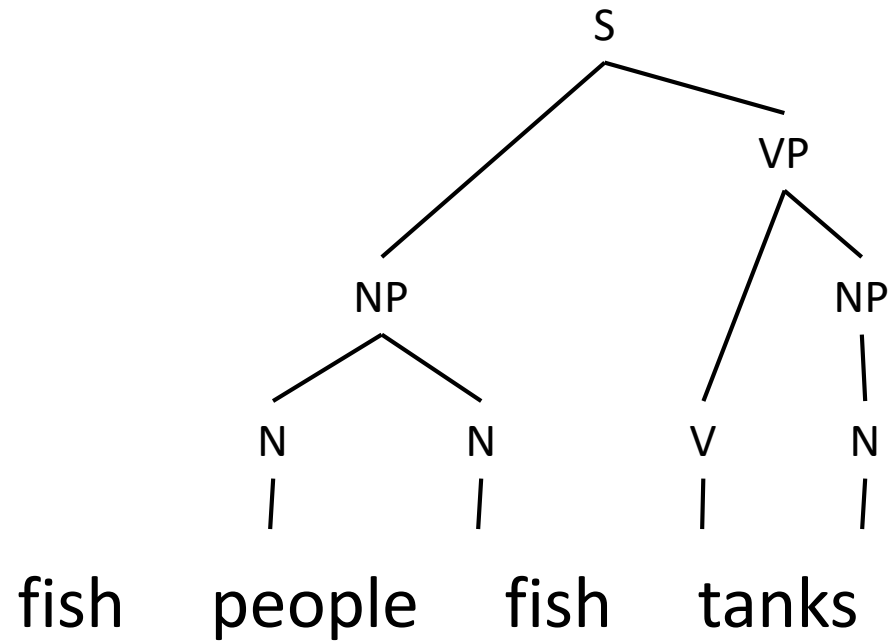
# CKY Parsing

time complexity is cubic for CKY parsing

Exact polynomial time parsing of (P)CFGs

# Constituency Parsing

find the constituents and how they combine to get the parse tree



## PCFG

### Rule Prob $\theta_i$

$S \rightarrow NP VP$   $\theta_0$

$NP \rightarrow NP NP$   $\theta_1$

...

$N \rightarrow \text{fish}$   $\theta_{42}$

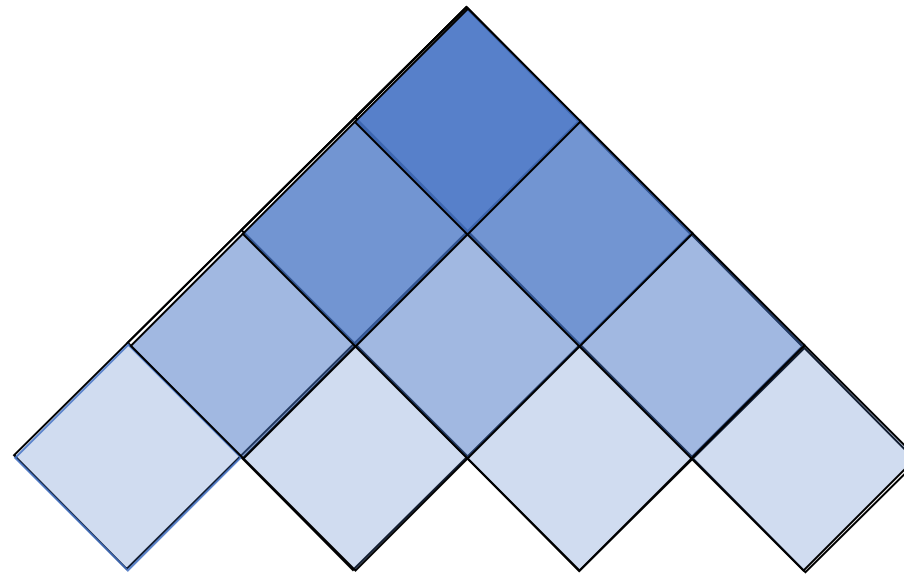
$N \rightarrow \text{people}$   $\theta_{43}$

$V \rightarrow \text{fish}$   $\theta_{44}$

...

