(a) X = no of tests.

$n = gK$ [$g \to$ groups, $K \Rightarrow$ size of each group]

Now for each group, one test has to be done.

Now suppose out of g groups r groups have tested true samples.

its probability is $\binom{g}{r} q^r (1-q)^r$

where q = probability of a group testing true

The sample of a group will test true even if anyone has disease so,

$P(\text{noone in grp has disease}) = (1-p)^K$

$P(\text{atleast one has disease})$

$= \boxed{q = 1-(1-p)^K}$

$$E(X) = g + \sum_{r=0}^{g} \binom{g}{r} q^r (1-q)^r \; (rK)$$

↓ if grp tests true all members have to be tested.

$$= g + \sum_{r=1}^{g} \binom{g-1}{r-1} (gK) (q^r)(1-q)^{g-r}$$

$$= g + n \sum_{t=0}^{g-1} \binom{g-1}{t} q^{t+1} (1-q)^{(g-1)-t}$$

$$= g + nq\,(q+1-q)^{g-1}$$

$= g + nq$

$= \left(\frac{n}{k} + nq\right)$

$E(X) = \frac{n}{k} + n\left(1 - (1-p)^k\right)$

(b) when p is small $(1-p)^k \approx 1 - kp$

$E(X) = \frac{n}{k} + n \cdot kp$

$= n\left(\frac{1}{k} + kp\right) \geq n\sqrt{p}$

$\frac{d}{dk}\left(\frac{1}{k} + kp\right)n = 0$

$-\frac{1}{k^2} + p = 0 \Rightarrow k = \sqrt{p}$ } min at

$E(X)_{min} = 2n\sqrt{p}$

$\sqrt{p} = \frac{1}{k} + kp$

$k\sqrt{p} = 1 + k^2 p$

$k^2 p + 1 - k\sqrt{p}$

$D = b^2 - 4ac = p - 4p = -3p$

It has no real roots.

which is clear because $E(X)_{min} = 2n\sqrt{p}$.

∴ no such k exist for which $n\sqrt{p}$.

(a) $\frac{d}{dk}\left(\frac{1}{k} + kp\right) \Rightarrow \left(\frac{-1}{k^2} + p\right) = 0$

$$k = \frac{1}{\sqrt{p}}$$

$$E(X)_{min} = 2n\sqrt{p} \quad \text{when} \quad \boxed{k = \frac{1}{\sqrt{p}}}$$

(c) when 1% are diseased

$$\Rightarrow p = \frac{1}{100}$$

$$E(X)_{min} = \frac{2n \cdot 1}{\sqrt{p}}$$

$$= \frac{2n}{\sqrt{\frac{1}{100}}} = \left(\frac{n}{5}\right)$$

So instead of n test we need to test expected $\frac{n}{5}$ test

so $\boxed{ans = \frac{1}{5}}$

d) we can make groups of groups recursively like n soldiers are divided into g groups If some group test true that group is divided into g' group and if any of them is tested true that is further divided into groups.

until there are only 2 people left to test which we can test individually.

In this way we are avoiding to test blood samples that are disease free.