INVESTIGATING THE DEPENDENCE ON MACHINE SIZE OF THE ENERGY CONFINEMENT IN TOKAMAKS USING DATA-DRIVEN METHODS

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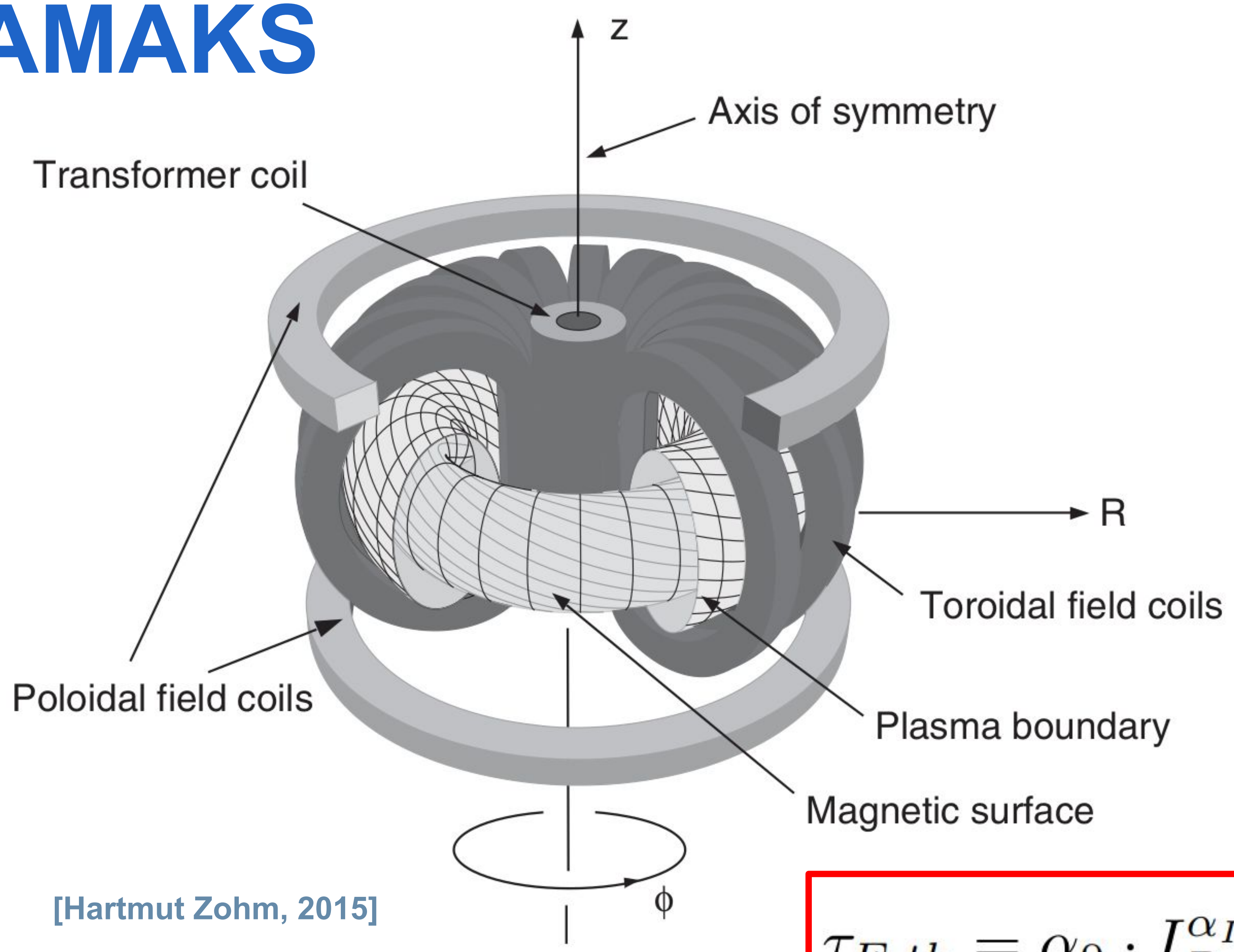
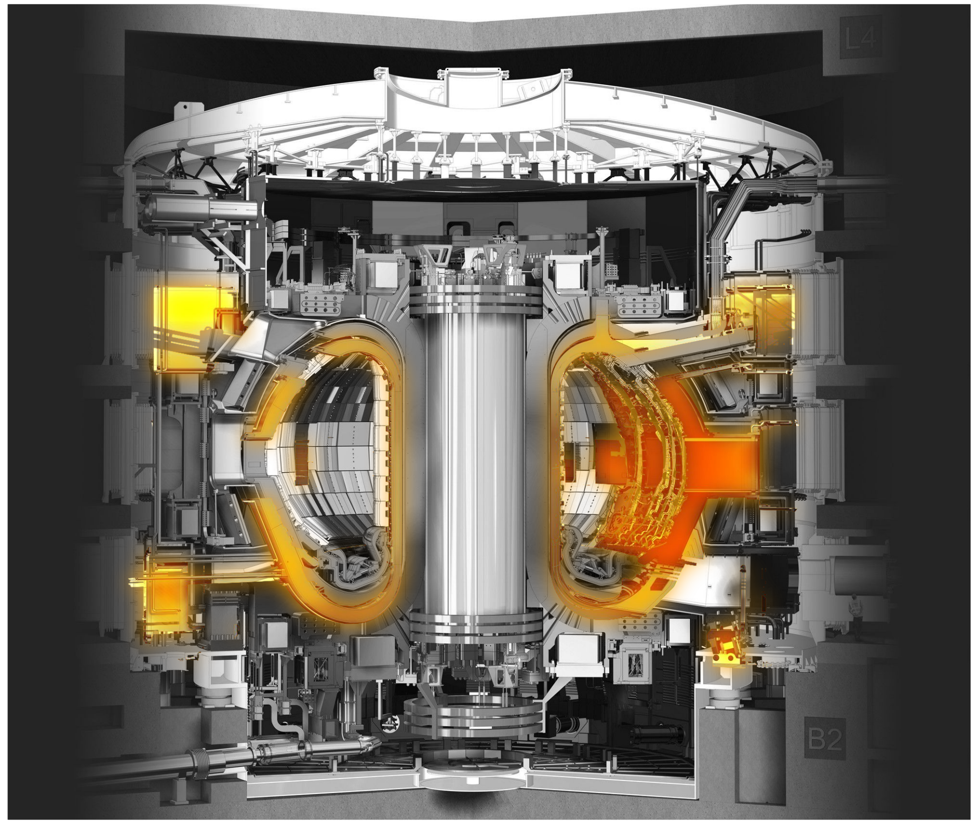
11/07/2023

TOKAMAKS



[Hartmut Zohm, 2015]

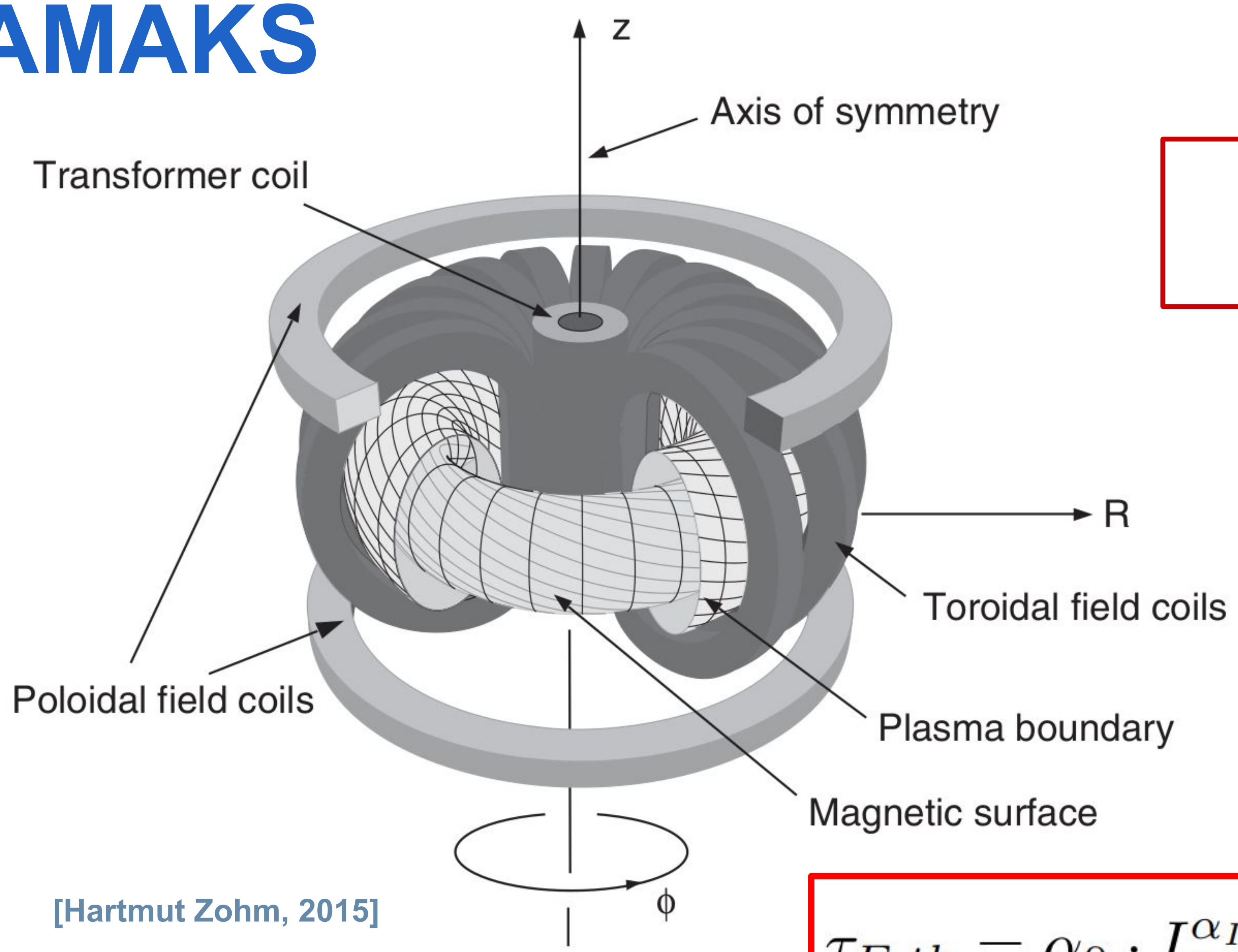


[ITER Organization, 2023]

$\log(\tau_{E,th})$ → OLS

$$\tau_{E,th} = \alpha_0 \cdot I_P^{\alpha_I} \cdot B_t^{\alpha_B} \cdot \bar{n}_e^{\alpha_n} \cdot P_{l,th}^{\alpha_P} \cdot R_{geo}^{\alpha_R} \cdot \kappa_a^{\alpha_\kappa} \cdot \epsilon^{\alpha_\epsilon} \cdot M_{eff}^{\alpha_M}$$

TOKAMAKS



[Hartmut Zohm, 2015]

H-mode (EMLy subset) confinement database

STDB2
1998

$\alpha_R \sim 2.22$

WHY?

STDB5
2021

$\alpha_R \sim 1.45$

DATA?

PHYSICS?

WHAT CAN WE LEARN?

$$\boxed{\tau_{E,th} = \alpha_0 \cdot I_P^{\alpha_I} \cdot B_t^{\alpha_B} \cdot \bar{n}_e^{\alpha_n} \cdot P_{l,th}^{\alpha_P} \cdot R_{geo}^{\alpha_R} \cdot \kappa_a^{\alpha_\kappa} \cdot \epsilon^{\alpha_\epsilon} \cdot M_{eff}^{\alpha_M}}$$

WORKFLOW

WHICH REGISTERS IN STDB5 ARE **DECREASING** α_R?

