2301 COL 202 Tutorial 11.4

Anubhav Pandey

TOTAL POINTS

1 / 2

QUESTION 1

1 Tutorial for Group 4 **1 / 2**

+ 0 pts InCorrect

✓ **+ 1 pts** *Partially Correct*

+ 2 pts Correct

No mention of induction base case, hypothesis and reasonning

1 How??

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Tutorial 11, problem 4

Problem 17.7.
We play a game with a deck of 52 regular playing cards, of which 26 are red and 26 are black. I randomly shuffle the cards and place the deck face down on a table. You have the option of "taking" or "skipping" the top card. If you skip the top card, then that card is revealed and we continue playing with the remaining deck. If you take the top card, then the game ends; you win if the card you took was revealed to be black, and you lose if it was red. If we get to a point where there is only one card left in the deck, you must take it. Prove that you have no better strategy than to take the top card—which means your probability of winning is 1/2.

Hint: Prove by induction the more general claim that for a randomly shuffled deck of n cards that are red or black—not necessarily with the same number of red cards and black cards—there is no better strategy than taking the top card.

Sol:

let at after k rounds of skipping number of black cards left is n and number of red cards left is m.

P(win) = Probability of win.

⇒ Case 1: we choose the top card

$$p(\text{win}) = \frac{n}{n+m}.$$

Case 2: we skip the top card and choose the next card.

$$\Rightarrow P(\text{win}) = \frac{m}{n+m} \cdot \frac{n}{n+m-1} + \frac{n}{n+m} \cdot \frac{n-1}{n+m-1}$$

$$= \frac{n}{n+m}$$

$\Rightarrow$ After K rounds, the probability of chosing top card or next card is same. (for all $K \geq 0$)

$\therefore$ if $K=0$, $n=m=26$

$P(\text{win})$, at 0 skips $= \frac{1}{2}$

$P(\text{win})$, at 1 skips $= \frac{1}{2}$

and so on, any two consecutive card selection would have $\frac{1}{2}$ winning probability.

$$\boxed{\because \ \underset{\text{at } K=K}{P(\text{win})} = \underset{K=K+1}{P(\text{win})}} \quad \forall K$$

$\therefore$ There's no better stratergy than choosing the first card.

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