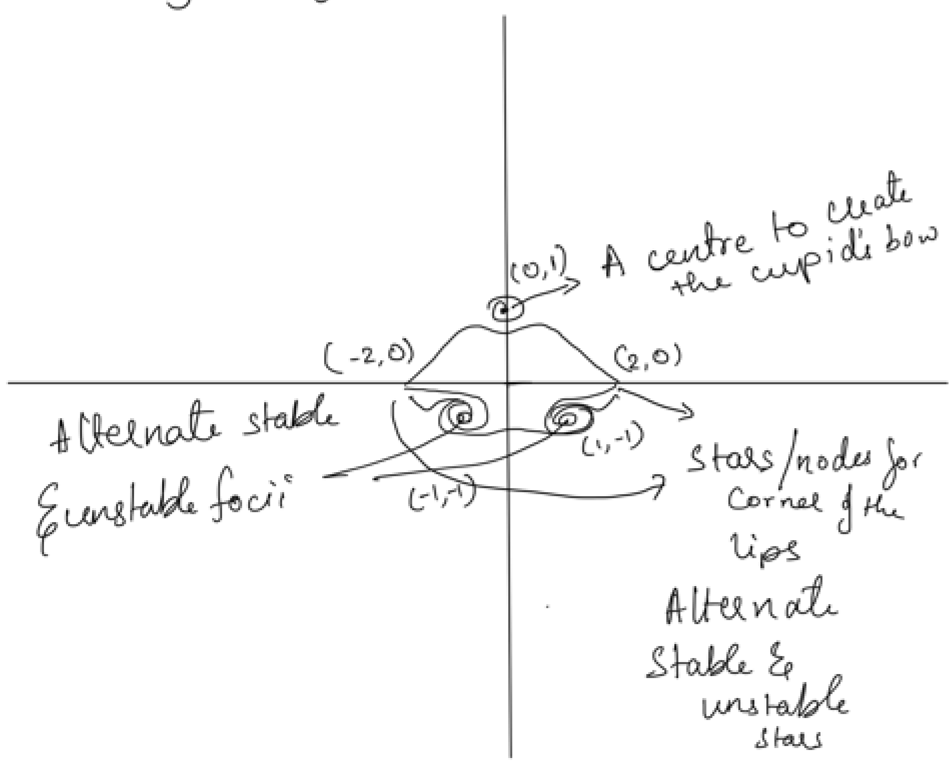
Designing a phase portrait

The initial, rather ambitious plan was to make a phase (face) portrait, but as I progressed, I realised it was too much work to do with the limited tools in the nolinear domain. So I contented myself with getting a phase portrait resemble lips.

Planning the system:



$$\dot{x} = f(x,y) \qquad A = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}$$
$$\dot{y} = g(x,y)$$

$$(f_x - \lambda)(g_y - \lambda) - f_y g_x = 0$$

$$\lambda^2 - (f_x + g_y)\lambda + f_x g_y - f_y g_x = 0$$

Stable star: $\lambda_1 = \lambda_2 = < 0$

Unstable " : $\lambda_1 = \lambda_2 > 0$

Stable focus: complex conjugate -ve realpart

Unstable " : " " +ve " "

Centre : purely imaginary

Saddle: real +ve & -ve

$$\lambda_{1,2} = \frac{f_x + g_y \pm \sqrt{(f_x+g_y)^2 - 4f_x g_y + 4f_y g_x}}{2}$$

$$= \frac{f_x + g_y \pm \sqrt{(f_x - g_y)^2 + 4f_y g_x}}{2}$$

USS @ -2,0

$\Rightarrow \lambda_1 = \lambda_2 = +ve$

SS @ 2,0

$\lambda_1 = \lambda_2 = -ve$

Centre @ 0,1.

