

13.  $Ax = \lambda x$

$$\max_x \{x^T A x\} = \max \{x_1^T A x_1, x_2^T A x_2, \dots, x_n^T A x_n\}$$

$$= \max \{\lambda_1^T A x_1, \lambda_2^T A x_2, \dots, \lambda_n^T A x_n\}$$

$$= \max \{\lambda_1 \|x_1\|^2, \lambda_2 \|x_2\|^2, \dots, \lambda_n \|x_n\|^2\} \text{ given } \|x_i\|=1$$

$$= \max \{\lambda_1, \lambda_2, \dots, \lambda_n\}$$

$\Rightarrow$  Solution is given by largest eigen value

14. Since the matrix is full rank square matrix  $A_{n \times n}$  of size  $n$  it has  $n$  eigen values

$$V = [x_1, \dots, x_n]$$

$$AV = A[x_1, \dots, x_n]$$

$$AV = [\lambda_1 x_1, \lambda_2 x_2, \dots, \lambda_n x_n]$$

$$= \begin{bmatrix} \lambda_1 x_{11} & \lambda_2 x_{12} & \dots & \lambda_n x_{1n} \\ \lambda_1 x_{21} & \lambda_2 x_{22} & \dots & \lambda_n x_{2n} \\ \vdots & \vdots & & \vdots \\ \lambda_1 x_{n1} & \lambda_2 x_{n2} & \dots & \lambda_n x_{nn} \end{bmatrix}$$

$$AV = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & & x_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

$$x \cdot y = \|x\| \|y\| \cos \theta$$

$$x' = Ax, \quad y' = Ay$$

$$x' \cdot y' = \|x'\| \|y'\| \cos \phi = \|Ax\| \|Ay\| \cos \phi$$

$$= |A|^2 \|x\| \|y\| \cos \phi$$

$$\Rightarrow \|x\| \|y\| \cos \theta = \|x\| \|y\| \cos \phi \Rightarrow \cos \theta = \cos \phi$$

$$\Rightarrow \theta = \phi$$