→ IS THIS DECREASE DUE TO **DATA ISSUES**?

→ CAN WE **PREDICT** WHETHER A NEW REGISTER WILL **DECREASE** α_R?

→ **RELATE** THE FINDINGS **TOKAMAK CHARs & PHYSICS**

WORKFLOW

WHICH REGISTERS IN STDB5 ARE **DECREASING** α_R?

→

IS THIS DECREASE DUE TO **DATA ISSUES**?

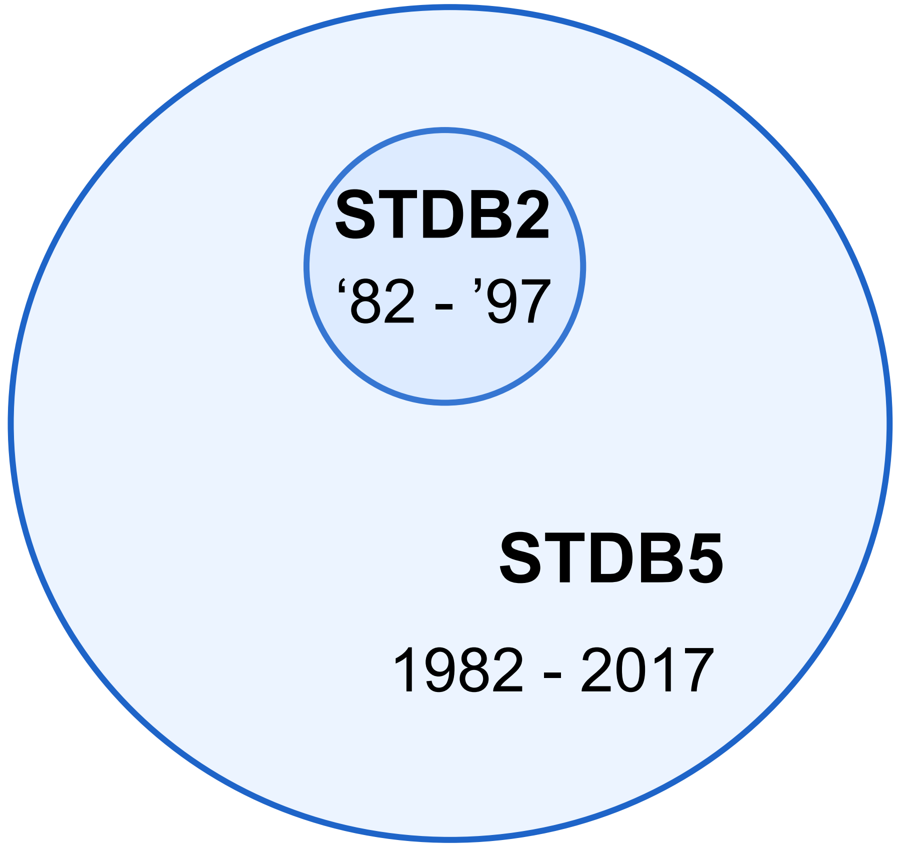
→

CAN WE **PREDICT** WHETHER A NEW REGISTER WILL **DECREASE** α_R?