$\lambda_1 = \lambda_2 = i$

Sf @ -1,-1

$\lambda_1, \lambda_2 = a \pm ib$
$a < 0$

Usf @ 1,-1

$\lambda_1 = \lambda_2 = a \pm ib$
$a > 0$

$f(x,y) = g(x,y) = 0$ at $(-2,0)$ $(2,0)$
$(0,1)$ $(-1,-1)$
$(1,-1)$

$f_x + g_y > 0$ at $(-2,0), (1,-1)$

< 0 at $(2,0)\ (-1,-1)$

$f_x + g_y = 0$ at $(0,1)$

$(f_x - g_y)^2 + 4 f_y g_x = 0$ at $(-2,0)\ (2,0)$

< 0 at $(1,-1)(-1,-1)(0,1)$

$x-2 / y$ $x+2/y$ $x/y-1$ $x+1/y+1$ (d)

a   b   c $x-1/y+1$ (e)

a_x	a_y	b_x	b_y	c_x	c_y	d_x	d_y
1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1

e_x	e_y
1	0
0	1

$f = (y+1)(y-1)(x^2-4)$

$-2,0 \quad 2,0$

$g_y = +4, -4$

$f_x = 2x(y^2-1)$ $-1,-1$

$1,-1$ $-ve$

$+ve$

$f_y = 2y(x^2-4)$

$6 \quad -3 \quad -4$

-8

-2,0	2,0	0,1	-1,-1	1,-1	
4	-4	0	0	0	f_x
0	0	-8	6	6	f_y
x	x	1	-1	-1	g_x
y	-y	0	< 0	> 0	g_y
x+2/y	x-2/y	x/y-1	x+1/y+1	x-1/y+1	

With these conditions for g, I created a dataset of points in and around the equilibrium points to match the conditions required and obtained a polynomial with which I got a semblance of the phase portrait I had planned but with a slight asymmetry owing to the stability of the equilibrium points at the corner of the lips.

