2301 COL 202 Tutorial 12.4

Anubhav Pandey

TOTAL POINTS

2 / 2

QUESTION 1

1 Problem for Group 4 **2 / 2**

- ✓ **+ 0.67 pts** *Part a correct*
- ✓ **+ 0.67 pts** *Part b correct*
- ✓ **+ 0.67 pts** *Part c correct*
- **+ 0 pts** Incorrect

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Tut. 12 Problem 4

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Problem 19.30.
A gambler bets on the toss of a fair coin: if the toss is Heads, the gambler gets back the amount he bet along with an additional the amount equal to his bet. Otherwise he loses the amount bet. For example, the gambler bets \$10 and wins, he gets back \$20 for a net profit of \$10. If he loses, he gets back nothing for a net profit of −\$10—that is, a net loss of \$10.

Gamblers often try to develop betting strategies to beat the odds is such a game. A well known strategy of this kind is *bet doubling*, namely, bet \$10 on red, and keep doubling the bet until a red comes up. So if the gambler wins his first \$10 bet, he stops playing and leaves with his \$10 profit. If he loses the first bet, he bets \$20 on the second toss. Now if the second toss is Heads, he gets his \$20 bet plus \$20 back and again walks away with a net profit of $20 - 10 = \$10$. If he loses the second toss, he bets \$40 on the third toss, and so on.

You would think that any such strategy will be doomed: in a fair game your expected win by definition is zero, so no strategy should have nonzero expectation. We can make this reasoning more precise as follows:

> Let W_n be a random variable equal to the amount won in the nth coin toss. So with the bet doubling strategy starting with a \$10 bet, $W_1 = \pm 10$ with equal probability. If the betting ends before the nth bet, define $W_n = 0$. So W_2 is zero with probability 1/2, is 10 with probability 1/4, and is −10 with probability 1/4. Now letting W be the amount won when the gambler stops betting, we have
>
> $$W = W_1 + W_2 + \cdots + W_n + \cdots.$$
>
> Furthermore, since each toss is fair,
>
> $$\text{Ex}[W_n] = 0$$
>
> for all $n > 0$. Now by linearity of expectation, we have
>
> $$\text{Ex}[W] = \text{Ex}[W_1] + \text{Ex}[W_2] + \cdots + \text{Ex}[W_n] + \cdots = 0 + 0 + \cdots + 0 + \cdots = 0, \quad (19.19)$$
>
> confirming that with fair tosses, the expected win is zero.

But wait a minute!

(a) Explain why the gambler is certain to win eventually if he keeps betting.

(b) Prove that when the gambler finally wins a bet, his net profit is \$10.

(c) Since the gambler's profit is always \$10 when he wins, and he is certain to win, his expected profit is also \$10. That is

$$\text{Ex}[W] = 10,$$

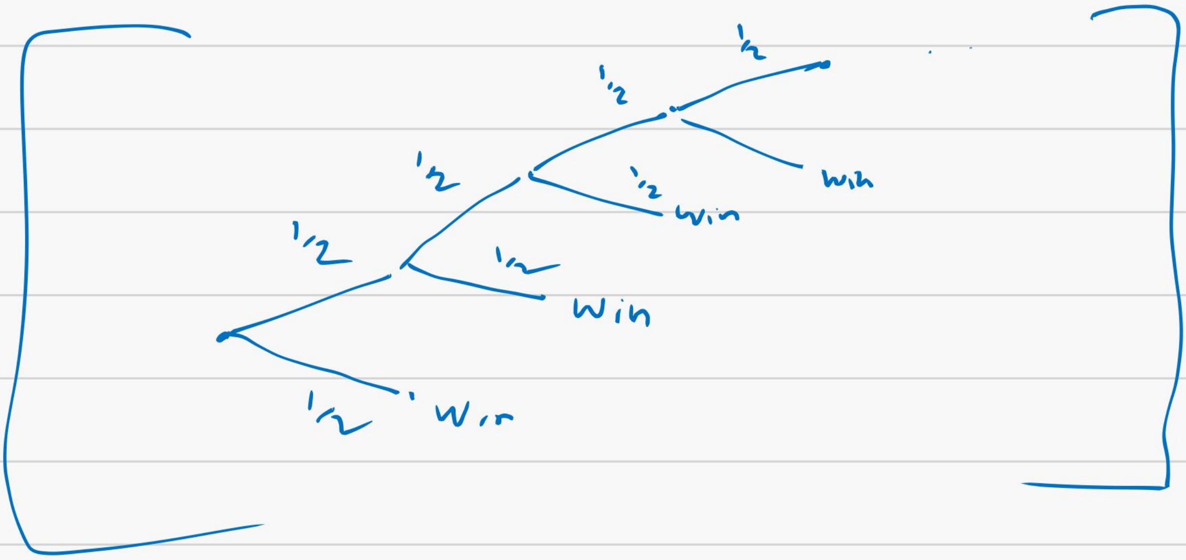
contradicting (19.19). So what's wrong with the reasoning that led to the false conclusion (19.19)?

(a) If the gambler wins at K^{th} step, the sum of all the money he lost till $(K-1)^{th}$ step is less than that of what he won at K^{th} step and net profit always = 10 \$ which is shown below

b) Let $P(n)$ denote that Gambler wins in n^{th} round.

$$\Rightarrow \quad P(1) = \frac{1}{2}, \qquad P(2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(3) = \left(\frac{1}{2}\right)^3, \quad \ldots$$



let $Ex(W)$ represents its net profit after winning.

$$E(W) = 10 \times \frac{1}{2} + (20-10)\frac{1}{2} \times \frac{1}{2}$$

$$+ (40-20-10) \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$+ (80-40-20-10) \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$E_K \equiv$ Net profit when won at K^{th} level.

$$E(w) = E_1 + E_2 + E_3 + \cdots$$

$$E(w) = 10 \times \frac{1}{2} + (20-10) \times \frac{1}{2} \times \frac{1}{2} + (40-20-10) \times \left(\frac{1}{2}\right)^3 + \cdots$$

$$= 10\left(\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots\right)$$

$$\boxed{E(w) = 10}$$

(C) The earlier argument is not valid because the series of possible wins has infite terms, which will not converge ~~to~~ 0

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