→

RELATE THE FINDINGS **TOKAMAK CHARs & PHYSICS**

FINDING THE DECREASING REGISTERS



Dataset size = 6252

FINDING THE DECREASING REGISTERS

New STDB5

1989 - 2017

Dataset size = 4942

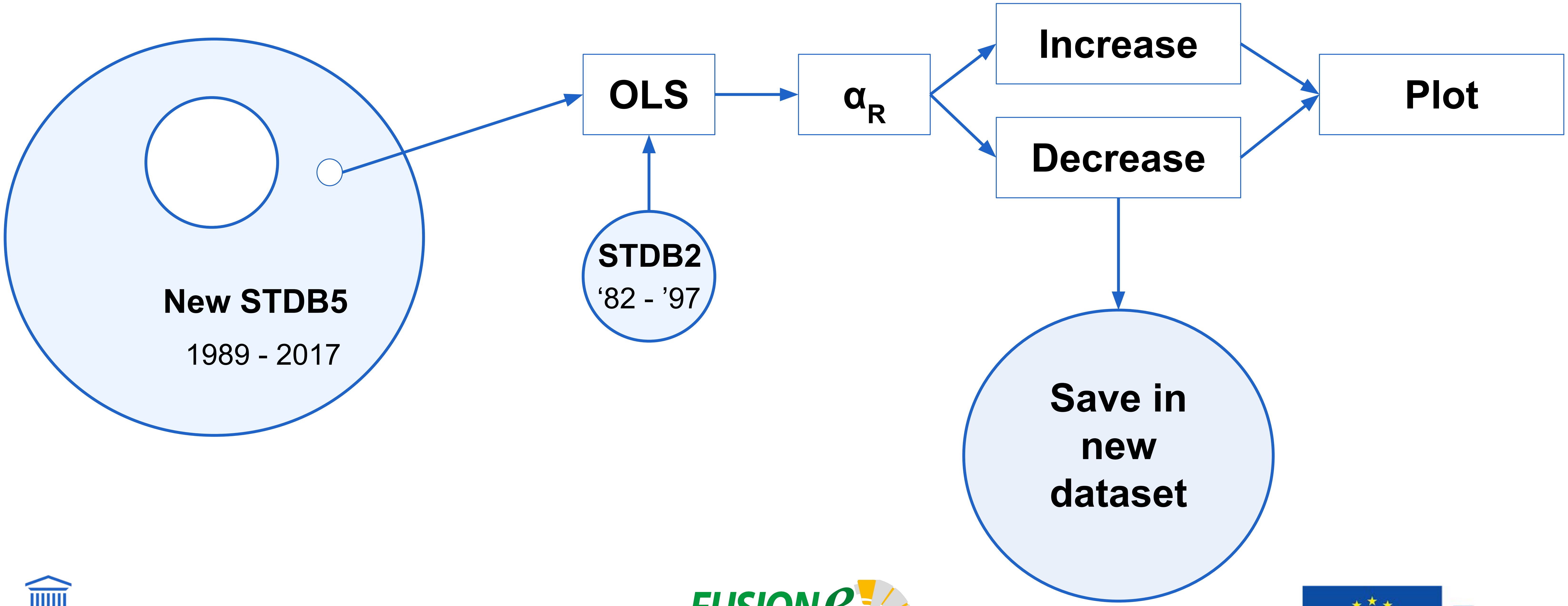
STDB2

‘82 - ’97

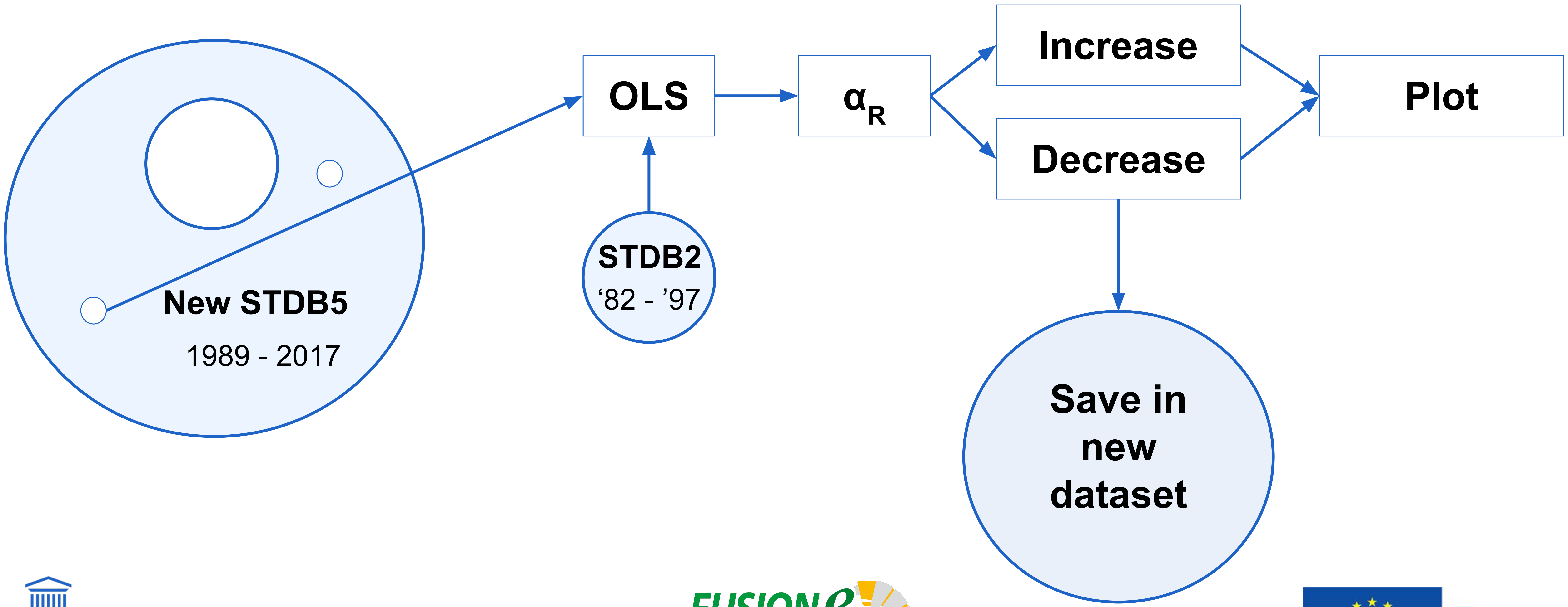
Dataset size = 1310

UNAFFECTING

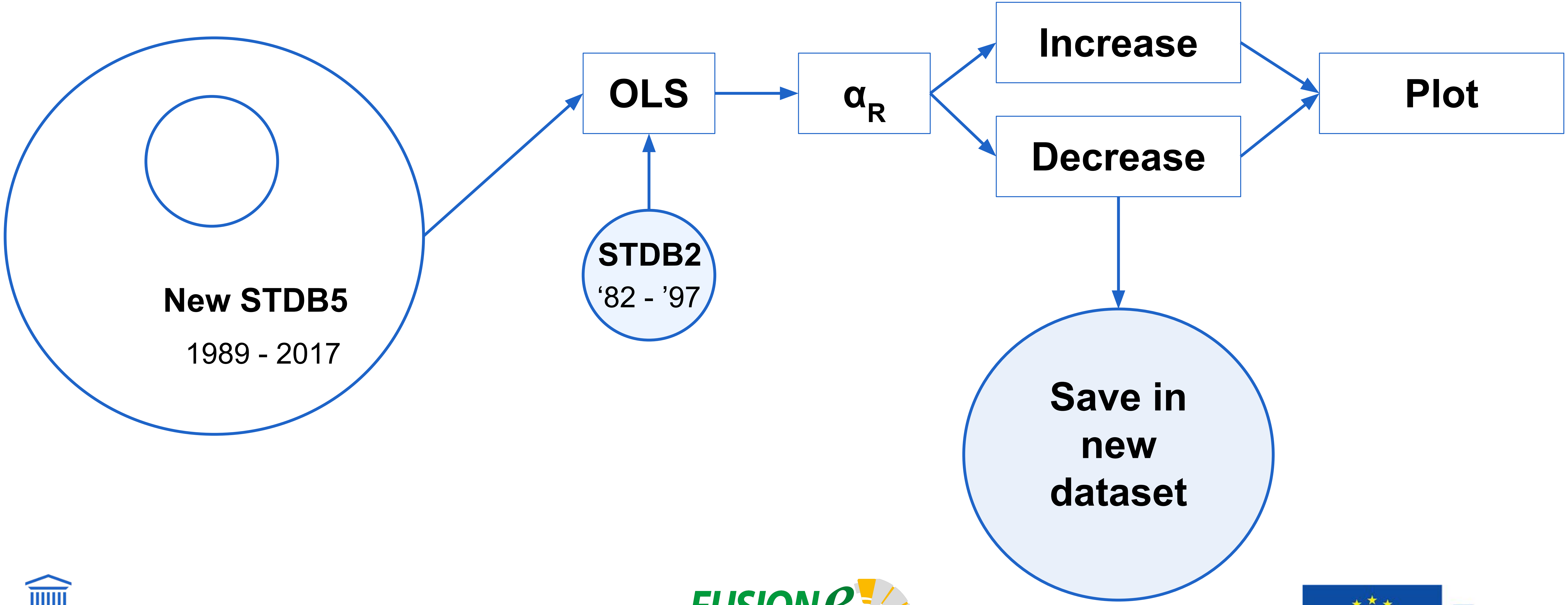
FINDING THE DECREASING REGISTERS



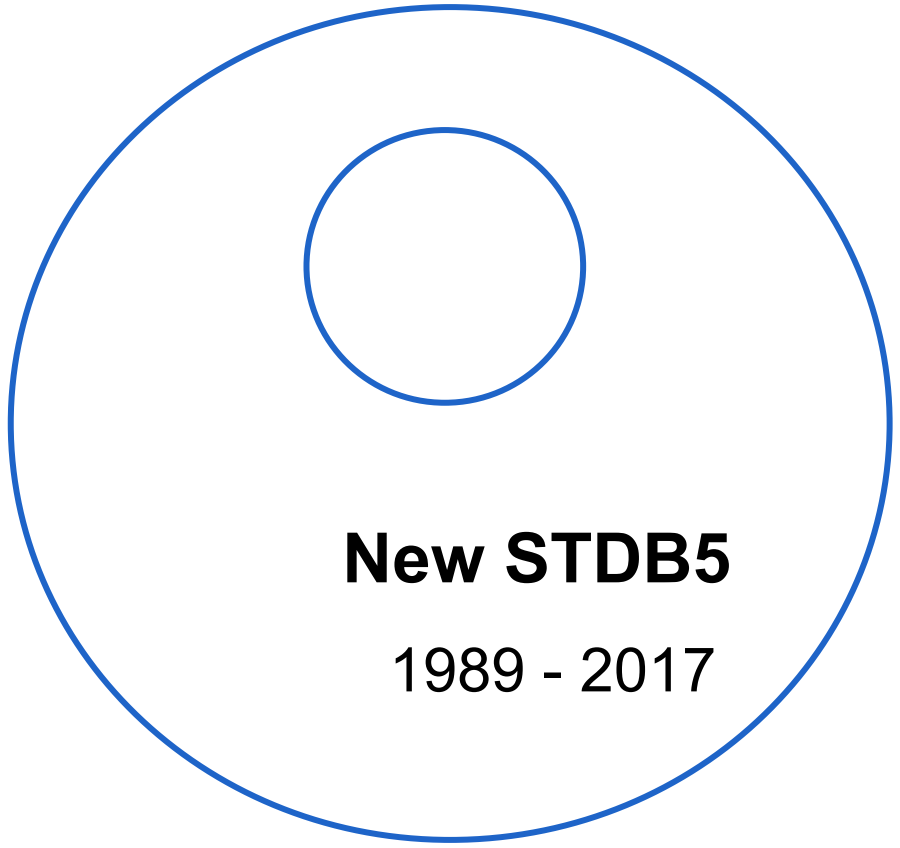
FINDING THE DECREASING REGISTERS

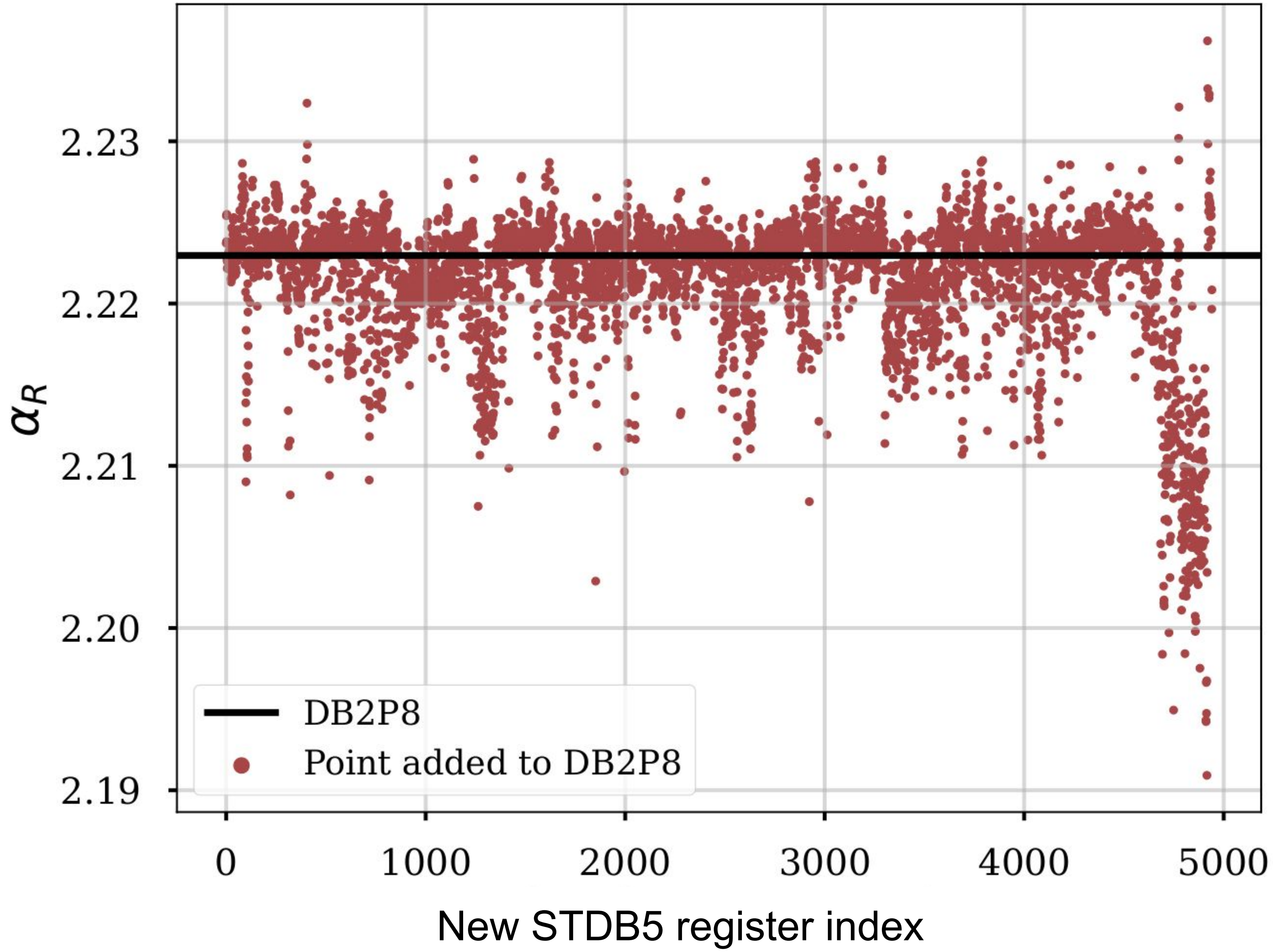


FINDING THE DECREASING REGISTERS



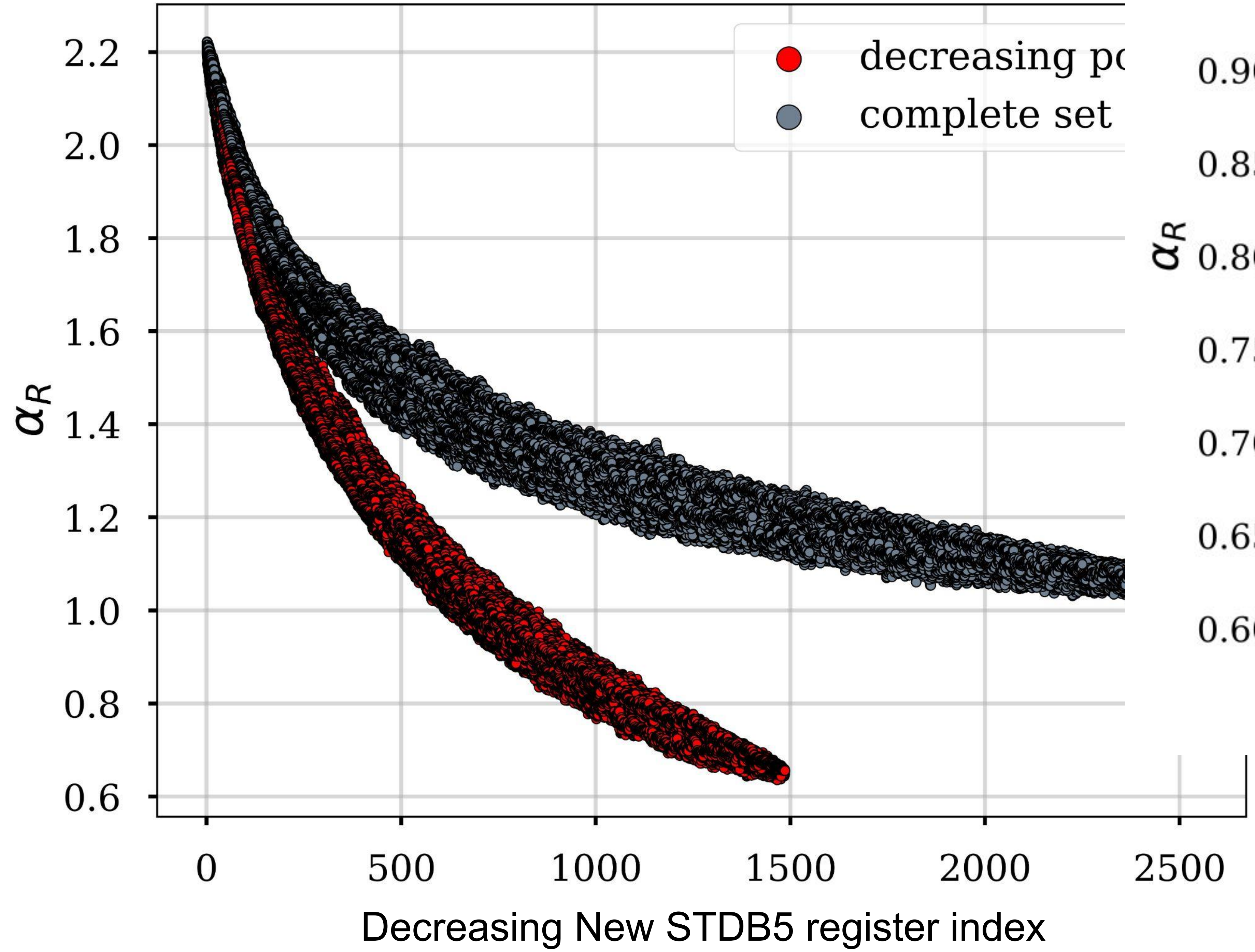
DECREASING REGISTERS

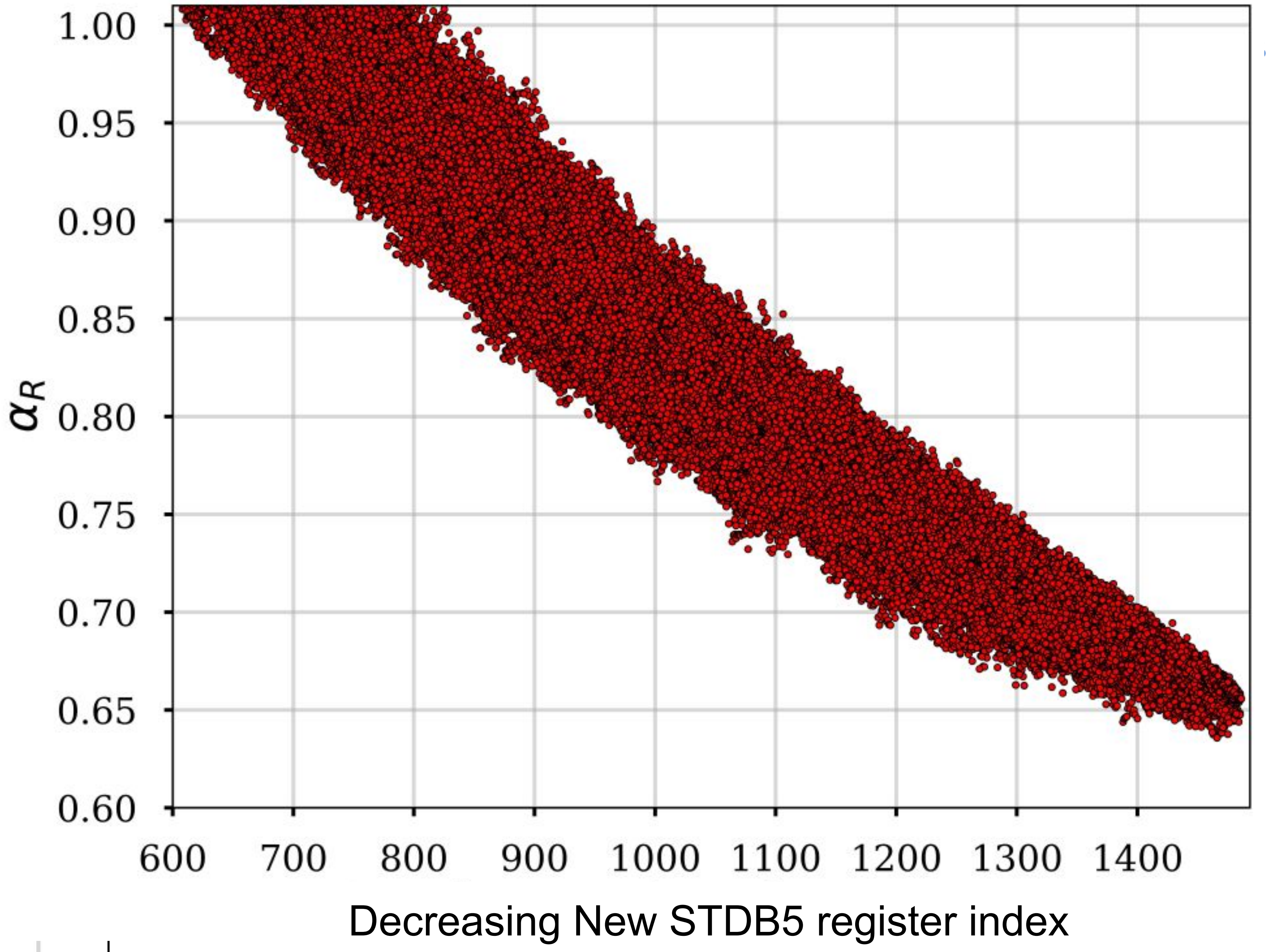




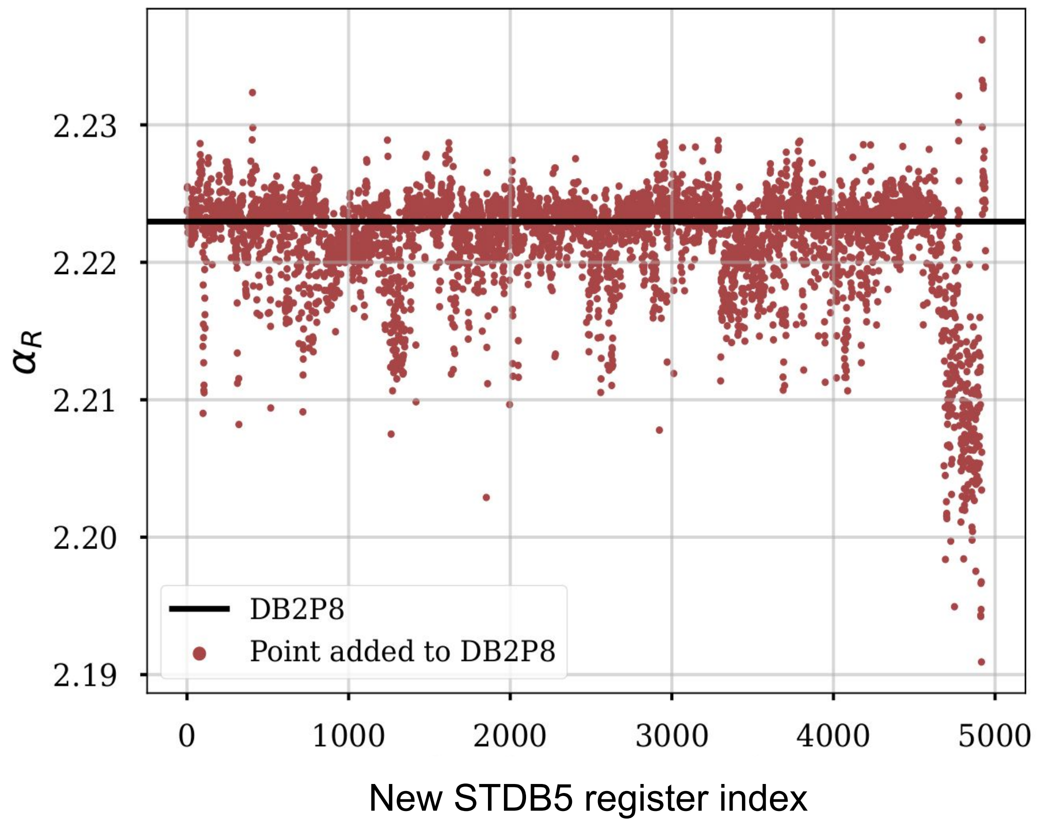
Plot

Random Sampling


decreasing pc
complete set
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
α_R
0
500
1000
1500
2000
2500
Decreasing New STDB5 register index


