1 Supplementary Methods: MBITES-de

Let the behavioral state variable name from MBITES denote the expected number of mosquitoes of a given chronological age (a), that are in each behavioral state. Similarly, let the waiting times to events be modeled as a rate that is dependent on the behavioral state (x), age, and time (t): $\xi_x(a, t)$. The proportion of mosquitoes that transition from state x to state y at the end of a bout is denoted P_{xy}. Death rates can be age dependent (*i.e.,* due to senescence), which affects the proportions transitioning to other states, so we write $P_{xy}(a)$. To deal with the event-driven nature of these bouts and the possibility some bouts may be repeated many times before transitioning to another state, we index mosquitoes by the i^{th} attempt to repeat the same event as a way of computing waiting times properly; for example, a mosquito repeating a blood feeding attempt bout transitions from $B_n(a)$ to $B_{n+1}(a)$. Finally, we let $\Lambda(t)$ represent the rate of emergence of adult female mosquitoes. The following system of coupled PDEs is homologous to MBITES:

$$
\begin{aligned}
F_1(0,t) = &\quad \Lambda(t) \\
\frac{\partial F_1(a,t)}{\partial t} + \frac{\partial F_1(a,t)}{\partial a} = &\quad \xi_O(a,t)P_{OF}(a)\sum_i O_i(a,t) + \xi_B(a,t)P_{BF}(a)\sum_i B_i(a,t) \\
&\quad +\xi_R(a,t)P_{RF}(a)R(a,t) - \xi_F(a,t)F_1(a,t) \\
\frac{\partial F_i(a,t)}{\partial t} + \frac{\partial F_i(a,t)}{\partial a} = &\quad \xi_F(a,t)P_{FF}(a)F_{i-1}(a,t) - \xi_F(a,t)F_i(a,t) \\
\frac{\partial B_1(a,t)}{\partial t} + \frac{\partial B_1(a,t)}{\partial a} = &\quad \xi_O(a,t)P_{OB}(a)\sum_i O_i(a,t) + \xi_F(a,t)P_{FB}(a)\sum_i F_i(a,t) \\
&\quad +\xi_R(a,t)P_{RB}(a)R(a,t) \\
&\quad -\xi_B(a,t)B_1(a,t) \\
\frac{\partial B_i(a,t)}{\partial t} + \frac{\partial B_i(a,t)}{\partial a} = &\quad \xi_B(a,t)P_{BB}(a)B_{i-1}(a,t) - \xi_B(a,t)B_i(a,t) \\
\frac{\partial R(a,t)}{\partial t} + \frac{\partial R(a,t)}{\partial a} = &\quad \xi_B(a,t)P_{BR}(a)\sum_i B_i(a,t) - \xi_R(a,t)R(a,t) \\
\frac{\partial L_1(a,t)}{\partial t} + \frac{\partial L_1(a,t)}{\partial a} = &\quad \xi_R(a,t)P_{RL}(a)R(a,t) + \xi_O(a,t)P_{OL}(a)\sum_i O_i(a,t) \\
&\quad -\xi_L(a,t)L_1(a,t) \\
\frac{\partial L_i(a,t)}{\partial t} + \frac{\partial L_i(a,t)}{\partial a} = &\quad \xi_L(a,t)P_{LL}(a)L_{i-1}(a,t) - \xi_L(a,t)L_i(a,t) \\
\frac{\partial O_1(a,t)}{\partial t} + \frac{\partial O_1(a,t)}{\partial a} = &\quad \xi_L(a,t)P_{LO}(a)\sum_i L_i(a,t) + \xi_R(a,t)P_{RO}(a)R(a,t) \\
&\quad -\xi_O(a,t)O_1(a,t) \\
\frac{\partial O_i(a,t)}{\partial t} + \frac{\partial O_i(a,t)}{\partial a} = &\quad \xi_O(a,t)P_{OO}(a)O_{i-1}(a,t) - \xi_O(a,t)O_i(a,t)
\end{aligned}
\tag{1}
$$

It is a nuisance to deal with an infinite set of equations, but if the state transitions are Markovian, then a change of variables to lump the the n^{th} states together: $x = \sum_i x_i$, with a rescaled rate variable, $\gamma_x(a,t) = \xi(a,t)(1 - P_{xx})$

<Insert John's cool proof>.

