7 $\{\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n\}$ set of linearly dependent vectors,

if this set is dependent, then

$$a_1\vec{u} + a_2\vec{u}_2 + \dots a_n\vec{u}_n = 0$$

where $a_i \in R$ and $a_j \neq 0 \ \forall i \in \{1 \dots n\}$

say we find j vectors that are independent (we can assume ~~it is first~~ they are first j vectors without loss of generality).

$$a_1\vec{u}_1 + a_2\vec{u}_2 + \dots a_j\vec{u}_j = 0$$

$$\vec{u}_j = -\frac{a_1\vec{u}}{a_j} - \frac{a_2\vec{u}_2}{a_j} \dots -\frac{a_{j-1}\vec{u}_{j-1}}{a_j}$$

$\therefore$ $(u_1, u_2 \dots u_{j-1})$ spans the entire space and $u_j \in$ span of $(u_1 \dots u_{j-1})$

Let $u \in \text{span}\{u_1, u_2 \dots u_n\}$

$$v = b_1u_1 + b_2u_2 + \dots + b_mu_m$$

$$u_j = -\frac{a_1u_1}{a_j} - \frac{a_2u_2}{a_j} \dots -\frac{a_{j-1}u_{j-1}}{a_j}$$

$$v = b_1u_1 + b_2u_2 + \dots + b_{j-1}u_{j-1} + \left(-\frac{a_1u_1}{a_j} - \frac{a_2u_2}{a_j} \dots -\frac{a_{j-1}u_{j-1}}{a_j}\right) + b_{j+1}u_{j+1} + \dots + b_mu_m$$

$$v = \left(b_1 - \frac{a_1}{a_j}\right)u_1 + \left(b_2 - \frac{a_2}{a_j}\right)u_2 + \dots + \left(b_{j-1} - \frac{a_{j-1}}{a_j}\right)u_{j+1} + b_{j+1}u_{j+1} + b_{j+2}u_{j+2} + b_mu_m$$

$\Rightarrow v \in \text{span}\{u_1, u_2 \dots u_{j-1}, u_{j+1} \dots u_m\}$

span of $\{u_1 \dots u_m\}$ = span of $\{u_1, u_2 \dots u_{j-1}, u_{j+1} \dots u_m\}$

$\Rightarrow$ list is unchanged

9.a) A is linearly independent ~~then~~ only when $a_i \neq 0$

$a_1 v_1 + a_2 v_2 + \ldots + a_n v_n = 0$ where $a_i \neq 0$ & $a_i \in \mathbb{R}$ $\forall i \in \{1 \ldots n\}$

B is linearly independent only when $b_i \neq 0$

in $b_1(v_1 - v_2) + b_2(v_2 - v_3) + \ldots b_{n-1}(v_{n-1} - v_n) + b_n v_n = 0$

given A & B are linearly independent vectors $\Rightarrow a_i = b_i = 0 \ \forall i \in \{1 \ldots n\}$

~~regrouping~~ ~~but~~ this also implies the other way around, so, if we can prove $b_i = 0$, then B is independent

$$b_1 v_1 + (b_2 - b_1) v_2 + (b_3 - b_2) v_3 + \ldots + (b_n - b_{n-1}) v_n = 0$$

$b_1 = a_1 \Rightarrow b_1 = 0$

$\Rightarrow b_2 - b_1 = 0 \Rightarrow b_2 = 0$

and so on,

$\Rightarrow$ B is linearly independent

b) A is spanning set, & $v \in \text{span}(A)$

$v = a_1 v_1 + a_2 v_2 + \ldots a_n v_n$ for $a_i \in \mathbb{R}$

$v = b_1(v_1 - v_2) + b_2(v_2 - v_3) + \ldots b_{n-1}(v_{n-1} - v_n) + b_n v_n$

$v = b_1 v_1 + (b_2 - b_1) v_2 + \ldots + (b_n - b_{n-1}) v_n$

WKT $v \in \text{span}(A) \Rightarrow v \in B$

$\Rightarrow v$ $\Rightarrow$ B ~~also~~ spans vector space