Tutorial 10

Problem 12.24

(a) The vertices of the graph that we will create will be all the variables used in the set of operations the user is giving.

Let ~~the~~ 'variable life' be defined as all operations ~~occurences~~ between the first and last appearance of the variable. ~~Variable~~ It will be denoted by

$L_x :=$ Variable life of x.

Note: If there is only one appearance than it would be counted as variable life

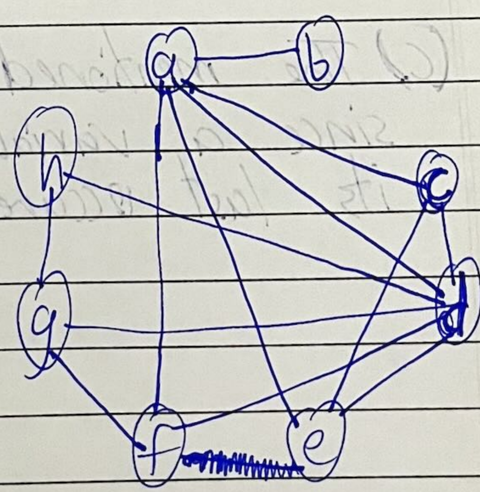
~~An edge exists between u and v if~~

$L_x = [m, n]$: m = first occurence of x, n = last occurence(x)

An edge exists between u and v if there is an overlap in $L_{\cancel{u}}$ and L_v i.e., $L_u \cup L_v \neq \phi$

In the given example:

0 Inputs: {a, b}	$L_a = [0, 5]$
1 $c = a+b$	$L_b = [0, 1]$
2 $d = a*c$	$L_c = [1, 4]$
3 $e = c+3$	$L_d = [2, 7]$
4 $f = c-e$	$L_e = [3, 4]$
5 $g = a+f$	$L_f = [4, 6]$
6 $h = f+1$	$L_g = [5, 7]$
7 Outputs: {d, g, h}	$L_h = [6, 7]$



ex. L_a overlaps with L_b, L_c, L_d, L_e, L_f hence 5 edges exist b/w a and each of {b, c, d, e, f}.

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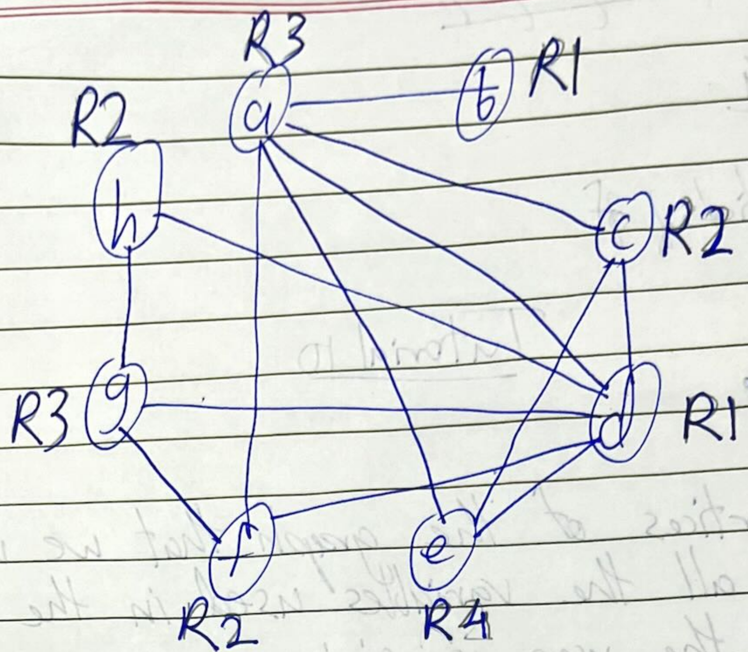
(b)



Each colour denotes usage of one colour.
If two variables get the same register (colour), the one appearing later overwrites the earlier one.
Running through set of given operations:

R1 = ~~b~~ d
R2 = ~~c~~ ~~f~~ h
R3 = ~~a~~ g
R4 = e

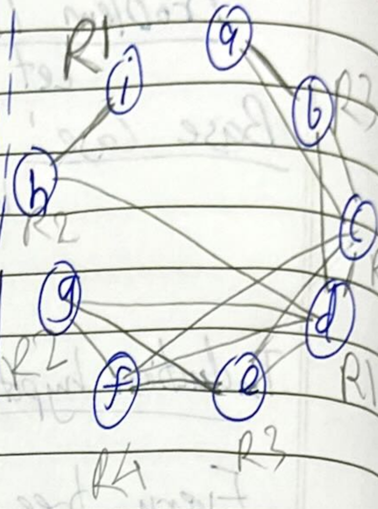
[Strike through a variable means it has been overwritten by the variable to its immediate right]

So, we require 4 registers for this example.