1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
α_R
600
700
800
900
1000
1100
1200
1300
1400
Decreasing New STDB5 register index

DECREASING REGISTERS



Random Sampling → **OLS** ← **STDB2**

"Small dataset"	"Big dataset"
Smallest subset for $\alpha_R < 1$	**Smallest α_R**
$\alpha_R \sim 0.9998$	$\alpha_R \sim 0.6379$
Subset size = 618	Subset size = 1459
9.88% decreased α_R 90.12% did not	23.34% decreased α_R 76.66% did not

STDB5

DECREASING AND UNAFFECTING REGISTERS

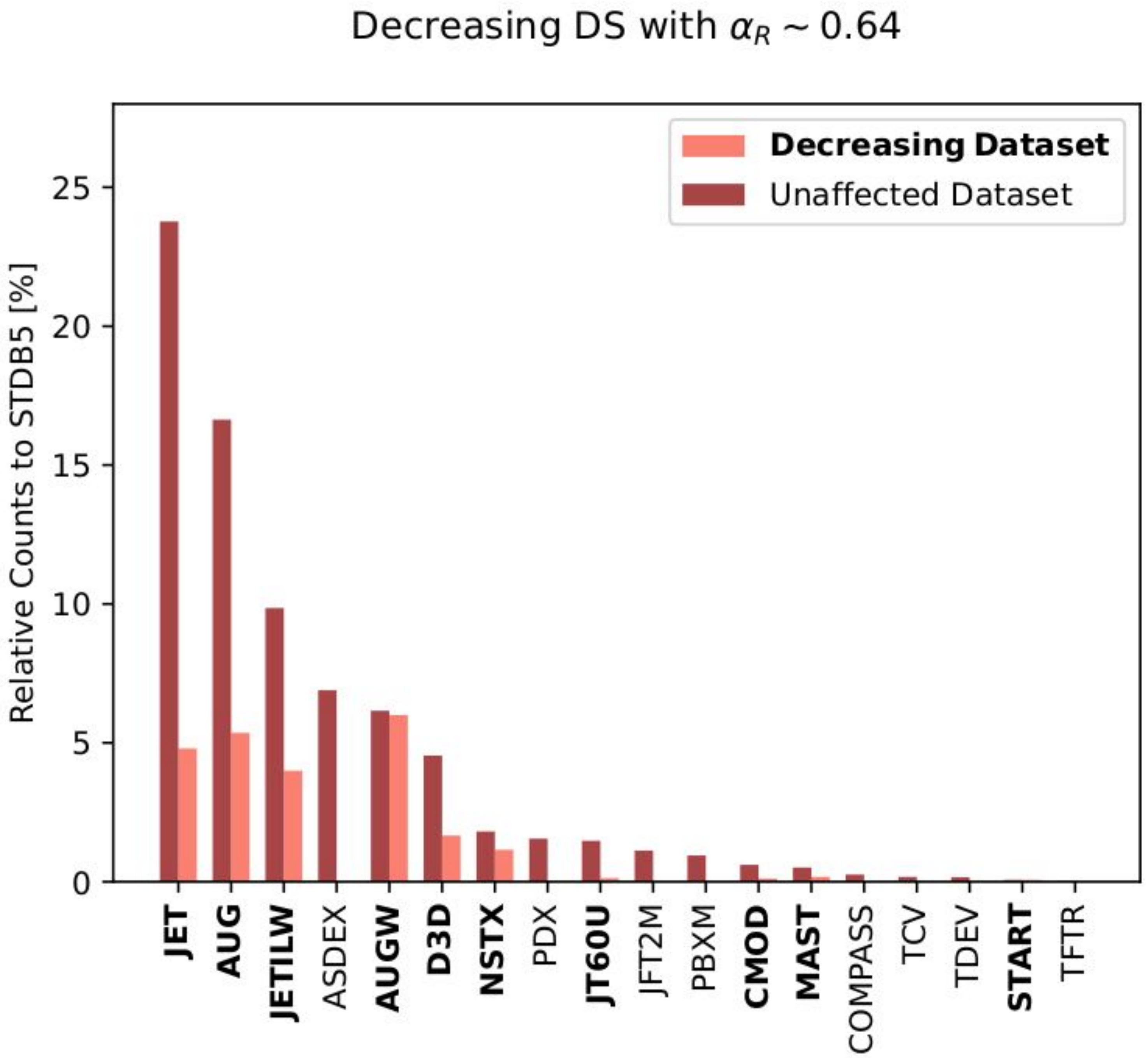
"Small dataset"	"Small dataset"	"Big dataset"	"Big dataset"
Smallest subset for $\alpha_R < 1$	**Complementary** for $\alpha_R < 1$	**Complementary** for smallest α_R	**Smallest α_R**
$\alpha_R \sim 0.99$	$\alpha_R \sim 1.71$	$\alpha_R \sim 2.16$	$\alpha_R \sim 0.64$
dataset size = 618	dataset size = 5634	dataset size = 4793	dataset size = 1459

