

Chinmay Bhagwat :- Homework 4

1) Bayes' Theorem:-

Proof:- $P(H/D) = \frac{P(D/H) \cdot P(H)}{P(D)}$ \rightarrow Bayes' law

Joint probability $\rightarrow P(A, B) = P(B/A) \cdot P(A) = P(A/B) \cdot P(B) \rightarrow$ Product rule

$$\rightarrow P(B/A) \cdot P(A) = P(A/B) \cdot P(B)$$

Therefore $P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$

Bayes' theorem is one of the famous method to calculate conditional probability or the likelihood of one event occurring if another has previously occurred. A conditional probability can lead to more accurate outcomes by including extra conditions. This method can also be used to see hypothetical new information effects the likelihood of an event. Bayes' theorem is used in many places like the Naive Bayes classifier.

2.)

$$\text{Here } P(\text{cancer}) = 0.008$$

$$P(\neg \text{cancer}) = 0.992$$

$$P(+/\text{cancer}) = 0.98$$

$$P(-/\text{cancer}) = 0.02$$

$$P(+/\neg \text{cancer}) = 0.03$$

$$P(-/\neg \text{cancer}) = 0.97$$

we need to find $P(\text{cancer}/+)$

$$= \frac{P(+/\text{cancer}) \cdot P(\text{cancer})}{P(+)}$$

$$\begin{aligned} \text{Here } P(+) &= P(+/\text{cancer}) \cdot P(\text{cancer}) + P(+/\neg \text{cancer}) \cdot P(\neg \text{cancer}) \\ &= (0.98)(0.008) + (0.03)(0.992) = \underline{0.0376} \end{aligned}$$

$$P(\text{cancer}/+) = (0.98) \cdot (0.008) / 0.0376 = \underline{\underline{0.208}}$$

$$\text{So } P(\text{cancer}/++) = P(+/\text{cancer}) \cdot P(\text{cancer}/+) / P(++)$$

$$\begin{aligned} P(++) &= P(+/\text{cancer}) \cdot P(\text{cancer}/+) + P(+/\neg \text{cancer}) \cdot P(\neg \text{cancer}/+) \\ &= (0.98) \cdot (0.208) + (0.02) (1 - 0.208) \\ &= \underline{\underline{0.21968}} \end{aligned}$$

$$\begin{aligned} P(\text{cancer}/++) &= (0.98) \cdot (0.208) / 0.21968 \\ &= \underline{\underline{0.91}} \end{aligned}$$

$$P(\neg \text{cancer}/++) = \underline{\underline{0.09}}$$

$$3) P(\text{Yes}) = 8/12$$

$$P(\text{No}) = 4/12$$

New instance = Outlook = sun, Temp = cool, Humidity = high, Wind = strong

$$P(\text{Sunny} / \text{Yes}) = 2/8$$

$$P(\text{Sunny} / \text{No}) = 3/4$$

$$P(\text{cool} / \text{Yes}) = 3/8$$

$$P(\text{cool} / \text{No}) = 1/4$$

$$P(\text{high} / \text{Yes}) = 3/8$$

$$P(\text{high} / \text{No}) = 3/4$$

$$P(\text{strong} / \text{Yes}) = 3/8$$

$$P(\text{strong} / \text{No}) = 2/4$$

$$\begin{aligned} \text{For target value (Yes)} &= 8/12 \times 2/8 \times 3/8 \times 3/8 \times 3/8 \\ &= 0.00878 \end{aligned}$$

$$\begin{aligned} \text{For target value (No)} &= 4/12 \times 3/4 \times 1/4 \times 3/4 \times 2/4 \\ &= 0.02 \end{aligned}$$

$$\text{Here } 0.02 > 0.0087$$

Therefore instance will be "NO"

4) Back Propagation:

$$\text{Sigmoid} \rightarrow \sigma(x) = \frac{1}{1+e^{-x}}$$

1st training:

$$\text{Output } c = \sigma((0.1 \times 1) + 0.1(1)) = \sigma(0.2) = 0.55$$

$$\text{Output } d = \sigma(0.1 \times 0.55 + 0.1 \times 1) = 0.54$$

$$\text{Error} \rightarrow e_d = 0.54 \times (1 - 0.54) \times (1 - 0.54) = 0.115$$

$$e_c = (0.55)(1 - 0.55)(0.1)(0.115) = 0.0028$$

$$\delta_{do} = (0.3)(0.1146)(1) = 0.0342 \rightarrow w_{do} = (0.1) + (0.0342) = 0.1342$$

$$\delta_{co} = (0.3)(0.0028)(1) = 0.00085 \rightarrow w_{co} = (0.1) + (0.00085) = 0.10085$$

$$\delta_{ca} = (0.3)(0.0028)(1) = 0.00085 \rightarrow w_{ca} = 0.10085$$

$$\delta_{dc} = (0.3)(0.115)(0.55) = 0.019 \rightarrow w_{dc} = (0.1) + (0.019) = 0.119$$

$$\delta_{cb} = (0.3)(0.0028) = 0 \rightarrow w_{cb} = 0.1$$

2nd training

$$\text{Output } c = \sigma(0.10085 + 0.1) = \sigma(0.20085) = 0.55$$

$$\text{Output } d = \sigma(0.119 \times 0.55 + 0.134) = \sigma(0.1996) = 0.5497$$

$$\text{error} \rightarrow e_d = (0.5497)(1 - 0.5497)(1 - 0.5497) = -0.1361$$

$$e_c = (0.55)(1 - 0.55)(0.119)(-0.1361) = -0.0009$$

So

$$d_{co} = (0.3)(-0.004) + (0.4)(0.00085) = -0.0004$$

$$d_{ca} = (0.3)(-0.004)(0) + (0.4)(0.00085) = -0.00086$$

$$d_{do} = (0.3)(-0.1361)(1) + (0.4)(0.034) = -0.01$$

$$d_{dc} = (0.3)(-0.1361)(0.55) + (0.4)(0.014) = -0.0055$$

$$d_{cb} = (0.3)(-0.004) \cdot 1 + (0.4)(0) = -0.0012$$

$$w_{eo} = 0.10085$$

$$w_{ca} = 0.10085 + 0.00086 = 0.1016$$

$$w_{do} = 0.11342 - 0.01 = 0.1242$$

$$w_{dc} = 0.1189 - 0.0055 = 0.1134$$

$$w_{cb} = 0.1 - 0.0012 = 0.0988$$