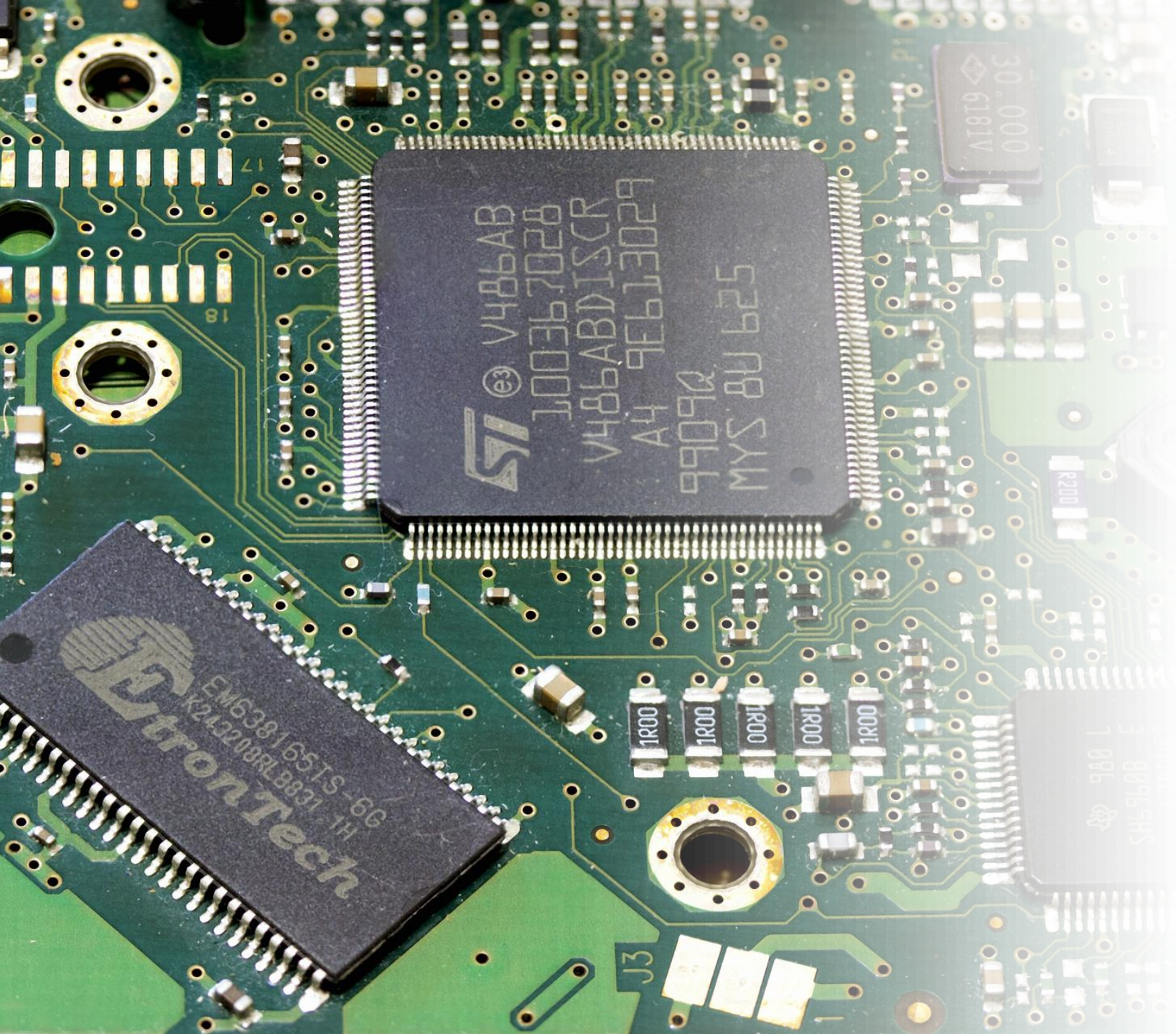

Digital Logic and System Design

4. Gate-level Minimisation

COL215, I Semester 2023-2024
Venue: LHC 111
'E' Slot: Tue, Wed, Fri 10:00-11:00

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Optimising a Gate-level Implementation

- Improving a gate-level design
- Objective?
  - Area
  - Delay
  - Power/Energy
  - Temperature
  - Testability/Reliability/Security/...
- Smaller is better
  - Always?