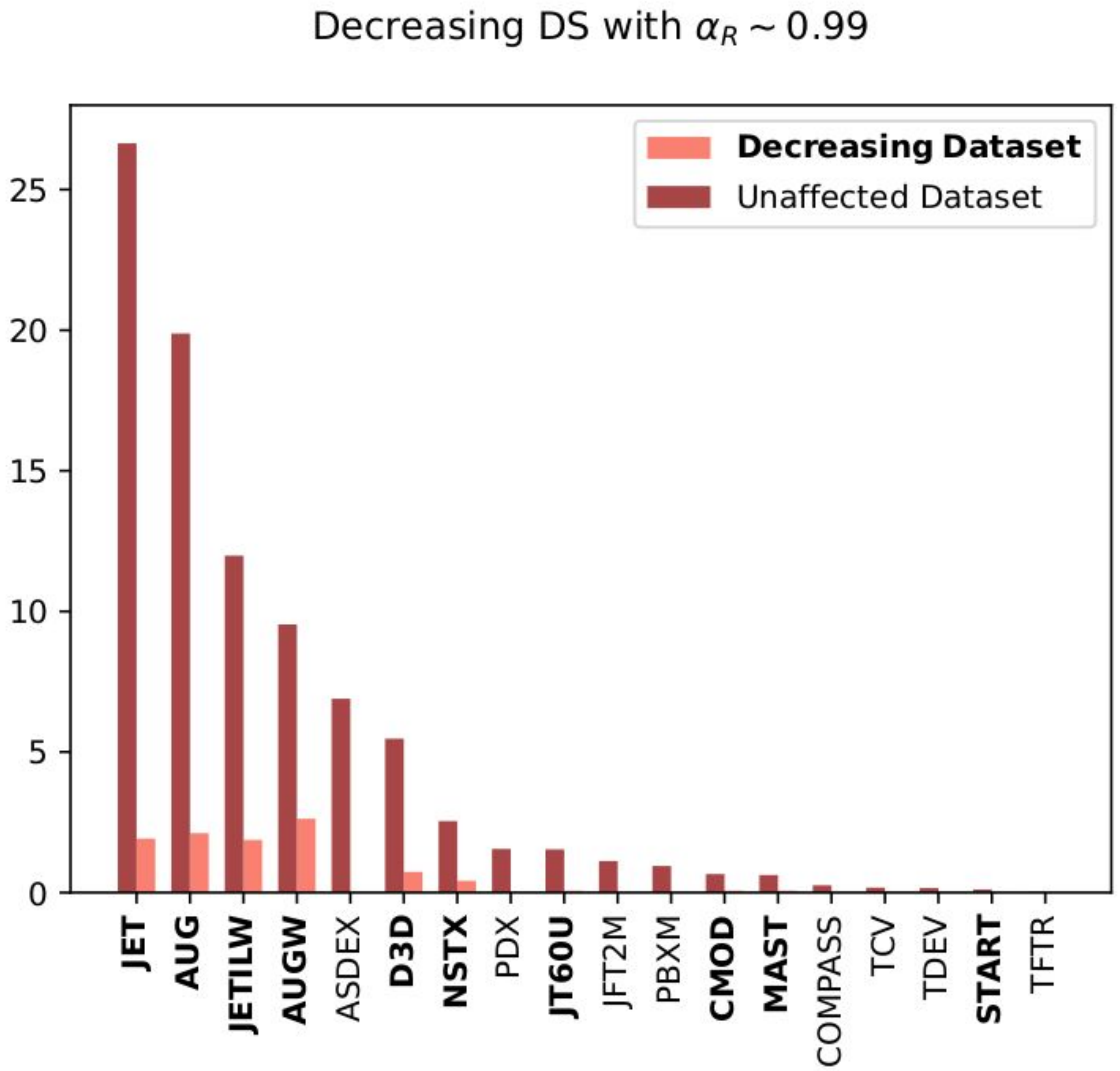
Together they make **STDB5**

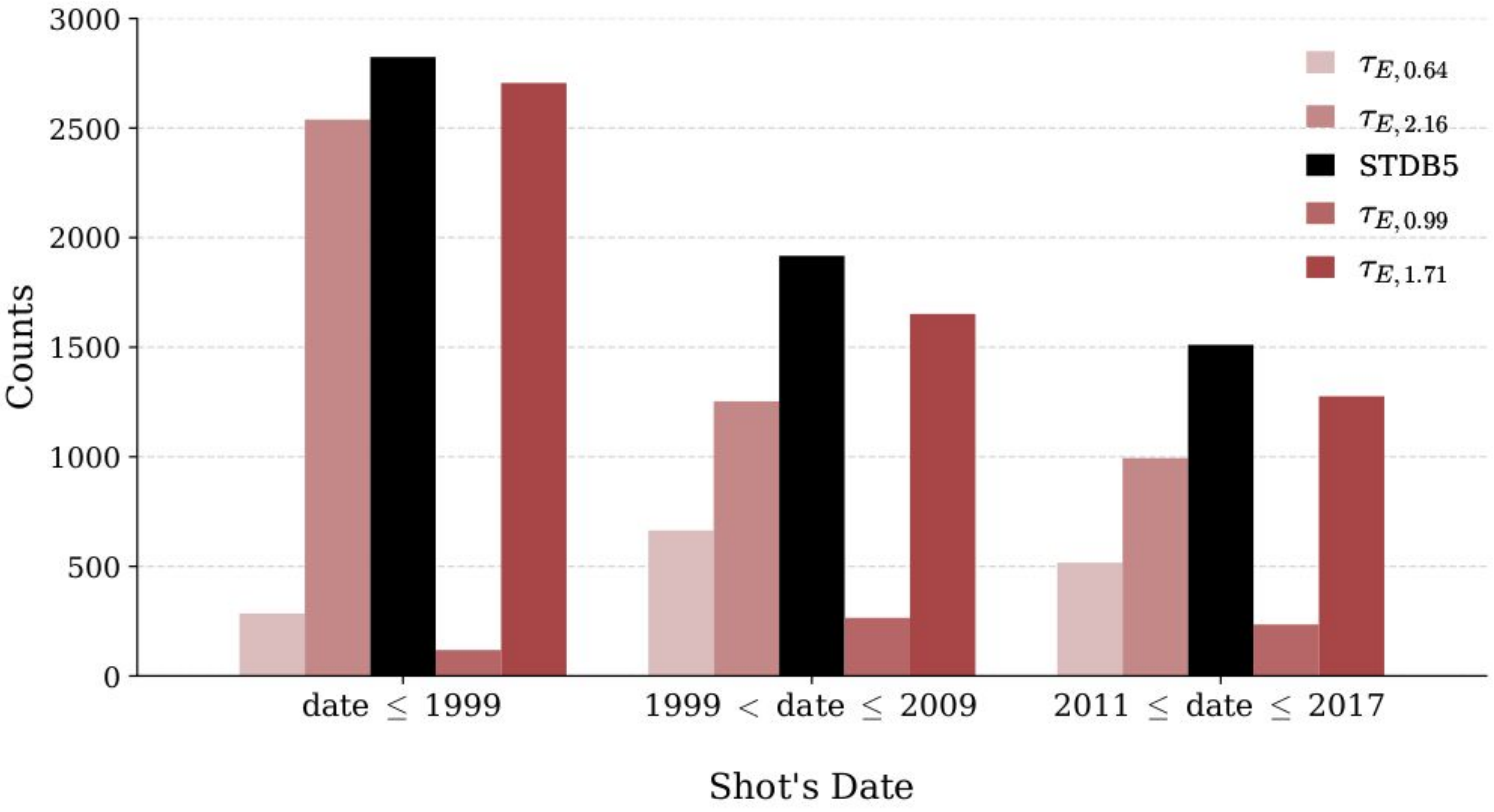
dataset size = 6252

DECREASING AND UNAFFECTING REGISTERS

	Scaling	α_0	I_p	B_t	$\bar{n}_e$	$P_{l,th}$	R_{geo}	κ_a	ϵ	M_{eff}
From big_ds	Decr.	0.10	1.42	0.02	-0.14	-0.53	0.64	-0.002	-0.56	0.13
	Unaff.	0.06	0.78	0.24	0.41	-0.75	2.16	0.5	0.79	0.22
From small_ds	Decr.	0.10	1.24	0.11	-0.02	-0.57	0.99	0.14	-0.19	0.12
	Unaff.	0.08	1.01	0.12	0.27	-0.71	1.71	0.34	0.32	0.23


Decreasing DS with $\alpha_R \sim 0.64$
Decreasing Dataset
Unaffected Dataset
Relative Counts to STDB5 [%]
0
5
10
15
20
25
JET
AUG
JETILW
ASDEX
AUGW
D3D
NSTX
PDX
JT60U
JFT2M
PBXM
CMOD
MAST
COMPASS
TCV
TDEV
START
TFTR


Decreasing DS with $\alpha_R \sim 0.99$
Decreasing Dataset
Unaffected Dataset
0
5
10
15
20
25
JET
AUG
JETILW
AUGW
ASDEX
D3D
NSTX
PDX
JT60U
JFT2M
PBXM
CMOD
MAST
COMPASS
TCV
TDEV
START
TFTR



3000
2500
2000
1500
1000
500
0
Counts
$\tau_{E,0.64}$
$\tau_{E,2.16}$
STDB5
$\tau_{E,0.99}$
$\tau_{E,1.71}$
date $\leq$ 1999
1999 $<$ date $\leq$ 2009
2011 $\leq$ date $\leq$ 2017
Shot's Date

WORKFLOW

WHICH REGISTERS IN STDB5 ARE **DECREASING** α_R?

IS THIS DECREASE DUE TO **DATA ISSUES**?

CAN WE **PREDICT** WHETHER A NEW REGISTER WILL **DECREASE** α_R?

RELATE THE FINDINGS **TOKAMAK CHARs & PHYSICS**

Depends on how you classify / label them.

MULTICOLLINEARITY?

Used as predictors for the **target**

Features / Variables

I_p [MA]	B_t [T]	….	ϵ	$T_{E,th}$ [s]
0.2959	2.205	….	0.242	0.051
….	….	….	….	….
0.9794	4.795	….	0.327	0.1731
0.3740	1.744	….	0.286	0.0340

CONSEQUENCES

- Increase in the standard errors of each feature
- Numerical instability
- Unreliable models

VIF | **Condition Index** → **VD**

Variance Inflation Factor

Variance Decomposition

[W.-M Lee, 2021]
[D.Besley, E. Kuh, and R. Welsch, 2004]
[R. M. O'Brien, 2007]

VARIANCE INFLATION FACTOR

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

$$\widehat{\mathbf{y}} = \mathbf{X}\beta \qquad \hat{\beta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$$

$$VIF_k = \frac{1}{1 - R_k^2}$$

I_p [MA]	B_t [T]	….	ϵ	~~$\tau_{E,th}$ [s]~~
0.2959	2.205	….	0.242	~~0.051~~
….	….	….	….	~~….~~
0.9794	4.795	….	0.327	~~0.1731~~
0.3740	1.744	….	0.286	~~0.0340~~

[D.Besley, E. Kuh, and R. Welsch, 2004]
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VARIANCE INFLATION FACTOR

$$\hat{\mathbf{y}} = \mathbf{X}\beta \qquad \hat{\beta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$$

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I_p [MA]	B_t [T]	….	ϵ	~~$\tau_{E,th}$ [s]~~
0.2959	2.205	….	0.242	~~0.051~~
….	….	….	….	~~….~~
0.9794	4.795	….	0.327	~~0.1731~~
0.3740	1.744	….	0.286	~~0.0340~~

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$$VIF_k = \frac{1}{1 - R_k^2}$$

$$Var[\beta_k] = \frac{\left(1 - R^2\right) \cdot \sum (y_i - \bar{y})^2}{n - M - 1} \cdot \frac{1}{\sum x_k^2} \cdot VIF_k$$

$\mathbf{I_p}$ [MA]	$\mathbf{B_t}$ [T]	….	**ϵ**	~~$\boldsymbol{\tau_{E,th}}$ [s]~~
0.2959	2.205	….	0.242	~~0.051~~
….	….	….	….	~~….~~
0.9794	4.795	….	0.327	~~0.1731~~
0.3740	1.744	….	0.286	~~0.0340~~

