Trigonometry

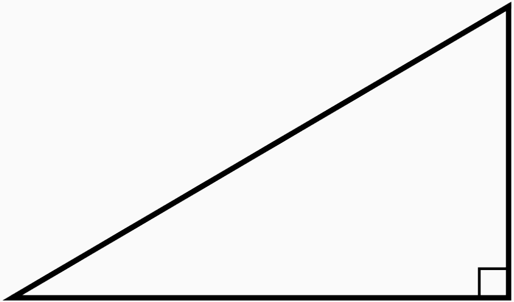
The Pythagorean Theorem

Lesson

Chipmunk Math

Sides of a Right Triangle

Sides of a Right Triangle — ANIMATION DUMMY



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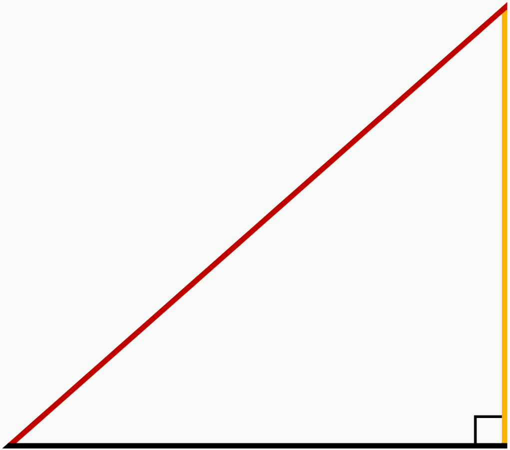

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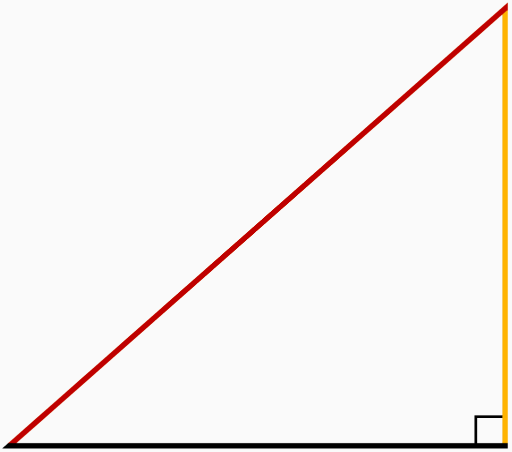

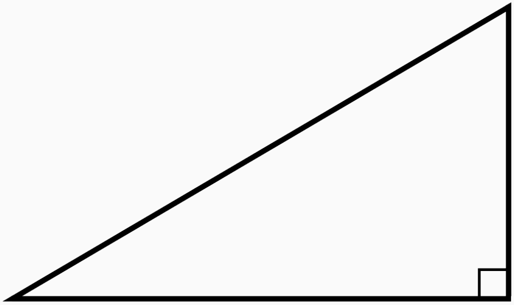

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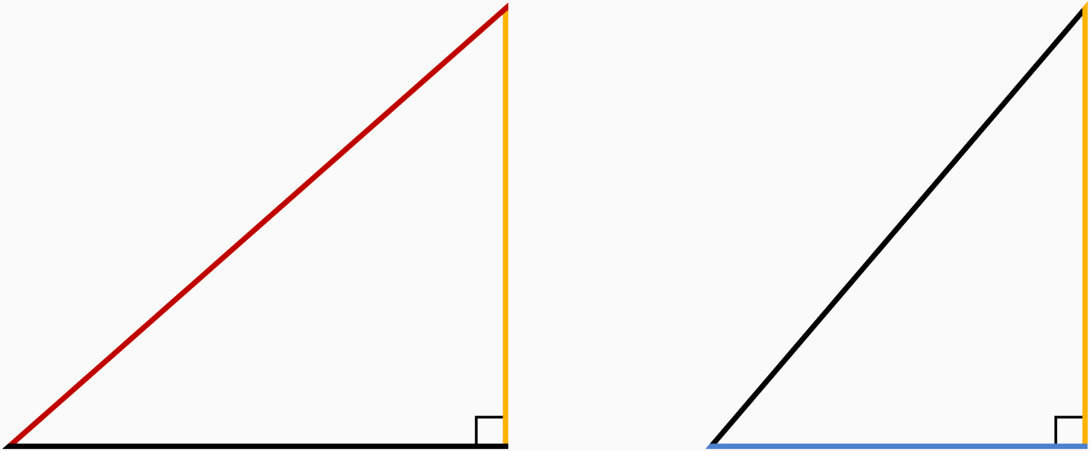
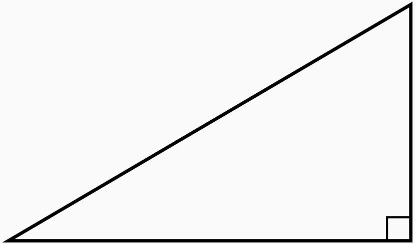