Literal Count

- Optimal circuit: **minimum number of literals**
- Focus on **Area**
  - **#literals** proportional to **#transistors**
- Simplification in using Literals:
  - Ignoring complement operations
  - Ignoring wires
  - Ignoring blank spaces
  - Ignoring transistor dimensions

Circular Design Flow

- Literal Count useful in spite of limitations
  - Fast decisions
  - Don’t need to generate full gate-level circuit, physical layout
- Gate-level decision
  - depends on geometry
  - ...which depends on gate-level decision 
- **Fast estimates** are critical!

Logic minimisation

- Optimization by applying Boolean Algebra theorems
  - Problem: in what order?
- **Map** method (**Karnaugh Map** or **K-Map**)
  - Pictorial representation of Truth Table
  - **2^n squares/cells** for n-variable function
    - one square for each input combination (truth table row)
    - value = 1 if corresponding minterm included in function
    - otherwise, value = 0

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Truth Table

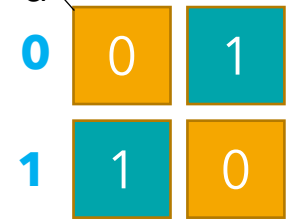
0	1	1	0
1	0	0	1

Map of function F

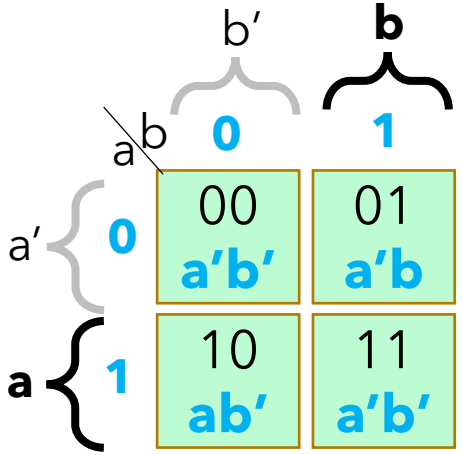
2-variable Karnaugh Map

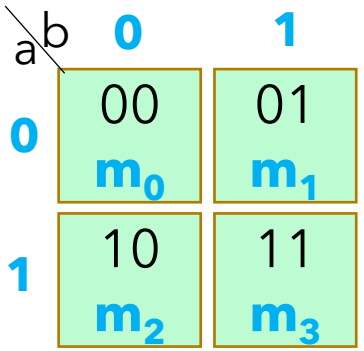
a	b	F
0	0	0
0	1	1
1	0	1
0	1	0

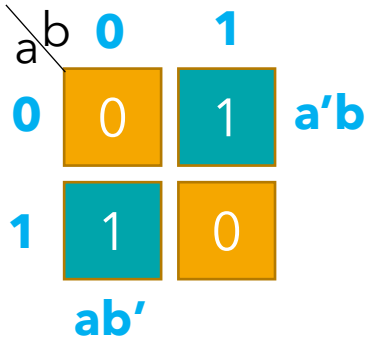
Truth Table



Karnaugh Map of F





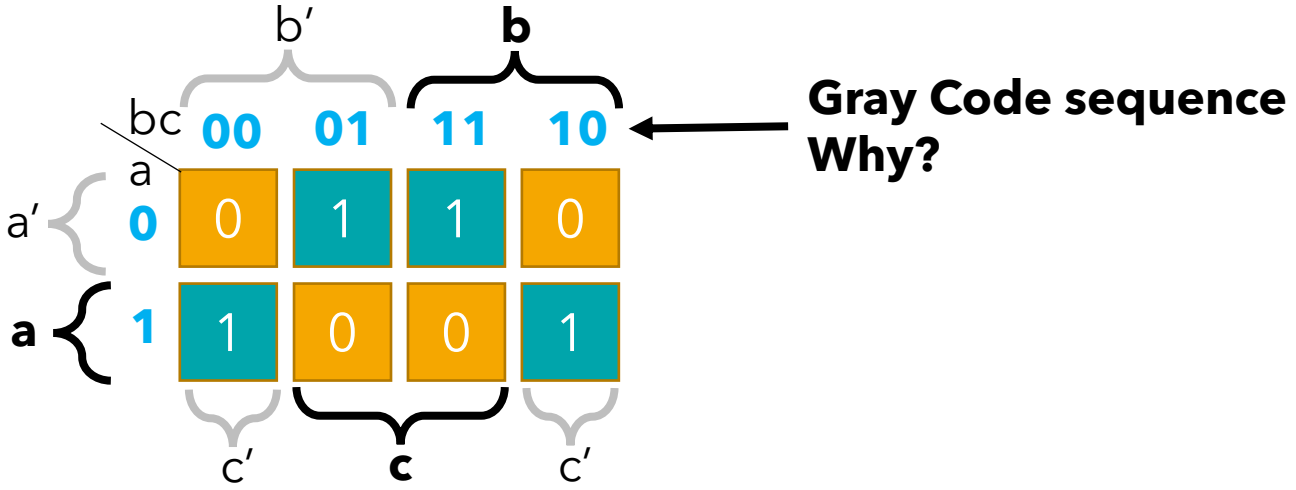


F = a′b + ab′

3-variable Karnaugh Map

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Truth Table

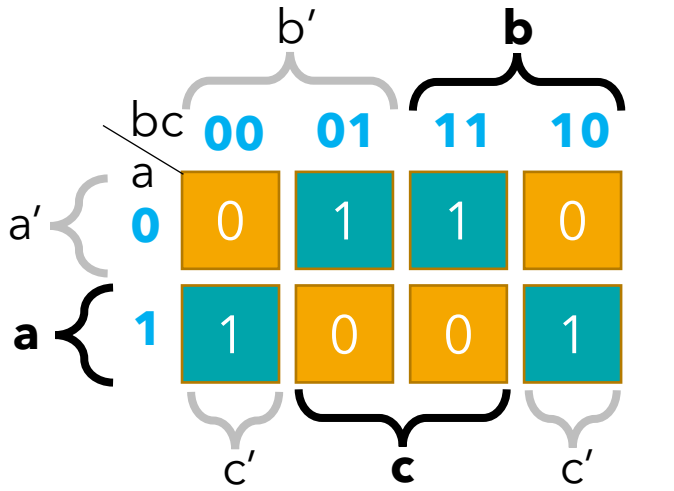


F = a'b'c + a'bc + ab'c' + abc'

