Trigonometry

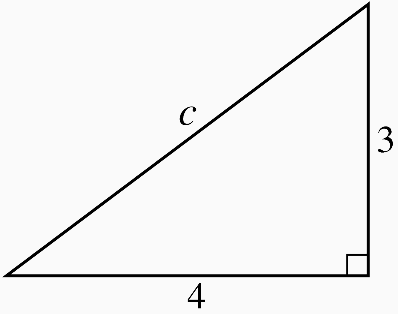
The Pythagorean Theorem

Extra—Pythagorean Triples

Chipmunk Math

"Clean" Answers

Solve for the unknown side in this right triangle:



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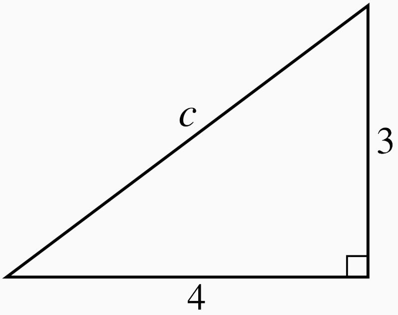

$$a^2 + b^2 = c^2$$

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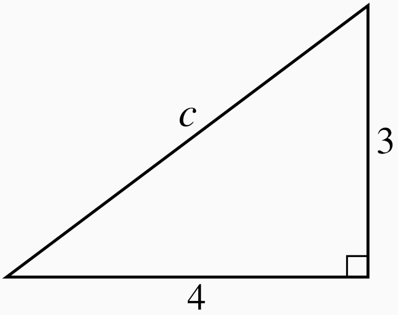

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$

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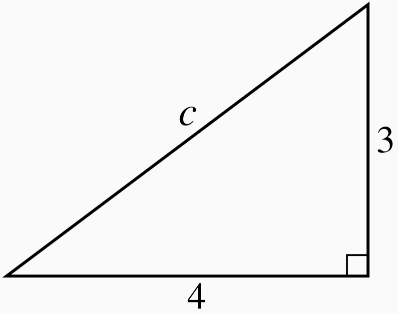

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$

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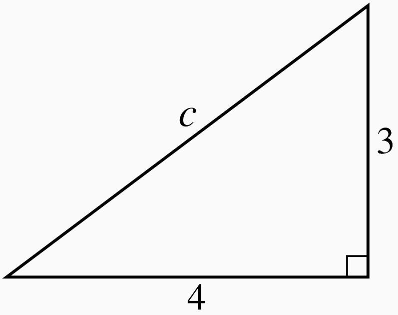
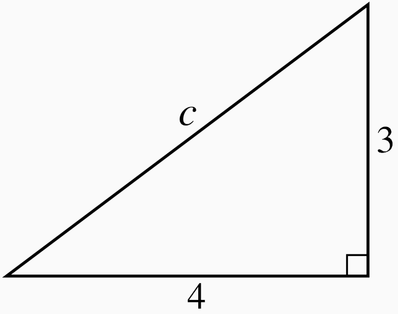


$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
3^2 + 4^2 &= c^2 \\
9 + 16 &= c^2 \\
25 &= c^2
\end{aligned}
$$

“Clean” Answers

Solve for the unknown side in this right triangle:



$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = c$$

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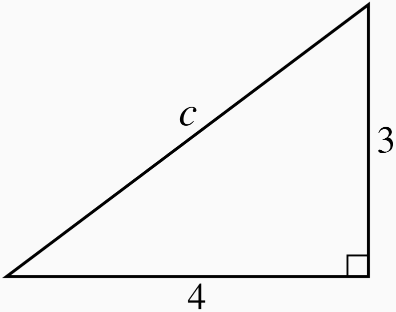

$$
\begin{aligned}
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\sqrt{25} &= c \\
5 &= c
\end{aligned}
$$

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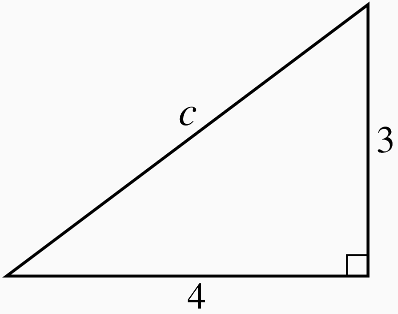

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What a “clean”, round answer!