Right Triangle Side Names



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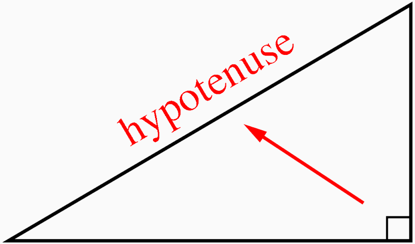
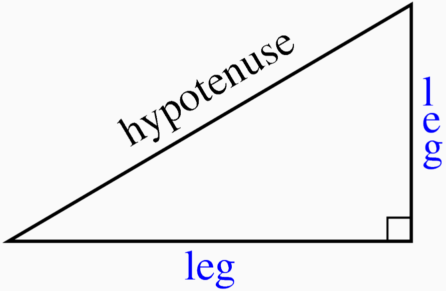


Definition

Hypotenuse: The side that's opposite the right angle. (This is the longest side).

Right Triangle Side Names

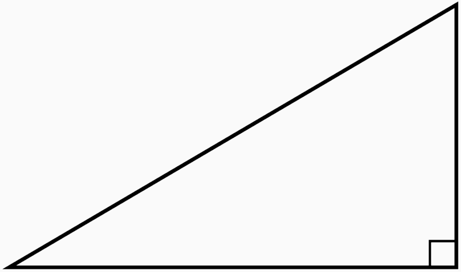


Definition

Hypotenuse: The side that's opposite the right angle. (This is the longest side).

Leg: The name for the other sides. Both legs are shorter than the hypotenuse and they are opposite the non-right angles.

Relationship Between Sides



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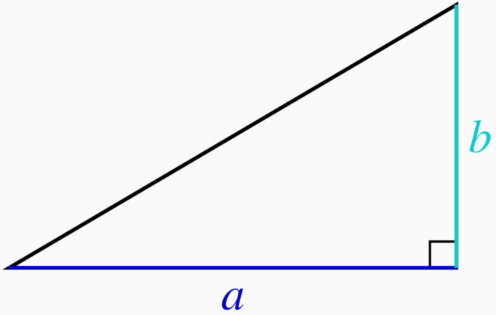

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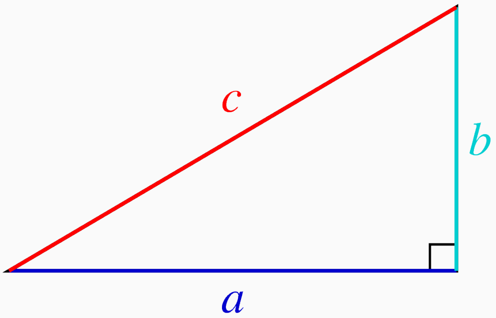

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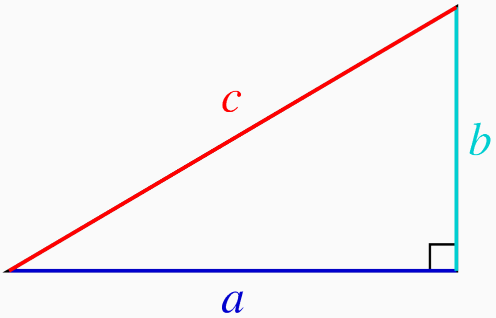
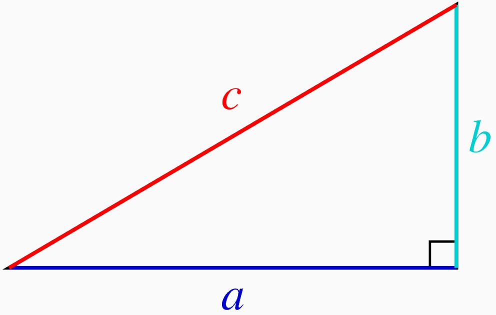


