Trigonometry

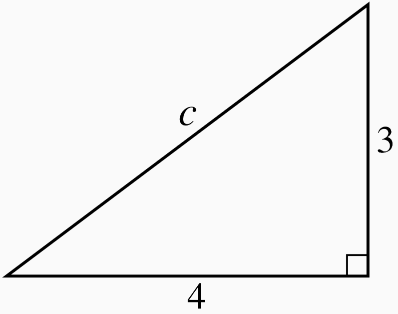
The Pythagorean Theorem

Extra—Pythagorean Triples

Chipmunk Math

“Clean” Answers

Solve for the unknown side in this right triangle:



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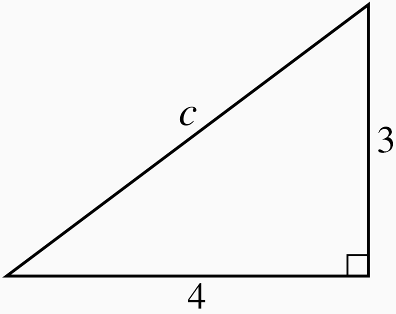

$$a^2 + b^2 = c^2$$

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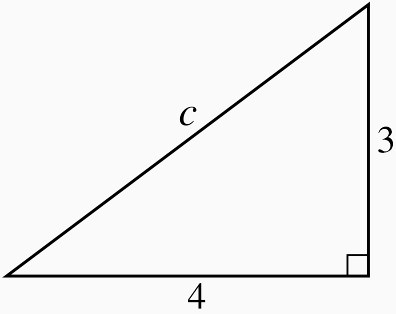

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = c^2$$

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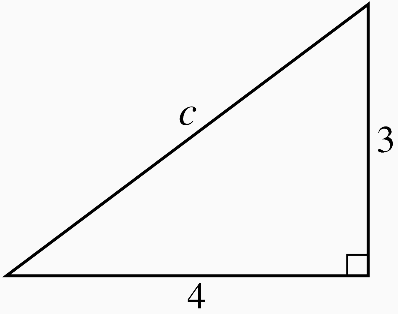

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$

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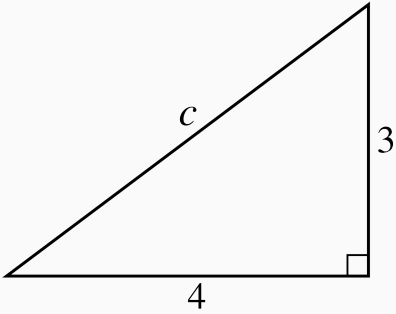

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$

"Clean" Answers

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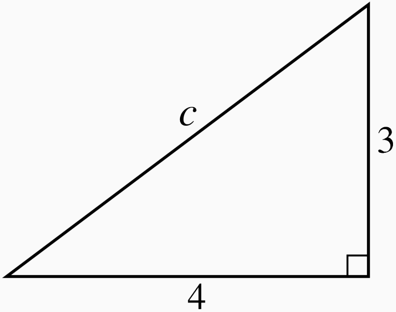

$$a^2 + b^2 = c^2$$
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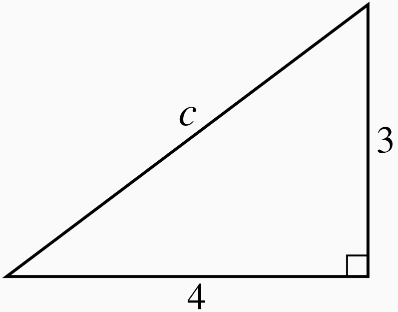

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
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$$\sqrt{25} = c^2$$
$$5 = c$$

“Clean” Answers

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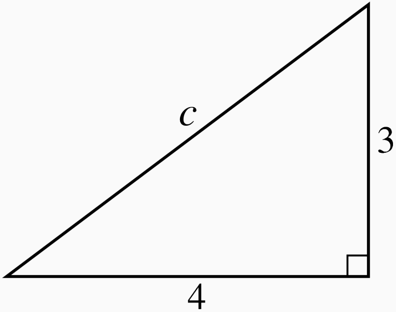

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
3^2 + 4^2 &= c^2 \\
9 + 16 &= c^2 \\
25 &= c^2 \\
\sqrt{25} &= c^2 \\
5 &= c
\end{aligned}
$$

What a “clean”, round answer!