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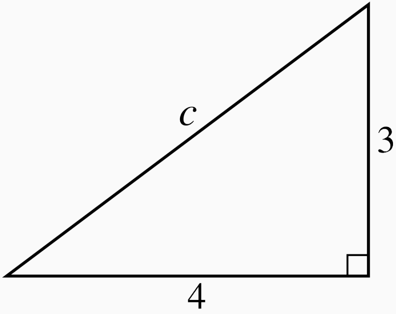

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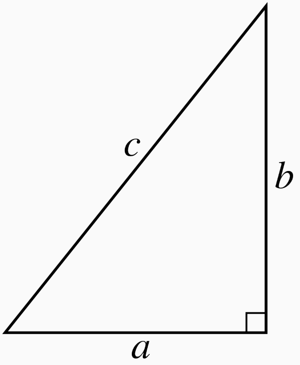
9 + 16 &= c^2 \\
25 &= c^2 \\
\sqrt{25} &= c \\
5 &= c
\end{aligned}
$$

What a "clean", round answer! In fact, all the sides are "nice"!

Pythagorean Triple

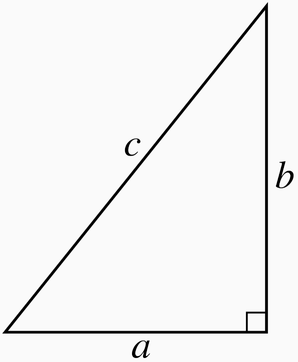


Definition (Pythagorean Triple)

A *Pythagorean triple* is a set of three whole numbers a, b, and c such that

$$a^2 + b^2 = c^2.$$

Pythagorean Triple



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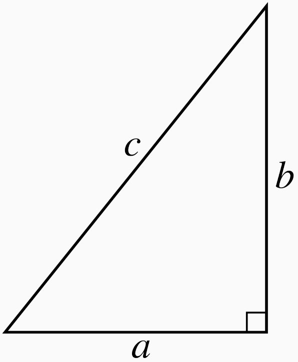

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$$a^2 + b^2 = c^2.$$

Most well-known example: $(3, 4, 5) \iff 3^2 + 4^2 = 5^2$

There are infinitely many Pythagorean triples out there.

Some Small Pythagorean Triples

$$(3, 4, 5)$$

$$(6, 8, 10)$$

$$(5, 12, 13)$$

$$(9, 12, 15)$$

$$(8, 15, 17)$$

Some Small Pythagorean Triples

$$(3, 4, 5)$$
$$(6, 8, 10)$$
$$(5, 12, 13)$$
$$(9, 12, 15)$$
$$(8, 15, 17)$$

Some Small Pythagorean Triples

$$(3, 4, 5)$$

$$(6, 8, 10) \iff 2 \cdot (3, 4, 5)$$

$$(5, 12, 13)$$

$$(9, 12, 15) \iff 3 \cdot (3, 4, 5)$$

$$(8, 15, 17)$$

Neat... but Is It *Useful*?

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Tests.

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Math teachers love “clean” answers.

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$\implies$ They show up often on tests.

Neat… but Is It *Useful*?

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Math teachers love "clean" answers.

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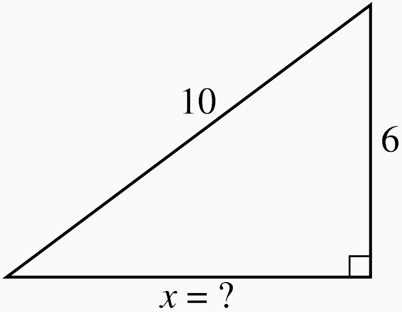
$\implies$ They show up often on tests.

Knowing the first few triples can give you an edge on tests.

Speeding Up with Triples



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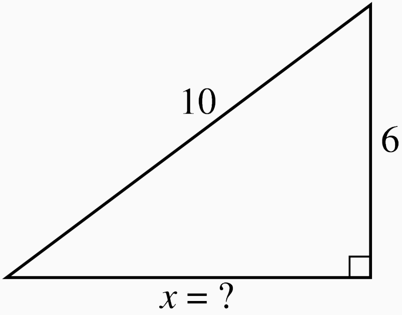
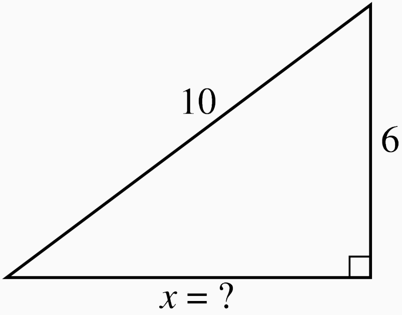