$$a^2 + b^2 = c^2$$

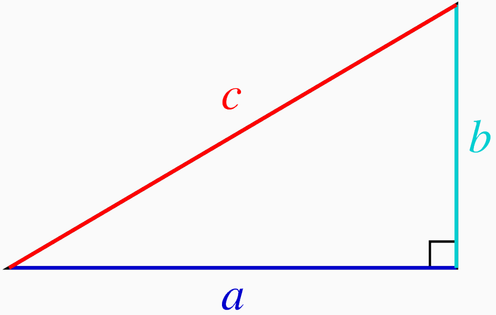
Relationship Between Sides



$$a^2 \quad + \quad b^2 \quad = \quad c^2$$

Legs (short sides) $\Longrightarrow$ Together on one side

Relationship Between Sides

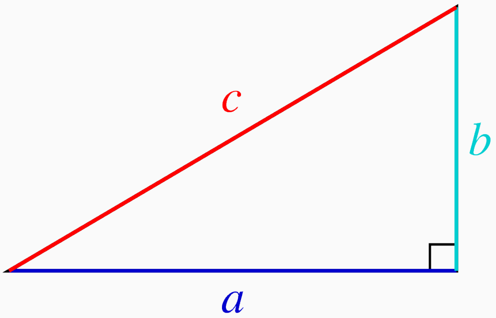


$$a^2 + b^2 = c^2$$

Legs (short sides) $\implies$ Together on one side

Hypotenuse (long side) $\implies$ Alone on other side

Relationship Between Sides



$$\underbrace{a^2}_{\text{leg}} + \underbrace{b^2}_{\text{leg}} = \underbrace{c^2}_{\text{hypotenuse}}$$

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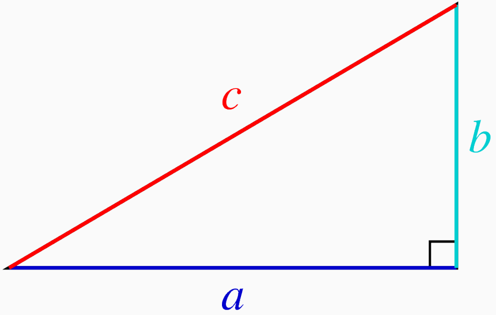

$$\underset{\text{leg}}{\underset{\uparrow}{a^2}} + \underset{\text{leg}}{\underset{\uparrow}{b^2}} = \underset{\text{hypotenuse}}{\underset{\uparrow}{c^2}}$$

Legs (short sides) $\implies$ Together on one side

Hypotenuse (long side) $\implies$ Alone on other side

We call this equation the Pythagorean Theorem.

Pythaga-who?

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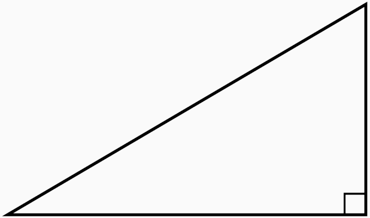
He actually was *not* the first person to notice the special relationship that right triangles have.

Still, he may have been the first to *prove* that it must always be true.

By tradition, we name it after him.

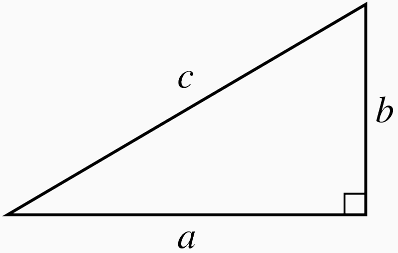
The Pythagorean Theorem



Theorem (Pythagorean)

For a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two legs.

