2301 COL 202 Tutorial 8.4

Anubhav Pandey

TOTAL POINTS

2 / 2

QUESTION 1

1 Problem for group 4 **2 / 2**

✓ **- 0 pts** *Correct*

- 2 pts Incorrect

- 1 pts Partly correct

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Tut 8 Problem 4

(a) $$\sum_{K=0}^{n} \binom{n}{K}^2 = \binom{2n}{n}$$

Combinatorial logic :

Let there be $2n$ objects kept in 2 boxes (n objects in each box) and we are to select n out of these $2n$ objects {$\binom{2n}{n}$ ways} which is equivalent to chose K objects from one box and $n-k$ from other, where $K = 0, 1, 2, \ldots, n$.

$$\therefore \binom{2n}{n} = \sum_{K=0}^{n} \binom{n}{K} \cdot \binom{n}{n-K}$$

but $$\binom{n}{K} = \frac{n!}{K!(n-K)!} = \binom{n}{n-K}$$

$$\boxed{\binom{2n}{n} = \sum_{K=0}^{n} \binom{n}{K}^2}$$

Algebraic Proof:

$$\binom{2n}{n} = \underbrace{[x^n]\ (1+x)^{2n}}_{\hookrightarrow \text{ coefficient of } x^n \text{ in } (1+x)^{2n}}$$

$$= [x^n]\ (1+x)^n \cdot (1+x^n)$$

$$= \sum_{k=0}^{n} \Big([x^k]\ (1+x)^n\Big)\Big([x^{n-k}]\ (1+x)^n\Big)$$

$$= \sum_{k=0}^{n} \binom{n}{k} \cdot \binom{n}{n-k}$$

$$= \sum_{k=0}^{n} \binom{n}{k}^2$$

(b) $\boxed{C_n = n^2}$

consider a generating function $A(x)$.

$$A(x) = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

$$B(x) = 1 + 2x + 3x^2 + 4x^3 + \cdots = \frac{1}{(1-x)^2}$$

$$M(x) = x\,B(x) = x + 2x^2 + 3x^3 + 4x^4 + \cdots$$

$\Rightarrow \quad M'(x) = 1 + 2^2 x + 3^2 x^2 + 4^2 x^3 + \cdots$

$= B(x) + x B'(x)$

$$C(x) = x M'(x)$$
$$= x B(x) + x^2 B'(x)$$

$$\boxed{C(x) = \frac{x}{(1-x)^2} + x^2 \cdot \frac{2}{(1-x)^3}}$$

$c_n = n^2$ has generating function $C(x)$ ✓

$$C(x) = \sum k^2 \cdot x^k$$

1 Problem for group 4 2 / 2

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