Without triples:

$$x^2 + 6^2 = 10^2$$

Speeding Up with Triples



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With triples:

$$(3, 4, 5)$$

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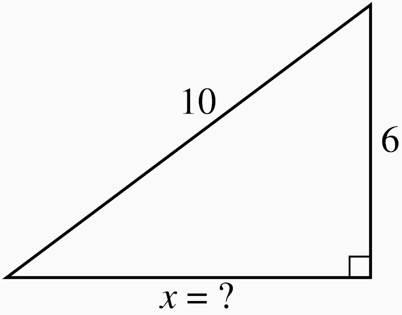


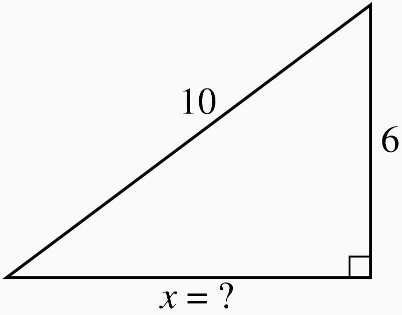
Without triples:

$$x^2 + 6^2 = 10^2$$

With triples:

$$(3, 4, 5) \xrightarrow{\times 2} (6, 8, 10)$$

Speeding Up with Triples



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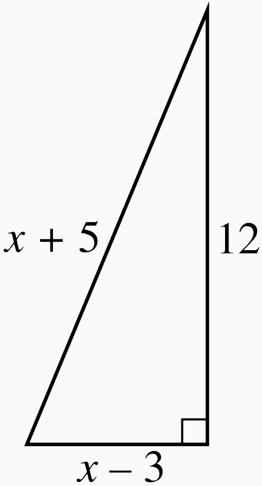
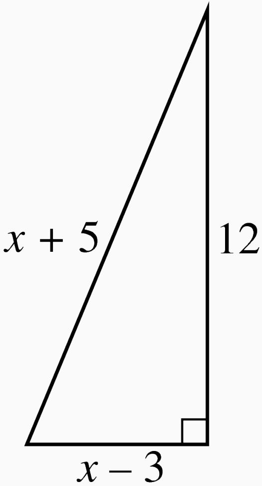
Speeding Up with Triples, cont.

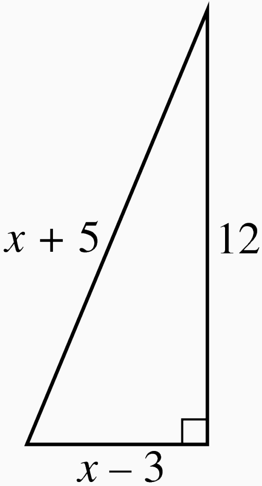


Speeding Up with Triples, cont.



Practice Problem #4

Speeding Up with Triples, cont.

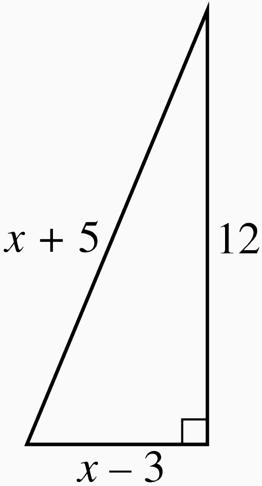


Without triples:

$$(x-3)^2 + 12^2 = (x+5)^2$$

Practice Problem #4

Speeding Up with Triples, cont.



Without triples:

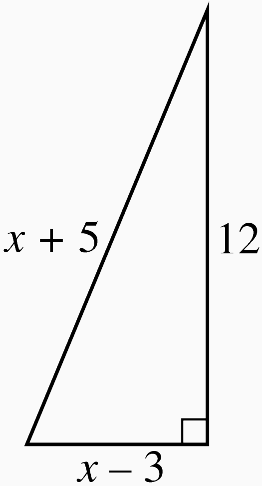
$$(x-3)^2 + 12^2 = (x+5)^2$$

With triples:

What triples involve a side of 12?

Practice Problem #4

Speeding Up with Triples, cont.