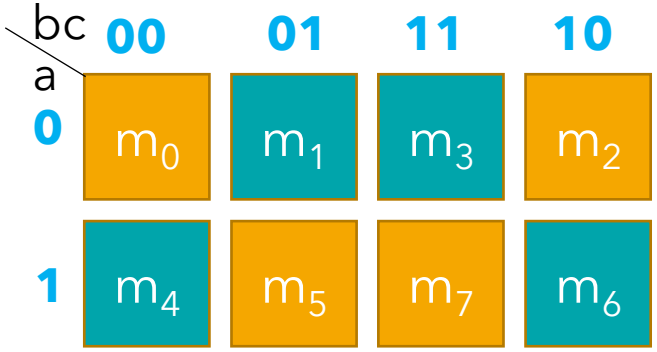
3-variable Karnaugh Map

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Truth Table

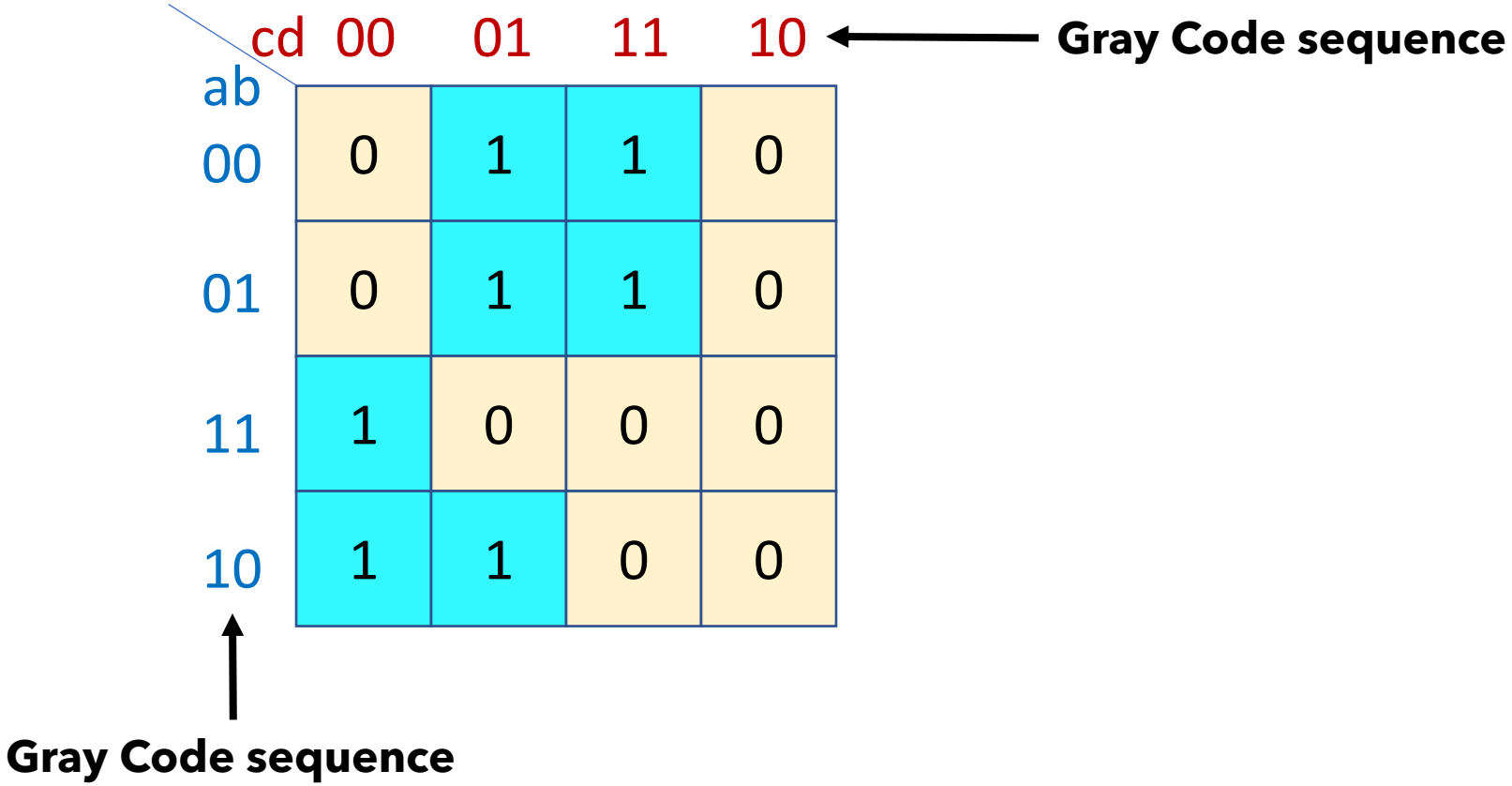


F = a′b′c + a′bc + ab′c′ + abc′

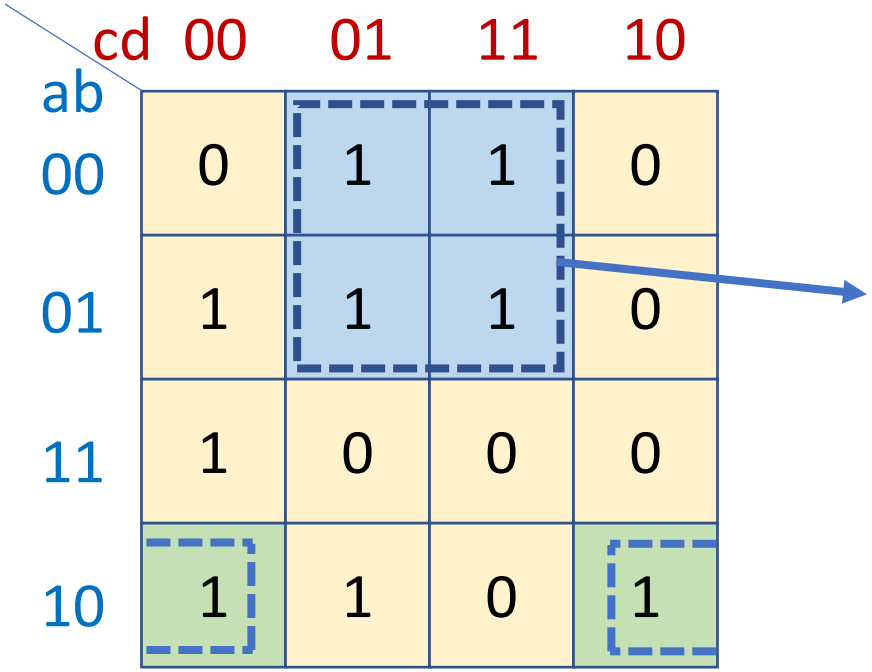


F = $m_1 + m_3 + m_4 + m_6$
= ∑ (1,3,4,6)