The Pythagorean Theorem



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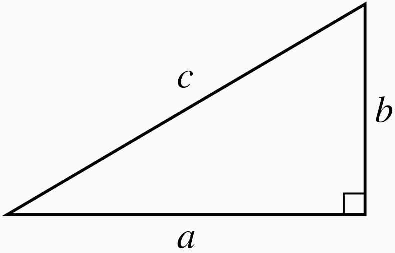
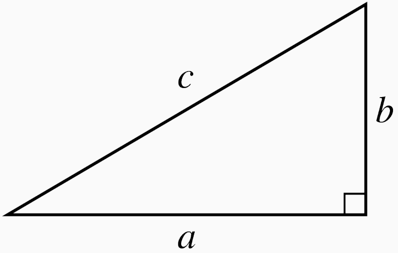


Theorem (Pythagorean)

For a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two legs.

$$c^2 = a^2 + b^2$$

The Pythagorean Theorem



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$$a^2 + b^2 = c^2$$

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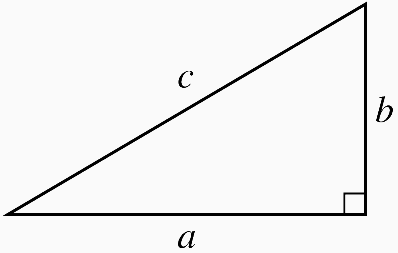

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For a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two legs.

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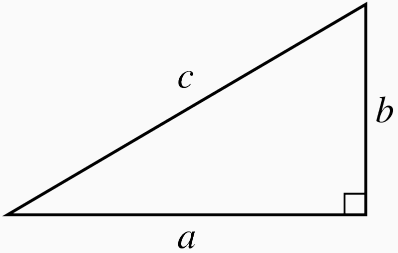
Remember, this theorem *only* applies to *right triangles*.

The Pythagorean Theorem



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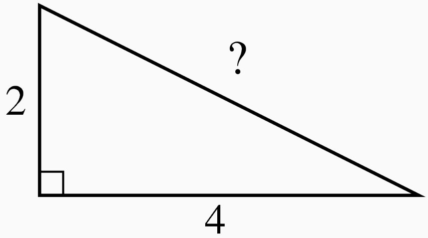
Remember, this theorem *only* applies to *right triangles*.

Memorize this thing. It just never stops showing up.

Example — Pythagorean Theorem



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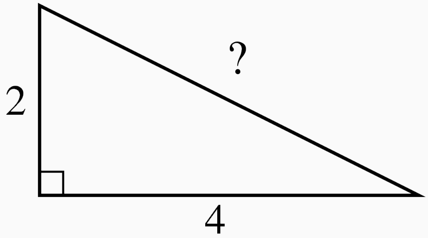
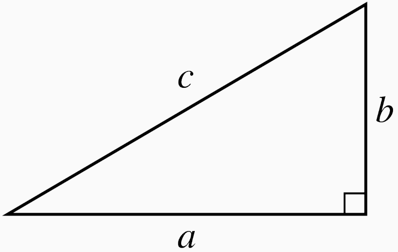


Pythagorean Theorem



$$a^2 + b^2 = c^2$$

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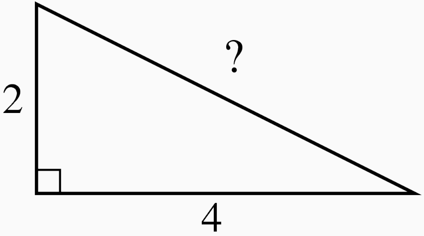
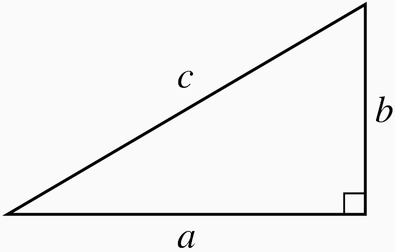


$$4^2 + 2^2 = ?^2$$

Pythagorean Theorem



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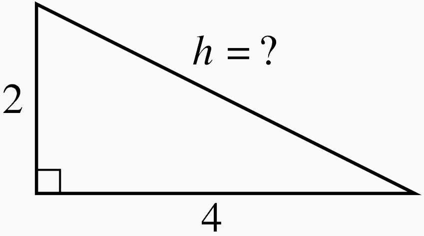
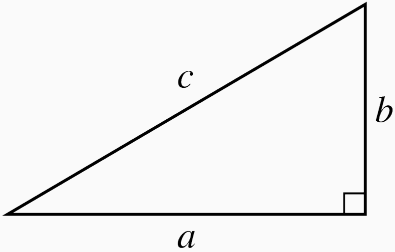


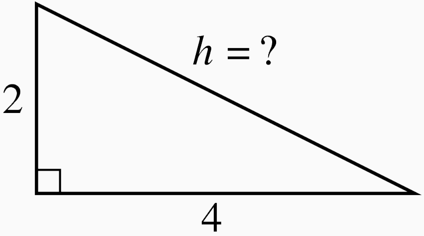
$$4^2 + 2^2 = h^2$$

Pythagorean Theorem



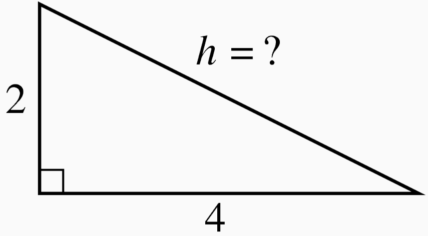
$$a^2 + b^2 = c^2$$

Example — Pythagorean Theorem, cont.



