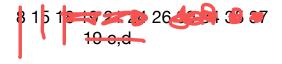
## Tutorial 10

- 1. [Submission Problems for Group 1] Problem 19.34 in LLM Book
- 2. [Submission Problems for Group 2] Problem 19.26 in LLM Book
- 3. [Submission Problems for Group 3] Problem 19.18 in LLM Book
- 4. [Submission Problems for Group 4] Problem 19.30 in LLM Book
- 5. [Bonus] Problem 19.8, 19.15, 19.19, 19.21, 19.24, 19.35, 19.37, in LLM Book
- 6. [Bonus] Let n be a random integer, chosen uniformly between 1 and N. What is the expected number of distinct prime divisors of n? Show that the result is asymptotically equal to  $\ln \ln N$  (as  $N \to \infty$ ).
- 7. [Bonus] For a permutation  $\pi$  of the set [n], let  $c_k(\pi)$  denote the number of k-cycles in the cycle decomposition of  $\pi$ . (For instance, if n=7 and  $\pi=(18)(256)(3)(47)(9)$  then  $c_1(\pi)=2, c_2(\pi)=2, c_3(\pi)=1$ , and  $c_k(\pi)=0$  for all  $k\neq 1,2,3$ .) Pick  $\pi$  at random from all permutations of [n].
  - (a) Calculate  $E[c_k(\pi)]$ . Your answer should be a very simple expression (no factorials, no binomial coefficients, no summation).
  - (b) Calculate the expected number of cycles (including cycles of length 1) in the cycle decomposition of a random permutation (This will be a simple sum, not a closed-from expression). Prove that this number is  $\sim \ln n$ .



<sup>&</sup>lt;sup>1</sup>You can read more about the cycle notation for a permutation here