4-variable Karnaugh Map



Grouping of cells in K-Map



a'b'c'd + a'b'cd + a'bc'd + a'bcd = a'd

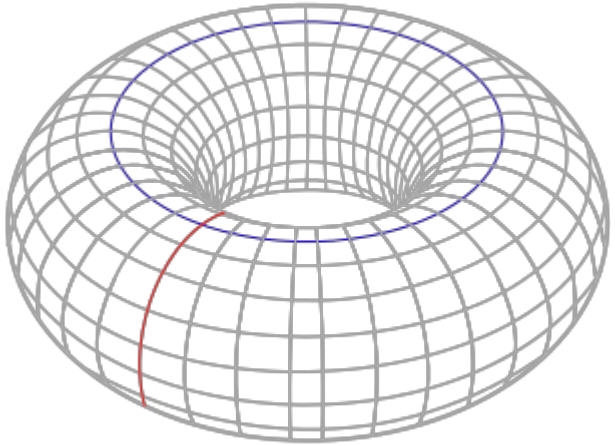
Group represents **term**
Larger group means **smaller term**

Wrapping around

ab \ cd	00	01	11	10
00	0	1	1	0
01	1	1	1	0
11	1	0	0	0
10	1	1	0	1

Grouped cells can wrap around the map border

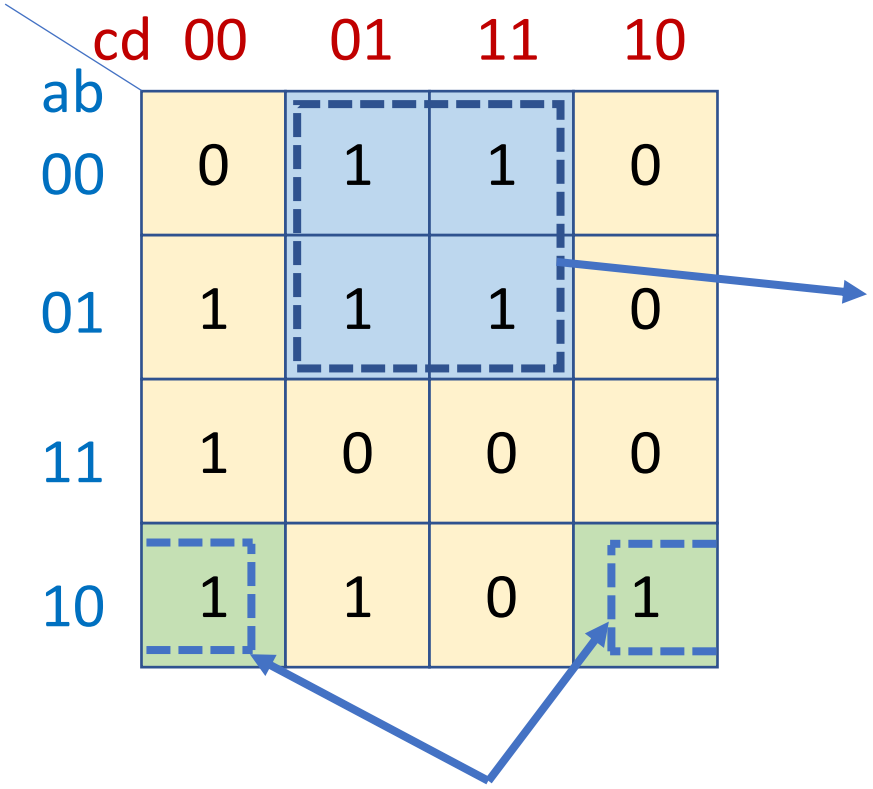
ab'c'd' + ab'cd' = ab'd'



Groups/Regions can wrap around, as in a **Torus**

[Figure Source: Wikimedia https://commons.wikimedia.org/wiki/File:Simple_torus_with_cycles.svg]