While this is one method, I also plan

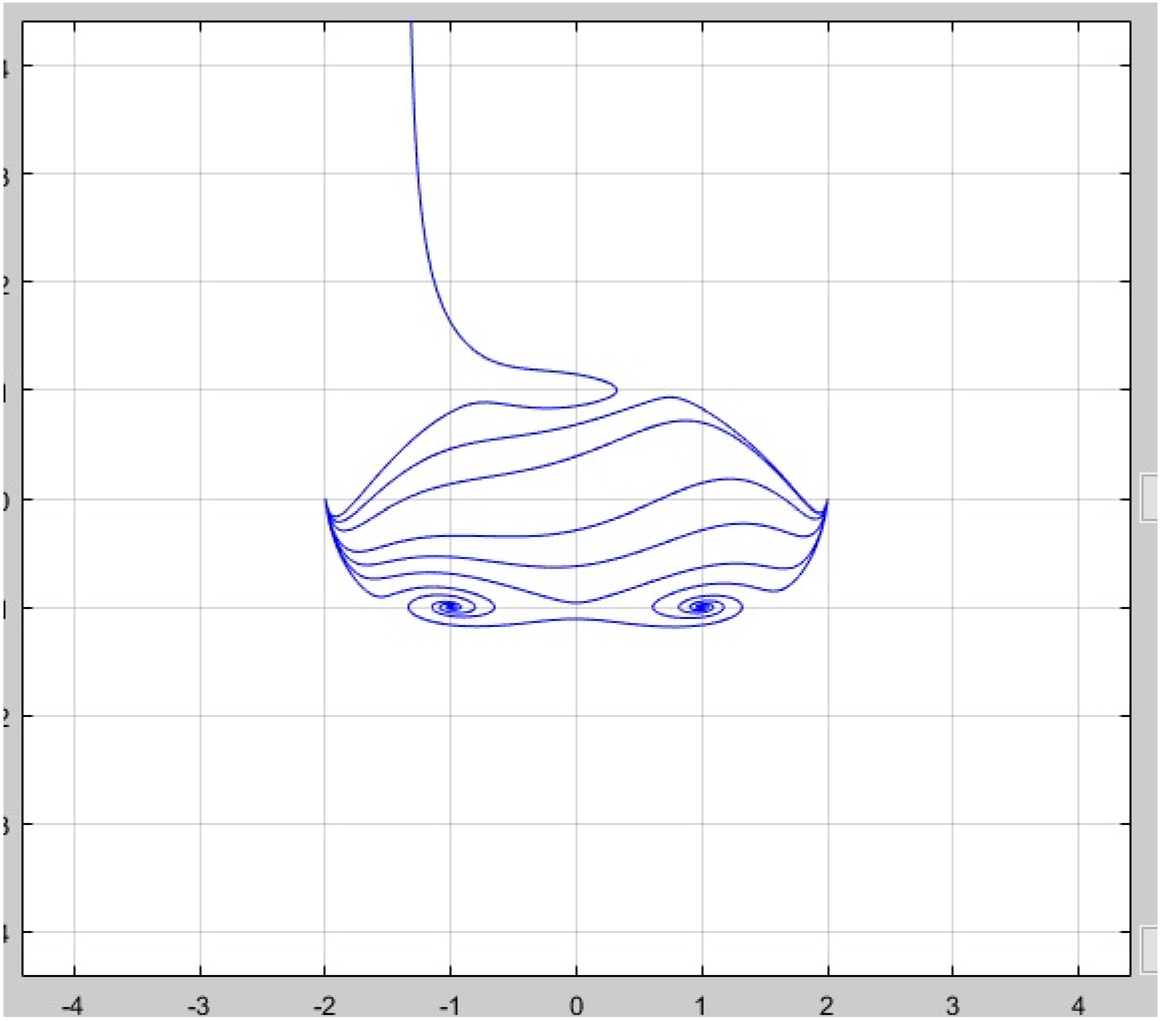
While this is one method, I also plan on exploring modelling phase portraits through the energy function. I could not complete the 'Portrait' as I had originally planned because it would require too many equilibrium points and manually calculating eigenvalues at all of them and then finding functions for them will be too hard.

```
X = [-2 2 0 -1 1 -2 -2 2 2 -0.1 0.1 0 0 -0.9 -1.1 0.9 1.1 -1 -1 1 1];
Y = [0 0 1 -1 -1 -0.1 0.1 -0.1 0.1 1 1 0.9 1.1 -1 -1 -1 -1 -0.9 -1.1 -0.9 -1.1];
Z = [0 0 0 0 0 -0.4 0.4 0.4 -0.4 -0.1 0.1 0.1 0.1 -0.1 0.1 0.1 -0.1 -0.1 0.1 0.1 -0.1];
W = [100000 100000 100000 100000 100000 50000 50000 50000 50000 50000 50000 50000 50000 50000 50000 50000 50000 50000 50000 50000 50000];
```

These were the data points and weights used for curve fitting to obtain g(x,y)

The polynomial I obtained for g(x,y) was : $1.45 + 1.825*x + 1.138*y + -0.7113*x*^2 + -0.5076*x*y + -2.856*y^2 + -1.975*x^3 + -0.6341*x^2*y + -1.318*x*y^2 + -2.247*y^3 + 0.08722*x^4 + -0.3745*x^3*y +0.7104*x^2*y^2 + 0.5715*x*y^3 + 1.406*y^4 + 0.3797*x^5 + 0.08722*x^4*y +0.3285*x^3*y^2 + 0.6332*x^2*y^3 + 0.45*x*y^4 + 1.109*y^5$. after fitting 21 points using the table above.

The phase portrait was admirably close to what I had imagined. You can try copy pasting the same f(x,y) and g(x,y) and running it in pplane8 to see the other unsightly trajectories that I've cleverly omitted for my purpose.



-4
-3
-2
-1
0
1
2
3
4