Colegio De Muntinlupa
Numerical Methods | Advanced Mathematics
Laboratory Exercise 3 (Take-Home)

General Instructions:

I. This take-home exercise is by pair.

Deadline of submission: March 12, 2023, Sunday.

A grade of zero will be credited to those who are to commit any form of cheating, may it be attempted. From here, further instructions on the details are provided.

II. Inputs:

- Coefficient Matrix A
- right-hand side b

III. Stopping criterion & initial conditions:

- Stopping criterion: $||x_{new} - x_{prev}|| < tol$.
- initial guess: $x = \begin{bmatrix} 0 \\ 0 \\ \ldots \\ \ldots \\ 0 \end{bmatrix}$
- tolerance: 10^{-6}
- maximum number of iterations: 100

IV. Output/s:

- For Gauss-Jordan:
  A. The final augmented matrix **M.**
  B. Solution vector/array **x**
- For Jacobi and Gauss-Seidel:
  A. **Table M** summarizing the
    i. iteration i starting from 0
    ii. updates for the solution
  B. Solution vector/array **x**

Note: That means the first line of your function/s should look like this:

`function [M,x]=functionname(A,b)`

V. Some helpful functions

- `eye(m,n)` or `eye(n)`
- `zeros(m,n)` or `zeros(n)`
- `diag(A)`
- `[m,n]=size(A)`
- `inv(A)`
- `tril(A)`
- `triu(A)`
- `norm(x)`
- `length(x)`

Note: **m** & **n** are dimensions, **A** is a matrix; **x** is an array/vector

VI. Documentation

- Each method should be saved as a **function**.
- Save your code **PER SUB-ITEM**. Save the function with the ff format for the filename:

  METHOD_itemnumber_Surnames.m

  e.g. GaussSeidel_1b_InauditoDelCarmen.m
- Submit a **pdf** file containing the answers for each items/sub-items. Properly screenshot the code and the results in the command window. Save your file using the ff format: 3rdLabExer_Surnames.pdf
- Attach your codes and pdf file in BB. <u>One submission per pair</u>.

Instructions: Do as indicated.

1. Consider the system given by:

$$
\begin{aligned}
4x_1 - x_2 - x_3 \qquad\qquad\qquad &= 18\\
-x_1 + 4x_2 - x_3 - x_4 \qquad\qquad &= 18\\
-x_2 + 4x_3 - x_4 - x_5 \qquad &= 4\\
-x_3 + 4x_4 - x_5 - x_6 &= 4
\end{aligned}
$$

$$\begin{aligned} -x_4 + 4x_5 - x_6 - x_7 \quad &= 26 \\ -x_5 + 4x_6 - x_7 - x_8 &= 16 \\ -x_6 + 4x_7 - x_8 &= 10 \\ -x_7 + 4x_8 &= 32 \end{aligned}$$

Numerically solve the system using

a. Gauss-Jordan Elimination

b. Jacobi

c. Is the convergence for Jacobi guaranteed? Why or why not?

2. Given $\begin{aligned} -4x_1 + 5x_2 &= 18 \\ x_1 + 2x_2 &= 3 \end{aligned}$.

a. Manually solve the system using Gauss-Jordan Elimination using an augmented matrix.

b. Numerically implement Gauss-Seidel to find an approximate solution. Does the method converge? Why or why not?

3. Givem $\begin{aligned} x_1 - 2x_2 &= -1 \\ 2x_1 + x_2 &= 3 \end{aligned}$.

a. Manually solve the system using Gauss-Jordan Elimination using an augmented matrix.

b. Numerically implement Gauss-Seidel to find an approximate solution. Does the method converge? Why or why not?

4. Consider the differential equation given by: $(D-1)\left(D^2+1\right)y = 0$.

a. **Manually** find the **general solution** of the homogeneous differential equation.

b. If $y(0) = 4$, $y'(0) = -1$ and $y''(0) = -2$, set-up the linear system using the constants c_1, c_2 and c_3.

c. Run Gauss-Jordan Elimination to solve the IVP.

5. Jessa has taken note that CDM's canteen has 20 tables with three classifications according to its capacity. These tables are classified as A, B and C with 4, 6 and 8 seating capacities, respectively. The total seating capacity is 108. If only half of table A, half of table B, and only one-fourth of table C are occupied, 46 students are seated.

a. Set-up the linear system that will find how many tables with 4, 6 and 8 seating capacities are there.

b. Run Gauss-Jordan Elimination to find solve the system. Conclude properly.

END