Major Solution 2023-I

1. (d) 2. (a) 3. (d) 4. (a) 5. (c)

6. (c) 7. (d)

(8) Shifted e^- charge is given by

$$\frac{-Ze\left(\frac{4}{3}\pi x^3\right)}{\frac{4}{3}\pi R^3} = \frac{-Zex^3}{R^3}$$

Coulomb Force of attraction b/w nuclear charge Ze & shifted e^- charge

$$F = \frac{(Ze)\left(-\frac{Zex^3}{R^3}\right)}{4\pi\epsilon_0 x^2}$$

$$= -\frac{Z^2e^2}{4\pi\epsilon_0 R^3}x \qquad \text{So } \beta = \frac{-Z^2e^2}{4\pi\epsilon_0 R^3}$$

Magnitude of this force is balanced by Force due to electric field

$$\frac{Z^2e^2}{4\pi\epsilon_0 R^3}x = ZeE$$

$$X = \frac{4\pi\epsilon_0 R^3}{Ze} E$$

Dipole moment $p = (Ze)X$

$$p = 4\pi\epsilon_0 R^3 E$$

p is also given by $\Rightarrow p = \alpha_e E$

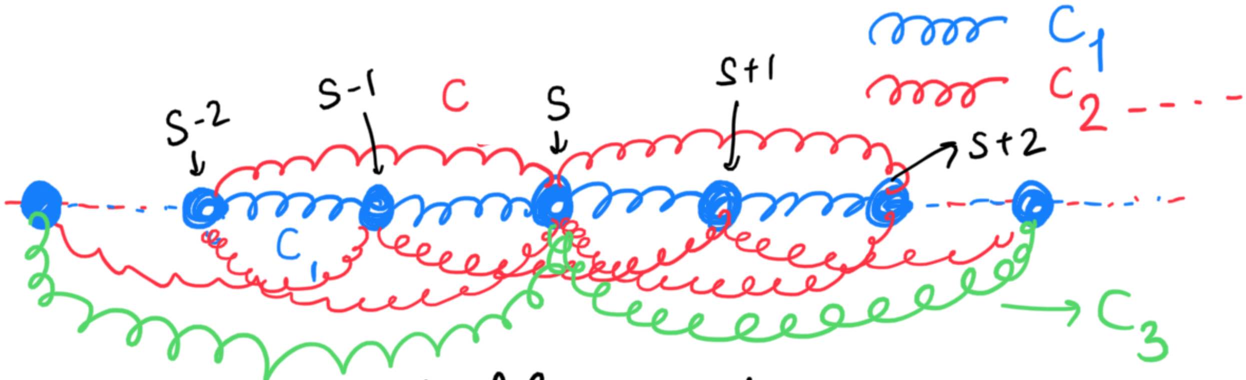
So $\alpha_e = 4\pi\epsilon_0 R^3$

For Ar :- $R = 1\,\mathring{A}$

$$\alpha_e = 4 \times \frac{22}{7} \times \underset{\left(\frac{\text{Farad}}{m}\right)}{8.87 \times 10^{-12}} \times \underset{m^3}{10^{-30}}$$

$$= 1.1 \times 10^{-40} \quad F m^2$$

(9)



