# 2301 COL 202 Tutorial 5.4

Anubhav Pandey

TOTAL POINTS

## 2 / 2

QUESTION 1

*1* Problem for Group 4 **2 / 2**

✓ **+ 2 pts** *Correct*

**+ 1 pts** Partially Correct

**+ 0 pts** Wrong

# COL 202 TUTORIAL

## Tutorial 5, Group 4

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**SOLUTION : Problem 4**

Let U be a set of all finite ordered subsets of positive integers.
Consider sequence of prime numbers
$p_1 = 2$,
$p_2 = 3$,
$p_3 = 5$,
$p_4 = 7$.... and so on
let,
$S = \{a_1, a_2, a_3, a_4, .....a_k\}$ be an ordered subset of natural numbers, such that S $\subseteq$ U.

Consider $X_S = p_1^{a_1} * p_2^{a_2} * ......p_k^{a_k}$

Claim 1 : $X_S$ is a function , such that $X_S : S- > N$
Proof : Suppose there is another set $S_1$ such that $X_{S_1} = X_S$
Which means that prime factorisation of both $X_S$ and $X_{S_1}$ is same, which means both S and $S_1$ has exactly K elements, and if we compare the power of prime numbers in prime factorization of $X_S$ and $X_{S_1}$ then we'll get that each element of both sets are equal that to in order.
Therefore $X_S$ is injective.

Claim 2 : $X_S$ is surjective.
Every Natural number except 1 can be written as product of prime numbers(it's prime factors) and $X_S = 1$ when $S = \phi$.
Which means $X_S$ transverses whole N after considering all possible $S \subseteq U$
Therefore $X_S$ is surjective.

Since there is both injective and surjective map, therefore it's a bijection , therefore set U of all possible countable subsets of natural numbers is also countable

□ **QED**

## *1* Problem for Group 4 **2 / 2**

- ✓ **+ 2 pts** *Correct*
- **+ 1 pts** Partially Correct
- **+ 0 pts** Wrong