# Cocke-Kasami-Younger (CKY) Constituency Parsing

Text

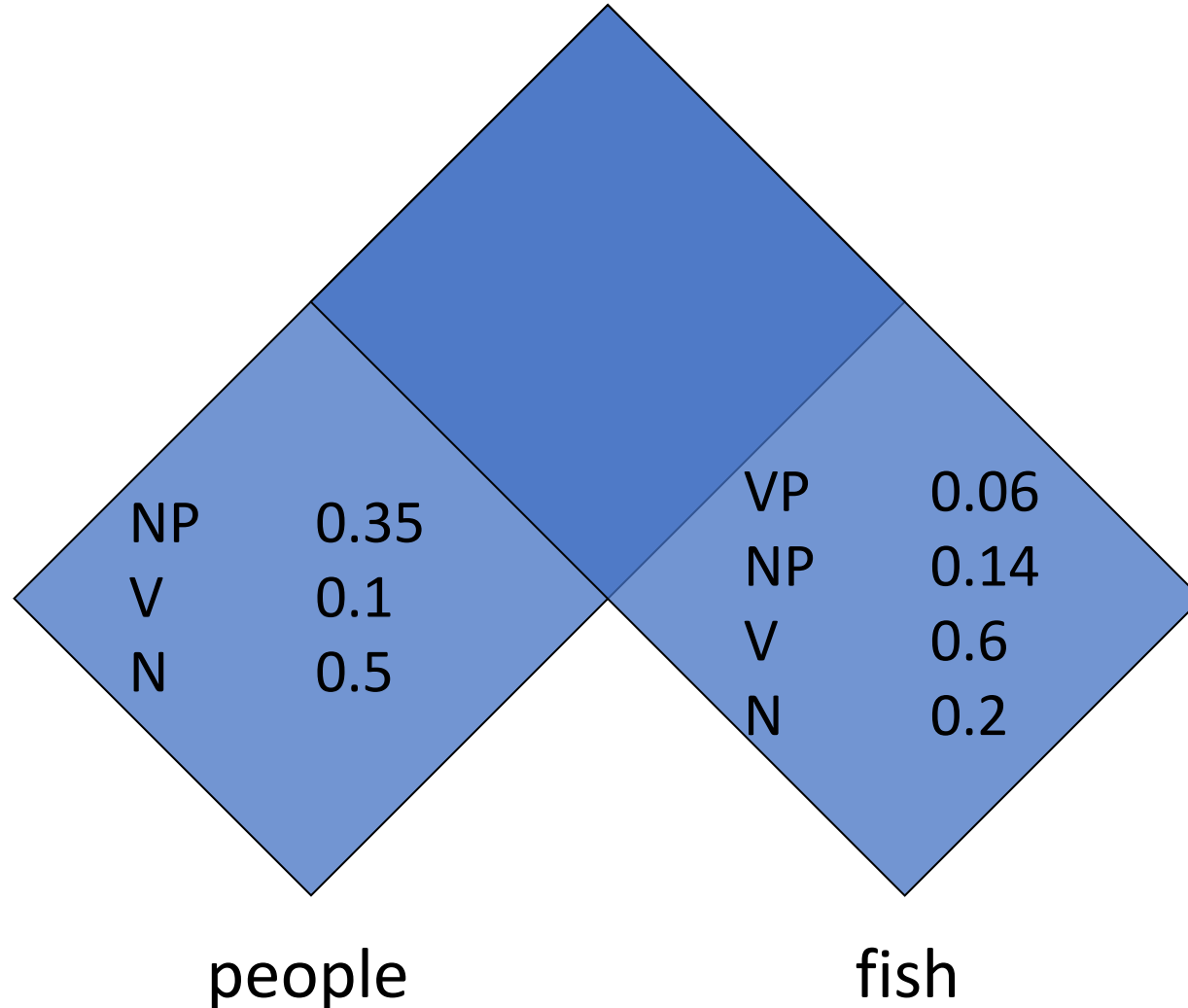


fish people fish tanks

Parse triangle/ chart

each cell stores  
the probabilities for producing the  
corresponding part of the sentence starting  
from a string