Force $\propto$ displacement

Force on s^{th} atom $= C_1(u_{s-1} - u_s) + C_1(u_{s+1} - u_s)$

$$+ C_2(u_{s-2} - u_s) + C_2(u_{s+2} - u_s)$$

$$+ C_3(u_{s-3} - u_s) + C_3(u_{s+3} - u_s)$$

$$- \; - \; - \; - \; - \; - \; -$$

$$+ C_p(u_{s-p} - u_s) + C_p(u_{s+p} - u_s)$$

$$= C(u \quad + u \quad - 2u \quad) + C(u \quad + u \quad - 2u_s)$$

$$+ C_1(u_{s+1} + u_{s-1} - 2u_s) + C_2(u_{s+2} + u_{s-2}$$

$$+ C_3(u_{s+3} + u_{s-3} - 2u_s) + \text{------}$$

$$+ C_p(u_{s-p} + u_{s+p} - 2u_s)$$

$$= -2(C_1 + C_2 + C_3 + \text{----} C_p)u_s$$

$$+ C_1(u_{s+1} + u_{s-1}) + C_2(u_{s+2} + u_{s-2})$$

$$+ C_3(u_{s+3} + u_{s-3}) + \text{------}$$

$$+ C_p(u_{s+p} + u_{s-p})$$

$C_1, C_2 \text{---} C_p =$ Force constants

Equation of motion for s^{th} atom

$$\boxed{\begin{aligned} m\frac{\partial^2 u_s}{\partial t^2} &= \\ &-2(C_1 + C_2 + \text{---} C_p)u_s \\ &+ C_1(u_{s+1} + u_{s-1}) + C_2(u_{s+2} + u_{s-2}) \\ &+ \text{-----} + C_p(u_{s+p} + u_{s-p}) \end{aligned}}$$

Let the solution be

$$u_s(t) = A e^{i(\omega t - K s a)}$$

L.H.S.

$$m\frac{\partial^2 u_s}{\partial t^2} = -m\omega^2 A e^{i(\omega t - K s a)}$$

$$\text{R.H.S.} = -2(C_1 + C_2 + \text{---} C_p) A e^{i(\omega t - K s a)}$$

$$+ c_1 A e^{i(\omega t - ksa)} (e^{-ika} + e^{ika})$$

$$+ c_2 A e^{i(\omega t - ksa)} (e^{-2ika} + e^{2ika})$$

$$+ - - - - + c_p A e^{i(\omega t - ksa)} (e^{-pika} + e^{pika})$$

$$= -2 \sum_{j=1}^{p} c_j A e^{i(\omega t - ksa)}$$

$$+ 2c_1 A e^{i(\omega t - ksa)} \cos ka$$

$$+ 2c_2 A e^{i(\omega t - ksa)} \cos 2ka$$

$$+ - - - - + 2c_p e^{i(\omega t - ksa)} \cos pka$$

L.H.S. = R.H.S.

$$-m\omega^2 = -2 \sum_{j=1}^{p} c_j + 2c_1 \cos ka$$

$$+ 2c_2 \cos 2ka + - - - - + 2c_p \cos pka$$

$$\boxed{\omega^2 = \frac{2}{m} \sum_{j=1}^{p} c_j (1 - \cos jka)}$$

Given

10) $L = 5\,mm$ $\quad D = 2\,mm \Rightarrow R = 1\,mm$

$V = 2\,kV$ $\quad d = 2.5 \times 10^{-10}\, m/V$

$\epsilon_x = 1000$

From Piezo electric effect

$$P = d\,T$$

$P \to$ Polarization

$d \to$ Piezoelectric coefficient

$T \to$ Stress

Stress $T = \frac{F}{A}$

$$P = \frac{dF}{A}$$

$$F = \frac{PA}{d}$$

$$P = \chi_e \epsilon_0 E = (\epsilon_r - 1)\epsilon_0 \frac{V}{L}$$

$$\boxed{F = \frac{(\epsilon_r - 1)\epsilon_0 V A}{dL}}$$

or $V = \frac{Q}{C} = \frac{AP}{\frac{\epsilon_0 \epsilon_r A}{L}} = \frac{LP}{\epsilon_0 \epsilon_r}$