“Clean” Answers

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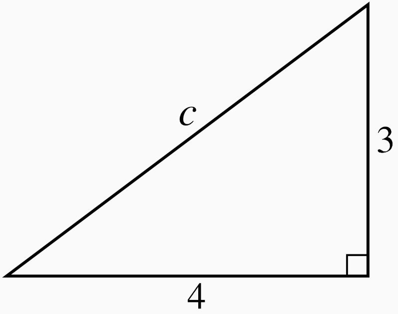

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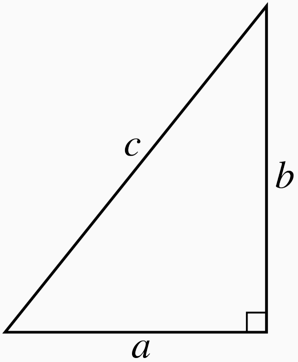
a^2 + b^2 &= c^2 \\
3^2 + 4^2 &= c^2 \\
9 + 16 &= c^2 \\
25 &= c^2 \\
\sqrt{25} &= c^2 \\
5 &= c
\end{aligned}
$$

What a “clean”, round answer! In fact, all the sides are “nice”!

Pythagorean Triple

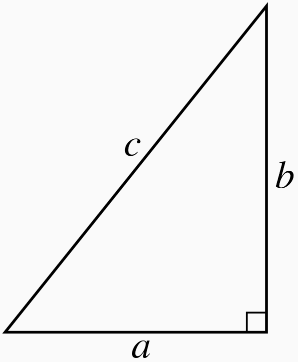


Definition (Pythagorean Triple)

A *Pythagorean triple* is a set of three whole numbers a, b, and c such that

$$a^2 + b^2 = c^2.$$

Pythagorean Triple



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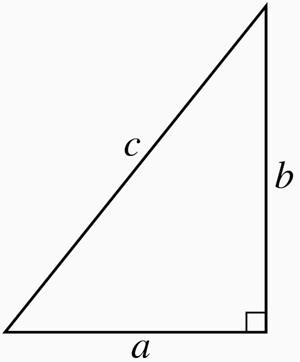

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A *Pythagorean triple* is a set of three whole numbers a, b, and c such that

$$a^2 + b^2 = c^2.$$

<u>Most well-known example:</u> $(3, 4, 5) \iff 3^2 + 4^2 = 5^2$

There are infinitely many Pythagorean triples out there.

Some Small Pythagorean Triples

$$(3, 4, 5)$$
$$(6, 8, 10)$$
$$(5, 12, 13)$$
$$(9, 12, 15)$$
$$(8, 15, 17)$$

Some Small Pythagorean Triples

$$(3, 4, 5)$$

$$(6, 8, 10)$$

$$(5, 12, 13)$$

$$(9, 12, 15)$$

$$(8, 15, 17)$$

Some Small Pythagorean Triples

$$
\begin{aligned}
&(3, 4, 5) \\
&(6, 8, 10) \iff 2 \cdot (3, 4, 5) \\
&(5, 12, 13) \\
&(9, 12, 15) \iff 3 \cdot (3, 4, 5) \\
&(8, 15, 17)
\end{aligned}
$$

Neat… but Is It *Useful*?

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Tests.

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Math teachers love “clean” answers.

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$\implies$ They show up often on tests.

Neat... but Is It *Useful*?

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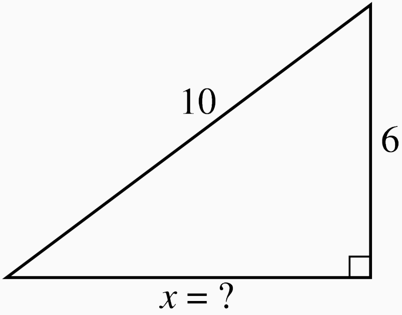
$\implies$ They show up often on tests.

Knowing the first few triples can give you an edge on tests.

Speeding Up with Triples



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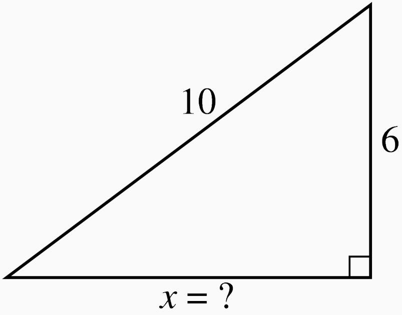
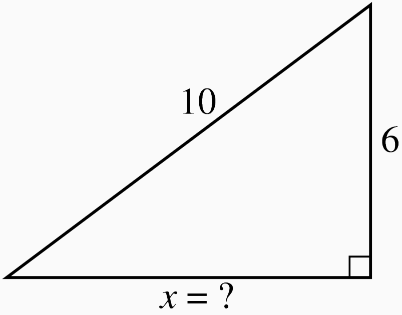


