Appendix

Chi Nguyen

August 2022

1 Vector calculus

1.1 $f : \mathrm{R}^n \to \mathrm{R}$

$$\begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{bmatrix} \to f(x) \Rightarrow \frac{\delta f}{\delta x} = \begin{bmatrix} \frac{\delta f(x)}{\delta x_1} \frac{\delta f(x)}{\delta x_2} ... \frac{\delta f(x)}{\delta x_n} \end{bmatrix}$$

1.2 $f : \mathrm{R} \to \mathrm{R}^n$

$$x \to \begin{bmatrix} f_1(x) \\ f_2(x) \\ ... \\ f_n(x) \end{bmatrix} \Rightarrow \frac{\delta f}{\delta x} = \begin{bmatrix} \frac{\delta f_1(x)}{\delta x_1} \\ \frac{\delta f_2(x)}{\delta x_2} \\ ... \\ \frac{\delta f_n(x)}{\delta x_n} \end{bmatrix}$$

1.3 $f : \mathrm{R}^m \to \mathrm{R}^n$

$$\begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_m \end{bmatrix} \to \begin{bmatrix} f_1(x) \\ f_2(x) \\ ... \\ f_n(x) \end{bmatrix} \Rightarrow \frac{\delta f}{\delta x} = \begin{bmatrix} \frac{\delta f_1(x)}{\delta x_1} & \frac{\delta f_1(x)}{\delta x_2} & ... & \frac{\delta f_1(x)}{\delta x_m} \\ \frac{\delta f_2(x)}{\delta x_1} & \frac{\delta f_2(x)}{\delta x_2} & ... & \frac{\delta f_2(x)}{\delta x_m} \\ ... & ... & ... & ... \\ \frac{\delta f_n(x)}{\delta x_1} & \frac{\delta f_n(x)}{\delta x_2} & ... & \frac{\delta f_n(x)}{\delta x_m} \end{bmatrix}$$

1.4 The Hessian matrix is the square matrix of second partial derivatives of a scalar valued function f(1.1)

$$H(f) = \begin{bmatrix} \frac{\delta f^2(x)}{\delta x_1^2} & \frac{\delta f^2(x)}{\delta x_1 \delta x_2} & ... & \frac{\delta f^2(x)}{\delta x_1 \delta x_n} \\ \frac{\delta f^2(x)}{\delta x_2 \delta x_1} & \frac{\delta f^2(x)}{\delta x_2^2} & ... & \frac{\delta f^2(x)}{\delta x_2 \delta x_n} \\ ... & ... & ... & ... \\ \frac{\delta f^2(x)}{\delta x_n \delta x_1} & \frac{\delta f^2(x)}{\delta x_n \delta x_2} & ... & \frac{\delta f^2(x)}{\delta x_n^2} \end{bmatrix}$$