$$P = \epsilon_0 \epsilon_r \frac{V}{L}$$

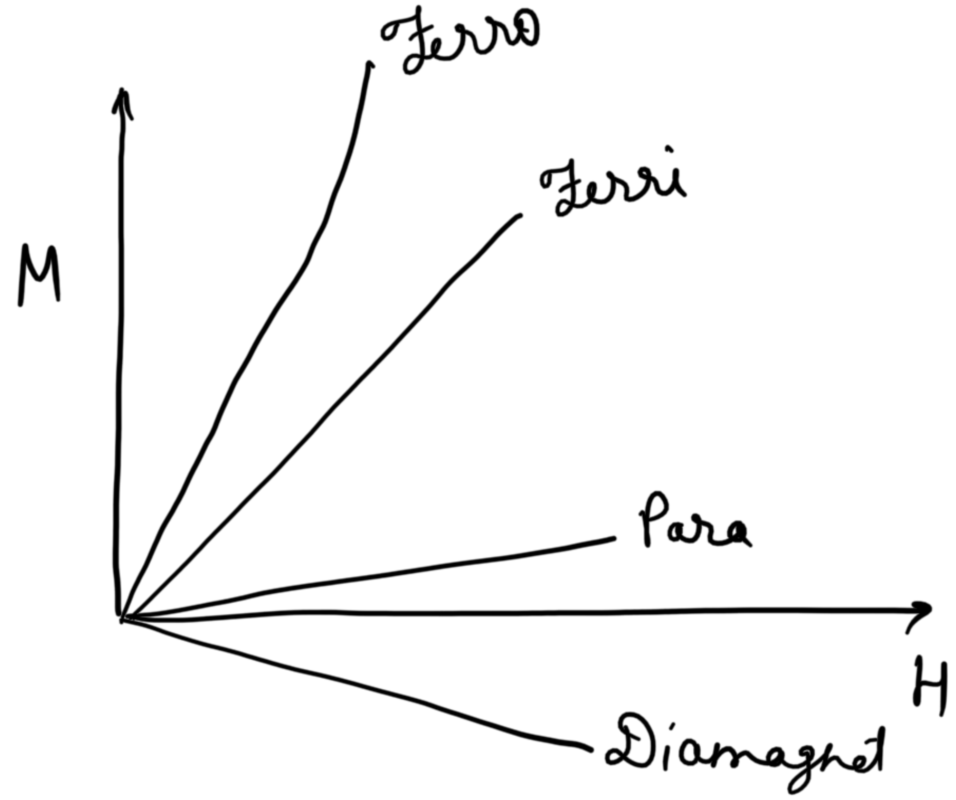
So $\boxed{F = \epsilon_r \epsilon_0 \frac{VA}{dL}}$

Both are fine as $\epsilon_r >>> 1$

$$\text{So } F = \frac{1000 \times 8.85 \times 10^{-12} \times 2 \times 10^{3} \times \pi \times 10^{-6}}{2.5 \times 10^{-10} \times 5 \times 10^{-3}}$$

$$= 4.4 \times 10 \simeq 44 \text{ N}$$

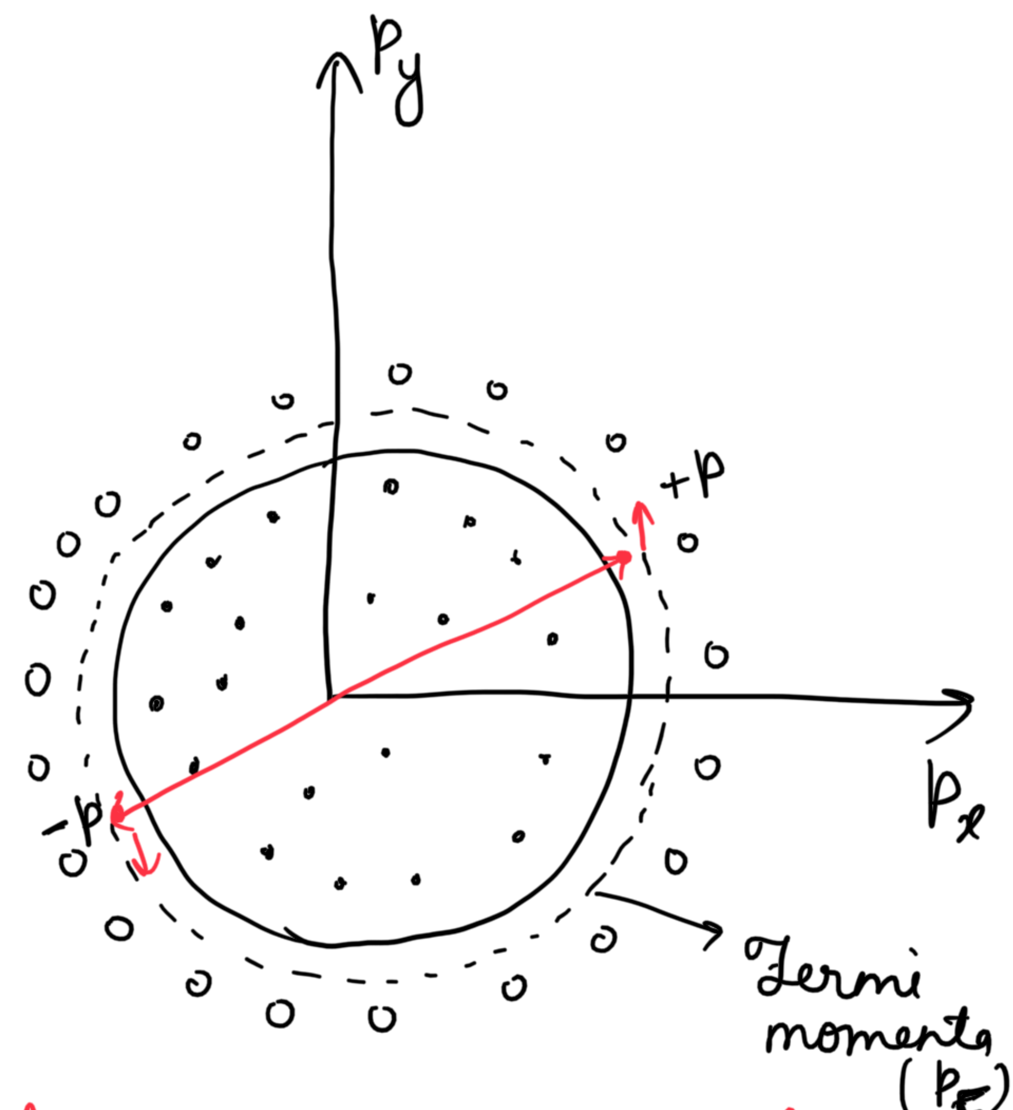
11)



