COL202 TUTORIAL 3

SUBMISSION FOR GROUP 2

PROBLEM 3.2

Jahnabi Roy
2022CS11094
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1 Question 2

Recall the stacking puzzle we encountered in class (see Section 5.2.5 in LLM Book).
Define the potential p(S) of a stack of blocks S to be $k(k-1)/2$ where k is the number of blocks in S. Define the potential p(A) of a set of stacks A to be the sum of the potentials of the stacks in A. Generalize Theorem 5.2.1 in the LLM Book about scores in the stacking game to show that for any set of stacks A if a sequence of moves starting with A leads to another set of stacks B then $p(A) \geq p(B)$, and the score for this sequence of moves is $p(A) - p(B)$.

Proof. **1.1**

> **Induction Hypothesis 1.1.** Set of stack B can be achieved from stack A in n steps, where $n \geq 0$ and the score for this sequence of moves is $p(B) - p(A)$.

1.1.1 Base Case:

If n = 0, then set A will be equal to set B. So, $A = B$. So, score will be $0 = p(B) - p(A)$. So, our Induction Hypothesis holds for the base case.

1.1.2 Inductive Step:

Let P : Induction Hypothesis.

We assume $\forall m \geq 0$, $P(m)$ holds. In this step, we prove, $P(m) \Rightarrow P(m+1)$. So, for $P(m+1)$, we say that it takes $m+1$ moves to go from set A to set B. But since, we already know that B is reachable in m moves too. So, set A reaches another set A' and then goes to set B, that is, $A \to A' \to B$ where $A \to A'$ occurs in 1 step and $A' \to B$ occurs in m steps. Since $P(m)$ holds, score for transition from A' to B will be $p(B) - p(A')$.

Calculating for transition from $p(A)$ to $p(A')$. Let's say a stack of k blocks is unstacked into k_1 and k_2. So, $k = k_1 + k_2$. So, in set A, $p(A) = (k_1 + k_2)(k_1 + k_2 - 1)/2 + a_0$ where a_0 is the sum of potentials of the other set of stacks. In set A', $p(A') = k_1(k_1 - 1)/2 + k_2(k_2 - 1)/2 + a_0$. The score of this move, according to rules would be $= k_1k_2 = p(A') - p(A)$.

So, total score in $m + 1$ moves becomes $= p(B) - p(A') + p(A') - p(A) = p(B) - p(A)$.

And $p(B) \geq p(A')$ and $p(A') \geq p(A)$, so, we get $p(B) \geq p(A)$.

So, $P(m + 1)$ holds true too. So, we have proved $P(m) \Rightarrow P(m + 1)$, thus our Induction Hypothesis holds true.

Hence, proved. □