# Viterbi (Max) Scores



NP -> NP NP =  $0.35 * 0.14 * 0.1 = 0.0049$

VP -> V NP =  $0.1 * 0.14 * 0.5 = 0.007$

S -> VP =  $0.007 * 0.1 = 0.0007$

S -> NP VP =  $0.35 * 0.06 * 0.9 = 0.0189$

$S \rightarrow NP VP$  0.9

$S \rightarrow VP$  0.1

$VP \rightarrow V NP$  0.5

$VP \rightarrow V$  0.1

$VP \rightarrow V @VP\_V$  0.3

$VP \rightarrow V PP$  0.1

$@VP\_V \rightarrow NP PP$  1.0

$NP \rightarrow NP NP$  0.1

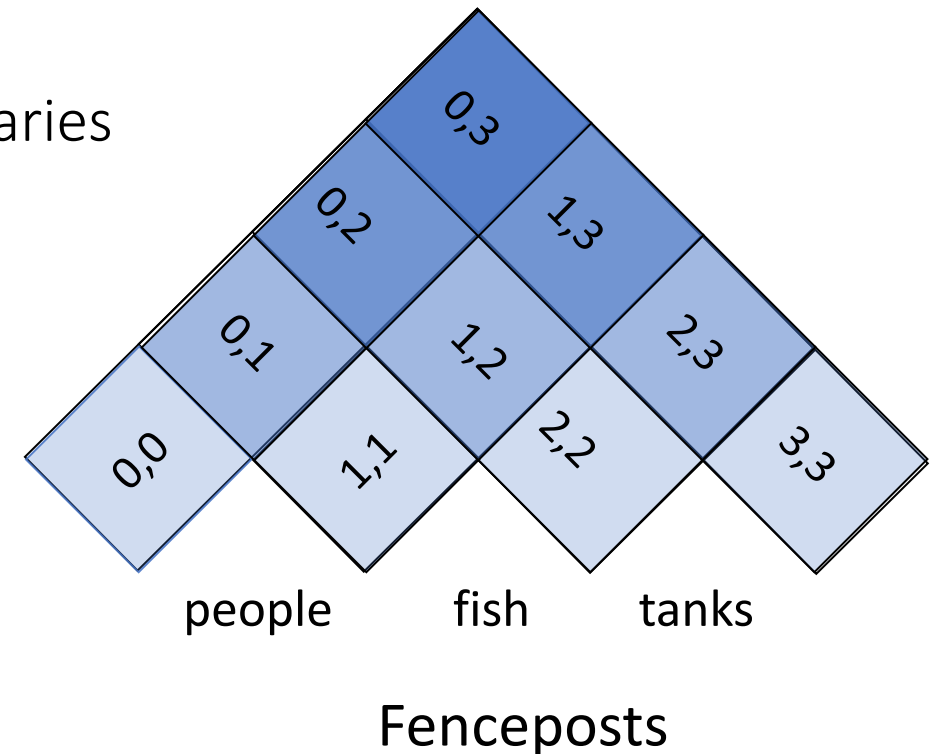
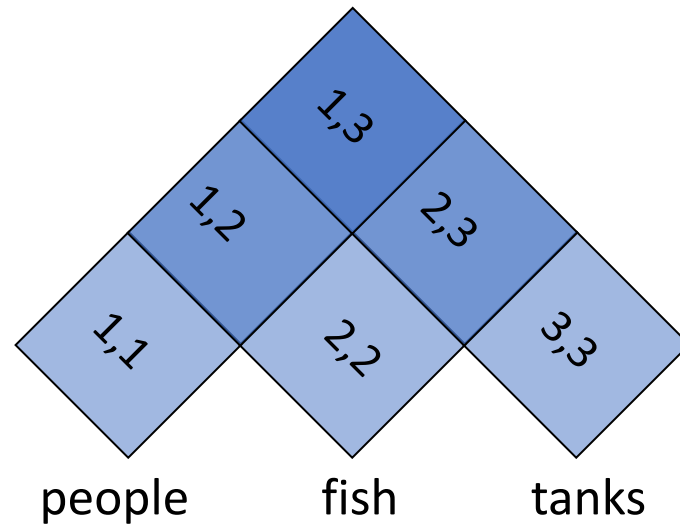
$NP \rightarrow NP PP$  0.2

$NP \rightarrow N$  0.7

$PP \rightarrow P NP$  1.0

# Extended CKY parsing

- Unaries can be incorporated into the algorithm
  - Messy, but doesn't increase algorithmic complexity
- Empties can be incorporated
  - Use fenceposts
  - Doesn't increase complexity; essentially like unaries



# Extended CKY parsing

- Unaries can be incorporated into the algorithm
  - Messy, but doesn't increase algorithmic complexity
- Empties can be incorporated
  - Use fenceposts
  - Doesn't increase complexity; essentially like unaries
- Binarization is *vital*
  - Without binarization, you don't get parsing cubic in the length of the sentence and in the number of nonterminals in the grammar
    - Binarization may be an explicit transformation or implicit in how the parser works (Earley-style dotted rules), but it's always there.



