COL202 TUTORIAL 5

SUBMISSION FOR GROUP 2

PROBLEM 5.2

Jahnabi Roy
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1 Question 2

Let $\{0,1\}^w$ be the set of infinite binary sequences. Call a sequence in $\{0,1\}^w$ lonely if it never has two 1s in a row. For example, the repeating sequence {0, 1, 0, 1, 0, 1, 0, 1, 0, . . . } is lonely, but the sequence {0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, . . . } is not lonely because it has two 1s next to each other. Let F be the set of lonely sequences. Show that F is uncountable.

Proof. We use diagonal argument to solve this problem.
Suppose there was a bijection between $\mathcal{N}$ and set of lonely sequences F. If such a relationship existed, then we would be able to display it as an infinite bit strings in some countable order. And in this list, any bit string in the set of lonely sequences will come up in finite sets just like we can reach any integer n in finite number of steps from 0. So, representing our list as list of lists or a matrix, we get the following figure :

$\mathcal{N}$	0	1	2	3	4	5	6 . . .
0	**1**	0	0	1	0	1	. . .
1	0	**0**	1	0	1	0	. . .
2	0	0	**1**	0	0	1	. . .
3	1	0	0	**0**	1	0	. . .
4	1	0	1	0	**1**	0	. . .
5	1	0	0	0	0	**0**	. . .
.	.	.	.	.	.	.	. . .
.	.	.	.	.	.	.	. . .
.	.	.	.	.	.	.	. . .

We now define a D where D is the sequence of all diagonal elements from the matrix above.
D = {1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1 . . .}
Then, let us define another sequence C such that for every element $e \in D$, the complement of the bit, $e' \in C$.
So, C = {0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0 . . .} According to definition of lonely sequences, C is also a lonely sequence. And, if a^{th} term of the N_a is 1, then a^{th} term of C will be 0. So, C has atleast one bit different from every sequence in our list. So, C is a lonely sequence but is not a part of the matrix. So, the matrix is not complete. So, there cannot exist a bijection between $\mathcal{N}$ and set of lonely sequences F. So, F is not countable.
Hence, F is uncountable.

Hence. proved. $\square$