COL 202 Discrete Mathematics **Diwali 2023**

Tutorial 3

1. [Submission Problem for Group 1] Use the Well Ordering Principle to prove that there is no solution over the positive integers for the following equation:

$$4a^3 + 2b^3 = c^3$$

2. [Submission Problem for Group 2] Recall the stacking puzzle we encountered in class (see Section 5.2.5 in LLM Book).

   Define the potential $p(S)$ of a stack of blocks S to be $k(k-1)/2$ where k is the number of blocks in S. Define the potential $p(A)$ of a set of stacks A to be the sum of the potentials of the stacks in A. Generalize Theorem 5.2.1 in the LLM Book about scores in the stacking game to show that for any set of stacks A if a sequence of moves starting with A leads to another set of stacks B then $p(A) \geq p(B)$, and the score for this sequence of moves is $p(A) - p(B)$.

3. [Submission Problem for Group 3] Use strong induction to prove that $n \leq 3^{n/3}$ for every integer $n \geq 0$.

4. [Submission Problem for Group 4] Use the Well Ordering Principle to prove that any integer greater than or equal to 8 can be represented as the sum of nonnegative integer multiples of 3 and 5.

5. [Bonus] Use the Well Ordering Principle to prove that any integer greater than or equal to 8 can be represented as the sum of nonnegative integer multiples of 3 and 5.

6. [Bonus] Find what is wrong with these bogus proofs given in LLM Book: Problem 2.2, 2.3, 5.26.

7. [Bonus] Problems 2.11, 2.12, 5.9, 5.13, 5.20, 5.23 from the LLM Book.

$n \leq 3^{n/3}$

$\underline{\underline{0, 1, 2, 3}}$

we know that $P(n)$ is true for $n = 0, 1, 2, \ldots k-1$

For $n = k$

Case 1 : k is even $= 2m$ (say) , $m \geq 2$

$\rightarrow \quad 3^{2m/3} = (3^{m/3})^2 \geq (m)^2 \geq 2m$

Case 2 : k is odd $= 2m+1$ (say) , $m \geq 2$

$\rightarrow \quad 3^{\frac{m+1}{3} + \frac{m}{3}} \geq (m+1)(m) \geq 2m+1$

$m^2 - m - 1 \geq 0$