Set A

Department of Mathematics

MTL 106 (Introduction to Probability Theory and Stochastic Processes)

Quiz 3 Examination

Time: 20 min **Date: 26/10/2021**
Max. Marks: 10

Note: The exam is closed-book, and all the questions are compulsory.

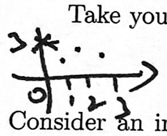
1. Consider the computer networks with three independent identical behaving switches. If a switch is working at the beginning of a day, it stays working at the beginning of the next day with probability p, and if it is not working at the beginning of a day, it stays not working at the beginning of the next day with probability q. Take $p = 0.4$ and $q = 0.3$.

   Let X_n be the number of working switches at the beginning of the nth day. Assume that, all three swiches are working initially.

   $= \{0, 1, 2, 3\}$ $= \{0, 1, 2, \dots\}$

   (a) Write the state space and parameter space for the stochastic process $\{X_n, n = 0, 1, \dots\}$.

   (b) Draw the sample path over time or realization for the the stochastic process $\{X_n, n = 0, 1, \dots\}$. Take your own possible number of working switches at any day.



(2 + 2 marks)

2. Consider an irreducible DTMC $\{X_n, n = 0, 1, \dots\}$ with state space $S = \{0, 1, 2, 3\}$ and one step transition probability matrix $\begin{pmatrix} 0.7 & 0.3 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0 \\ 0 & 0 & 0.7 & 0.3 \\ 0.3 & 0 & 0 & 0.7 \end{pmatrix}$. $f_{00} = 1$; $\mu_0 < \infty$

   (a) Classify the states as ~~transient~~, + recurrent or ~~null recurrent~~. all are

   (b) Suppose $X_0 = i$ with probability 1. The sojourn time T_i of the DTMC in state i is said to be k if $\{X_0 = X_1 = \cdots = X_{k-1} = i, X_k \neq i\}$. Take $i = 0$. Find the probability mass function of the random variable T_0. [redacted] $= \{(0.7)^{k-1}\, 0.3,\ k = 1, 2, 3 \dots$; 0 oth

   (c) Find the stationary distribution, $\pi = (\pi_0, \pi_1, \pi_2, \pi_3)$, if it exists.

   $= \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

(2 + 2 + 2 marks)

Set B

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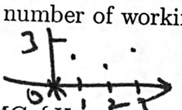
1. Consider the computer networks with three independent identical behaving switches. If a switch is working at the beginning of a day, it stays working at the beginning of the next day with probability p, and if it is not working at the beginning of a day, it stays not working at the beginning of the next day with probability q. Take $p = 0.4$ and $q = 0.7$.

   Let X_n be the number of working switches at the beginning of the nth day. Assume that, all three swiches are not working initially.

   $= \{0, 1, 2, 3\}$ $\quad = \{0, 1, 2, \ldots\}$

   (a) Write the state space and parameter space for the stochastic process $\{X_n, n = 0, 1, \ldots\}$.

   (b) Draw the sample path over time or realization for the the stochastic process $\{X_n, n = 0, 1, \ldots\}$. Take your own possible number of working switches at any day.



(2 + 2 marks)

2. Consider an irreducible DTMC $\{X_n, n = 0, 1, \ldots\}$ with state space $S = \{0, 1, 2, 3\}$ and one step transition probability matrix $\begin{pmatrix} 0.3 & 0.7 & 0 & 0 \\ 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 0.3 & 0.7 \\ 0.7 & 0 & 0 & 0.3 \end{pmatrix}$. $\quad f_{00} = 1;\ \mu_0 < \infty$

   all are

   (a) Classify the states as ~~transient~~, + recurrent or ~~null recurrent~~.

   (b) Suppose $X_0 = i$ with probability 1. The sojourn time T_i of the DTMC in state i is said to be k if $\{X_0 = X_1 = \cdots = X_{k-1} = i, X_k \neq i\}$. Take $i = 2$. Find the probability mass function of the random variable T_2.

   $= \begin{cases} (0.3)^{k-1}\, 0.7, & k = 1, 2, 3, \ldots \\ 0, & \text{oth} \end{cases}$

   (c) Find the stationary distribution, $\pi = (\pi_0, \pi_1, \pi_2, \pi_3)$, if it exists.

