M T W T F S S

Page No.:

Date:

YOUVA

# MLSP
## Assignment 3

Chinmaya Udupa
SC19B081

2) $h_\theta(x) = \theta_1 x \qquad J(\theta_1) = \frac{1}{2}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

$x = [1, -1, 2]^T$
$y = [3, -2, 4]^T$

$$\Rightarrow J(\theta_1) = \frac{1}{2}\left[(\theta_1 - 3)^2 + (-\theta_1 + 2)^2 + (2\theta_1 - 4)^2\right]$$

optimal $J(\theta)$ can be found be differentiating $J(\theta_1)$ WRT $\theta_1$ & setting it to 0

$$\Rightarrow \frac{\partial J(\theta_1)}{\partial \theta} = (\theta_1 - 3) + -(-\theta_1 + 2) + 2(2\theta_1 - 4) = 0$$

$\Rightarrow$ ~~$4\theta_1 = 9$~~ $\qquad \Rightarrow 6\theta_1 = 13$

~~$\Rightarrow \theta_1 = \frac{9}{4}$~~ $\qquad \Rightarrow \boxed{\theta_1 = \frac{13}{6}}$

3) $$J(\theta_1) = \frac{1}{2}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

$$\frac{\partial J(\theta_1)}{\partial \theta_1} = \sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right) x^{(i)} = 0$$

$$\theta_1\left(\sum_{i=1}^{m}\left(x^{(i)}\right)^2\right) = \sum_{i=1}^{m} x^{(i)} y^{(i)}$$

$$\Rightarrow \theta_1 = \frac{\sum_{i=1}^{m} x^{(i)} y^{(i)}}{\sum_{i=1}^{m}\left(x^{(i)}\right)^2}$$

Substituting $x = [1, -1, 2]^T$

$y = [3, -2, 4]^T$

$$\Rightarrow \theta_1 = \frac{(1\times 3) + (-1\times -2) + (2\times 4)}{1^2 + (-1)^2 + 2^2} = \frac{13}{6}$$

answers match

4) $$J(\theta_0, \theta_1) = \frac{1}{2}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} + \theta_0 - y^{(i)}\right)^2$$

$$\frac{\partial J}{\partial \theta_1} = \sum_{i=1}^{m}\left(\theta_1 x^{(i)} + \theta_0 - y^{(i)}\right)x^{(i)} = 0$$

$$\frac{\partial J}{\partial \theta_0} = \sum_{i=1}^{m}\left(\theta_1 x^{(i)} + \theta_0 - y^{(i)}\right) = 0$$

when $m = 2$

$$\theta_1 x_1^2 + \theta_0 x_1 - y_1 x_1 + \theta_1 x_2^2 + \theta_0 x_2 - y_2 x_2 = 0$$

$$\& \quad \theta_1 x_1 + \theta_0 - y_1 + \theta_1 x_2 + \theta_0 - y_2 = 0$$

$$\Rightarrow (x_1^2 + x_2^2)\theta_1 + (x_1 + x_2)\theta_0 = x_1 y_1 + x_2 y_2$$

$$\& \quad (x_1 + x_2)\theta_1 + (1+1)\theta_0 = y_1 + y_2$$

$$\Rightarrow \begin{bmatrix} x_1^2 + x_2^2 & x_1 + x_2 \\ x_1 + x_2 & 2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} x_1 y_1 + x_2 y_2 \\ y_1 + y_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \theta_1 \\ \theta_0 \end{bmatrix} = \frac{1}{2x_1^2 + 2x_2^2 - x_1^2 - x_2^2 - 2x_1 x_2}\begin{bmatrix} 2 & -x_1 - x_2 \\ -x_1 - x_2 & x_1^2 + x_2^2 \end{bmatrix}\begin{bmatrix} x_1 y_1 + x_2 y_2 \\ y_1 + y_2 \end{bmatrix}$$

$$= \frac{1}{(x_1 - x_2)^2}\begin{bmatrix} (x_1 - x_2)(y_1 - y_2) \\ (x_2 y_1 - x_1 y_2)(x_1 - x_2) \end{bmatrix}$$

$$\Rightarrow \boxed{\theta_1 = \frac{y_1 - y_2}{x_1 - x_2}} \quad \& \quad \boxed{\theta_0 = \frac{x_2 y_1 - x_1 y_2}{x_1 - x_2}}$$

1) a) $\begin{bmatrix} 3 & -1 \\ 2 & 5 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u & a \\ v & b \end{bmatrix} = \begin{bmatrix} 3u-v & 3a-b \\ 2u+5v & 2a+5b \\ -2u+2v & -2a+2b \end{bmatrix}$

b) $A \in \mathbb{R}^{2\times 2}$ ; $B \in \mathbb{R}^{2\times 4}$

AB exists & size of $\mathbb{R}^{2\times 4}$

c) $A \in \mathbb{R}^{3\times 5}$ ; $B \in \mathbb{R}^{4\times 1}$

AB doesnot exist

d) $A \in \mathbb{R}^{3\times 2}$, $y \in \mathbb{R}^3$

$y^T A$ - row vector of size $\mathbb{R}^2$

e) $A \in \mathbb{R}^{3\times 2}$, $x \in \mathbb{R}^2$

$Ax$ — column vector of size $\mathbb{R}^3$

f) $(Bx+y)^T A^T = 0$

taking transpose on both sides

$\boxed{(CD)^T = D^T C^T}$

$A(Bx+y) = 0$

$\Rightarrow ABx = -Ay$

~~taking A⁻¹ on both sides~~

$\Rightarrow (A^{-1}A)Bx = -(A^{-1}A)y$

$CC^{-1} = C^{-1}C = I$

$IC = CI = C$

$\Rightarrow Bx = -y$

$\Rightarrow (B^{-1}B)x = -(B^{-1})y$

$\Rightarrow \boxed{x = -B^{-1}y}$