# CKY Parsing

Cocke–Younger–Kasami algorithm

A worked example

# The grammar:

## Binary, no epsilons,

binarization is essential for this algorithm removing unaries is not necessary => binarization ensures cubic time parsing

$S \rightarrow NP VP$  0.9

$S \rightarrow VP$  0.1

$VP \rightarrow V NP$  0.5

$VP \rightarrow V$  0.1

$VP \rightarrow V @VP\_V$  0.3

$VP \rightarrow V PP$  0.1

$@VP\_V \rightarrow NP PP$  1.0

$NP \rightarrow NP NP$  0.1

$NP \rightarrow NP PP$  0.2

$NP \rightarrow N$  0.7

$PP \rightarrow P NP$  1.0

$N \rightarrow people$  0.5

$N \rightarrow fish$  0.2

$N \rightarrow tanks$  0.2

$N \rightarrow rods$  0.1

$V \rightarrow people$  0.1

$V \rightarrow fish$  0.6

$V \rightarrow tanks$  0.3

$P \rightarrow with$  1.0

	fish	1	people	2	fish	3	tanks	4
0	score[0][1]	score[0][2]	score[0][3]	score[0][4]				
1		score[1][2]	score[1][3]	score[1][4]				
2			score[2][3]	score[2][4]				
3				score[3][4]				
4								

$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0
$N \rightarrow \textit{people}$	0.5
$N \rightarrow \textit{fish}$	0.2
$N \rightarrow \textit{tanks}$	0.2
$N \rightarrow \textit{rods}$	0.1
$V \rightarrow \textit{people}$	0.1
$V \rightarrow \textit{fish}$	0.6
$V \rightarrow \textit{tanks}$	0.3
$P \rightarrow \textit{with}$	1.0

	fish	1	people	2	fish	3	tanks	4
0								
1								
2								
3								
4								

for i=0; i<#(words); i++  
 for A in nonterms  
 if A -> words[i] in grammar  
 score[i][i+1][A] = P(A -> words[i]);

$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0
$N \rightarrow \textit{people}$	0.5
$N \rightarrow \textit{fish}$	0.2
$N \rightarrow \textit{tanks}$	0.2
$N \rightarrow \textit{rods}$	0.1
$V \rightarrow \textit{people}$	0.1
$V \rightarrow \textit{fish}$	0.6
$V \rightarrow \textit{tanks}$	0.3
$P \rightarrow \textit{with}$	1.0

	fish	1	people	2	fish	3	tanks	4
0	$N \rightarrow \textit{fish}$ 0.2 $V \rightarrow \textit{fish}$ 0.6							
1			$N \rightarrow \textit{people}$ 0.5 $V \rightarrow \textit{people}$ 0.1					
2				$N \rightarrow \textit{fish}$ 0.2 $V \rightarrow \textit{fish}$ 0.6			$N \rightarrow \textit{tanks}$ 0.2 $V \rightarrow \textit{tanks}$ 0.1	
3								
4								

```
// handle unaries
boolean added = true
while added
    added = false
    for A, B in nonterms
        if score[i][i+1][B] > 0 && A->B in grammar
            prob = P(A->B)*score[i][i+1][B]
            if(prob > score[i][i+1][A])
                score[i][i+1][A] = prob
                back[i][i+1][A] = B
            added = true
```