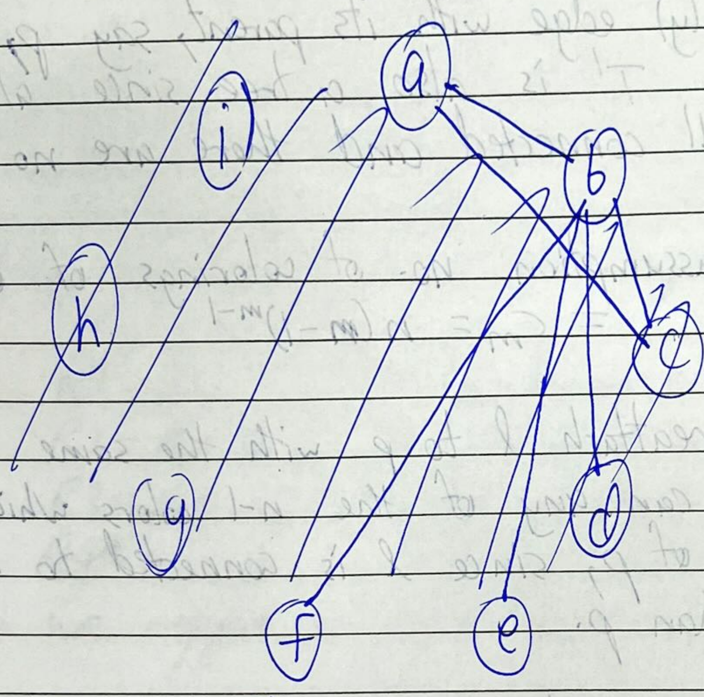
(c) The mentioned allocation takes care of this case since a variable will not be overwritten until its last occurence.
However, it may not lead to an optimal case. So, we can treat a reassignment as making a new variable altogether. This frees up its old register so that a variable can be assigned to it.
Consider the following example

Without creating new variable for reassignment

0	Inputs: {a, b, c}	$L_a = [0,1]$
1	$d = a * c$	$L_b = [0,2]$
2	$e = b - 3$	$L_c = [0,4]$
3	$f = d + 2$	$L_d = [1,7]$
4	$g = e + c$	$L_e = [2,6]$
5	$d = f * 4$	$L_f = [3,6]$
6	$h = e \div f$	$L_g = [4,4]$
7	$i = d + h$	$L_h = [6,8]$
8	Outputs: {i, h}	$L_i = [7,8]$



4 registers required



Creating new variable on reassignment

At operation 5,

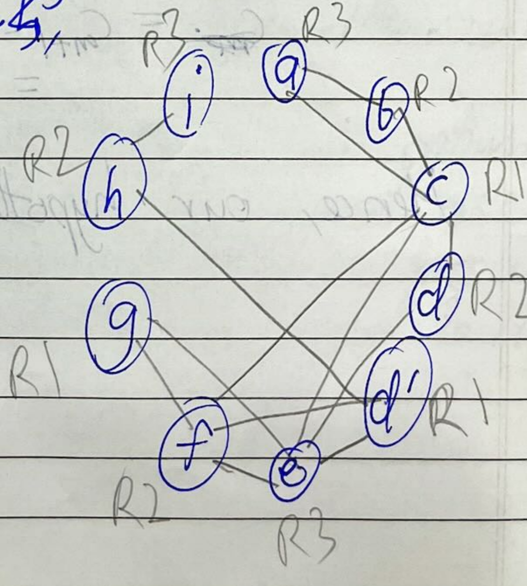
5. $d' = f * 4$
6. $h = e \div f$
7. $i = d' + h$
8. {i, h}

$L_d = [1, 3]$

$L_{d'} = [5, 7]$

3 registers required

∴ More optimal.



Problem 12.65

Let no. of colourings of $G^{(V,E)} = C_m$ ($m =$ ~~no. of~~ $|V|$)

Base Case: $m = 1$ (Single vertex in tree)

We can assign any of the n colors to it

$$C_1 = n \left[= n(n-1)^{1-1}\right]$$

Induction hypothesis for $m \geq 2$ assume $C_m = n(n-1)^{m-1}$

Consider $m+1$ vertex ~~graph~~ tree, T.

Every tree with no. vertices ≥ 2 has at least two leaves. Pick a leaf l in T. Remove it from T along with its (only) edge with its parent, say p, to obtain T'. Now, T' is also a tree since all nodes of T' are still connected and there are no cycles.

By our assumption no. of colorings of T'

$$= C_m = n(n-1)^{m-1}$$

Now, we reattach l to p with the same edge as before. We can any of the $n-1$ colors which are not the color of p, since l is connected to no other node than p.

So total no. of colorings of T

$$= C_{m+1} = n(n-1)^{m-1} \cdot (n-1)$$

$$= n(n-1)^{(m+1)-1}$$

Hence, our hypothesis is true. Q.E.D.