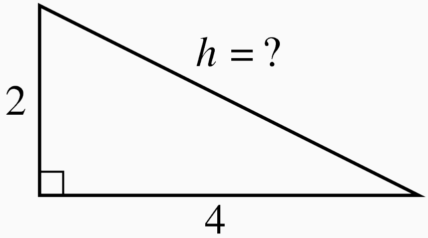
$$4^2 + 2^2 = h^2$$

Example — Pythagorean Theorem, cont.



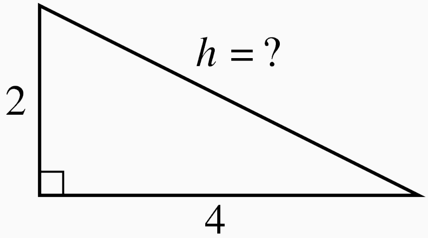
$$4^2 + 2^2 = h^2$$
$$16 + 4 = h^2$$

Example — Pythagorean Theorem, cont.



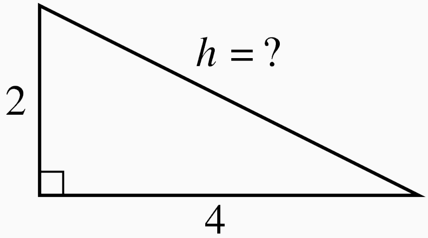
$$4^2 + 2^2 = h^2$$
$$16 + 4 = h^2$$
$$20 = h^2$$

Example — Pythagorean Theorem, cont.



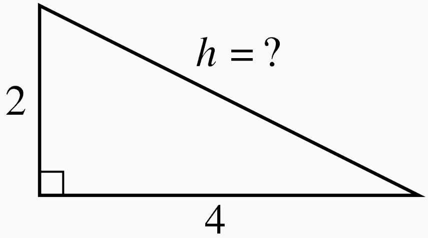
$$\begin{aligned} 4^2 + 2^2 &= h^2 \\ 16 + 4 &= h^2 \\ 20 &= h^2 \\ \sqrt{20} &= h \end{aligned}$$

Example — Pythagorean Theorem, cont.



$$
\begin{aligned}
4^2 + 2^2 &= h^2 \\
16 + 4 &= h^2 \\
20 &= h^2 \\
\sqrt{20} &= h \\
\sqrt{4 \cdot 5} &= h
\end{aligned}
$$

Example — Pythagorean Theorem, cont.



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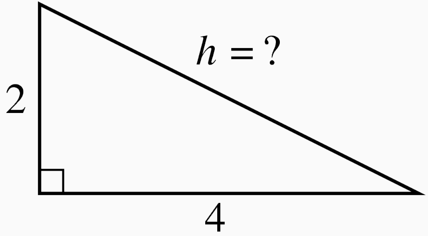

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Refresher: Working with Square Roots

Simplify square roots by breaking numbers into their factors.

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Look for squares to “pull out” of the radical.

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Example

$$\sqrt{63}$$

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Refresher: Working with Square Roots, Calculators

Alternatively, if you're allowed, you can use a calculator...

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$$\sqrt{}(63)$$

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7.9372539

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Careful: It's **not** exactly $\sqrt{63}$.

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Careful: It's **not** exactly $\sqrt{63}$.

It's a very, very good *approximation:*

$$\sqrt{63} \approx 7.9372539$$

Refresher: Working with Square Roots, Calculators

Alternatively, if you're allowed, you can use a calculator...