			fish	1	people	2	fish	3	tanks	4
$S \rightarrow NP VP$	0.9	0	$N \rightarrow fish\ 0.2$ $V \rightarrow fish\ 0.6$ $NP \rightarrow N\ 0.14$ $VP \rightarrow V\ 0.06$ $S \rightarrow VP\ 0.006$							
$S \rightarrow VP$	0.1									
$VP \rightarrow V NP$	0.5									
$VP \rightarrow V$	0.1									
$VP \rightarrow V @VP\_V$	0.3									
$VP \rightarrow V PP$	0.1	1	$N \rightarrow people\ 0.5$ $V \rightarrow people\ 0.1$ $NP \rightarrow N\ 0.35$ $VP \rightarrow V\ 0.01$ $S \rightarrow VP\ 0.001$							
$@VP\_V \rightarrow NP PP$	1.0									
$NP \rightarrow NP NP$	0.1									
$NP \rightarrow NP PP$	0.2									
$NP \rightarrow N$	0.7									
$PP \rightarrow P NP$	1.0	2	$N \rightarrow fish\ 0.2$ $V \rightarrow fish\ 0.6$ $NP \rightarrow N\ 0.14$ $VP \rightarrow V\ 0.06$ $S \rightarrow VP\ 0.006$							
$N \rightarrow people$	0.5									
$N \rightarrow fish$	0.2									
$N \rightarrow tanks$	0.2									
	0.2									
$N \rightarrow rods$	0.1	3	<div>prob=score[begin][split][B]*score[split][end][C]*P(A-&gt;BC) if (prob &gt; score[begin][end][A])   score[begin][end][A] = prob   back[begin][end][A] = new Triple(split,B,C)</div>							$N \rightarrow tanks\ 0.2$ $V \rightarrow tanks\ 0.1$ $NP \rightarrow N\ 0.14$ $VP \rightarrow V\ 0.03$ $S \rightarrow VP\ 0.003$
$V \rightarrow people$	0.1									
$V \rightarrow fish$	0.6									
$V \rightarrow tanks$	0.3									
$P \rightarrow with$	1.0									
		4								

this cell captures that how NP / VP / S can generate the sentence from 1 to 2 s -> token\_1 token\_2, another possibility is that  
s -> vp -> token\_1 token\_2  
out of all these possibilities from s we take the sequence with the maximum probability

			fish	1	people	2	fish	3	tanks	4
S → NP VP	0.9	0								
S → VP	0.1									
VP → V NP	0.5									
VP → V	0.1									
VP → V @VP_V	0.3									
VP → V PP	0.1									
@VP_V → NP PP	1.0									
NP → NP NP	0.1									
NP → NP PP	0.2									
NP → N	0.7									
PP → P NP	1.0									
N → people	0.5									
N → fish	0.2									
N → tanks	0.2									
N → rods	0.1									
V → people	0.1									
V → fish	0.6									
V → tanks	0.3									
P → with	1.0									
		1								
		2								
		3								
		4								

$S \rightarrow NP VP$  0.9  
 $S \rightarrow VP$  0.1  
 $VP \rightarrow V NP$  0.5  
 $VP \rightarrow V$  0.1  
 $VP \rightarrow V @VP\_V$  0.3  
 $VP \rightarrow V PP$  0.1  
 $@VP\_V \rightarrow NP PP$  1.0  
 $NP \rightarrow NP NP$  0.1  
 $NP \rightarrow NP PP$  0.2  
 $NP \rightarrow N$  0.7  
 $PP \rightarrow P NP$  1.0  
  
 $N \rightarrow people$  0.5  
 $N \rightarrow fish$  0.2  
 $N \rightarrow tanks$  0.2  
  
 $N \rightarrow rods$  0.1  
 $V \rightarrow people$  0.1  
 $V \rightarrow fish$  0.6  
 $V \rightarrow tanks$  0.3  
 $P \rightarrow with$  1.0

	fish	1	people	2	fish	3	tanks	4
0								
1	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105					
2			N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189				
3				N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042		
4							N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	

```

for split = begin+1 to end-1
  for A,B,C in nonterms
    prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = new Triple(split,B,C)
  
