Note that $S = a/g$, x = prevalence of infection in humans; y = proportion infected mosquitoes (not infectious); at equilibrium, the proportion infections is $e^{-gn}\bar{y} = P\bar{y}$:

$$\begin{aligned} r^{-1}\dot{x} &= \tfrac{R_C x}{1+cSx}(1-x) - x \\ g^{-1}\dot{y} &= cSx(1-y) - y \end{aligned} \tag{1}$$

then for humans:

$$\begin{aligned} R_C(1-\bar{x}) &= 1 + cS\bar{x} \\ R_C - 1 &= \bar{x}(R_C + cS) \\ \bar{x} &= \tfrac{R_C-1}{R_C+cS} \end{aligned} \tag{2}$$

and for mosquitoes:

$$\begin{aligned} \bar{y}(1+cS\bar{x}) &= cS\bar{x} \\ \bar{y} &= \tfrac{cS\bar{x}}{1+cS\bar{x}} \\ \bar{y} &= \tfrac{cS(R_C-1)}{R_C+cSR_C} \\ \bar{y} &= \tfrac{R_C-1}{R_C}\tfrac{cS}{1+cS} \end{aligned} \tag{3}$$

now the force of infection h at equilibrium is:

$$\begin{aligned} \bar{h} &= \tfrac{R_C x}{1+cSx} \\ &= \tfrac{rR_C(R_C-1)}{R_C+cS+cS(R_C-1)} \\ &= \tfrac{r(R_C-1)}{1+cS} \end{aligned} \tag{4}$$