This means we can rewrite the infinite system of equations as a set of 5 differential equations:

$$
\begin{aligned}
F_o(0,t) = & \quad \Lambda(t) \\
\frac{\partial F(a,t)}{\partial t} + \frac{\partial F(a,t)}{\partial a} = & \quad \gamma_O(a,t)P_{OF}(a)O(a,t) + \gamma_B(a,t)P_{BF}(a)B(a,t) \\
& \quad +\gamma_R(a,t)P_{RF}(a)R(a,t) + \gamma_F(a,t)P_{FF}(a)F_e(a,t) \\
& \quad -\gamma_F(a,t)F(a,t) \\
\frac{\partial B(a,t)}{\partial t} + \frac{\partial B_o(a,t)}{\partial a} = & \quad \gamma_O(a,t)P_{OB}(a)O(a,t) + \gamma_F(a,t)P_{FB}(a)F(a,t) \\
& \quad +\gamma_R(a,t)P_{RB}(a)R(a,t) + \gamma_B(a,t)P_{BB}(a)B_e(a,t) \\
& \quad -\gamma_B(a,t)B(a,t) \qquad (2) \\
\frac{\partial R(a,t)}{\partial t} + \frac{\partial R(a,t)}{\partial a} = & \quad \gamma_B(a,t)P_{BR}(a)B(a,t) - \gamma_R(a,t)R(a,t) \\
\frac{\partial L(a,t)}{\partial t} + \frac{\partial L(a,t)}{\partial a} = & \quad \gamma_R(a,t)P_{RL}(a)R(a,t) + \gamma_O(a,t)P_{OL}(a)O(a,t) \\
& \quad +\gamma_L(a,t)P_{LL}(a)L_e(a,t) - \gamma_L(a,t)L(a,t) \\
\frac{\partial O(a,t)}{\partial t} + \frac{\partial O(a,t)}{\partial a} = & \quad \gamma_L(a,t)P_{LO}(a)L(a,t) + \gamma_R(a,t)P_{RO}(a)R(a,t) \\
& \quad +\gamma_O(a,t)P_{OO}(a)O_e(a,t) - \gamma_O(a,t)O_o(a,t)
\end{aligned}
$$

1.1 The MBITES-de for Cohorts

Finally, we want a version of these equations to model changes in a cohort of individuals with respect to age (assuming all the mosquitoes emerge from aquatic habitats at the same time of day):

$$
\begin{aligned}
F_o(0) &= 1 \\
\dot{F}_o &= \gamma_O(a)P_{OF}(a)O(a) + \gamma_B(a)P_{BF}(a)B(a) + \gamma_R(a,t)P_{RF}(a)R(a) \\
&\quad +\gamma_F(a)P_{FF}(a)F_e(a) - \gamma_F(a)F_o(a) \\
\dot{F}_e &= \gamma_F(a)P_{FF}(a)F_o(a) - \gamma_F(a)F_e(a) \\
\dot{B}_o &= \gamma_O(a)P_{OB}(a)O(a) + \gamma_F(a)P_{FB}(a)F(a) + \gamma_R(a)P_{RB}(a)R(a) \\
&\quad +\gamma_B(a)P_{BB}(a)B_e(a) - \gamma_B(a)B_o(a) \\
\dot{B}_e &= \gamma_B(a)P_{BB}(a)B_o(a) - \gamma_B(a)B_e(a) \\
\dot{R} &= \gamma_B(a)P_{BR}(a)\textstyle\sum_i B_i(a) - \gamma_R(a)R(a) \\
\dot{L}_o &= \gamma_R(a)P_{RL}(a)R(a) + \gamma_O(a)P_{OL}(a)O(a) \\
&\quad +\gamma_L(a)P_{LL}(a)L_e(a) - \gamma_L(a)L_o(a) \\
\dot{L}_e &= \gamma_L(a)P_{LL}(a)L_o(a) - \gamma_L(a)L_e(a) \\
\dot{O}_o &= \gamma_L(a)P_{LO}(a)L(a) + \gamma_R(a)P_{RO}(a)R(a) \\
&\quad +\gamma_O(a)P_{OO}(a)O_e(a) - \gamma_O(a)O_o(a) \\
\dot{O}_e &= \gamma_O(a)P_{OO}(a)O_o(a) - \gamma_O(a)O_e(a)
\end{aligned}
\qquad (3)
$$

1.2 Infection Dynamics in the MBITES-de Equations

To simulate infection dynamics in MBITES-de, we subdivide each variable X into new variables X_x, $x \in \{U, Y, Z\}$, to represent the fraction of mosquitoes in behavioral state X that are uninfected, U, infected, Y, or infected and infectious Z. These lead to the following systems of coupled differential equations that remain unchanged, but for the equation describing resting mosquitoes. We let $Q\kappa(t)$ the proportion of mosquitoes becoming infected after blood feeding at time t.

