

# 2301 COL 202 Tutorial 12.3

Abhinav Rajesh Shripad

TOTAL POINTS

**2 / 2**

QUESTION 1

1 Problem for Group 3 **2 / 2**

✓ - **0 pts** Correct

- **2 pts** Incorrect

# COL 202 Assignment 12

Abhinav Shripad(2022CS11596)

November 2023

## 1 Problem Statement

We are given a random vector of  $n$  distinct numbers. We then determine the maximum of these numbers using the following procedure: Pick the first number. Call it the current maximum. Go through the rest of the vector (in order) and each time we come across a number (call it  $x$ ) that exceeds our current maximum, we update the current maximum with  $x$ . What is the expected number of times we update the current maximum?

## 2 Solution

Let  $T_n$  denote the number of updates we need. We proceed by recursion on the sequence. Consider the sequence  $a_i$   $i = 1, 2, \dots, n$ . Thus the sub-sequence from  $a_1$  to  $a_{n-1}$  also has  $n - 1$  distinct numbers. Thus it has  $T_{n-1}$  expected number of updates. Now the last number will be counted as an update if and only if it is the largest of the sequence till now. It has probability  $\frac{1}{n}$ . We can see it as if a random de-arrangement of  $n$  numbers, the probability that largest will come at last will be  $\frac{(n-1)!}{n!} = \frac{1}{n}$ . Thus we get the recurrence

$$T_n = T_{n-1} + \frac{1}{n}$$

. Now we can easily see that  $T_2 = \frac{1}{2}$  because the second number will be maximum with chances 0.5 (we don't count initialization of max to the first element). Thus we can easily see that

$$T_n = \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

or  $T_n = H_n - 1$  where  $H_n$  is the  $n^{th}$  harmonic number.

1 Problem for Group 3 2 / 2

✓ - 0 pts Correct

- 2 pts Incorrect