Without triples:

$$x^2 + 6^2 = 10^2$$

Speeding Up with Triples



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With triples:

$$(3, 4, 5)$$

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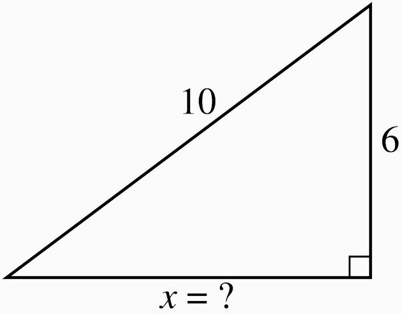


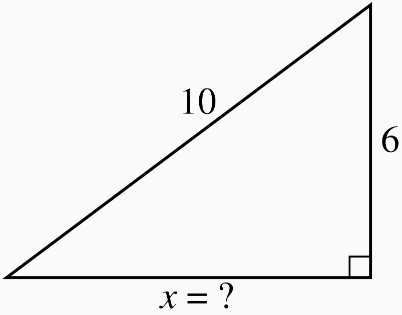
<u>Without triples:</u>

$$x^2 + 6^2 = 10^2$$

<u>With triples:</u>

$$(3, 4, 5) \xRightarrow{\times 2} (6, 8, 10)$$

Speeding Up with Triples



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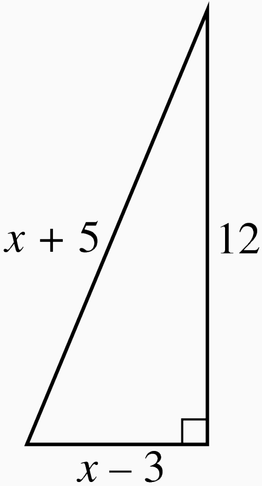
Speeding Up with Triples, cont.



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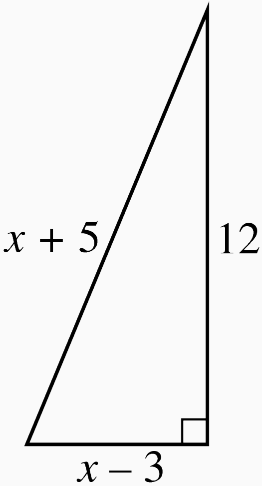
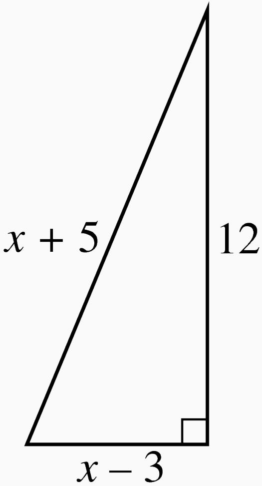


Practice Problem #4

Speeding Up with Triples, cont.

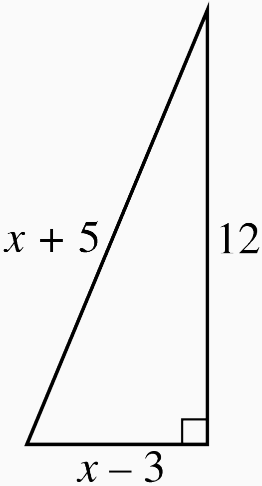


Without triples:

$$(x-3)^2 + 12^2 = (x+5)^2$$

Practice Problem #4

Speeding Up with Triples, cont.



Without triples:

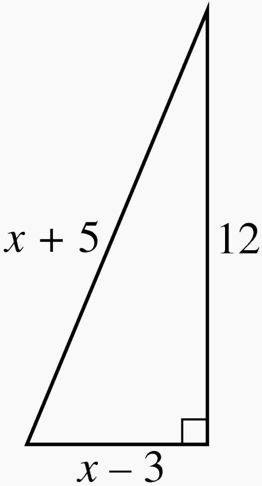
$$(x-3)^2 + 12^2 = (x+5)^2$$

With triples:

What triples involve a side of 12?

Practice Problem #4

Speeding Up with Triples, cont.



