2022CS11113
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Group - 01

a) Let G be a digraph.

Given that the vertices are on linear partial order let us prove that:
for any set of vertices $\exists$ a path through them.

Proof: Let $A = \{a_1, a_2 \dots a_n\}$
be a set of vertices.

$\forall$ i, j s.t $i \neq j$

$\exists$ a relation b/w a_i & a_j

$\Rightarrow$ either $a_i R a_j$ or $a_j R a_i$

$\Rightarrow$ $a_i \rightarrow a_j$ or $a_j \rightarrow a_i$

Now we need to prove that the path is unique.

Proof:
Suppose
$a_1 \rightarrow a_2 \rightarrow a_3 \dots \rightarrow a_n$ is one
of the paths

Now let us assume that there exists another path
in which a_m & a_n occur in diff. order.

i.e. In our 1st path we have something like
$a_m \rightarrow \dots \rightarrow a_n$

And in 2nd path we have something like
$a_n \rightarrow \dots - a_m$

For 1st path
Now, $a_m \rightarrow a_k \rightarrow a_{k+1} \cdots a_{k+r} \rightarrow a_n$

$\Rightarrow$ $a_m R a_k$, $a_k R a_{k+1} \cdots a_{k+r} R a_n$

$\therefore$ $a_m R a_n$ (by transitive property) — (1)

For 2nd (new) path.

$a_n \rightarrow a_p \rightarrow a_{p+1} \cdots a_{p+q} \rightarrow a_m$

$\Rightarrow a_n R a_p$, $a_p R a_{p+1} \cdots a_{p+q} R a_m$.

$\therefore$ $a_n R a_m$ (by transitive property) — (2)

(1) & (2) violates the asymmetric property of strict order.

Hence proved path is unique

b) Claim: ~~Proof~~ we need to prove that $\forall i,j$ st. $i \neq j$
$a_i R a_j$ OR $a_j R a_i$

~~Proof~~: By unique path property we can ~~prove~~ say that $a_i R a_j$ or $a_j R a_i$. hence proved.

Claim: We need to prove transitivity

Proof: Given: $a_i R a_j$ & $a_j R a_k$.
combining these walks there is a walk from ~~form~~ a_i to a_k
$\Rightarrow a_i R a_k$.

Also, positive length paths are there
$\Rightarrow$ $\cancel{a_i R a_i}$ $\therefore$ irreflexive

$\therefore$ G is a linear strict partial order.