# 2301 COL 202 Quiz 1

Abhinav Rajesh Shripad

TOTAL POINTS

**11.5 / 12**

QUESTION 1

*1* Predicate Formula **4 / 4**

✓ **- 0 pts** *Correct*

**- 0.5 pts** Minor mistake in part (a)

**- 1 pts** Partially correct in part (a)

**- 2 pts** Incomplete/Incorrect part (a)

Most common mistake:
Arguing existence of distinct $$x,s_1,s_2,\ldots s_k$$ for some $$k \leq n$$, such that $$ \land^{k}_{i = 1} E(x,s_i)$$ and $$x \neq s_i, \forall i \in [n]$$ is incorrect.
Even if everyone emails more than n people, for k=1, this formula will become true, which is incorrect.

**- 0.5 pts** Minor mistake in part (b)

**- 1 pts** Partially correct in part (b)

**- 2 pts** Incomplete/Incorrect part (b)

QUESTION 2

*2* WOP **3.5 / 4**

✓ **+ 1.5 pts** *WOP correctly stated*

✓ **+ 2.5 pts** *Correct Proof*

**+ 0 pts** Wrong Proof

✓ **- 0.5 pts** *Minor Mistakes*

1. This should be $$9,10,11 \notin S$$

2. The actual contradiction would be that $$n \in S$$

QUESTION 3

*3* Induction **4 / 4**

✓ **+ 1 pts** *Base Case*

✓ **+ 1 pts** *Induction Hypothesis*

✓ **+ 2 pts** *Correct Induction*

**+ 0 pts** Wrong solution

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**(COL 202)** Discrete Mathematics

18 August, 2023

Quiz 1

**Duration: 45 minutes** **(12 marks)**

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- Be clear in your writing.
- If you use a statement proved in class or in the problem set, then write down the entire statement before using it.
- **You will not get a new sheet, so make sure you are certain when you write something.** Make a judicious decision of which tool(s) to use to get a clean and short answer that fits in the space.

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1. **($2 \times 2 = 4$ points)** Translate the following sentences into a predicate formula. The domain of discourse should be the set of students in the class; in addition, the only predicates that you may use are (1) Equality, (2) $E(x, y)$ meaning that "x has sent e-mail to y."

   (a) There is a student who has e-mailed *at most n* other people in the class, besides possibly himself.

   (b) There is a student who has emailed *at least n* other people in the class, besides possibly himself.

Let $S$ be the set of students.

(a) $\exists x \in S$ such that $\forall$ tuples $(x_1, x_2 \cdots, x_{n+1})$ with $x_i \in S$ $(E(x,x_1) \wedge E(x,x_2) \cdots \wedge E(x, x_{n+1}))$ is always false

(b) $\exists$ ~~x~~, $(x_1, x_2, x_3 \cdots x_n) \in S$ and $S^n$ respectively such that $(E(x,x_1) \wedge E(x,x_2) \cdots \wedge E(x,x_n))$ is true

2. **(4 points)** Use the Well Ordering Principle to prove that any integer greater than or equal to 8 can be represented as the sum of nonnegative integer multiples of 3 and 5.

Let S be set of integers greater than <u>OR</u> equal to 8 which cannot be represested as sum of 3's multiple and 5 multiple.

For sake of contradiction, assume S is non-empty. $\rightarrow |S| \neq 0$

Observe that $8 = 5\times1 + 3\times1 \rightarrow 8 \notin S$

Illy $9 = 3\times3 + 5\times0 \rightarrow 9 \notin S$

Illy $10 = 3\times0 + 5\times2 \rightarrow 10 \notin S$

Illy $11 = 3\times2 + 5\times1 \rightarrow 11 \notin S$

Since S is non-empty sub set of positive integers.

$\rightarrow \exists$ a smallest element $n \in S$, which is not sum of 3 and 5 multiples.

Observe that $n > 11$

Since $n-3 > 8$ and $n-3 < n$,

$n-3 = 3\times a + 5\times b$ for some $a, b \in \mathbb{Z}^+$

by <u>minimality</u> of n

$\rightarrow n = 3\times(a+1) + 5\times b$, and $a+1 \in \mathbb{Z}^+$ $(a \in \mathbb{Z}^+)$

and $b \in \mathbb{Z}^+$

contradiction $\rightarrow$ n is not smallest

$\rightarrow$ S is empty

$\rightarrow \forall n \geq 8, \in \mathbb{Z}^+$ can be respresented as sum of 3 and 5 multiple

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3. **(4 points)** Prove by induction that

$$\sqrt{1\sqrt{2\sqrt{3\ldots\sqrt{n}}}} < 2$$

**Claim:-** For all integer $n, m \in \mathbb{N}$

$\forall m \in \mathbb{N}$ $\sqrt{n\sqrt{(n+1)\sqrt{\cdots\sqrt{(n+m)}}}} < n+1$

**Proof:-** we prove this by induction on m.

For $m=1$

$\rightarrow \sqrt{n\sqrt{(n+1)}} = n^{1/2}(n+1)^{1/4} < (n+1)^{1/2}(n+1)^{1/2}$

$= n+1$

Let the predicate of claim be $P(n,m)$.

$\rightarrow P(n,1)$ is tautology $\forall n$

Inductive hypothesis:- Let $P(n,m)$ be tautology $\forall n$, and $m = 1, 2, 3 \cdots m_0-1$

Inductive Step:-

$$\sqrt{n\underbrace{\sqrt{(n+1)\sqrt{\cdots\sqrt{n+m_0}}}}_{\wedge\; n+2}} < \sqrt{n(n+2)} < \sqrt{n^2+2n+1} = n+1$$

$n+2$ by $P(n+1, m_0-1)$ which is true by inductive hypothesis

Hence Claim is proved

$\rightarrow$ $P(n, m)$ is tautology

$\rightarrow$ $P(1, n-1)$ is true

$\rightarrow$ $\sqrt{1\sqrt{2\sqrt{3 \cdots \sqrt{n}}}} < 2$

Hence Proved