2301 COL 202 Tutorial 3.3

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TOTAL POINTS

2 / 2

QUESTION 1

1 Problem for Group 3 **2 / 2**

✓ **- 0 pts** *Correct*

COL 202 Assignment 3

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1 Problem Statement

Use strong induction to prove that $n \leq 3^{n/3}$ for every integer $n \geq 0$.

2 Solution

Let $P(n)$ denote the predicate to be proven in the problem.

For proving by strong induction we proceed by manually checking for $n = 0, 1, 2$ and 3 and then use the strong inductive hypothesis.
For $n = 0$ the inequality becomes $0 \leq 1$ which is true.
For $n = 1$ the inequality becomes $1 \leq 3^{1/3}$ which is equivalent to $1^3 \leq 3$ which is true.
For $n = 2$ the inequality becomes $2 \leq 3^{2/3}$ which is equivalent to $2^3 \leq 3^2$ which is true.
For $n = 3$ the inequality becomes $3 \leq 3^{3/3}$ which is equivalent to $3 \leq 3$ which is true.

Base Case:- $P(n)$ is true for $n = 0, 1, 2, 3$

Inductive Hypothesis:- If $P(n)$ is true for $n = 0, 1....k - 1$ for $k - 1 \geq 3$ then $P(n)$ is true for $n = k$

Inductive Step:- We prove this by doing casework based on parity of k.
Case 1:- k is an even number, say $k = 2m$ with $m \in N$ and $m \geq 2$ as $m \leq 1$ is covered in base case

$$\begin{aligned} 3^{2m/3} &= (3^{m/3})^2 \\ &\geq m^2 \\ &\geq 2m \end{aligned}$$

Where the 1st inequality follows from the strong inductive hypothesis and 2nd inequality from the fact $m \geq 2$.

Case 2:- k is an odd number, say $k = 2m + 1$ with $m \in N$ and $m \geq 2$ as $m \leq 1$ is covered in base case

$$3^{2m+1/3} = (3^{m+1/3})(3^{m/3})$$
$$\geq m(m+1)$$
$$= m^2 + m$$
$$\geq 2m + 1$$

Where the 1st inequality follows from the strong inductive hypothesis and last inequality from the fact $m^2 \geq 2m$ and $m \geq 1$.

Thus we can conclude that $P(n)$ is a tautology for for $n \in \mathbf{N^0}$

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✓ **- 0 pts** *Correct*