COL202 TUTORIAL 11

SUBMISSION FOR GROUP 2

PROBLEM 11.2

Jahnabi Roy
2022CS11094
8th November, 2023

1 Question 2

There is a subject—naturally not *Math for Computer Science*—in which 10% of the assigned problems contain errors. If you ask a Teaching Assistant (TA) whether a problem has an error, then they will answer correctly 80% of the time, regardless of whether or not a problem has an error. If you ask a lecturer, he will identify whether or not there is an error with only 75% accuracy.
We formulate this as an experiment of choosing one problem randomly and asking a particular TA and Lecturer about it. Define the following events:

$$\begin{aligned} \text{E} &::= [\text{the problem has an error}], \\ \text{T} &::= [\text{the TA says the problem has an error}], \\ \text{L} &::= [\text{the lecturer says the problem has an error}]. \end{aligned}$$

a) Translate the description above into a precise set of equations involving conditional probabilities among the events E, T and L.
We are given that 10% of the assigned problems have an error. So, on choosing a problem, it is 10% likely to be wrong. So, $Pr(E) = 0.1$. We can write the conditional probabilities among the events can be written as :
$Pr(T|E) = Pr(T'|E') = 0.8$ and $Pr(L|E) = Pr(L'|E') = 0.75$.

b) Suppose you have doubts about a problem and ask a TA about it, and they tell you that the problem is correct. To double-check, you ask a lecturer, who says that the problem has an error. Assuming that the correctness of the lecturer's answer and the TA's answer are independent of each other, regardless of whether there is an error, what is the probability that there is an error in the problem?.
Here, we want to calculate the following expression :
$Pr(E|T' \cap L)$
By Bayes' theorem :
$Pr(E|T' \cap L) = Pr(E) * (\frac{Pr(T' \cap L|E)}{Pr(T' \cap L)})$
Since we are to assume TAs correctness and lecturer's answer to be independent of each other, we can thus compute the following :
$Pr(T' \cap L|E) = Pr(T'|E)Pr(L|E) = 0.2 * 0.75 = 0.15$.
$Pr(T' \cap L)$ can be computed as the Pr(question being wrong and TA being correct and Lecturer being wrong) + Pr(question being correct and the TA being wrong and the lecturer being correct).
$Pr(T' \cap L)$
$= Pr(E)Pr(T' \cap L|E) + Pr(E')Pr(L \cap T'|E')$
$= 0.1 * 0.2 * 0.75 + 0.9 * 0.8 * 0.25 = 0.195$.
So, $Pr(E|T' \cap L)$
$= 0.1 * \frac{0.15}{0.195}$
$= 0.769$

c) Is event T independent of event L (that is, $Pr(L|T) = Pr(L)$)? First, give an argument based on intuition, and then calculate both probabilities to verify your intuition. Intuitively, the answer should be no since TAs are usually correct and can report a question as wrong but Lecturers are also usually correct so it increases the chances an error to be reported in the question. Also, TAs and Lecturers marking a question as wrong might overlap too more often than being separate.

Mathematically, we can do the following computation. We are given that 10% of the assigned problems have an error. So, on choosing a problem, it is 10% likely to be wrong. So, $Pr(E) = 0.1$. Since, the TA is correct 80% of the time, the probability that the TA says the problem is wrong would be the times when problem is actually wrong or when the problem is not wrong but he does not answer correctly. Let Pr(Y) be the probability the TA answers correctly. So, $Pr(Y) = 0.8$. Since, it is given in the question that Y and E are independent events, $Pr(Y \cap E)$ will be $Pr(Y)Pr(E)$. By this, we have, $Pr(T) = Pr(E)Pr(Y) + Pr(E')Pr(Y') = 0.1 * 0.8 + 0.9 * 0.2 = 0.26$. Similarly, since the lecturer is right 75% of the time, let $Pr(X)$ be the probability that Lecturer answers correctly. $Pr(X) = 0.75$. Since, it is given in the question that X and E are independent events, $Pr(X \cap E)$ will be $Pr(X)Pr(E)$. So, $Pr(L) = Pr(X)Pr(E) + Pr(X')Pr(E') = 0.1 * 0.75 + 0.9 * 0.25 = 0.3$.

$Pr(T) = 0.26$
$Pr(L) = 0.3$
$Pr(L \cap T) = Pr(L \cap T|E)Pr(E) + Pr(L \cap T|E')Pr(E')$
$= Pr(L|E)Pr(T|E)Pr(E) + Pr(L|E')Pr(T|E')Pr(E')$
$= 0.1 * 0.8 * 0.75 + 0.9 * 0.2 * 0.25$
$= 0.06 + 0.045$
$= 0.105.$

But, $Pr(L)Pr(T) = 0.26 * 0.3 = 0.078$.

Since, $Pr(L \cap T|L) \neq Pr(L)Pr(T) \implies Pr(L|T) \neq Pr(L)$, thus, L and T are not independent.