2301 COL 202 Quiz 2

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TOTAL POINTS

8.5 / 12

QUESTION 1

1 Complement **4 / 4**

✓ **- 0 pts** *Correct*

- 4 pts Incorrect or unattempted

QUESTION 2

2 Planar Bipartite **4 / 4**

✓ **- 0 pts** *Completely correct*

- 4 pts Incorrect

- 3 pts Very little progress towards the correct solution

- 2 pts Partially Correct

- 1 pts Mostly correct

QUESTION 3

3 Chord **0.5 / 4**

+ 0 pts Incorrect

✓ **+ 0.5 pts** *Proved graph is cyclic*

+ 1 pts Cyclic and base and induction hypothesis

+ 2 pts Partially Correct

+ 4 pts Completely Correct

1 The graph is not necessarily planar

2 0.5 marks given for this equation

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(COL 202) Discrete Mathematics 3 November, 2023

Quiz 2

Duration: 45 minutes **(12 marks)**

- Be clear in your writing. If you use a statement proved in class or in the problem set, then write down the entire statement before using it.
- **You will not get a new sheet, so make sure you are certain when you write something.** Make a judicious decision of which tool(s) to use to get a clean and short answer that fits in the space.

1. **(4 points)** The *complement* of a graph G is a graph $\overline{G}$ on the same vertices such that two distinct vertices of $\overline{G}$ are adjacent if and only if they are not adjacent in G. Prove that at least one among G and $\overline{G}$ is *connected.*

If G is connected → we are done

If G is not connected, let the connected components of G be $G_1, G_2 \ldots G_n$, such that $\bigcup_{i=1}^{n} G_i = G$ and $G_i \cap G_j = \phi$ for $i \neq j$ and

~~$|V(G_i)|$~~ $|V(G_i)| \geq 1 \ \forall i \in 1, 2, \ldots n$.

observe that $n \geq 2$, otherwise G is connected

let $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$.

$\forall v \in V(G_1)$, $(v_1, v_2) \in E(\overline{G})$ as G_1 and G_2 are disconnected in G. → $(v_1, v_2) \in E(\overline{G})$ and $(v, v_2) \in E(\overline{G})$

→ (v_1, v) are connected to each other.

→ each vertex of G_1 is connected with eachother in $\overline{G}$

$\|^{ly}$ of each G_i for $i = 1, 2 \ldots n$. let $v_i \in V(G_i)$

→ $(v_i v_j) \in E(\overline{G})$ for $i \neq j$ as $v_i v_j$ no edge exists in G

→ Every G_i has a vertex connected to other G_j and each G_i is connected to its vertices in $\overline{G}$. → $\overline{G}$ is connected. Hence Proved

True only for $n \geq 3$

2. (4 points) Prove that a planar bipartite graph on n nodes has at most $2n - 4$ edges.

Claim :- The number of faces of the graph f is bounded by $n-2$, ie $f \leq n-2$.

soln) Since the graph is bipartite, we can see that each vertex on a cycle is part of cycle of even length, ~~fase~~ induct on cycle we

Base case - $n = 2$, Trivial $f = 1$ (infinite face)

consider the case when graph has maximal possible faces for fixed n, on adding a vertex v, it increase f by one and n by 1

thus $f \leq n-2$

Thus using euler relation we get

$v - e + f = 2 \rightarrow f = 2+e-n \leq n-2$

$\rightarrow \boxed{e \leq 2n-4}$

Hence Proved

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3. **(4 points)** If C is a cycle, and e is an edge connecting two non-adjacent nodes of C, then we call e a *chord* of C. Prove that if every node of a graph G has degree at least 3, then G contains a cycle with a chord.

Let v, e, f denote the number of vertex, edge and face in this graph.

Claim 1:- $v \geq 4$. Proof) If $v \leq 3$, take a vertex v_1, it doesn't have 3 vertices to make a edge with. Contradiction to $\deg(v) \geq 3$. $\rightarrow \boxed{v \geq 4}$

Claim 2:- $2e \geq 3v$

, we know that $\deg(v_i) \geq 3 \rightarrow \sum_{v_i \in V(G)} \deg(v_i) \geq 3v$

but $\sum \deg(v_i) = 2e \rightarrow \boxed{2e \geq 3v}$

Euler's Formula gives, $f = 2+e-v \geq 2 + \frac{3v}{2} - v$

$$= 2 + \frac{v}{2} \geq 2 + \frac{4}{2} = 4$$

$\rightarrow \boxed{f \geq 4}$ WLOG, we assume there is only one connected component of the graph. If not we investigate each component alone. we

Let F_∞ be the face extending to infinity.

Out of the other 3 face, 2 must share a edge if not they belong to different connected component let the faces be F_1 and F_2 with edge e.

Cycle containing F_1 and F_2 union has e as a chord.

Hence Proved