12) Magetic Anisotrophy meaning magnetic properties are direction dependent.
The spin-orbit (H_{SO}) coupling plays crucial role in the direction dependence.
Due to H_{SO}, spin is coupled to the orbit then if one wants to rotate the spin, its orbit is also getting shifted. orbit is coupled to lattice & it wants to minimise the

electrostatic energy therefore certain direction are preferred over others giving rise to notion of easy & hard axis.

(13)



Cooper pair : $(+p\uparrow \ -p\downarrow)$

• : Occupied electronic state

o : Empty " "

Cooper pair have really small binding energy of the order of 1meV, that why low temperature

is required for their formation
High temperature will break
Cooper pair

14) $E = 1000 \; \frac{V}{m}$

$P = 4 \times 10^{-8} \; C/m^2$

$P = (\epsilon_r - 1)\, \epsilon_0 \, E$

$\epsilon_r - 1 = \frac{P}{\epsilon_0 E}$

$\epsilon_r = 1 + \frac{P}{\epsilon_0 E}$

$= 1 + \frac{4 \times 10^{-8}}{8.85 \times 10^{-12} \times 1000}$

$= 1 + \frac{40}{8.85}$

$\boxed{\epsilon_r = 5.5}$

15 Phonons in two dimension:-
@ Density of State :-
No. of k points in area $dk_x \, dk_y$

$= \frac{dk_x \, dk_y}{\quad}$

$$\left(\frac{2\pi}{L}\right)^2$$

$$= \frac{2\pi k\, dk}{4\pi^2} L^2$$

$$= L^2 \frac{k\, dk}{2\pi}$$

For linear dispersion $(k \to 0)$

$$\omega = v_s k \quad \Rightarrow k = \frac{\omega}{v_s}$$

No. of k points $= \dfrac{L^2 \omega\, d\omega}{2\pi v_s^2}$

$$= D(\omega)\, d\omega$$

$$D(\omega) = \frac{L^2 \omega}{2\pi v_s^2}$$ (density of state per mode)

Now for 1 Transverse & 1 longitudinal we have different velocities

Total DOS :-

$$\boxed{D(\omega) = \frac{L^2 \omega}{2\pi}\left(\frac{1}{v_t^2} + \frac{1}{v_l^2}\right)}$$