```



to consider all the splits which can get us to the particular split again take the viterbi scores of these possibilities for a given non terminal we want the best path to the end sentence if multiple paths exist

			fish	1	people	2	fish	3	tanks	4																	
S → NP VP	0.9	0	<table><tr><td>N → fish 0.2</td><td>NP → NP NP 0.0049</td><td>NP → NP NP 0.0000686</td><td></td></tr><tr><td>V → fish 0.6</td><td>VP → V NP 0.105</td><td>VP → V NP 0.00147</td><td></td></tr><tr><td>NP → N 0.14</td><td>S → VP 0.0105</td><td>S → NP VP 0.000882</td><td></td></tr><tr><td>VP → V 0.06</td><td></td><td></td><td></td></tr><tr><td>S → VP 0.006</td><td></td><td></td><td></td></tr></table>	N → fish 0.2	NP → NP NP 0.0049	NP → NP NP 0.0000686		V → fish 0.6	VP → V NP 0.105	VP → V NP 0.00147		NP → N 0.14	S → VP 0.0105	S → NP VP 0.000882		VP → V 0.06				S → VP 0.006							
N → fish 0.2	NP → NP NP 0.0049	NP → NP NP 0.0000686																									
V → fish 0.6	VP → V NP 0.105	VP → V NP 0.00147																									
NP → N 0.14	S → VP 0.0105	S → NP VP 0.000882																									
VP → V 0.06																											
S → VP 0.006																											
S → VP	0.1	1		<table><tr><td>N → people 0.5</td><td>NP → NP NP 0.0049</td><td></td></tr><tr><td>V → people 0.1</td><td>VP → V NP 0.007</td><td></td></tr><tr><td>NP → N 0.35</td><td>S → NP VP 0.0189</td><td></td></tr><tr><td>VP → V 0.01</td><td></td><td></td></tr><tr><td>S → VP 0.001</td><td></td><td></td></tr></table>	N → people 0.5	NP → NP NP 0.0049		V → people 0.1	VP → V NP 0.007		NP → N 0.35	S → NP VP 0.0189		VP → V 0.01			S → VP 0.001										
N → people 0.5	NP → NP NP 0.0049																										
V → people 0.1	VP → V NP 0.007																										
NP → N 0.35	S → NP VP 0.0189																										
VP → V 0.01																											
S → VP 0.001																											
VP → V NP	0.5	2			<table><tr><td>N → fish 0.2</td><td>NP → NP NP 0.00196</td></tr><tr><td>V → fish 0.6</td><td>VP → V NP 0.042</td></tr><tr><td>NP → N 0.14</td><td>S → VP 0.0042</td></tr><tr><td>VP → V 0.06</td><td></td></tr><tr><td>S → VP 0.006</td><td></td></tr></table>	N → fish 0.2	NP → NP NP 0.00196	V → fish 0.6	VP → V NP 0.042	NP → N 0.14	S → VP 0.0042	VP → V 0.06		S → VP 0.006													
N → fish 0.2	NP → NP NP 0.00196																										
V → fish 0.6	VP → V NP 0.042																										
NP → N 0.14	S → VP 0.0042																										
VP → V 0.06																											
S → VP 0.006																											
VP → V	0.1	3					<table><tr><td>N → tanks 0.2</td></tr><tr><td>V → tanks 0.1</td></tr><tr><td>NP → N 0.14</td></tr><tr><td>VP → V 0.03</td></tr><tr><td>S → VP 0.003</td></tr></table>	N → tanks 0.2	V → tanks 0.1	NP → N 0.14	VP → V 0.03	S → VP 0.003															
N → tanks 0.2																											
V → tanks 0.1																											
NP → N 0.14																											
VP → V 0.03																											
S → VP 0.003																											
VP → V @VP_V	0.3	4																									
VP → V PP	0.1																										
@VP_V → NP PP	1.0																										
NP → NP NP	0.1																										
NP → NP PP	0.2																										
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N → people	0.5																										
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N → tanks	0.2																										
N → rods	0.1																										
V → people	0.1																										
V → fish	0.6																										
V → tanks	0.3																										
P → with	1.0																										

$S \rightarrow NP VP$  0.9  
 $S \rightarrow VP$  0.1  
 $VP \rightarrow V NP$  0.5  
 $VP \rightarrow V$  0.1  
 $VP \rightarrow V @VP\_V$  0.3  
 $VP \rightarrow V PP$  0.1  
 $@VP\_V \rightarrow NP PP$  1.0  
 $NP \rightarrow NP NP$  0.1  
 $NP \rightarrow NP PP$  0.2  
 $NP \rightarrow N$  0.7  
 $PP \rightarrow P NP$  1.0  
  
