Trigonometry

The Pythagorean Theorem

Advanced Understanding

Chipmunk Math

Proving the Pythagorean Theorem

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<u>Let's prove it!</u>

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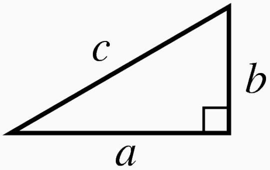
Let's prove it!

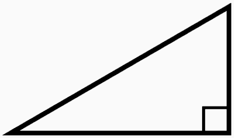
(So many ways to prove: what follows is just one of many.)

Proof: Pythagorean Theorem — Set-Up

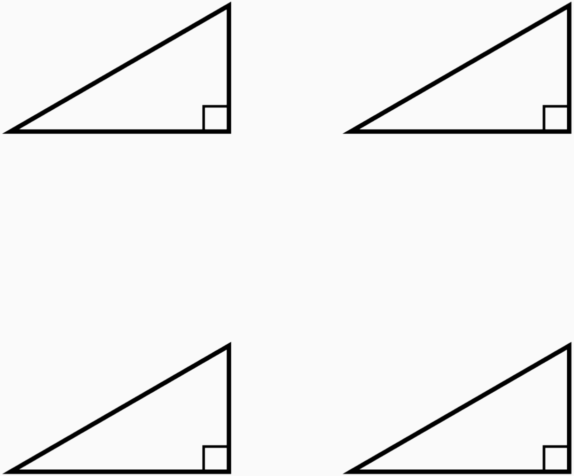
Proof — ANIMATION DUMMY



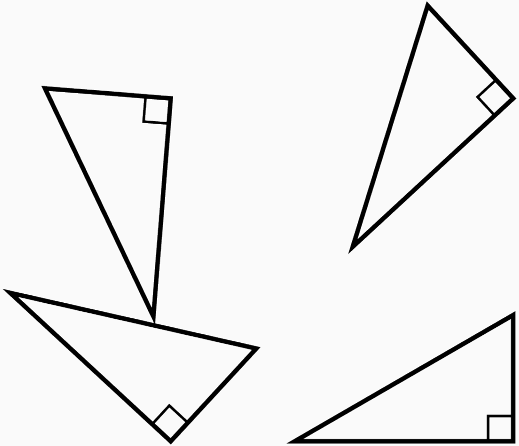
Proof — ANIMATION DUMMY



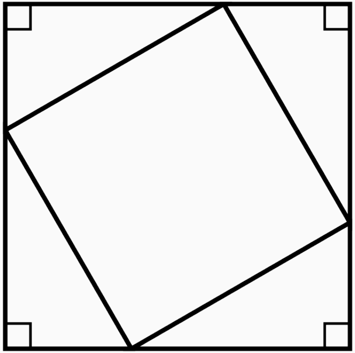
Proof — ANIMATION DUMMY



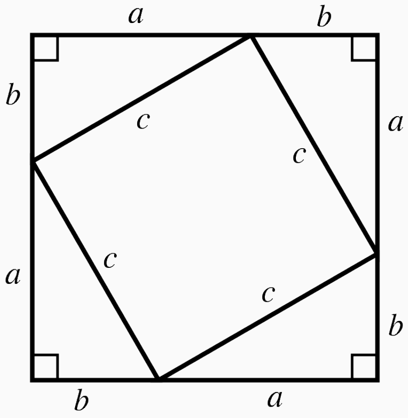
Proof — animation dummy



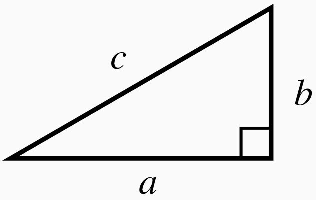
Proof — ANIMATION DUMMY



Proof — ANIMATION DUMMY



Proof: Pythagorean Theorem



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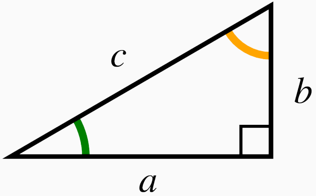
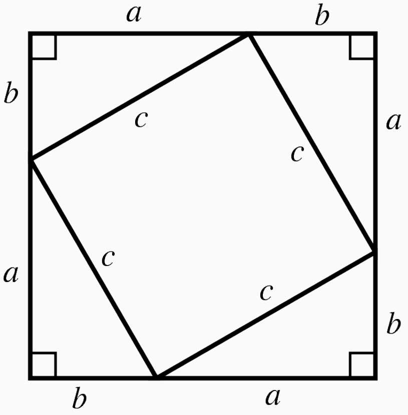
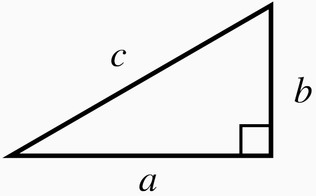


$$\measuredangle + \measuredangle = 90°$$

Proof: Pythagorean Theorem – ANIMATION DUMMY





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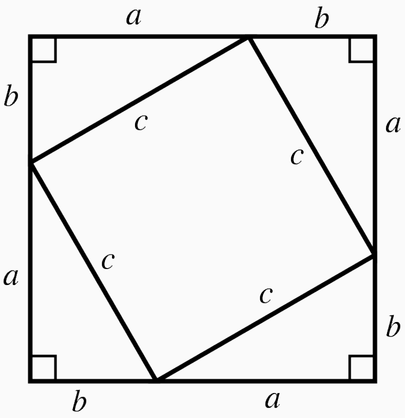