<u>Without triples:</u>

$$(x-3)^2+12^2=(x+5)^2$$

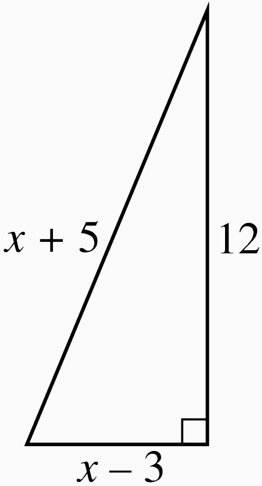
<u>With triples:</u>

What triples involve a side of 12?

$$(5, 12, 13) \qquad (9, 12, 15)$$

<u>Practice Problem #4</u>

Speeding Up with Triples, cont.



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Try one out and see what happens:

Practice Problem #4

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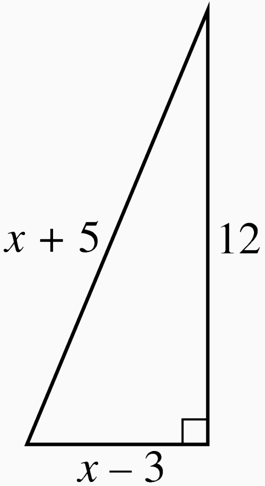

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Try one out and see what happens:

$$x - 3 = 5$$

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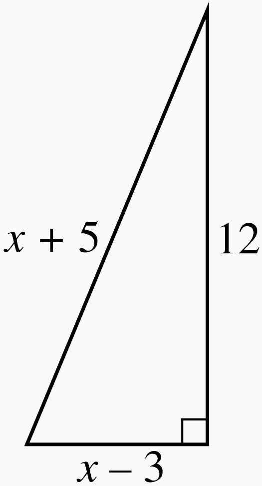

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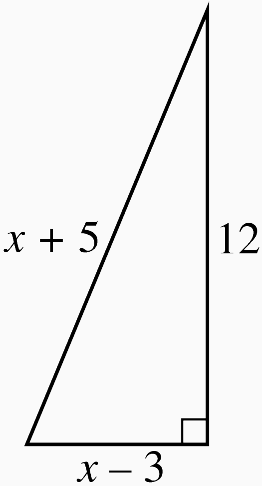
Try one out and see what happens:

$$x - 3 = 5 \quad \Longrightarrow \quad \boxed{x = 8}$$

Practice Problem #4

Speeding Up with Triples, cont.



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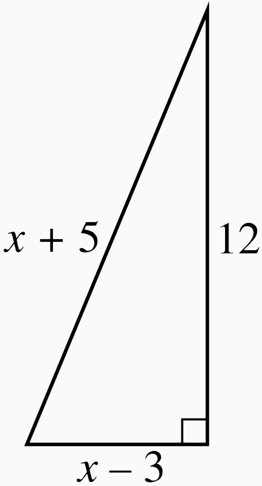
Try one out and see what happens:

$$x - 3 = 5 \implies \boxed{x = 8}$$

Check: $x + 5 \implies 8 + 5$

Practice Problem #4

Speeding Up with Triples, cont.



Practice Problem #4

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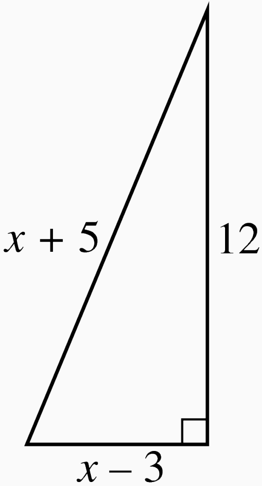

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Practice Problem #4

Scaling a Pythagorean Triple

Any **multiple** of a Pythagorean triple is also a triple.

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(This is connected to the concept of ***similarity***, which we talk about in a different lesson.)

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Let's prove it! Let's show that a multiple always works!

Proof That Scaling Works

Given:

(a, b, c) is a triple.

Proof That Scaling Works

<u>Given:</u>

(a, b, c) is a triple.

<u>Claim:</u>

$k \cdot (a, b, c) \Leftrightarrow (ka, kb, kc)$

is also a triple.

Proof That Scaling Works

Given:

(a, b, c) is a triple.

$$\implies a^2 + b^2 = c^2$$

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Want to show:

$(ka)^2 + (kb)^2 = (kc)^2$

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Proof:

Consider this expression:

$$(ka)^2 + (kb)^2$$

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<u>Want to show:</u>

$(ka)^2 + (kb)^2 = (kc)^2$

<u>Proof:</u>

Consider this expression:

$$(ka)^2 + (kb)^2$$

$$k^2a^2 + k^2b^2$$

Proof That Scaling Works

Given:

(a, b, c) is a triple.

