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2022CS11113

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(1) (a)

algebrically:

$$\binom{n}{m}\binom{m}{k} = \frac{n!}{\cancel{m!}(n-m)!}\frac{\cancel{m!}}{k!(m-k)!}$$

$$= \frac{n!}{(n-m)!\,k!\,(m-k)!} \quad -①$$

$$\binom{n}{k}\binom{n-k}{m-k} = \frac{n!}{k!\,\cancel{(n-k)!}} \times \frac{\cancel{(n-k)!}}{(m-k)!\,(n-m)!}$$

$$= \frac{n!}{(n-m)!\,(m-k)!\,k!} \quad -②$$

we can see ① = ②

combinatorically:

Suppose there are n objects.
We need to pick K and $m-k$ objects out of n objects.

~~Ca~~ Method 1: Choose m objects first
Then choose K objects from these m objects
so you end up with m objects and $m-k$ objects.

No. of ways $= \binom{n}{m} \cdot \binom{m}{k}$

Method 2:

Select k objects from n: $\binom{n}{k}$ ways

Select $m-k$ from remaining $n-k$: $\binom{n-k}{m-k}$ ways.

Total ways $= \binom{n}{k}\binom{n-k}{m-k}$

$\therefore \quad \binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$

(b) We know that

$$[x^n] \text{ in } \frac{1}{(1-x)^k} = \binom{n+k-1}{n}$$

$$= \binom{n+k-1}{k-1}$$

For $k = 3$

$$[x^n] \text{ in } \frac{1}{(1-x)^3} = \binom{n+2}{2}$$

$$\therefore \quad \frac{1}{(1-x)^3} = \binom{2}{2}x^0 + \binom{3}{2}x^1 + \binom{4}{2}x^2 \dots$$

Multiply by x^2 both sides

$$\frac{x^2}{(1-x)^3} = \binom{2}{2}x^2 + \binom{3}{2}x^3 + \binom{4}{2}x^4 \dots$$

$$[x^n] \text{ in } \frac{x^2}{(1-x)^3} = \binom{n}{2}$$

$$\therefore \quad g(x) = \frac{x^2}{(1-x)^3}.$$