Cell grouping: minimizing Boolean expressions



REPLACE: a'b'c'd + a'b'cd + a'bc'd + a'bcd

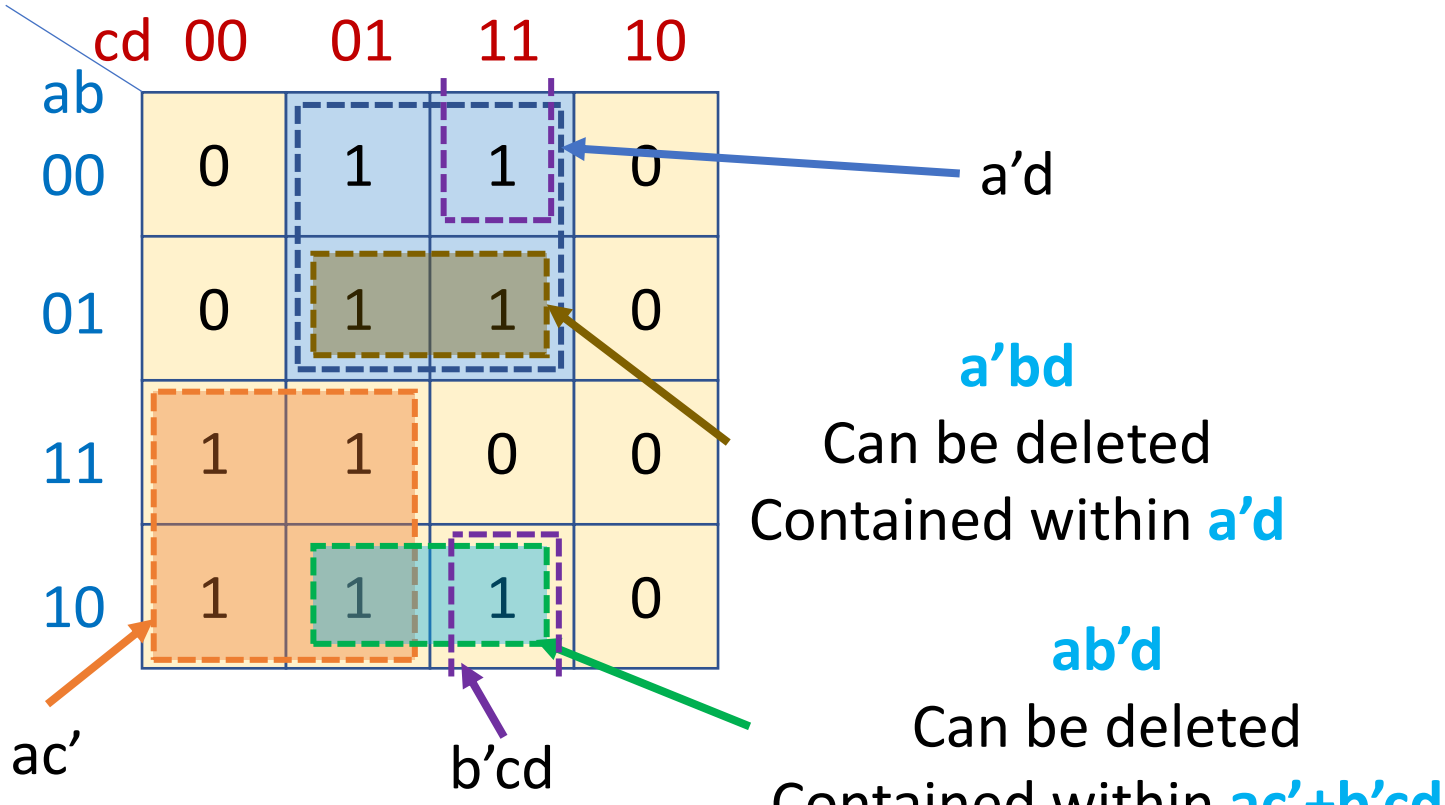