Use a calculator to check your simplification by comparing:

$$\sqrt{63} = 3\sqrt{7}$$

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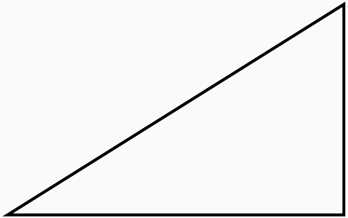
The Other Way Around

What if we had a triangle where we knew the sides, but didn't know if it was a right triangle?



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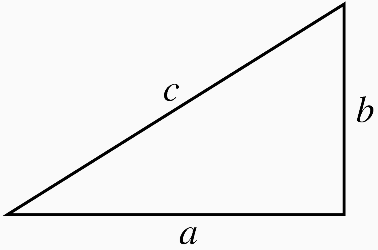

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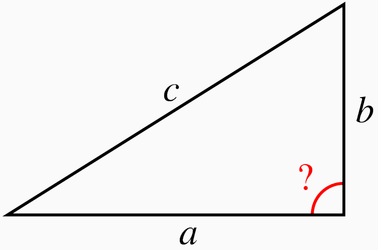

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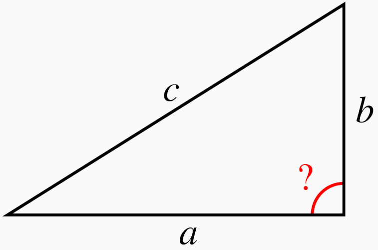

Great news! The Pythagorean Theorem works "in reverse"!

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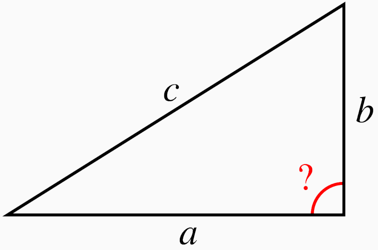
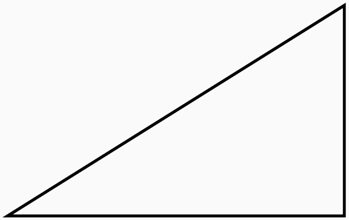


Great news! The Pythagorean Theorem works "in reverse"!

(When a theorem works "in reverse", we call the "backwards" version the ***converse*** of the theorem.)

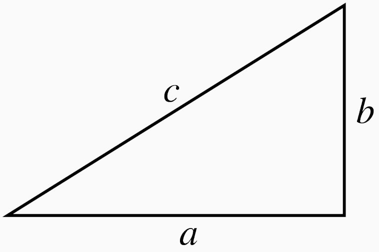
Converse of Pythagorean Theorem



Theorem (Converse of Pythagorean)

Given a triangle, if the square of the length of the longest side equals the sum of the squares of the other two side lengths, then the triangle is a right triangle.

Converse of Pythagorean Theorem



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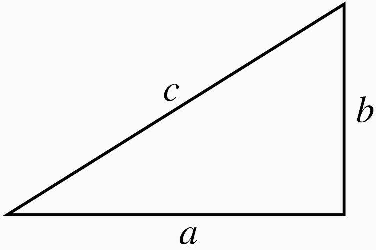

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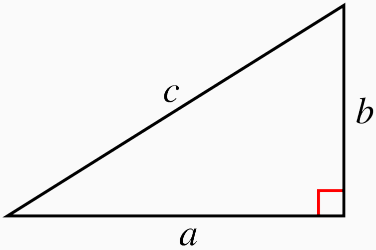
<u>If:</u> $c^2 = a^2 + b^2$

Converse of Pythagorean Theorem



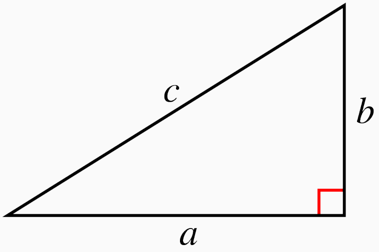
Theorem (Converse of Pythagorean)

Given a triangle, if the square of the length of the longest side equals the sum of the squares of the other two side lengths, then the triangle is a right triangle.

If: $c^2 = a^2 + b^2$

Then: Right triangle

Converse of Pythagorean Theorem



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Given a triangle, if the square of the length of the longest side equals the sum of the squares of the other two side lengths, then the triangle is a right triangle.