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$k \cdot (a, b, c) \Leftrightarrow (ka, kb, kc)$ is also a triple.

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<u>Claim:</u>
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<u>Want to show:</u>
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<u>Proof:</u>
Consider this expression:

$$(ka)^2 + (kb)^2$$
$$k^2a^2 + k^2b^2$$
$$k^2 \cdot (a^2 + b^2)$$
$$k^2 \cdot (c^2)$$
$$k^2c^2$$
$$(kc)^2$$

$$\therefore (ka)^2 + (kb)^2 = (kc)^2$$

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Given:
(a, b, c) is a triple.
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Claim:
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Consider this expression:

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$$k^2c^2$$
$$(kc)^2$$

$$\therefore (ka)^2 + (kb)^2 = (kc)^2 \quad \square$$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

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$$\frac{\text{Known triple:}}{(3, 4, 5)}$$

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Scale by any whole number for another Pythagorean triple!

$$\frac{\text{Known triple:}}{(3, 4, 5)} \quad \xRightarrow{\times 100}$$

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$$\frac{\text{Known triple:}}{(3,4,5)} \xRightarrow{\times 100} \frac{\text{Another triple:}}{(300,400,500)}$$

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$$\frac{\text{Known triple:}}{(3,4,5)} \quad \xrightarrow{\times 100} \quad \frac{\text{Another triple:}}{(300,400,500)}$$

Can even scale by **non**-whole numbers.

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$$\frac{\text{Known triple:}}{(3,4,5)} \quad \xrightarrow{\times 100} \quad \frac{\text{Another triple:}}{(300,400,500)}$$

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(Note: Result is no longer a Pythagorean triple, but it will still make a right triangle.)

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$$\frac{\text{Known triple:}}{(3,4,5)} \quad \xRightarrow{\times 0.9} \quad \frac{\text{Right triangle:}}{}$$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

$$\underline{\text{Known triple:}} \quad (3, 4, 5) \quad \xRightarrow{\times 100} \quad \underline{\text{Another triple:}} \quad (300, 400, 500)$$

Can even scale by **non**-whole numbers.

(Note: Result is no longer a Pythagorean triple, but it will still make a right triangle.)

$$\underline{\text{Known triple:}} \quad (3, 4, 5) \quad \xRightarrow{\times 0.9} \quad \underline{\text{Right triangle:}} \quad (2.7, 3.6, 4.5)$$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

$$\frac{\text{Known triple:}}{(3,4,5)} \xrightarrow{\times 100} \frac{\text{Another triple:}}{(300,400,500)}$$

Can even scale by **non**-whole numbers.

(Note: Result is no longer a Pythagorean triple, but it will still make a right triangle.)

$$\begin{array}{ccc} \underline{\text{Known triple:}} & & \underline{\text{Right triangle:}} \\ (3,4,5) & \xrightarrow{\times 0.9} & (2.7,3.6,4.5) \\ (3,4,5) & \xrightarrow{\times \sqrt{7}} & (3\sqrt{7},4\sqrt{7},5\sqrt{7}) \end{array}$$

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$(3, 4, 5)$	$\xrightarrow{\times \pi}$	$(3\pi, 4\pi, 5\pi)$

Memorizing Pythagorean Triples

Primitive Pythagorean triples where the hypotenuse < 100:

$$\begin{array}{cccc} (3,4,5) & (5,12,13) & (8,15,17) & (7,24,25) \\ (20,21,29) & (12,35,37) & (9,40,41) & (28,45,53) \\ (11,60,61) & (16,63,65) & (33,56,65) & (48,55,73) \\ (13,84,85) & (36,77,85) & (39,80,89) & (65,72,97) \end{array}$$

Memorizing Pythagorean Triples

The triples you actually should **memorize**:

$$(3, 4, 5) \qquad (5, 12, 13) \qquad (8, 15, 17) \qquad (9, 40, 41)$$

Memorizing Pythagorean Triples

$$(3,4,5) \quad (5,12,13) \quad (8,15,17) \quad (9,40,41)$$
$$(6,8,10) \quad (10,24,36) \quad (16,30,34)$$
$$(9,12,15)$$
$$(12,16,20)$$

And it helps to familiarize yourself with their **multiples**.

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- Most important—know the Pythagorean theorem:

$$\text{Right triangle} \iff a^2 + b^2 = c^2$$

Thanks for Watching!

Watch the rest of the videos on this topic!

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