BY: a'd

Reduces literal count by dropping variables from terms

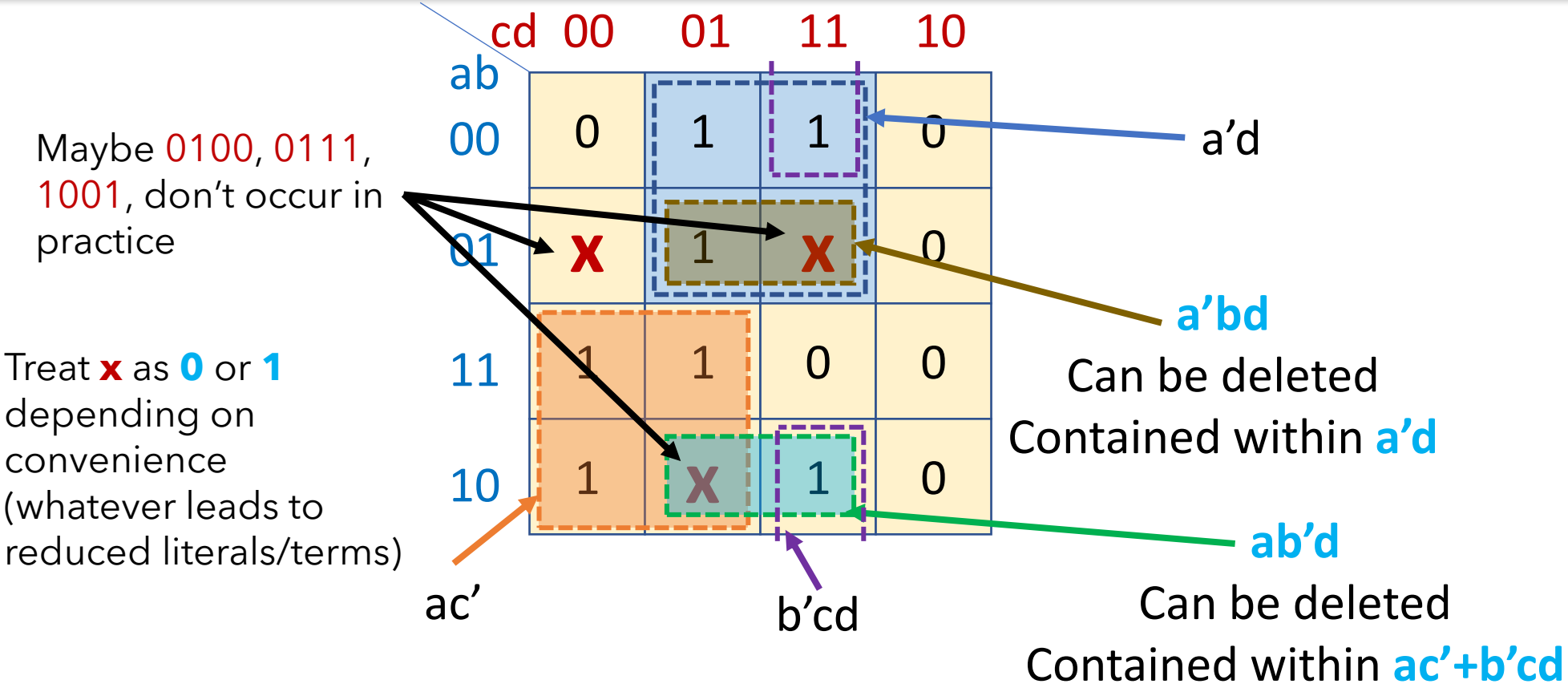
ab'c'd' + ab'cd' = ab'd'

