2301 COL 202 Tutorial 6.3

Abhinav Rajesh Shripad

TOTAL POINTS

2 / 2

QUESTION 1

1 Problem for Group 3 **2 / 2**

✓ **- 0 pts** *Correct*

- 2 pts Incorrect

1 Problem for Group 3 2 / 2

✓ **- 0 pts** *Correct*

- 2 pts Incorrect

COL 202 Assignment 6

Abhinav Shripad(2022CS11596)

September 2023

1 Problem Statement

Let $f_n = (1 + \frac{1}{\sqrt{n}})^n$ and $g_n = e^{\sqrt{n}}$. Prove: $f_n = \Theta(g_n)$ but $f_n \not\asymp g_n$. What is $\lim_{n\to\infty} \frac{f_n}{g_n}$?.

2 Solution

We first find the limit and then proceed towards proving the other part.

2.1 Limit

Let us denote l as the limit of fraction. Knowing that $ln()$ is a continuous function we can write

$$ln(l) = \lim_{n\to\infty} nln(1 + \frac{1}{\sqrt{n}}) - \sqrt{n}$$

Using the expansion of $ln(1 + x) = x - \frac{x^2}{2} + \Theta(x^3)$ we get that

$$ln(l) = \lim_{n\to\infty} \frac{-1}{2} + \mathcal{O}(\frac{1}{n^{\frac{3}{2}}})$$

thus $ln(l) = \frac{-1}{2}$. Hence we can write $f_n \approx e^{\frac{-1}{2}} g_n$ for large enough n. Since $l \neq 1$ we can say that $f_n \not\asymp g_n$.

2.2 Big-Theta

Using definition of limit we know that $\forall \epsilon \geq 0\ \exists n \geq N$ such that

$$l - \epsilon \leq \frac{f_n}{g_n} \leq l + \epsilon$$

where $l = e^{\frac{-1}{2}}$. We take $\epsilon = 0.5$,we can check that $l - \epsilon \geq 0$, for the corresponding N we get for $n \geq N$

$$(l - \epsilon)g_n \leq f_n \leq (l + \epsilon)g_n$$

Hence we can conclude that $f_n = \Theta(g_n)$