2301 COL 202 Tutorial 10.4

Anubhav Pandey

TOTAL POINTS

4 / 4

QUESTION 1

1 12.17 **2 / 2**

✓ **- 0 pts** *Correct*

- 0.5 pts Partially Correct

- 2 pts Incorrect

- 0.5 pts Partially correct

- 0.5 pts Partially Correct

QUESTION 2

2 12.38 **2 / 2**

✓ **- 0 pts** *Correct*

- 0.5 pts Partially Correct

- 0.5 pts Partially Correct

- 0.5 pts Partially Correct

- 0.5 pts Partially Correct

- 2 pts Incorrect

COL 202 Tutorial 10

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1 Problem Statement 1

Let R be the "implies" binary relation on propositional formulas defined by the rule that $F\ R\ G$ iff $\exists\ .F \Rightarrow G$ is a valid formula:
(a) Let A and B be the sets of formulas listed below. Explain why R is not a weak partial order on the set $A \bigcup B$.
(b) Fill in the R arrows from A to B.
c) The diagram in part (b) defines a bipartite graph G with $L(G) = A$, $R(G) = B$ and an edge between F and G iff $F\ R\ G$. Exhibit a subset S of A such that both S and $A - S$ are nonempty, and the set $N(S)$ of neighbors of S is the same size as S, that is, $|N(S)| = |S|$.
(d) Let G be an arbitrary, finite, bipartite graph. For any subset $S \subseteq L(G)$, let $\bar{S} = L(G) - S$, and likewise for any $M \subseteq R(G)$, $\bar{M} = L(G) - M$ Suppose S is a subset of $L(G)$ such that $|N(S)| = |S|$, and both S and $\bar{S}$ are nonempty. Circle the formula that correctly completes the following statement:
There is a matching from $L(G)$ to $R(G)$ if and only if there is both a matching from S to its neighbors,$|N(S)|$, and also a matching from $\bar{S}$ to

$$N(\bar{S}), N\bar{(S)}, N^{-1}(N(S)), N^{-1}(N(\bar{S})), N(S) - N\bar{(S)}$$

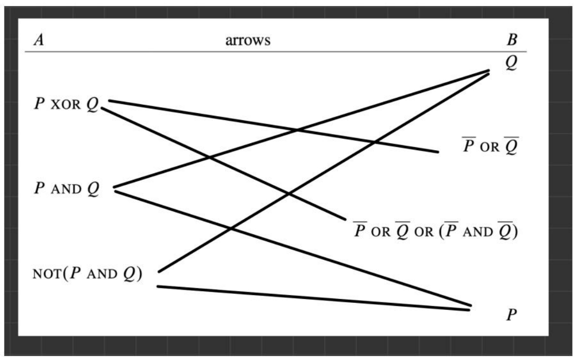
A	arrows	B
		Q
P XOR Q		
		$\overline{P}$ OR $\overline{Q}$
P AND Q		
		$\overline{P}$ OR $\overline{Q}$ OR ($\overline{P}$ AND $\overline{Q}$)
NOT(P AND Q)		
		P

2 Solution Problem 1

2.1 Part a

For set $A \bigcup B$ we can see that NOT(P AND Q) $\Rightarrow$ ($\bar{P}$ OR $\bar{Q}$) is a valid formula for the values and so is ($\bar{P}$ OR $\bar{Q}$) $\Rightarrow$ NOT(P AND Q). Thus it violates the anti-symmetry property of partial order relation.

2.2 Part b



By using brute force, we can check that above are the R arrow from A to B.

2.3 Part c

Take set S to comprise of the vertices (P XOR Q) and NOT(P AND Q). We can see that $|S| = |N(S)|$.

2.4 Part d

Since $|N(S)| = |S|$, each vertex in S is mapped to one vertex in $N(S)$ and no vertices are left in it. Thus the vertices from $\bar{S}$ must be mapped to what are not in $N(S)$ thus compliment of it, ie $N\bar{(S)}$ (Compliment of $N(S)$).

1 12.17 **2 / 2**

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3 Problem Statement 2

The n-dimensional hypercube H_n is a graph whose vertices are the binary strings of length n. Two vertices are adjacent if and only if they differ in exactly 1 bit. For example, in H_3, vertices 111 and 011 are adjacent because they differ only in the first bit, while vertices 101 and 011 are not adjacent because they differ at both the first and second bits.

(a) Prove that it is impossible to find two spanning trees of H3 that do not share some edge.
(b) Verify that for any two vertices x and y of H_3, there are 3 paths from x to y in H_3, such that, besides x and y, no two of those paths have a vertex in common.
(c) Conclude that the connectivity of H_3 is 3.
(d)Try extending your reasoning to H_4. (In fact, the connectivity of H_n is n for all $n \geq 1$. (A proof appears in the problem solution.)

4 Solution Problem 2

We assume that both the spanning trees are minimum, as if they are not, we can always make them.

4.1 Part a

Let us name the 2 spanning trees as red and blue tree and there edges also similarly. If both the tree's do not share a edge, each edge of H_3 is either red or blue or uncoloured. Since each tree is minimum spanning tree, they have 7 ($2^n - 1$) edges, thus H_3 has total 14 coloured edges. H_3 has total 12 ($n2^{n-1}$) edges, thus it is impossible that each edge is coloured with only 1 colour.

4.2 Part b

We use brute force depending upon the relation of x and y.
Case 1. Neighbours
Let x=000 and y=001,
p1=000-001. p2=000-100-101-001. p3=000-010-011-001.
Case 2. Face diagonal
Let x=000 and y=101,
p1=000-001-101. p2=000-100-101. p3=000-010-110-111-101.
Case 3. Body diagonal
Let x=000 and y=111,
p1=000-001-011-111. p2=000-010-011-111. p3=000-100-101-111.

4.3 Part c

If we remove 2 parts from H_3, we can still reach one vertex to other, using the 3rd path which does not contain these 2 points. But if we remove 3 points, one from each path, we cannot reach every vertex from other vertex. Thus connectivity of H_3 is 3. Although we need to show that for any 3 choices of vertex to be removed, there exists vertices x and y such that these 3 are on 3 different paths from x to y.

4.4 Part d

To prove that the connectivity of the n-dimensional hypercube, denoted as $H_(n)$, is n, you can use mathematical induction. Here's the proof:

Base Case (n = 1):
In the 1-dimensional hypercube $H_(1)$, there are two vertices connected by a single edge. Removing any one vertex will disconnect the graph, so the connectivity is 1.
Inductive Hypothesis:
Assume that the connectivity of the n-dimensional hypercube $H_(n)$ is n for some positive integer n, i.e., it takes the removal of n vertices to disconnect $H_(n)$.
Inductive Step (n+1):
We need to show that the connectivity of the (n+1)-dimensional hypercube $H_(n+1)$ is n+1. To do this, consider $H_(n+1)$ as two copies of H_n connected by edges.
$H_(n+1)$ can be thought of as $H_(n)$ on one side and $H_(n)$ on the other side, connected by edges. Now, to disconnect $H_(n+1)$, you need to remove vertices from both $H_(n)$ on one side and $H_(n)$ on the other side. Since the connectivity of $H_(n)$ is n (by our inductive hypothesis), you need to remove n vertices from one side to disconnect it.

2 12.38 **2 / 2**

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