Debye Frequency :-

$$\int_0^{\omega_D} D(\omega)\, d\omega = 2N \longrightarrow (1T + 1L)$$

$$\frac{L^2}{2\pi}\left(\frac{1}{v_t^2} + \frac{1}{v_l^2}\right) \frac{\omega_D^2}{2} = 2N$$

$$\boxed{\omega_D^2 = \frac{8\pi N}{L^2\left(\frac{1}{V_t^2}+\frac{1}{V_l^2}\right)}}$$

(b) : Total Energy :

$$E = \int^{\omega_D} d\omega \, \hbar\omega \langle n \rangle D(\omega)$$

$$= \int_0^{\omega_D} d\omega \, \hbar\omega \times \frac{1}{e^{\hbar\omega/k_BT}-1} \, \frac{L^2\omega}{2\pi}\left(\frac{1}{V_t^2}+\frac{1}{V_l^2}\right)$$

$$= \frac{\hbar L^2}{2\pi}\left(\frac{1}{V_t^2}+\frac{1}{V_l^2}\right)\int_0^{\omega_D} \frac{\omega^2 \, d\omega}{e^{\hbar\omega/k_BT}-1}$$

$$\frac{\hbar\omega}{k_BT} = x \qquad \frac{\hbar\omega_D}{k_BT} = x_D$$

$$= \frac{\hbar L^2}{2\pi}\left(\frac{1}{V_t^2}+\frac{1}{V_l^2}\right)\int_0^{x_D} \frac{x^2 \, dx}{e^x-1}\left(\frac{k_BT}{\hbar}\right)^3$$

$$= \frac{L^2}{2\pi\hbar^2} k_B^3 T^3 \left(\frac{1}{V_t^2}+\frac{1}{V_l^2}\right)\int_0^{x_D} \frac{x^2 \, dx}{e^x-1}$$

(c) :

$$C_V = \frac{\partial U}{\partial T}$$

$$= \frac{\hbar L^2}{\ldots}\left(\frac{1}{\ldots}+\frac{1}{\ldots}\right)\frac{\partial}{\ldots}\int^{\omega_D}\frac{\omega^2 \, d\omega}{\ldots}$$

$$\frac{\quad}{2\pi}\left(\frac{1}{V_t^2}+\frac{1}{V_l^2}\right)\frac{\partial}{\partial T}\int_0 \frac{\quad}{e^{\hbar\omega/K_BT}-1}$$

$$= \frac{\hbar L^2}{2\pi}\left(\frac{1}{V_t^2}+\frac{1}{V_l^2}\right)\int_0^{\omega_D}\frac{\omega^2 d\omega\, e^{\frac{\hbar\omega}{K_BT}}}{\left(e^{\hbar\omega/K_BT}-1\right)^2}\times\frac{\hbar\omega}{K_BT^2}$$

$$= \frac{\hbar^2 L^2}{2\pi K_B T^2}\left(\frac{1}{V_l^2}+\frac{1}{V_t^2}\right)\int_0^{\omega_D}\frac{\omega^3 e^{\frac{\hbar\omega}{K_BT}}}{\left(e^{\frac{\hbar\omega}{K_BT}}-1\right)^2}d\omega$$

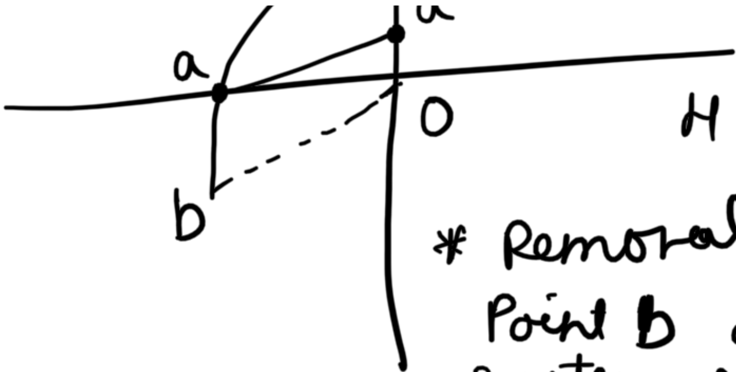
$$\frac{\hbar\omega}{K_BT} = x$$

$$C_V = \frac{\hbar^2 L^2}{2\pi K_B T^2}\left(\frac{1}{V_l^2}+\frac{1}{V_t^2}\right)\left(\frac{K_BT}{\hbar}\right)^4\int_0^{x_D}\frac{x^3 e^x}{(e^x-1)^2}dx$$

$$= \frac{L^2 K_B^3 T^2}{2\pi\hbar^2}\left(\frac{1}{V_l^2}+\frac{1}{V_t^2}\right)\int_0^{x_D}\frac{x^3 e^x}{(e^x-1)^2}dx$$