   $= \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

(2 + 2 + 2 marks)

Set C

Department of Mathematics

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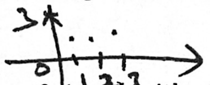
1. Consider the computer networks with three independent identical behaving switches. If a switch is working at the beginning of a day, it stays working at the beginning of the next day with probability p, and if it is not working at the beginning of a day, it stays not working at the beginning of the next day with probability q. Take $p = 0.3$ and $q = 0.4$.

   Let X_n be the number of not working switches at the beginning of the nth day. Assume that, all three swiches are not working initially.

   $=\{0,1,2,3\}$ $=\{0,1,2,\ldots\}$

   (a) Write the state space and parameter space for the stochastic process $\{X_n, n = 0, 1, \ldots\}$.

   (b) Draw the sample path over time or realization for the the stochastic process $\{X_n, n = 0, 1, \ldots\}$. Take your own possible number of working switches at any day.



(2 + 2 marks)

2. Consider an irreducible DTMC $\{X_n, n = 0, 1, \ldots\}$ with state space $S = \{0, 1, 2, 3\}$ and one step transition probability matrix $\begin{pmatrix} 0.4 & 0.6 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 \\ 0 & 0 & 0.4 & 0.6 \\ 0.6 & 0 & 0 & 0.4 \end{pmatrix}$. $f_{00}=1$; $M_0<\infty$

   all are

   (a) Classify the states as ~~transient~~, + recurrent or null ~~recurrent~~.

   (b) Suppose $X_0 = i$ with probability 1. The sojourn time T_i of the DTMC in state i is said to be k if $\{X_0 = X_1 = \cdots = X_{k-1} = i, X_k \neq i\}$. Take $i = 1$. Find the probability mass function of the random variable T_1.

   $= \begin{cases} (0.4)^{k-1}\, 0.6, & k=1,2,\ldots \\ 0, & \text{oth} \end{cases}$

   (c) Find the stationary distribution, $\pi = (\pi_0, \pi_1, \pi_2, \pi_3)$, if it exists.

   $=\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

(2 + 2 + 2 marks)

Set D

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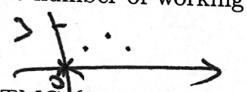
1. Consider the computer networks with three independent identical behaving switches. If a switch is working at the beginning of a day, it stays working at the beginning of the next day with probability p, and if it is not working at the beginning of a day, it stays not working at the beginning of the next day with probability q. Take $p = 0.7$ and $q = 0.3$.

   Let X_n be the number of not working switches at the beginning of the nth day. Assume that, all three swiches are working initially.

   $= \{0,1,2,3\}$ $\quad = \{0,1,2,\cdots\}$

   (a) Write the state space and parameter space for the stochastic process $\{X_n, n = 0, 1, \ldots\}$.

   (b) Draw the sample path over time or realization for the the stochastic process $\{X_n, n = 0, 1, \ldots\}$. Take your own possible number of working switches at any day.



(2 + 2 marks)

2. Consider an irreducible DTMC $\{X_n, n = 0, 1, \ldots\}$ with state space $S = \{0, 1, 2, 3\}$ and one step transition probability matrix $\begin{pmatrix} 0.6 & 0.4 & 0 & 0 \\ 0 & 0.6 & 0.4 & 0 \\ 0 & 0 & 0.6 & 0.4 \\ 0.4 & 0 & 0 & 0.6 \end{pmatrix}$. $\quad f_{00} = 1 \; ; \; M_0 < \infty$

   all are

   (a) Classify the states as ~~transient~~, + recurrent or ~~null recurrent~~.

   (b) Suppose $X_0 = i$ with probability 1. The sojourn time T_i of the DTMC in state i is said to be k if $\{X_0 = X_1 = \cdots = X_{k-1} = i, X_k \neq i\}$. Take $i = 3$. Find the probability mass function of the random variable T_3.

   $= \begin{cases} (0.6)^{k-1}\, 0.4, & k = 1,2,3,\ldots \\ 0, & \text{oth} \end{cases}$

   (c) Find the stationary distribution, $\pi = (\pi_0, \pi_1, \pi_2, \pi_3)$, if it exists.

   $= \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

(2 + 2 + 2 marks)