# 2301 COL 202 Tutorial 5.3

Abhinav Rajesh Shripad

TOTAL POINTS

## 2 / 2

QUESTION 1

*1* Q 3 **2 / 2**

✓ **- 0 pts** *Correct*

**- 1 pts** First part incorrect

**- 1 pts** Second part incorrect

Thanks for using Latex!

# COL 202 Assignment 5

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## 1 Problem Statement

Prove that if $A_0, A_1, ..., A_n, ...$ is an infinite sequence of countable sets, then so is

$$\bigcup_{n=0}^{\infty} A_n$$

A complex number $\alpha$ is called algebraic if there exists a uni-variate polynomial $p(x)$ with rational coefficients such that $p(\alpha) = 0$. Conclude using the first part of the question that the set of algebraic numbers is countable.

## 2 Solution

### 2.1 Part 1

Let $A_i(j)$ denote the $j^{th}$ element of the set $A_i$. We define a mapping $f : \bigcup_{n=0}^{\infty} A_n \to \mathbf{N}$ as follows:

$$f(A_i(j)) = 2^i 3^j$$

To prove that $f$ is into, let's consider an arbitrary natural number $n \in \mathbf{N}$. Consider the element $A_i(j)$, by construction, $f(A_i(j)) = 2^i 3^j$, thus by using the argument of highest power of 2 and 3 dividing $f(A_i(j))$ implies that $f$ is a into function.

Since the range of $f$ is a subset of $\mathbf{N}$ and $f$ is into, the domain of $f$, that is, the set $\bigcup_{n=0}^{\infty} A_n$ is countable.

### 2.2 Part 2

For a particular integer $n$, the set of uni-variate polynomials with degree $n$ and rational coefficients is countable. This is because each coefficient of the polynomial is rational, and the number of coefficients is finite ($n$ coefficients as it is uni-variate). The set of rational numbers is countable, and the Cartesian product of countable sets is also countable. Therefore, the set of such polynomials is countable.

Denote $A_i$ as the set of roots of uni-variate rational polynomials of degree $i$. We aim to demonstrate the countability of each $A_i$.

Consider any $A_i$. Since a polynomial of degree $i$ has at most $i$ distinct roots, the set $A_i$ can be at most a countable collection of elements. Furthermore, we have previously established that the set of uni-variate rational coefficient polynomials of degree $i$ is countable. This implies that for each specific degree $i$, the set $A_i$—which consists of the roots of such polynomials—is indeed countable.

Using the above notation, each $A_i$ is countable, and the number of different $A_i$ is countable as well, since each $A_i$ corresponds to a degree of polynomial which is a natural number. Thus, the union:

$$\bigcup_{n=0}^{\infty} A_n$$

is a countable union of countable sets, and therefore countable.

This completes the proof that the set of algebraic numbers is countable, as it is a subset of $\bigcup_{n=0}^{\infty} A_n$.

*1* Q 3 **2 / 2**

✓ **- 0 pts** *Correct*

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**- 1 pts** Second part incorrect

Thanks for using Latex!