2301 COL 202 Tutorial 8.3

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TOTAL POINTS

2 / 2

QUESTION 1

1 Problem for Group 3 **2 / 2**

✓ **- 0 pts** *Correct*

COL 202 Assignment 8

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1 Problem

Let r_n be the number of strings of length n over the alphabet A, B without consecutive $A's$ (so $r_0 = 1$,$r_1 = 2$,$r_2 = 3$). Prove: $r_n \approx c\gamma^n$ for a real number γ. Determine the constant c and γ precisely. Prove your answers.

2 Solution

Let a_n and b_n be the number of sequence following the problem conditions with additional condition of a_n has last term of A and b_n does not end in A. Thus we can say that

$$r_n = a_n + b_n$$

For the sequences in a_n the second last term must be not be A. Thus there exists a one to one mapping from a_n and b_{n-1} for $n \geq 2$. Thus we can write $a_n = b_{n-1}$.

Similarly for b_n we can make no deduction about the second last term, it can be either A or B. Thus we can write $b_n = a_{n-1} + b_{n-1}$. Thus $b_n = r_{n-1}$.

Thus we write the equations as

$$a_n + b_n = b_n + b_{n-1}$$

$$r_n = b_n + b_{n-1} = r_{n-1} + r_{n-2}$$

Thus the recurrence relation for r_n $n \geq 2$ is

$$r_n = r_{n-1} + r_{n-2}$$

Let $f(x)$ be power series corresponding to r_n as

$$f(x) = \sum_{n=0}^{\infty} r_n x^n$$

Using the recurrence relation we can write

$$(1 - x - x^2)f(x) = (r_1 - r_0)x + r_0 = x + 1$$

Partial fraction decomposition gives

$$f(x) = \frac{2\alpha+1}{(\alpha+2)(1-\alpha x)} + \frac{2\beta+1}{(\beta+2)(1-\beta x)}$$

Where $\alpha = \frac{1+\sqrt{5}}{2}$ $\beta = \frac{1-\sqrt{5}}{2}$
Comparing the co-efficient of x^n from both the sides gives

$$r_n = \frac{2\alpha+1}{\alpha+2}\alpha^n + \frac{2\beta+1}{\beta+2}\beta^n$$

Using the fact that $|\beta| \leq 1$ we get that

$$r_n \approx \frac{5+3\sqrt{5}}{10}\alpha^n$$

Thus we can conclude that $r_n \approx c\gamma^n$ for some real numbers γ and c where

$$c = \frac{5+3\sqrt{5}}{10} \qquad \gamma = \frac{1+\sqrt{5}}{2}$$

1 Problem for Group 3 2 / 2

✓ **- 0 pts** *Correct*