Without triples:

$$(x-3)^2 + 12^2 = (x+5)^2$$

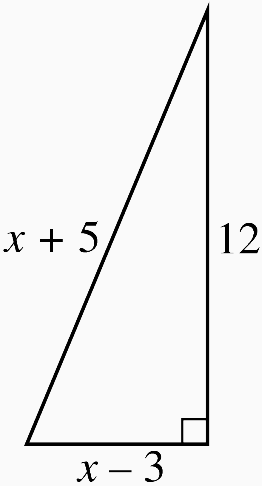
With triples:

What triples involve a side of 12?

$$(5, 12, 13) \qquad (9, 12, 15)$$

Practice Problem #4

Speeding Up with Triples, cont.



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Try one out and see what happens:

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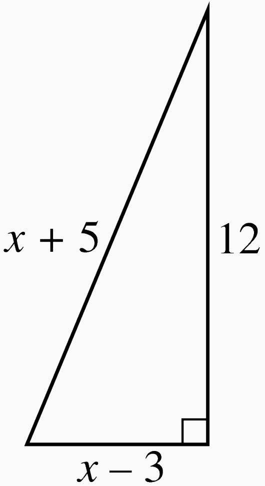

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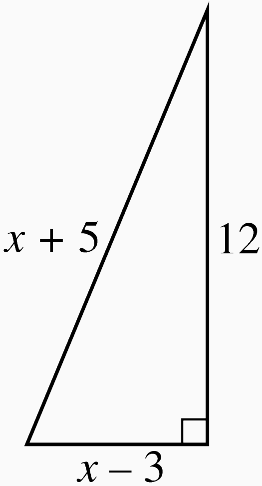

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Try one out and see what happens:

$$x - 3 = 5 \implies \boxed{x = 8}$$

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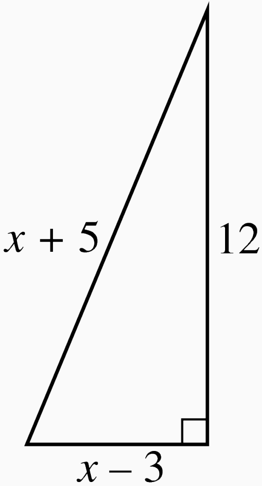

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Check: $x + 5 \implies 8 + 5$

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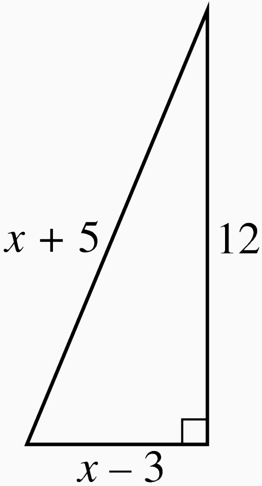

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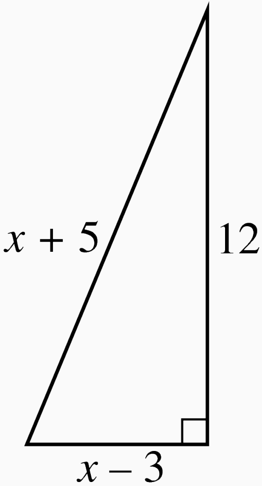

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Scaling a Pythagorean Triple

Any **multiple** of a Pythagorean triple is also a triple.

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(This is connected to the concept of ***similarity***, which we talk about in a different lesson.)

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Let's prove it! Let's show that a multiple always works!

Proof That Scaling Works

Given:

(a, b, c) is a triple.

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(a, b, c) is a triple.

Claim:

$k \cdot (a, b, c) \Leftrightarrow (ka, kb, kc)$

is also a triple.

Proof That Scaling Works

Given:

(a, b, c) is a triple.

$\implies a^2 + b^2 = c^2$

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Want to show:

$(ka)^2 + (kb)^2 = (kc)^2$

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Proof:

Consider this expression:

$$(ka)^2 + (kb)^2$$

Proof That Scaling Works

Given:

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$k \cdot (a, b, c) \Leftrightarrow (ka, kb, kc)$

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Proof:

Consider this expression:

$$(ka)^2 + (kb)^2$$
$$k^2a^2 + k^2b^2$$

Proof That Scaling Works

Given:

(a, b, c) is a triple.