 $N \rightarrow people$  0.5  
 $N \rightarrow fish$  0.2  
 $N \rightarrow tanks$  0.2  
  
 $N \rightarrow rods$  0.1  
 $V \rightarrow people$  0.1  
 $V \rightarrow fish$  0.6  
 $V \rightarrow tanks$  0.3  
 $P \rightarrow with$  1.0

	0	1	2	3	4
	fish	people	fish	tanks	
0	$N \rightarrow fish$ 0.2 $V \rightarrow fish$ 0.6 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.06 $S \rightarrow VP$ 0.006	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.105 $S \rightarrow VP$ 0.0105	$NP \rightarrow NP NP$ 0.0000686 $VP \rightarrow V NP$ 0.00147 $S \rightarrow NP VP$ 0.000882		
1		$N \rightarrow people$ 0.5 $V \rightarrow people$ 0.1 $NP \rightarrow N$ 0.35 $VP \rightarrow V$ 0.01 $S \rightarrow VP$ 0.001	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.007 $S \rightarrow NP VP$ 0.0189	$NP \rightarrow NP NP$ 0.0000686 $VP \rightarrow V NP$ 0.000098 $S \rightarrow NP VP$ 0.01323	
2			$N \rightarrow fish$ 0.2 $V \rightarrow fish$ 0.6 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.06 $S \rightarrow VP$ 0.006	$NP \rightarrow NP NP$ 0.00196 $VP \rightarrow V NP$ 0.042 $S \rightarrow VP$ 0.0042	
3				$N \rightarrow tanks$ 0.2 $V \rightarrow tanks$ 0.1 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.03 $S \rightarrow VP$ 0.003	
4					

```

for split = begin+1 to end-1
  for A,B,C in nonterms
    prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = new Triple(split,B,C)
  
```

we need to consider the probability of the start symbol in the corner most cell, which gives us the most probable way to generate the sentence

$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0
$N \rightarrow people$	0.5
$N \rightarrow fish$	0.2
$N \rightarrow tanks$	0.2
$N \rightarrow rods$	0.1
$V \rightarrow people$	0.1
$V \rightarrow fish$	0.6
$V \rightarrow tanks$	0.3
$P \rightarrow with$	1.0

	fish	1	people	2	fish	3	tanks	4
0	<div>N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006</div>		<div>NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105</div>		<div>NP → NP NP 0.0000686 VP → V NP 0.00147 S → NP VP 0.000882</div>		<div>NP → NP NP 0.0000009604 VP → V NP 0.00002058 S → NP VP 0.00018522</div>	
1			<div>N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 <span>Text</span> S → VP 0.001</div>		<div>NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189</div>		<div>NP → NP NP 0.0000686 VP → V NP 0.000098 S → NP VP 0.01323</div>	
2					<div>N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006</div>		<div>NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042</div>	
3							<div>N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003</div>	
4								