Deleting redundant groups

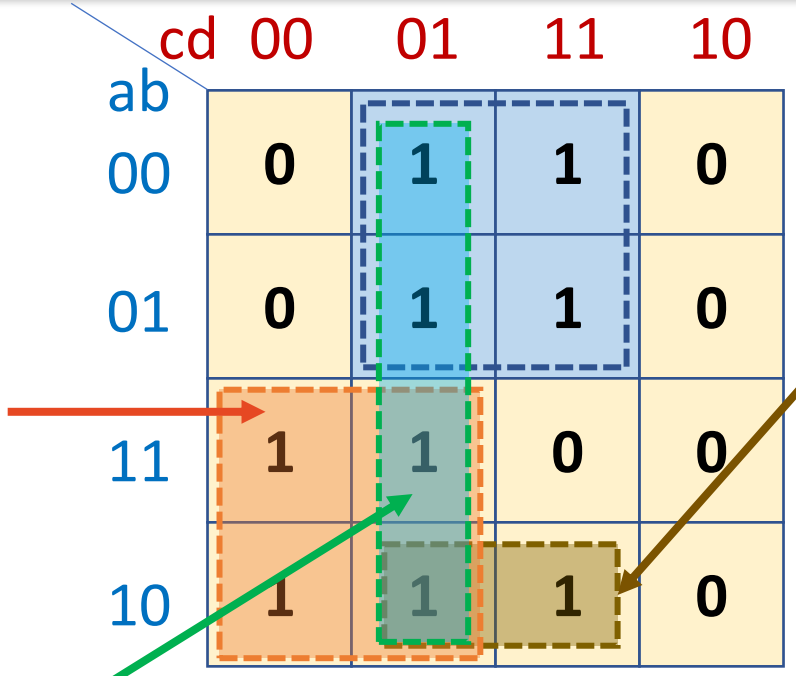
Second optimization: dropping terms



Don’t Care: Input combination ignored



Terminology: Implicants

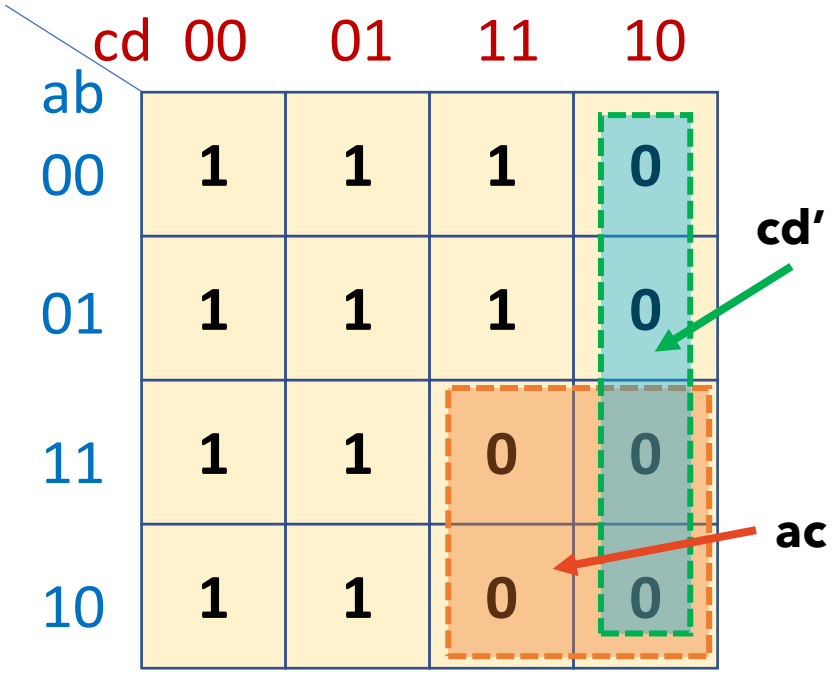


Essential Prime Implicant
Must be contained in any optimal covering of function

Implicant
If term is 1, then function is 1

Prime Implicant
Not contained in any other implicant

Product of Sums Simplification

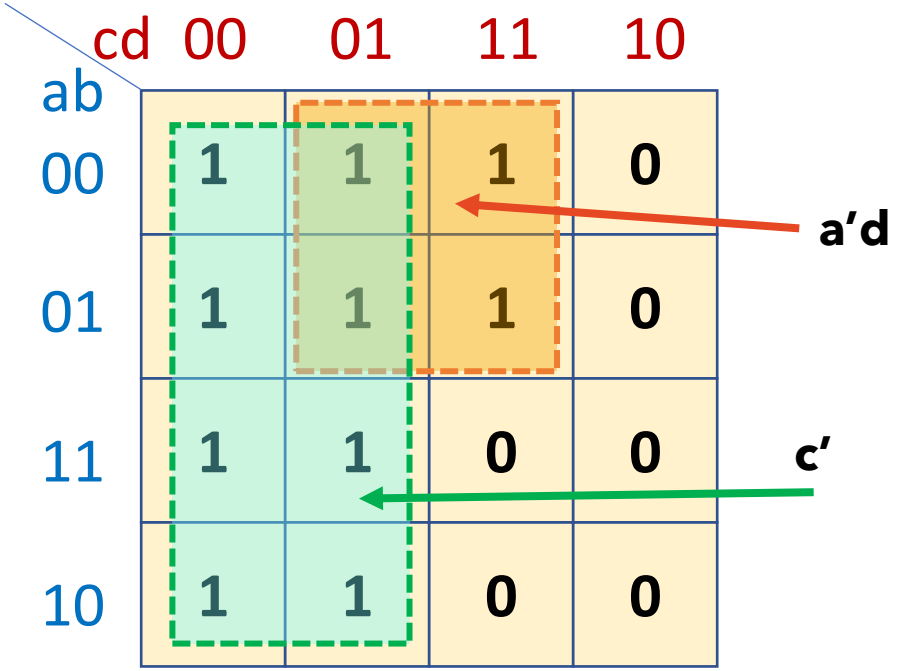


F' = ac + cd'

F = (c' + a')(c' + d)

= c' + a'd

Should be able to read this directly from the K-map



Confirm:

F = c' + a'd

The Exclusive-OR (XOR) Function

- XOR: $x \oplus y = xy' + x'y$
- $x \oplus 0 = ?$
- $x \oplus 1 = ?$
- $x \oplus x = ?$
- $x \oplus x' = ?$
- Commutative: $x \oplus y = y \oplus x$
- Associative: $(x \oplus y) \oplus z = x \oplus (y \oplus z)$

XOR as ODD Function

- XOR is 1 if there are an **ODD number of 1s** in input variables
- Could be used for **Parity Function**
  - **generation** (add Parity bit)
  - **checking** (even #1s including Parity bit)

ab \ cd	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$a \oplus b \oplus c \oplus d$

XOR with NAND gates