2301 COL 202 Tutorial 11.3

Abhinav Rajesh Shripad

TOTAL POINTS

2 / 2

QUESTION 1

1 Problem for Group 3 **2 / 2**

 ✓ **- 0 pts** *Correct*

COL 202 Assignment 11

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1 Problem Statement

We play a game with a deck of 52 regular playing cards, of which 26 are red and 26 are black. I randomly shuffle the cards and place the deck face down on a table. You have the option of "taking" or "skipping" the top card. If you skip the top card, then that card is revealed and we continue playing with the remaining deck. If you take the top card, then the game ends; you win if the card you took was revealed to be black, and you lose if it was red. If we get to a point where there is only one card left in the deck, you must take it. Prove that you have no better strategy than to take the top card—which means your probability of winning is $1/2$.

2 Solution

Let a shuffled deck of cards have b black and r red cards. If we decide to pick the first card, we have $\frac{b}{b+r}$ probability of winning. We now show that this is the best strategy by inducting upon $b+r$.

Base Case=b+r=1
This is obvious case as when only one card exists, we cant do anything. Thus winning probability is precisely $\frac{b}{b+r}$.

Inductive Hypothesis:-
Let for all b and r such that $b+r < n$ the probability of winning is $\frac{b}{b+r}$.
Consider a shuffled deck with b blacks and r reds. If we do NOT pick the top card we have following ways of winning.
First card is black and then we win or First card is red and then we win. The total probability of winning will be

$$(\frac{b}{b+r})(\frac{b-1}{b+r-1}) + (\frac{r}{b+r})(\frac{b}{b+r-1}) = \frac{b}{b+r}$$

, where the first part of each term is the probability of getting a particular colour card and the second part is probability of winning from that position (which we know based on inductive hypothesis).
Thus we can conclude that letting go of the first card doesn't change our probability of winning thus we should choose it as it has probability of winning $\frac{26}{26+26} = 1/2$.

1 Problem for Group 3

✓ **- 0 pts** *Correct*

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