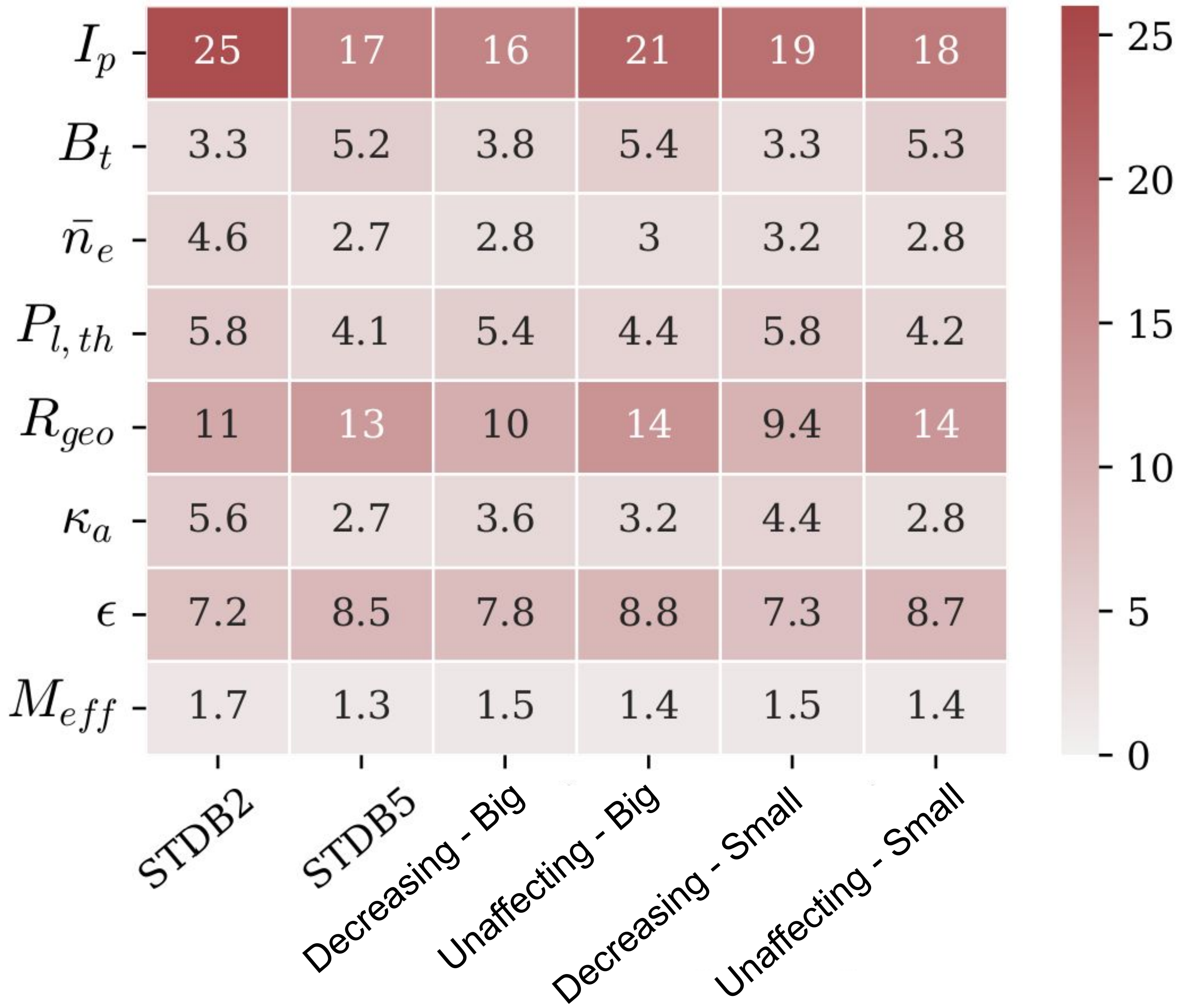
[D.Besley, E. Kuh, and R. Welsch, 2004]
[R. M. O'Brien, 2007]

VIF: RESULTS

$$\widehat{\mathbf{y}} = \mathbf{X}\beta \qquad \hat{\beta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$$

$$VIF_k = \frac{1}{1-R_k^2}$$

$$Var[\beta_k] = \frac{\left(1-R^2\right)\cdot\sum\left(y_i-\bar{y}\right)^2}{n-M-1}\cdot\frac{1}{\sum x_k^2}\cdot VIF_k$$



VIF: RESULTS

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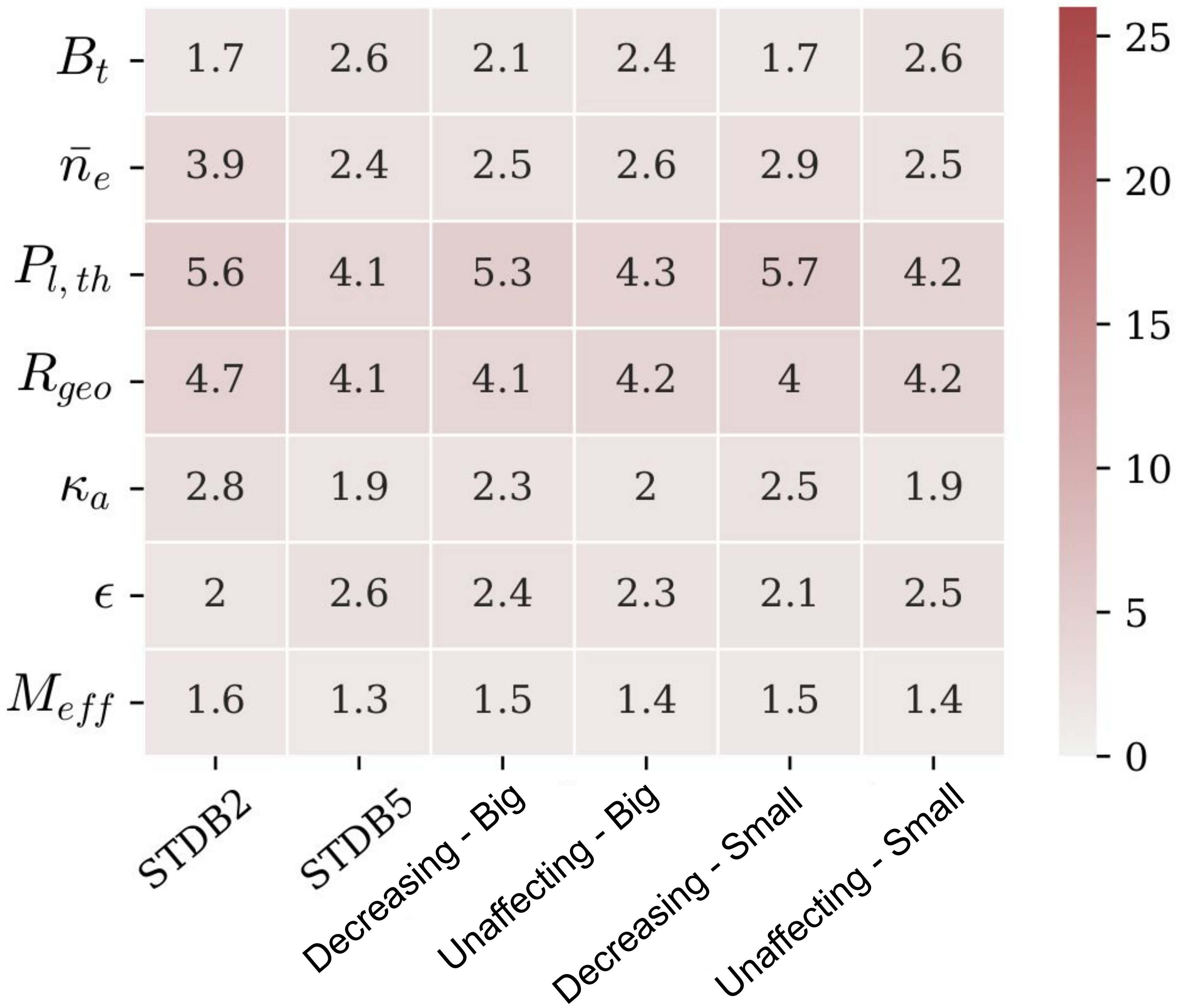
$$VIF_k = \frac{1}{1-R_k^2}$$

$$Var[\beta_k] = \frac{\left(1-R^2\right)\cdot\sum(y_i-\bar{y})^2}{n-M-1}\cdot\frac{1}{\sum x_k^2}\cdot VIF_k$$



WORKFLOW

WHICH REGISTERS IN STDB5 ARE **DECREASING** α_R?

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→ CAN WE **PREDICT** WHETHER A NEW REGISTER WILL **DECREASE** α_R?

→ **RELATE** THE FINDINGS **TOKAMAK CHARs & PHYSICS**

Depends on how you classify / label them.

Yes, but not entirely.

FEATURES OF INTEREST: Entropy

47 columns

35 | 12

$$E = -\sum_{\substack{i,j=1\\ i\neq j}}^{N} \left[S_{ij}\log(S_{ij}) + \left(1 - S_{ij}\right)\log(1 - S_{ij}) \right]$$

$$S_{ij} = \exp\left(-\gamma \cdot D_{ij}\right), \quad D_{ij} = \left[\sum_{k=1}^{M} \left(\frac{x_{ik} - x_{jk}}{max(F_k) - min(F_k)} \right)^2 \right]^{1/2}$$

$$S_{ij} = \frac{1}{M} \sum_{k=1}^{M} \delta_{ij}(x^k); \quad \text{with } \delta_{ij}(x^k) = \begin{cases} 1, \text{ if } x_i^k = x_j^k \\ 0, \text{ if } x_i^k \neq x_j^k \end{cases}$$

IDEA

Keep features that increased the entropy **when removed**.

[M. Dash and H. Liu, 2020]

FEATURES OF INTEREST: Entropy + Low MCL

47 columns

35 | 12

$$E = -\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \left[S_{ij} \log(S_{ij}) + \left(1 - S_{ij}\right) \log(1 - S_{ij}) \right]$$

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MULTICOLLINEARITY

Removing features with high VIF and causing high condition indices.

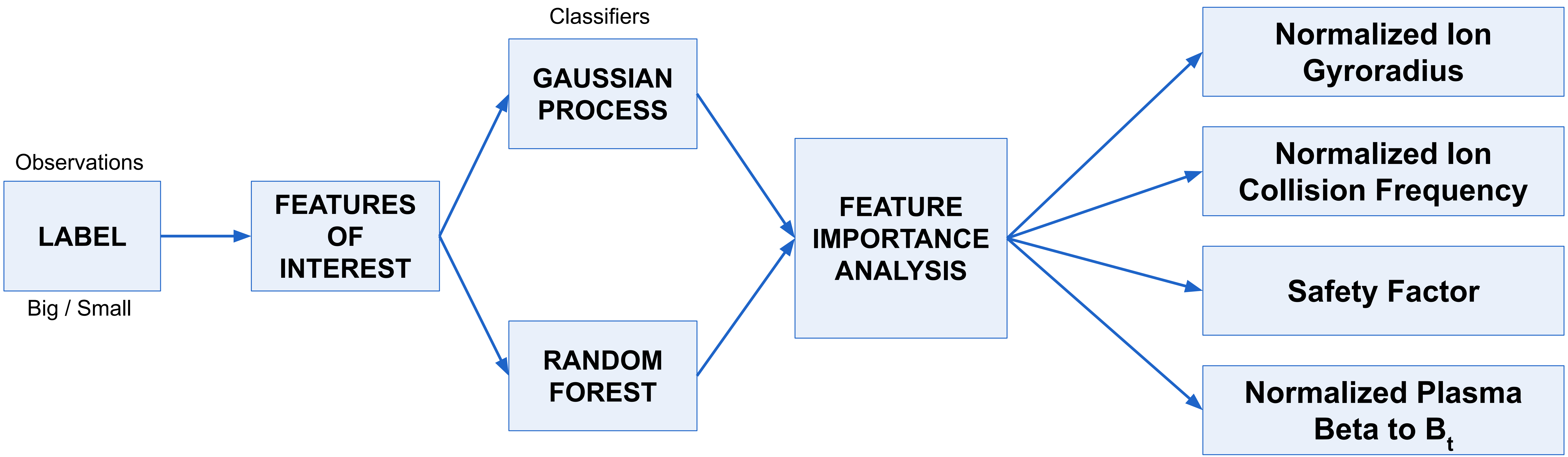
FEATURES OF INTEREST: Entropy + Low MCL

FEATURE	DESCRIPTION	FEATURE	DESCRIPTION
BETA	Plasma pressure normalised to $\mathbf{B}_t$	WFICFORM	Total fast ion energy due to ICRH estimated from approximate formula
RHOSTAR	Normalised Ion Gyroradius	WFFORM	Total fast ion energy due to NBI
NUSTAR	Normalised Ion collision frequency	ZEFFNEO	Line average plasma effective charge, from Bremsstrahlung
Q95	Plasma safety factor at the 95% poloidal flux surface	DWDIA	Time rate of change of the total plasma stored energy
PFLOSS	NBI power that is lost from the plasma through charge exchange and unconfined orbits		