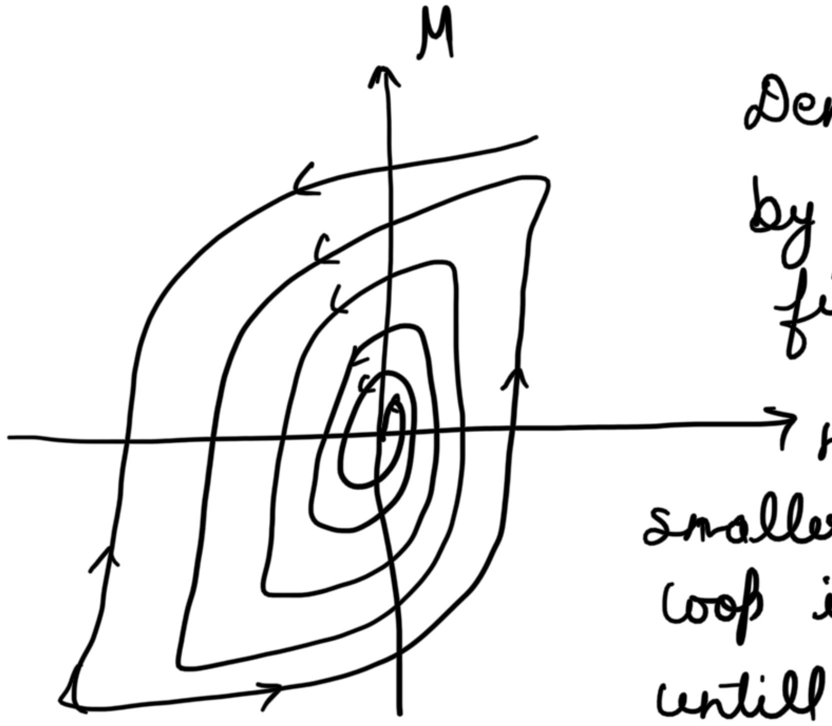
16) <u>Demagnetization</u> :- Demagnetization means obtaining zero value of M for zero H as in Ferromagnet due to hysteresis it seems impossible to achieve. There are two ways to demagnetize the Ferromagnetic material

① * High tunning of parameter is required



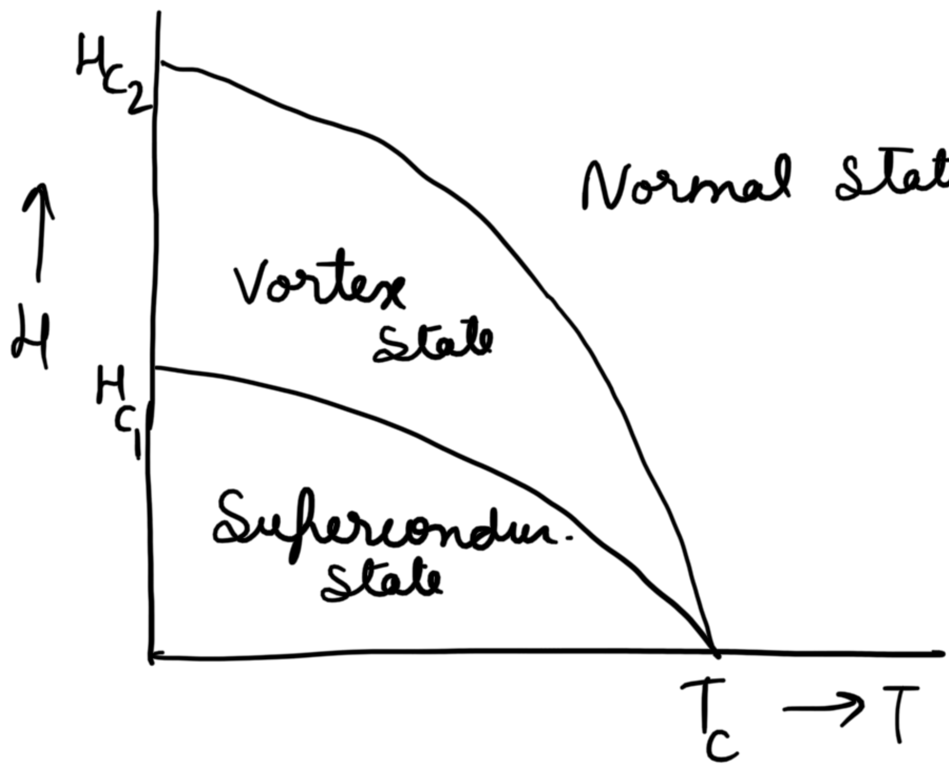
* Removal of H at Point b such that system recovers along bO

