

# Appendix

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## 1 Vector calculus

### 1.1 $f : \mathbb{R}^n \rightarrow \mathbb{R}$

$$\begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} \rightarrow f(x) \Rightarrow \frac{\delta f}{\delta x} = \left[ \frac{\delta f(x)}{\delta x_1} \frac{\delta f(x)}{\delta x_2} \dots \frac{\delta f(x)}{\delta x_n} \right]$$

### 1.2 $f : \mathbb{R} \rightarrow \mathbb{R}^n$

$$x \rightarrow \begin{bmatrix} f_1(x) \\ f_2(x) \\ \dots \\ f_n(x) \end{bmatrix} \Rightarrow \frac{\delta f}{\delta x} = \begin{bmatrix} \frac{\delta f_1(x)}{\delta x_1} \\ \frac{\delta f_2(x)}{\delta x_2} \\ \dots \\ \frac{\delta f_n(x)}{\delta x_n} \end{bmatrix}$$

### 1.3 $f : \mathbb{R}^m \rightarrow \mathbb{R}^n$

$$\begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_m \end{bmatrix} \rightarrow \begin{bmatrix} f_1(x) \\ f_2(x) \\ \dots \\ f_n(x) \end{bmatrix} \Rightarrow \frac{\delta f}{\delta x} = \begin{bmatrix} \frac{\delta f_1(x)}{\delta x_1} & \frac{\delta f_1(x)}{\delta x_2} & \dots & \frac{\delta f_1(x)}{\delta x_m} \\ \frac{\delta f_2(x)}{\delta x_1} & \frac{\delta f_2(x)}{\delta x_2} & \dots & \frac{\delta f_2(x)}{\delta x_m} \\ \dots & \dots & \dots & \dots \\ \frac{\delta f_n(x)}{\delta x_1} & \frac{\delta f_n(x)}{\delta x_2} & \dots & \frac{\delta f_n(x)}{\delta x_m} \end{bmatrix}$$

### 1.4 The Hessian matrix is the square matrix of second partial derivatives of a scalar valued function f(1.1)

$$H(f) = \left( \frac{\delta f}{\delta x} \right)^T \frac{\delta f}{\delta x} = \begin{bmatrix} \frac{\delta^2 f(x)}{\delta x_1^2} & \frac{\delta^2 f(x)}{\delta x_1 \delta x_2} & \dots & \frac{\delta^2 f(x)}{\delta x_1 \delta x_n} \\ \frac{\delta^2 f(x)}{\delta x_2 \delta x_1} & \frac{\delta^2 f(x)}{\delta x_2^2} & \dots & \frac{\delta^2 f(x)}{\delta x_2 \delta x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\delta^2 f(x)}{\delta x_n \delta x_1} & \frac{\delta^2 f(x)}{\delta x_n \delta x_2} & \dots & \frac{\delta^2 f(x)}{\delta x_n^2} \end{bmatrix}$$