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$k \cdot (a, b, c) \Leftrightarrow (ka, kb, kc)$
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Consider this expression:

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Given:
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Claim:
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Consider this expression:

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$$\therefore (ka)^2 + (kb)^2 = (kc)^2$$

Proof That Scaling Works

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(a, b, c) is a triple.
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Claim:
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Proof:
Consider this expression:

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$$k^2a^2 + k^2b^2$$
$$k^2 \cdot (a^2 + b^2)$$
$$k^2 \cdot (c^2)$$
$$k^2c^2$$
$$(kc)^2$$

$$\therefore (ka)^2 + (kb)^2 = (kc)^2 \quad \square$$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

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Known triple:

$$(3, 4, 5)$$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

$$\frac{\text{Known triple:}}{(3,4,5)} \quad \xrightarrow{\times 100}$$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

Known triple:		Another triple:
$(3, 4, 5)$	$\xRightarrow{\times 100}$	$(300, 400, 500)$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

$$\begin{array}{ccc} \underline{\text{Known triple:}} & & \underline{\text{Another triple:}} \\ (3, 4, 5) & \xRightarrow{\times 100} & (300, 400, 500) \end{array}$$

Can even scale by **non**-whole numbers.

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

$$\frac{\text{Known triple:}}{(3, 4, 5)} \xrightarrow{\times 100} \frac{\text{Another triple:}}{(300, 400, 500)}$$

Can even scale by **non**-whole numbers.

(Note: Result is no longer a Pythagorean triple, but it will still make a right triangle.)

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$$\frac{\text{Known triple:}}{(3,4,5)} \xRightarrow{\times 0.9} \frac{\text{Right triangle:}}{}$$

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$$\frac{\text{Known triple:}}{(3, 4, 5)} \xRightarrow{\times 100} \frac{\text{Another triple:}}{(300, 400, 500)}$$

Can even scale by **non**-whole numbers.

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$$\frac{\text{Known triple:}}{(3, 4, 5)} \xRightarrow{\times 0.9} \frac{\text{Right triangle:}}{(2.7, 3.6, 4.5)}$$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

Known triple:		Another triple:
$(3,4,5)$	$\xrightarrow{\times 100}$	$(300,400,500)$

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(Note: Result is no longer a Pythagorean triple, but it will still make a right triangle.)

Known triple:		Right triangle:
$(3,4,5)$	$\xrightarrow{\times 0.9}$	$(2.7,3.6,4.5)$
$(3,4,5)$	$\xrightarrow{\times \sqrt{7}}$	$(3\sqrt{7},4\sqrt{7},5\sqrt{7})$

Scale by Whatever!

Scale by any whole number for another Pythagorean triple!

Known triple:		Another triple:
$(3, 4, 5)$	$\xRightarrow{\times 100}$	$(300, 400, 500)$

Can even scale by **non**-whole numbers.

(Note: Result is no longer a Pythagorean triple, but it will still make a right triangle.)

Known triple:		Right triangle:
$(3, 4, 5)$	$\xRightarrow{\times 0.9}$	$(2.7, 3.6, 4.5)$
$(3, 4, 5)$	$\xRightarrow{\times \sqrt{7}}$	$(3\sqrt{7}, 4\sqrt{7}, 5\sqrt{7})$
$(3, 4, 5)$	$\xRightarrow{\times \pi}$	$(3\pi, 4\pi, 5\pi)$

Memorizing Pythagorean Triples

Primitive Pythagorean triples where the hypotenuse < 100:

$(3, 4, 5)$	$(5, 12, 13)$	$(8, 15, 17)$	$(7, 24, 25)$
$(20, 21, 29)$	$(12, 35, 37)$	$(9, 40, 41)$	$(28, 45, 53)$
$(11, 60, 61)$	$(16, 63, 65)$	$(33, 56, 65)$	$(48, 55, 73)$
$(13, 84, 85)$	$(36, 77, 85)$	$(39, 80, 89)$	$(65, 72, 97)$

Memorizing Pythagorean Triples

The triples you actually should **memorize**:

$$(3,4,5) \qquad (5,12,13) \qquad (8,15,17) \qquad (9,40,41)$$

Memorizing Pythagorean Triples

$(3, 4, 5)$	$(5, 12, 13)$	$(8, 15, 17)$	$(9, 40, 41)$
$(6, 8, 10)$	$(10, 24, 36)$	$(16, 30, 34)$	
$(9, 12, 15)$			
$(12, 16, 20)$			

And it helps to familiarize yourself with their **multiples**.

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- Don't stress over memorizing lots of triples.
  - Knowing them gives you a bit of an edge, but they're **never necessary**.
- Most important—know the Pythagorean theorem:

$$\text{Right triangle} \iff a^2 + b^2 = c^2$$

Thanks for Watching!

Watch the rest of the videos on this topic!

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