Description of all columns in STDB5 found in [Princeton, 2021, http://arks.princeton.edu/ark:/88435/dsp01m900nx49h]

FEATURES FOR PREDICTION

Best Performance with “**big dataset**”



PREDICTIONS IN NEW DEVICES

	I_P	B_t	$\bar{n}_e$	$P_{l,th}$	R_{geo}	κ_a	ϵ	M_{eff}	ρ_*	β_t	ν_*	q_{95}	$\tau_{E,th}$
ITER	15	5.3	1.03	87	6.2	1.8	0.32	2.5	0.002	2.24	0.014	3	3.5
SPARC	8.7	12.2	3.1	25	1.85	1.97	0.31	2.5	0.003	1.20	0.03	3.2	0.77

UNAFFECTED

$$\tau_{E,2.16} = 0.06 \cdot I_p^{0.78} \cdot B_t^{0.24} \cdot \bar{n}_e^{0.41} \cdot P_{l,th}^{-0.75} \cdot R_{geo}^{2.16} \cdot \kappa_a^{0.5} \cdot \epsilon^{0.79} \cdot M_{eff}^{0.22}$$

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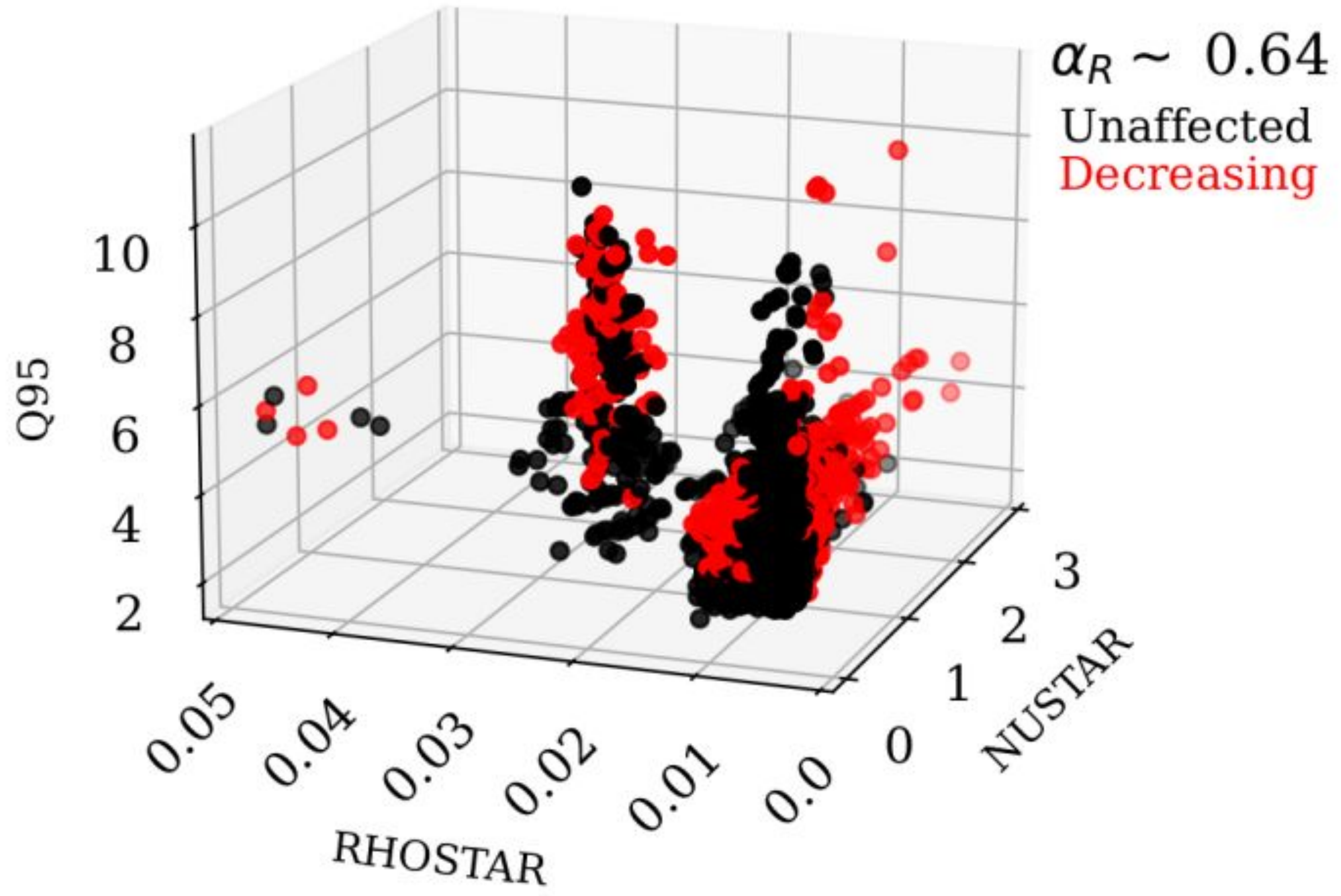
Depends on how you classify / label them.

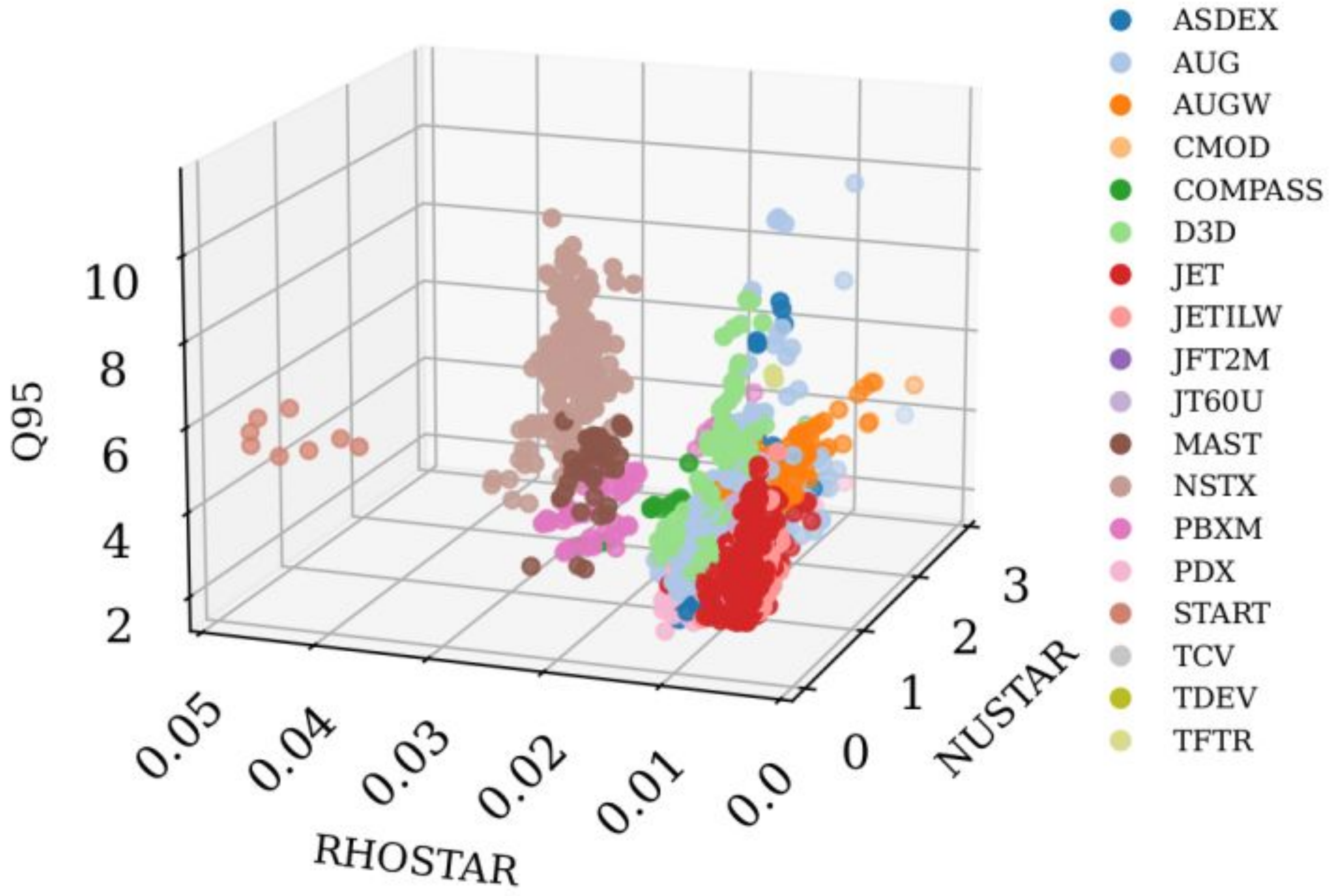
Yes, but not entirely.

Yes, with only four variables.

Characteristic Clusters: main features

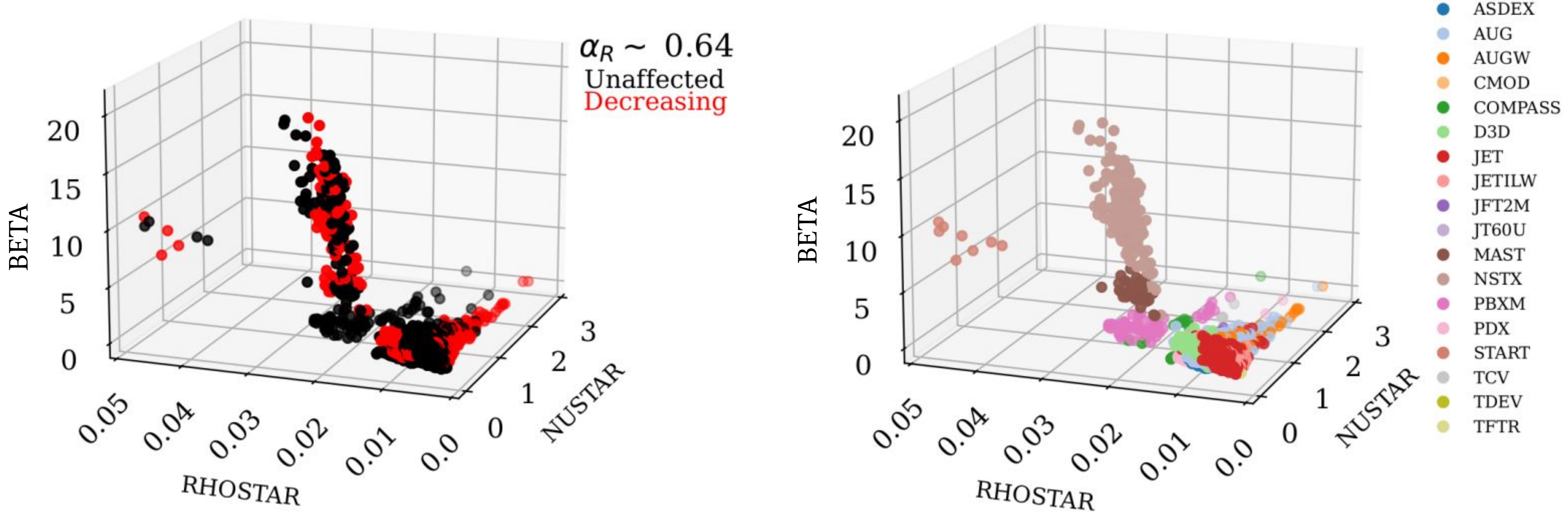
New STDB5





Characteristic Clusters: main features

New STDB5



Dimensionless Scaling and the Normalized Ion Gyroradius

$$\omega_i \cdot \tau_{E,the} \propto \rho_*^{-(2+\chi'_\rho)} \cdot F(\nu_*, \beta_t, \{p_i\})$$ with $0 \leq \chi'_\rho \leq 1$