$$\begin{aligned}
\frac{\partial R_U(a,t)}{\partial t} + \frac{\partial R_U(a,t)}{\partial a} &= (1 - Q\kappa(t))\,\gamma_B(a,t)P_{BR}(a)B_U(a,t) - \gamma_R(a,t)R_U(a,t) \\
\frac{\partial R_Y(a,t)}{\partial t} + \frac{\partial R_Y(a,t)}{\partial a} &= Q\kappa(t)\gamma_B(a,t)P_{BR}(a)B_U(a,t) \\
&\quad +\gamma_B(a,t)P_{BR}(a)B_Y(a,t) - \gamma_R(a,t)R_Y(a,t)
\end{aligned} \tag{4}$$

We let $\tau(t)$ denote the (possibly time-dependent) extrinsic incubation period. Because $\tau(t)$ is time dependent, we let $\hat{t}$ denote that point in the past when the mosquito became infected in order to become infectious at time t: *i.e.,* $t = \hat{t} + \tau(\hat{t})$. Let $\rho(t)$ the proportion of mosquitoes surviving through the extrinsic incubation period (*i.e.,* from $\hat{t}$ to $t = \hat{t} + \tau(\hat{t})$). An equation describing the proportion of infectious mosquitoes is:

$$\begin{aligned}
\frac{\partial R_Z(a,t)}{\partial t} + \frac{\partial R_Z(a,t)}{\partial a} &= \rho(t)Q\kappa\left(\hat{t}\right)\gamma_B(a,t)P_{BR}(a)B_U(a,t) \\
&\quad +\gamma_B(a,t)P_{BR}(a)B_Z(a,t) - \gamma_R(a,t)R_Z(a,t)
\end{aligned} \tag{5}$$

The remaining equations remain as follows:

$$\begin{aligned}
F_{o,x}(0,t) &= \Lambda(t) \\
\frac{\partial F_{o,x}(a,t)}{\partial t} + \frac{\partial F_{o,x}(a,t)}{\partial a} &= \gamma_O(a,t)P_{OF}(a)O_x(a,t) + \gamma_B(a,t)P_{BF}(a)B_x(a,t) \\
&\quad +\gamma_R(a,t)P_{RF}(a)R_x(a,t) + \gamma_F(a,t)P_{FF}(a)F_{e,x}(a,t) \\
&\quad -\gamma_F(a,t)F_{o,X}(a,t) \\
\frac{\partial F_{e,x}(a,t)}{\partial t} + \frac{\partial F_{e,x}(a,t)}{\partial a} &= \gamma_F(a,t)P_{FF}(a)F_{o,X}(a,t) - \gamma_F(a,t)F_{e,x}(a,t) \\
\frac{\partial B_{o,x}(a,t)}{\partial t} + \frac{\partial B_{o,x}(a,t)}{\partial a} &= \gamma_O(a,t)P_{OB}(a)O_x(a,t) + \gamma_F(a,t)P_{FB}(a)F(a,t) \\
&\quad +\gamma_R(a,t)P_{RB}(a)R_x(a,t) + \gamma_B(a,t)P_{BB}(a)B_{e,x}(a,t) \\
&\quad -\gamma_B(a,t)B_{o,x}(a,t) \\
\frac{\partial B_{e,x}(a,t)}{\partial t} + \frac{\partial B_{e,x}(a,t)}{\partial a} &= \gamma_B(a,t)P_{BB}(a)B_{o,x}(a,t) - \gamma_B(a,t)B_{e,x}(a,t) \\
\frac{\partial L_{o,x}(a,t)}{\partial t} + \frac{\partial L_{o,x}(a,t)}{\partial a} &= \gamma_R(a,t)P_{RL}(a)R_x(a,t) + \gamma_O(a,t)P_{OL}(a)O_x(a,t) \\
&\quad +\gamma_L(a,t)P_{LL}(a)L_{e,x}(a,t) - \gamma_L(a,t)L_{o,x}(a,t) \\
\frac{\partial L_{e,x}(a,t)}{\partial t} + \frac{\partial L_{e,x}(a,t)}{\partial a} &= \gamma_L(a,t)P_{LL}(a)L_{o,x}(a,t) - \gamma_L(a,t)L_{e,x}(a,t) \\
\frac{\partial O_{o,x}(a,t)}{\partial t} + \frac{\partial O_{o,x}(a,t)}{\partial a} &= \gamma_L(a,t)P_{LO}(a)L(a,t) + \gamma_R(a,t)P_{RO}(a)R_x(a,t) \\
&\quad +\gamma_O(a,t)P_{OO}(a)O_{e,x}(a,t) - \gamma_O(a,t)O_{o,x}(a,t) \\
\frac{\partial O_{e,x}(a,t)}{\partial t} + \frac{\partial O_{e,x}(a,t)}{\partial a} &= \gamma_O(a,t)P_{OO}(a)O_{o,x}(a,t) - \gamma_O(a,t)O_{e,x}(a,t)
\end{aligned} \tag{6}$$