If: $a^2 + b^2 = c^2$

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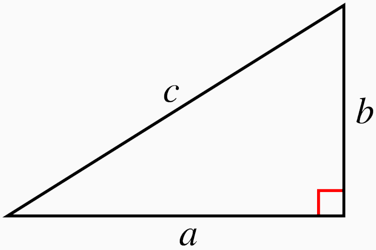
Converse of Pythagorean Theorem



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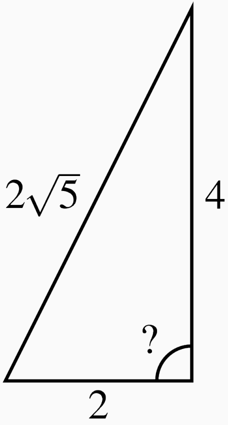
Given a triangle, if the square of the length of the longest side equals the sum of the squares of the other two side lengths, then the triangle is a right triangle.

If: $a^2 + b^2 = c^2$

Then: Right triangle

Also: $a^2 + b^2 \neq c^2 \iff$ **Not** a right triangle

Example — Pythagorean Converse



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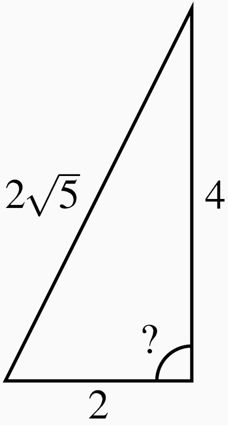

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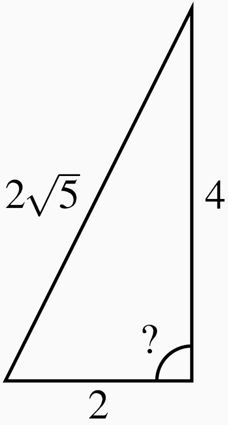
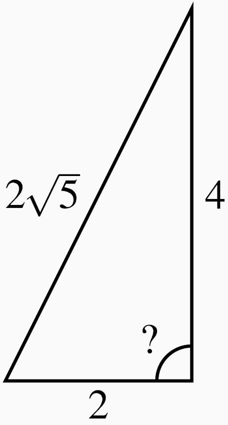


Converse: <u>**If**</u> $a^2 + b^2 = c^2$, <u>**then**</u> right triangle.

$$2^2 + 4^2 \stackrel{?}{=} \left(2\sqrt{5}\right)^2$$

Example — Pythagorean Converse



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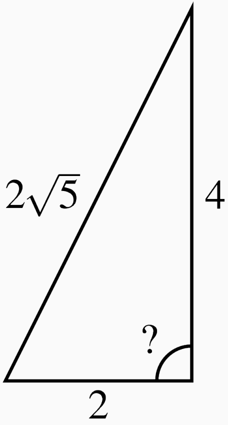

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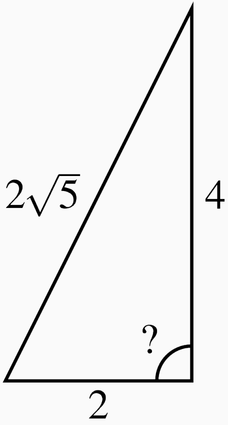

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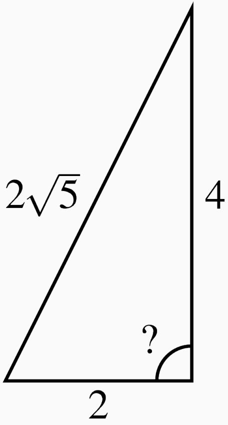
Example — Pythagorean Converse



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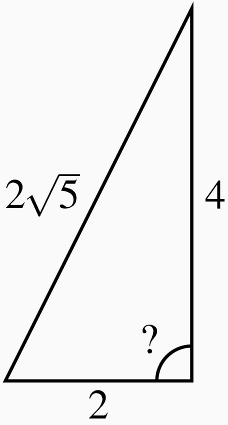

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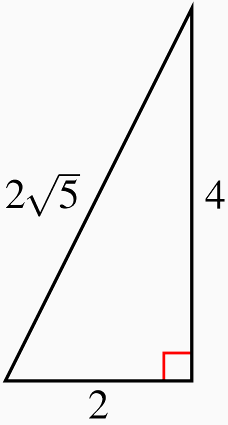
Example — Pythagorean Converse



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$$20 = 20 \checkmark$$

Therefore, it is a right triangle.

Thanks for Watching!

Watch the rest of the videos on this topic!

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