$$\boxed{\chi'_\rho = 0.9 \pm 0.3}$$

H-mode

characteristic turbulence scale length $\ell \approx \rho_*^{\chi'_\rho} \cdot a^{1-\chi'_\rho}$

			big_ds		small_ds	
	STDB2	STDB5	Decr.	Unaff.	Decr.	Unaff.
	χ_ρ	-3.09	-1.80	-1.31	-2.63	-1.5	-2.08
	χ'_ρ	1.09	0.2	0.69	0.63	-0.5	0.08
ITER	ℓ [m]	0.0012	7.99	227.43	0.026	64.99	1.15
SPARC	ℓ [m]	0.0017	1.67	22.33	0.019	8.46	0.37

[G. Verdoolaege et al., 2021]
[I. P. E. G. et al., 1999]

Dimensionless Scaling and the Normalized Ion Gyroradius

$$\omega_i \cdot \tau_{E,the} \propto \rho_*^{-(2+\chi'_\rho)} \cdot F(\nu_*, \beta_t, \{p_i\})$$

$$\boxed{\omega_i \cdot \tau_{E,th} = \chi_0 \cdot B_t^{\chi_B} \cdot \rho_*^{\chi_\rho} \cdot \beta_t^{\chi_\beta} \cdot \nu_*^{\chi_\nu} \cdot q_{95}^{\chi_q} \cdot \kappa_a^{\chi_\kappa} \cdot \epsilon^{\chi_\epsilon} \cdot M_{eff}^{\chi_M}}$$

$$\chi_B = 0,$$

$$\chi_\rho = \frac{2(-3\alpha_R - 3\alpha_I - 9\alpha_P + \alpha_n)}{5(1+\alpha_P)},$$

$$\chi_\beta = \frac{\alpha_R + \alpha_I + 8\alpha_P + 3\alpha_n}{5(1+\alpha_P)},$$

$$\chi_\nu = \frac{-\alpha_R - \alpha_I - 3\alpha_P + 2\alpha_n}{5(1+\alpha_P)},$$

$$\chi_q = \frac{\alpha_R - 4\alpha_I + 3\alpha_P - 2\alpha_n}{5(1+\alpha_P)},$$

$$\chi_\kappa = \frac{\alpha_k + \alpha_P}{1+\alpha_P},$$

$$\chi_\epsilon = \frac{2\alpha_\epsilon - 3\alpha_R + \alpha_I - 5\alpha_P + 2\alpha_n}{2(1+\alpha_P)},$$

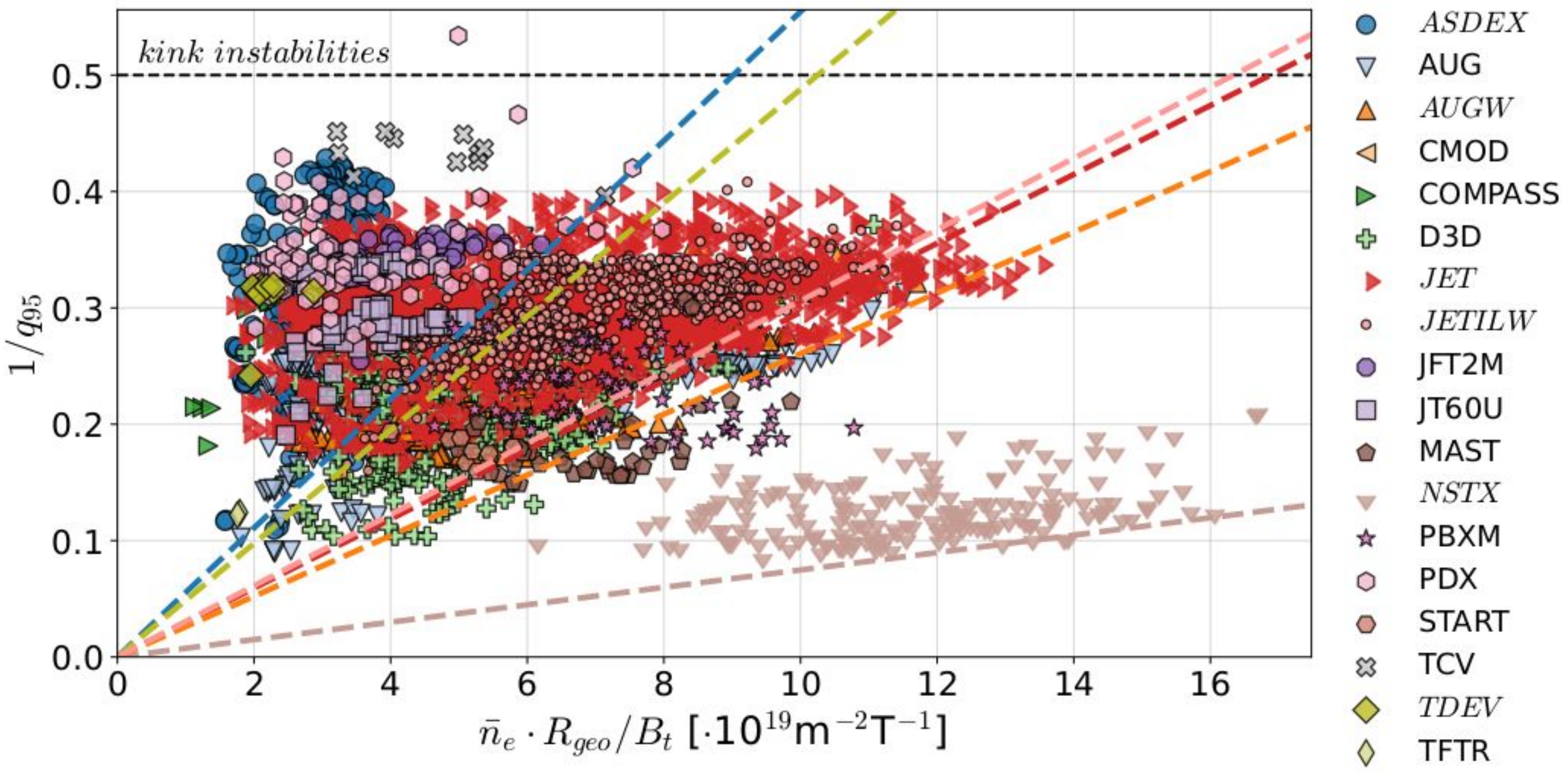
$$\chi_M = \frac{5\alpha_M + 3\alpha_R + 3\alpha_I + 4\alpha_P - \alpha_n - 5}{5(1+\alpha_P)}$$

[G. Verdoolaege et al., 2021]
[I. P. E. G. et al., 1999]

DENSITY LIMIT

STDB5

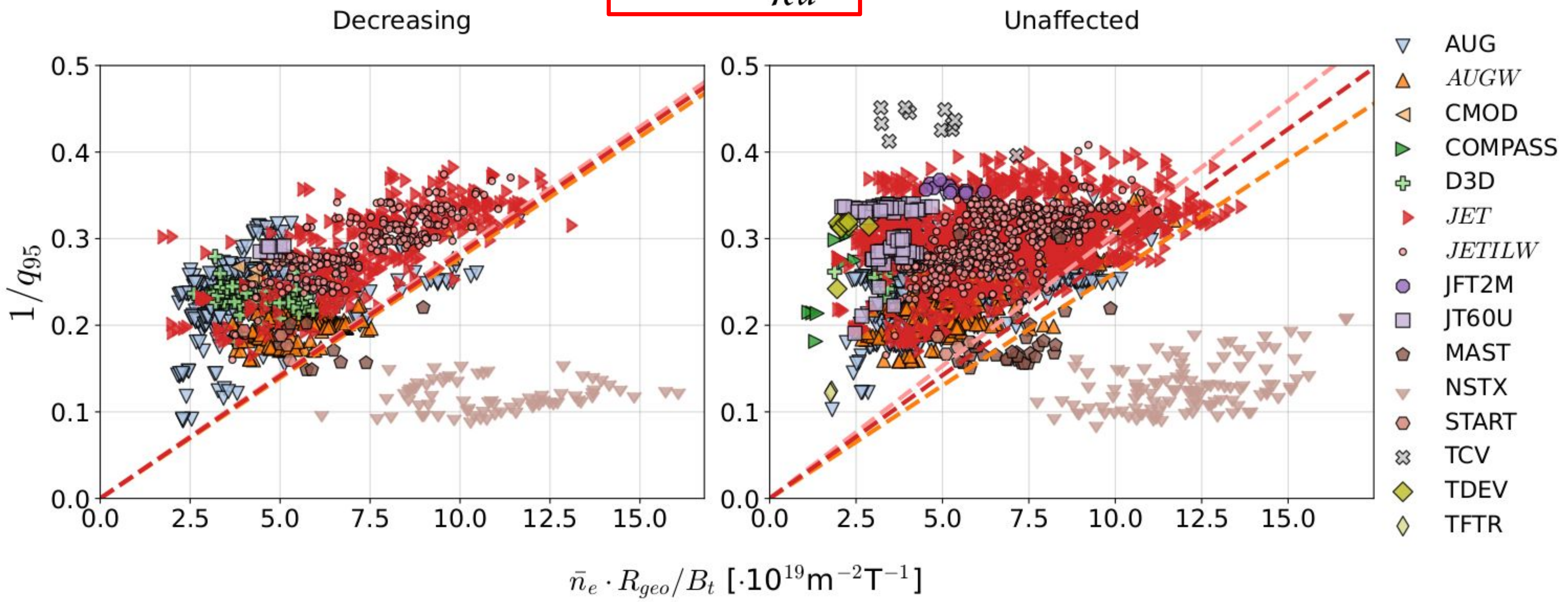
$$n_{GW} = \frac{10 \cdot I_p}{\pi a^2}$$



DENSITY LIMIT

$$n_{GW} = \frac{10 \cdot I_p}{\pi a^2}$$

New STDB5



Conclusions and Further Work

CONCLUSIONS

- Multicollinearity is **not** the sole factor influencing α_R.
- It is possible to predict influencing observations with . $\rho_* \quad \beta_t \quad \nu_* \quad q_{95}$
- Characteristic clusters for spherical tokamaks.
- It is possible to tell if a dataset lacks of representative physics.
- Most of the observations that surpassed the GW limit are classified as unaffecting.
- ITER and SPARC are expected to follow a scaling law similar to the 1998 scaling (great news!).

FURTHER WORK

- Obtain more data on spherical tokamaks.
- Discern between spherical and non spherical.
- Take time-series data of relevant shots and subject to other ML algorithms; e.g. **surrogate modelling**.
  - Underlying Physics
  - Plasma Facing Components

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Thank you! Questions?

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