②



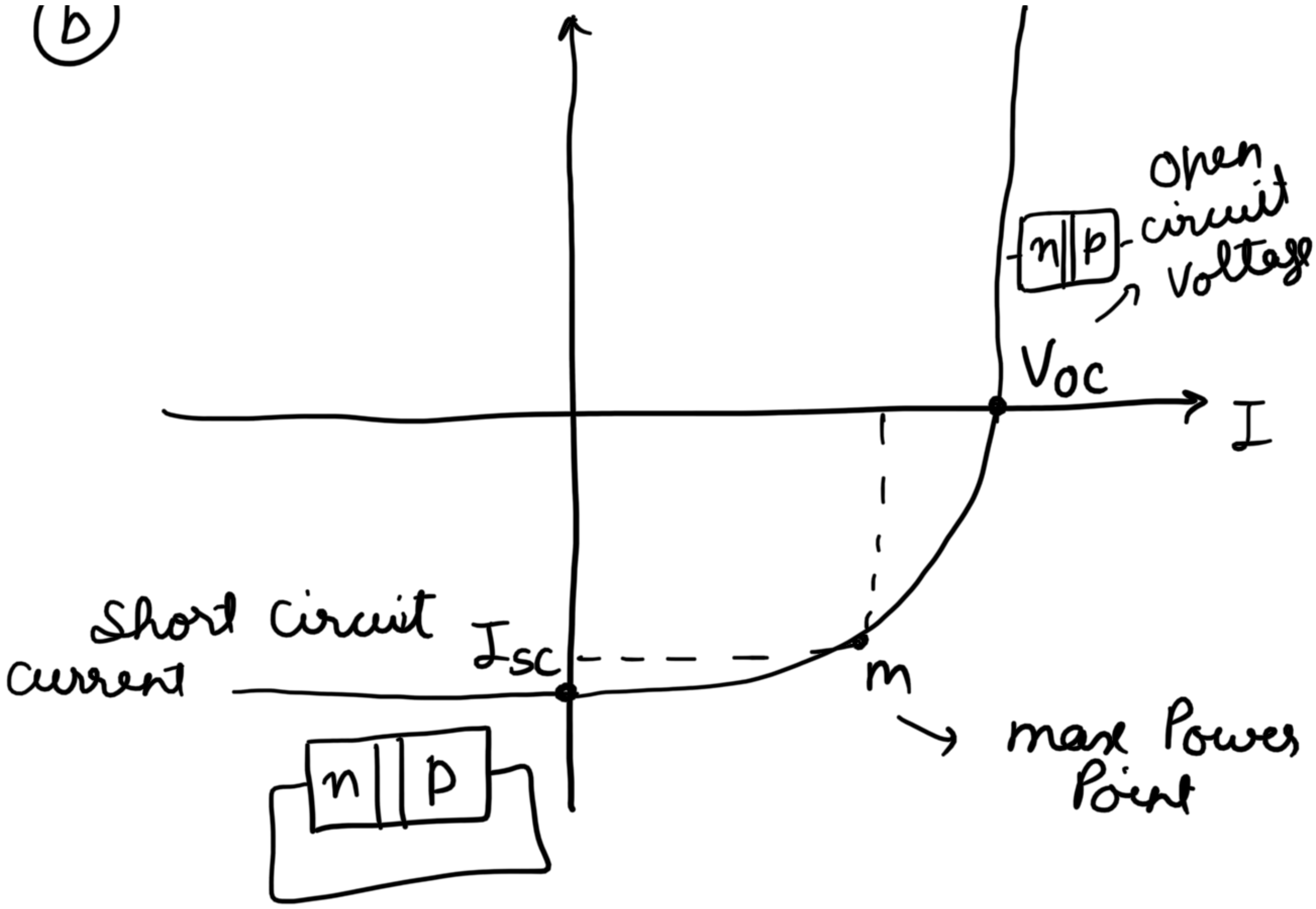
Demagnetizing by cycling the field magnetude decreasing & getting smaller hysteresis loop in each cycle. untill origin is reached

