COL 202 Discrete Mathematics **Diwali 2023**

Tutorial 5

1. [Submission Problem for Group 1] A collection $\mathcal{C}$ of sets is called a chain when, given any two sets in $\mathcal{C}$, one is a subset of the other. Prove that if $\mathcal{F}$ is chain of finite sets, then $\cup\mathcal{F}$ is countable. (Notice that without the chain condition, every set is the union of its finite subsets.) Problem 8.10 in LLM Book)

2. [Submission Problem for Group 2] Let $\{0,1\}^\omega$ be the set of infinite binary sequences. Call a sequence in $\{0,1\}^\omega$ lonely if it never has two 1s in a row. For example, the repeating sequence $\{0,1,0,1,0,1,0,1,0,\dots\}$ is lonely, but the sequence $\{0,0,1,1,0,1,0,1,0,0,0,1,\dots\}$ is not lonely because it has two 1s next to each other. Let F be the set of lonely sequences. Show that F is uncountable. (Problem 8.17 in LLM Book)

3. [Submission Problem for Group 3] Prove that if $\{A_0, A_1, \dots, A_n, \dots\}$ is an infinite sequence of countable sets, then so is
$$\bigcup_{n=0}^{\infty} A_n$$
.

   A complex number α is called algebraic if there exists a univariate polynomial $p(x)$ with rational coefficients such that $p(\alpha) = 0$. Conclude using the first part of the question that the set of *algebraic numbers* is countable.

4. [Submission Problem for Group 4] Prove that the set of all finite subsets of positive integers is countable. (Problem 8.9 in LLM Book)

5. [Bonus] Problems 8.14 (Schroeder- Berstein Theorem), 8.19, 8.20, 8.22.

6. Watch Veritasium's amazing video about foundations of mathematics.