

Tutorial 9

1. [Submission Problems for Group 1] Problem 10.48 in [LLM Book](#)
2. [Submission Problems for Group 2] Problem 10.24 in [LLM Book](#)
3. [Submission Problems for Group 3] Problem 10.7 in [LLM Book](#)
4. [Submission Problems for Group 4] Problem 10.6 in [LLM Book](#)
5. [Bonus] Some of the problems could be hard and maynot fit into the tutorial slot - use piazza for discussing those.
 - (a) A digraph G is strongly connected if there is a (directed) path between each pair of vertices, i. e., all vertices belong to the same strong component. (There is just one strong component.) An undirected walk (path, cycle, etc.) in a digraph is a walk (path, cycle, etc.) in the undirected graph obtained by ignoring orientation. A digraph is weakly connected if there is an undirected path between each pair of vertices.
Prove: if $(\forall v \in V)(\text{outdegree}(v) = \text{indegree}(v))$ and G is weakly connected then G is strongly connected.
 - (b) Problem 10.13 in [LLM Book](#)
 - (c) Tournaments are orientations of complete graphs. So in a tournament $G = (V, E)$, for every pair of vertices $u, v \in V$, exactly one of the following holds: (a) $u = v$; (b) $u \rightarrow v$; (c) $v \rightarrow u$. We can think of the vertices of a tournament as players in a round-robin tournament without ties or rematches. Each player plays against every other player exactly once; $u \rightarrow v$ indicates that player u beat player v .
A **Hamilton cycle** in a digraph is a (directed) cycle of length n , i. e., a cycle that passes through all vertices. G is **Hamiltonian** if it has a Hamilton cycle. A **Hamilton path** in a digraph is a (directed) path of length $n - 1$, i. e., a path that passes through all vertices.
 - i. Count the tournaments on a given set of n vertices.
 - ii. Let T be a tournament with n vertices. Prove: if all vertices have the same out-degree then n is odd.
 - iii. Prove that every tournament has a Hamilton path.
 - iv. Prove that every strongly connected tournament is Hamiltonian.