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2022CS11113
Group - 1

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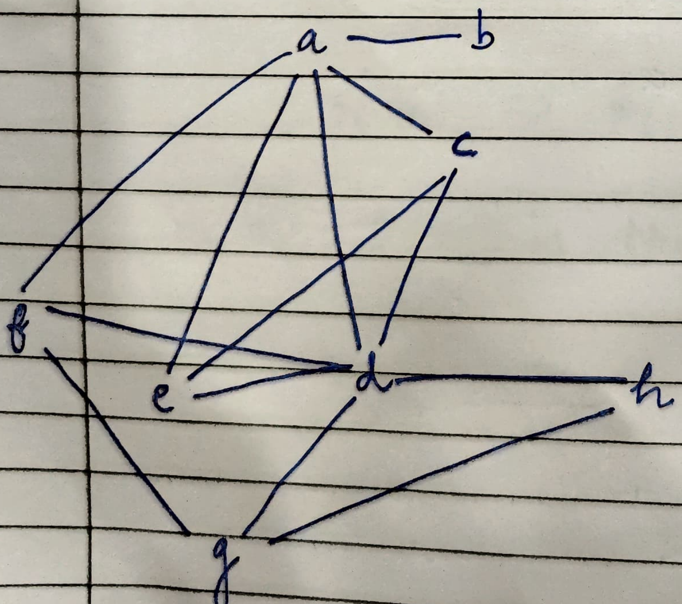
12.24

(a) Each vertex represents each variable.
Each edge between 2 vertices indicates that variables must be stored in different registers.

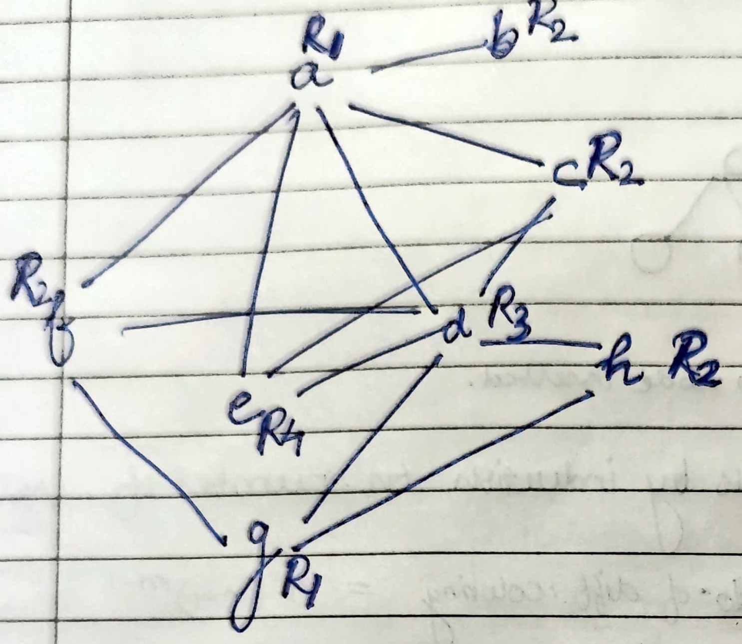
let us define: assignment ~~is~~ : The first appearance of a variable on the left side of an equation.

~~Output~~: Access: ~~The~~ ~~appe~~ If a variable appears on right side of an equation it is accessed.
Or it is an output ~~also~~ then also it is in access.
Duration: The duration of a variable is the segment of code extending from initial assignment and last access.
There is an edge between 2 variables if their duration overlaps.

This creates the following graph:



(b) One of the possible way of getting the graph colored with minimum colors is :-



min 4 registers are required.

(c) Each time a variable is reassigned we will consider it as a completely new variable.

for example: $t = r+s$
$u = t*3$
$t' = m-k$
$v = t'+u$
...

if reallocation of registers are allowed t' will be alloted the same register as t by our algorithm.

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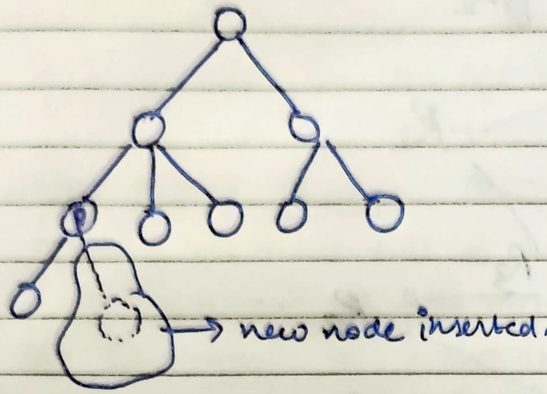
No. of colors available : n
No. of vertices = m



We will prove this by induction on number of vertices m.

$P(m)$: No. of diff. coloring $= n(n-1)^{m-1}$

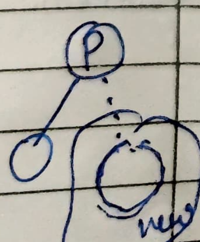
Base condition: $m=1$

No. of ways of diff. coloring $= n$
also $n(n-1)^{1-1} = n$

$\therefore$ $P(1)$ holds true.

Inductive step: Let us assume we have m nodes
We insert another node : $(m+1)^{th}$ node.

Assume $P(m)$ holds :
$\Rightarrow$ No. of diff coloring $= n\cdot(n-1)^{m-1}$
Now the new node that is inserted will have a parent node say P.



The new inserted node cannot have the same color as P.

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further, the new inserted node will have a single parent since it is a tree.

$\therefore$ no. of possible options for coloring the new node $= \underline{(n-1)}$

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No. of ways of coloring $(m+1)$ nodes

$=$ No. of ways of coloring m nodes

$\times$ no. of ways of coloring $(m+1)^{th}$ node.

$\Rightarrow$ No. of ways of coloring $(m+1)$ nodes $= n(n-1)^{m-1} \times (n-1)$

$= n(n-1)^{m}$

$\therefore$ $P(m+1)$: No. of ways of coloring $(m+1)$ nodes $= n(n-1)^{m}$ holds true.

$P(m) \Rightarrow P(m+1)$

$\therefore$ $P(m)$ is true $\forall$ $m \in \mathbb{N}$.