2301 COL 202 Tutorial 12.2

Jahnabi Roy

TOTAL POINTS

2 / 2

QUESTION 1

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✓ **- 0 pts** *Correct*

1 Question 2

A *literal* is a propositional variable or its negation. A k-clause is an OR of k literals, with no variable occurring more than once in the clause. For example,

P OR Q' OR R' OR V;

is a 4-clause, but

V OR Q' OR X' OR V';

is not, since V appears twice. Let S be a set of n distinct k-clauses involving v variables. The variables in different k-clauses may overlap or be completely different, so $k \le v \le nk$. A random assignment of true/false values will be made independently to each of the v variables, with true and false assignments equally likely. Write formulas in n, k and v in answer to the first two parts below.

(a) What is the probability that any particular k-clause in S is true under the random assignment?
Every k-clause has v variables. And each variable in the k-clause can take either true or false value. The k-clause will be true if at least one of the values is true. So, it becomes false when all the variables are false. The probability of all the variables to be false is $\frac{1}{2^k}$. So, probability of k-clause being true becomes $1 - \frac{1}{2^k}$.

(b) What is the expected number of true k-clauses in S?
Let T be the random variable which denotes the number of true k-clauses in set S. Then to calculate the expectation value of T, if $f(t) = 1 - \frac{1}{2^k}$, then $E[T] = \Sigma_{i=1}^{n} t * f(t) = \frac{n(n+1)}{2} * (1 - \frac{1}{2^k})$.

(c) A set of propositions is *satisfiable* iff there is an assignment to the variables that makes all of the propositions true. Use your answer to part (b) to prove that if n ¡ 2k, then S is *satisfiable*. .
It is clear that for a set to be *satisfiable* here, the above expectation value should be equal to n. Only then can we expect to get a *satisfiable* set. So, doing the same, we get,
$E[T] = \Sigma_{i=1}^{n} t * f(t) = \frac{n(n+1)}{2} * (1 - \frac{1}{2^k}) = n.$
$=> \frac{n+1*(1-\frac{1}{2^k})}{2} = 1$
$=> 1 - \frac{1}{2^k} = \frac{2}{n+1}$
note that $k \ge 2$ and $\frac{1}{2^{k-1}} < 1 - \frac{1}{2^k}$
$=> \frac{1}{2^{k-1}} < \frac{2}{n+1}$
$=> \frac{1}{2^k} < \frac{1}{n+1}$
$=> n + 1 < 2^k$
$=> n < n + 1 < 2^k$
$=> n < 2^k$

So, if we have $n < 2^k$, then we know that S is satisfiable.

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✓ **- 0 pts** *Correct*

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SUBMISSION FOR GROUP 2

PROBLEM 12.2

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2022CS11094
16th November, 2023