Call buildTree(score, back) to get the best parse

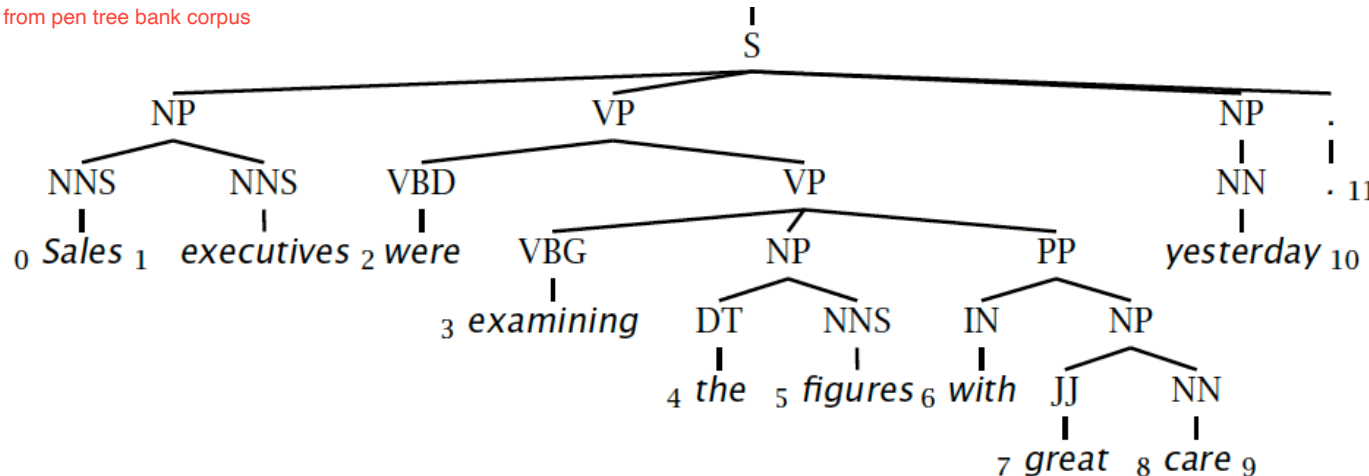
# Constituency Parser Evaluation

we only write the bracketed notation before the preterminals

# Evaluating constituency parsing

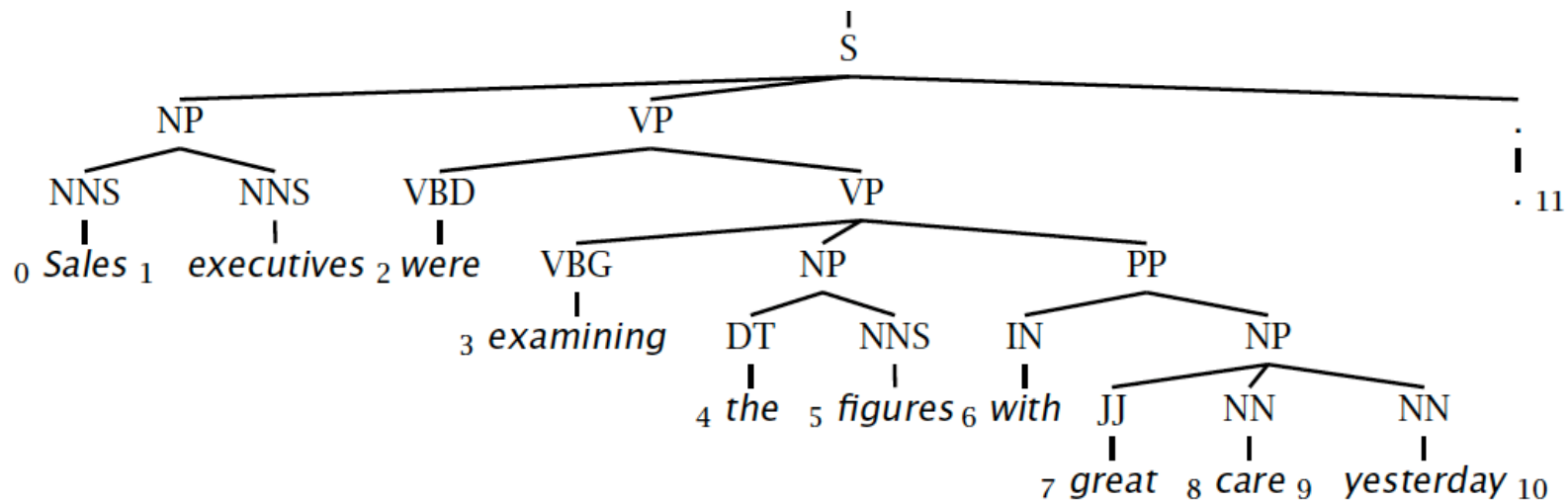
Gold standard brackets: S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), NP-(4:6), PP-(6:9), NP-(7,9), NP-(9:10)

ground truth from pen tree bank corpus



in order traversal of the tree

Candidate brackets: S-(0:11), NP-(0:2), VP-(2:10), VP-(3:10), NP-(4:6), PP-(6:10), NP-(7,10)



# Evaluating constituency parsing

## Gold standard brackets:

S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), NP-(4:6), PP-(6-9), NP-(7,9), NP-(9:10)

## Candidate brackets:

S-(0:11), NP-(0:2), VP-(2:10), VP-(3:10), NP-(4:6), PP-(6-10), NP-(7,10)

Labeled Precision                   $3/7 = 42.9\%$

Labeled Recall                     $3/8 = 37.5\%$

LP/LR F1                            40.0%

Tagging Accuracy                 $11/11 = 100.0\%$

fraction of preterminal symbols given correctly to  
the end words