COL202 TUTORIAL 5

SUBMISSION FOR GROUP 2

PROBLEM 6.2

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1 Question 2

Consider the statement: "if $a_n = \Omega(c_n)$ and $b_n = \Omega(c_n)$ then $a_n + b_n = \Omega(c_n)$". Show that this statement is false. Show that if we additionally assume $a_n b_n > 0$ then the statement becomes true.

Proof. Given that : $a_n = \Omega(c_n)$ and $b_n = \Omega(c_n)$. According to the list of definitions given above, we also have the following result that $|c_n/a_n|$ is bounded and similarly, $|c_n/b_n|$ is also bounded.
Proving by counter example.
Let $a_n = n$, $b_n = -n + k$ where $k \epsilon \mathcal{R}$ and $c_n = n$, where $n \geq N_0, n, N_0 \in \mathcal{N}$. Clearly, $|c_n/a_n|$ and $|c_n/b_n|$ are both bounded.
$a_n + b_n =$ k.
So, $|c_n/(a_n + b_n)| = n/k$. But as $n \to \infty$, $|c_n/(a_n + b_n)|$ also tends to infinity. So, $|c_n/(a_n + b_n)|$ is no longer bounded. So, our statement is not correct for a general overview. It is thus proven false by counterexample provided.

However, if $a_n b_n > 0$, then we know that this implies that $|(a_n + b_n)| = |a_n| + |b_n|$. We know $|c_{n_1}/a_{n_1}|$ is bounded. So,
$|c_{n_1}/a_{n_1}| \leq M_1$ .....(I)
Similarly, $|c_{n_2}/b_{n_2}|$ is bounded. So,
$|c_{n_2}/b_{n_2}| \leq M_2$ ..... (II)
(where M_1,$M_2 \geq 0$ and $n = max(n_1, n_2)$.

So, $|c_n n| \leq M_1 |a_n n|$ and $|c_n| \leq M_2 |b_n|$. Adding the two equations, we get
$2|c_n| \leq M_1|a_n| + M_2|b_n|$
$|c_n| \leq M_1|a_n|/2 + M_2|b_n|/2$

Let $M = max(M_1, M_2)$.
$|c_n| \leq M(|a_n| + |b_n|)/2$
Since we have already established earlier, $|(a_n + b_n)| = |a_n| + |b_n|$ for $a_n b_n > 0$,
$|c_n| \leq M(|a_n + b_n|)/2$
$|c_n|/|a_n + b_n| \leq M/2$

Thus, we get that $|c_n|/|a_n + b_n|$ is bounded. So, $a_n + b_n = \Omega(c_n)$ is justified if $a_n = \Omega(c_n)$ and $b_n = \Omega(c_n)$ for $a_n b_n > 0$.
Hence, Proved. $\square$