Problem 1

Let H denote the even that has Hansen's disease Let P denote the even that test positive The probability is:

$$
\begin{aligned}
P(H \mid P) &= \frac{P(PH)}{P(P)} \\
&= \frac{P(PH)}{P(PH) + P(PH^{\complement})} \\
&= \frac{(0.5)(0.98)}{(0.05)(0.98) + (0.03)(0.95)} \\
&= 0.6323
\end{aligned}
$$

Problem 2

1. Univariate normal distribution

$$
\begin{aligned}
\int_{-\infty}^{\infty} p(x)dx &= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx \\
&= \int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2} dx \frac{1}{\sigma\sqrt{2\pi}}
\end{aligned}
$$

$$
I = \int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2} dx = \int_{-\infty}^{\infty} e^{-y^2/2\sigma^2} dy
$$

$$
I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} e^{-y^2/2\sigma^2} dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2\sigma^2} dxdy
$$

With

$$
\begin{aligned}
x &= r\cos\theta \\
y &= r\sin\theta
\end{aligned}
$$

Apply Jacobian of the change of variables, we have:

$$
\begin{aligned}
I^2 &= \int_0^{2\pi}\int_0^{\infty} e^{-r^2/2\sigma^2} r dr d\theta \\
&= 2\pi \int_0^{\infty} e^{-r^2/2\sigma^2} r dr \\
&= 2\pi(-\sigma^2)^{-r^2/\sigma^2}\Big|_0^{\infty} \\
&= 2\pi\sigma^2
\end{aligned}
$$

$\Rightarrow I = \sqrt{2\pi\sigma^2} \Rightarrow \int_{-\infty}^{\infty} p(x)dx = 1 \Rightarrow p(x)$ *normalized*

MEAN: E[X] = $\int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} xe^{-(x-\mu)^2/2\sigma^2}dx\frac{1}{\sqrt{2\pi\sigma^2}}$
Let $y = x - \mu \Rightarrow x = y + \mu$, dx = dy

$$
\begin{aligned}
E(X) &= \int_{-\infty}^{\infty}(y+\mu)e^{-y^2/2\sigma^2}dy\frac{1}{\sqrt{2\pi}\sigma} \\
&= \int_{-\infty}^{\infty} ye^{-y^2/2\sigma^2}dy\frac{1}{\sqrt{2\pi}\sigma} + \int_{-\infty}^{\infty} e^{-y^2/2\sigma^2}dy\frac{\mu}{\sqrt{2\pi}\sigma} \\
&= 0 + \mu \\
&= \mu
\end{aligned}
$$

VAR: V[X] = $\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx = \int_{-\infty}^{\infty} y^2 e^{-y^2/2\sigma^2}dy\frac{1}{\sqrt{2\pi}\sigma}$

$$
\begin{aligned}
u &= y & du &= dy \\
dv &= ye^{-y^2/2\sigma^2}dy & v &= -\sigma^2 e^{-y^2/2\sigma^2}
\end{aligned}
$$

$$
\begin{aligned}
V[X] &= \Big[-\sigma^2 ye^{-y^2/2\sigma^2}\Big|_{-\infty}^{\infty} + \sigma^2\int_{-\infty}^{\infty} e^{-y^2/2\sigma^2}\Big]\frac{1}{\sqrt{2\pi}\sigma} \\
&= 0 + \sigma^2 \\
&= \sigma^2
\end{aligned}
$$