17) ⓐ



V

(b)



open circuit voltage
n
p
Voc
I
Short circuit current
Isc
m
max Power Point
n
p

1) $p = (5-2x)\,10^{10}\ cm^{-3}$

$\mu_n = 700\ cm^2/Vs$, $\mu_p = 500\ cm^2/Vs$.

a) Diffusion Current $\Rightarrow$ $J_{diff} = -qD_p \frac{dp}{dx}$.

$D_p = \frac{K_B T \mu_p}{q}$ $\longrightarrow$ From Einstein relation.

$\frac{dp}{dx} = -2\times10^{10}\ cm^{-4}$.

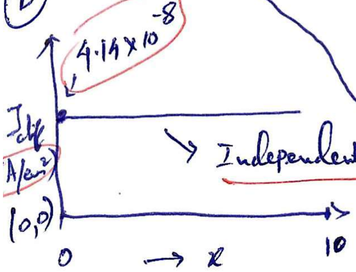
So, $J_{diff} = (-q)\frac{K_B T \mu_p}{q}\frac{dp}{dx} = +(K_B T\mu_p)\times 2\times10^{10}\ A/cm^2$

$= (1.38\times10^{-23})\times 300 \times 500 \times 2\times10^{10}$

$= 4.14\times10^{-8}\ A/cm^2$

0.5

b)



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c) Electric field $\Rightarrow$

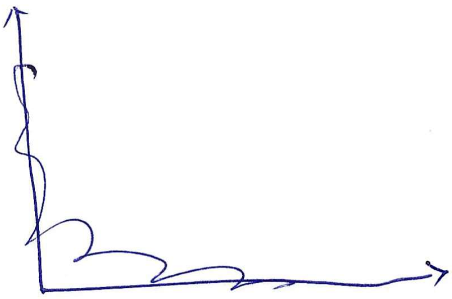
At thermal eqm: $J_{total} = 0 = J_{diff} + J_{drift}$.

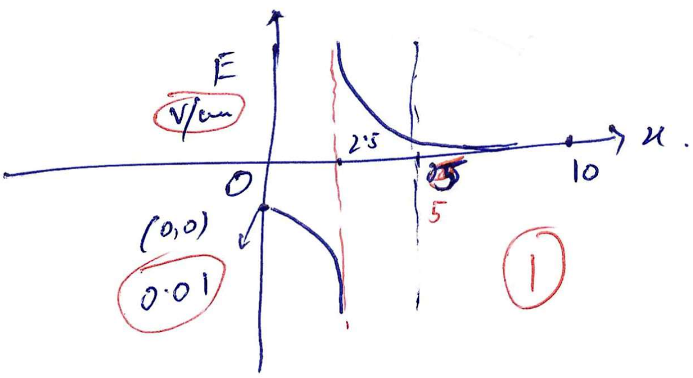
$J_{drift} = -J_{diff} = q\mu_p p E$.

or $E = \frac{-J_{diff}}{q\mu_p p} =$

0.5

$= \frac{-4.14\times10^{-8}}{1.6\times10^{-19}\times500\times(5-2x)10^{10}} = -\frac{0.052}{(5-2x)}\ V/cm$.





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