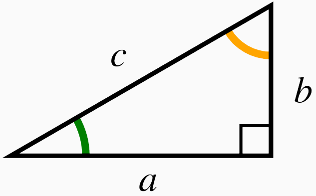

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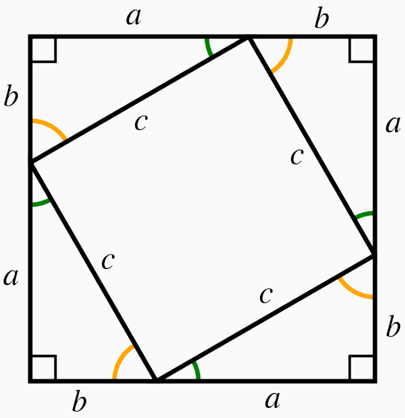
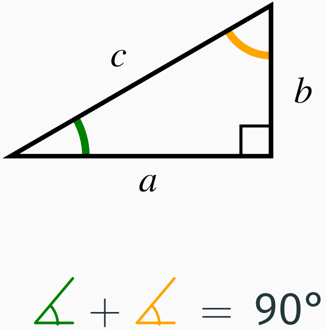
Proof: Pythagorean Theorem – ANIMATION DUMMY





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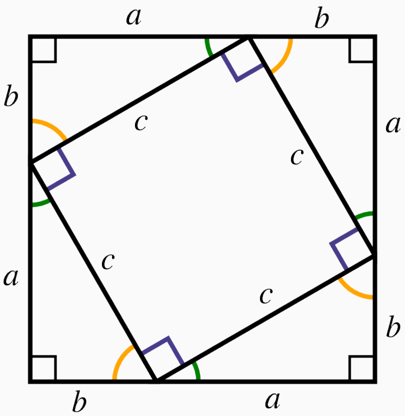

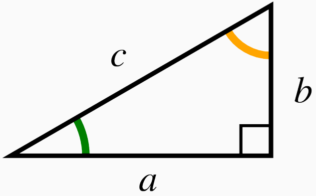

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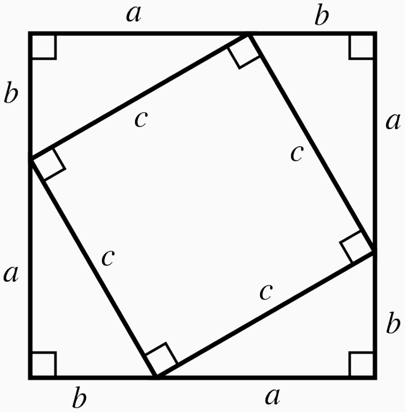

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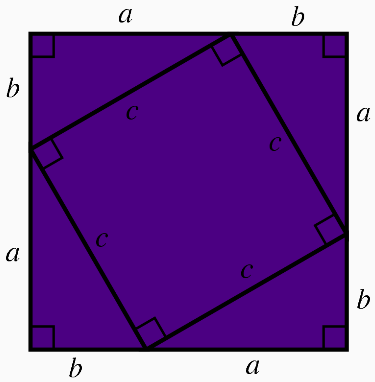

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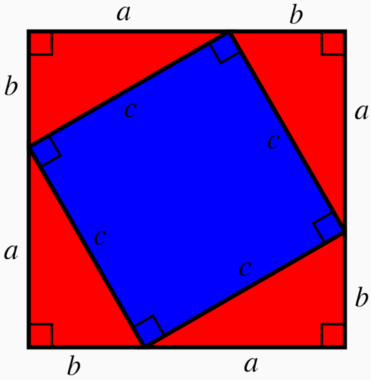

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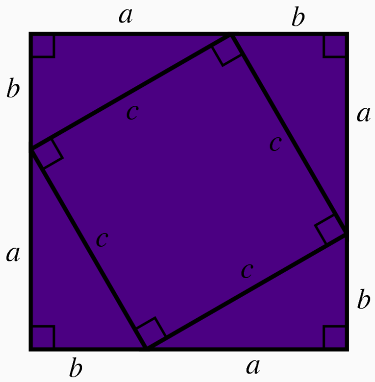
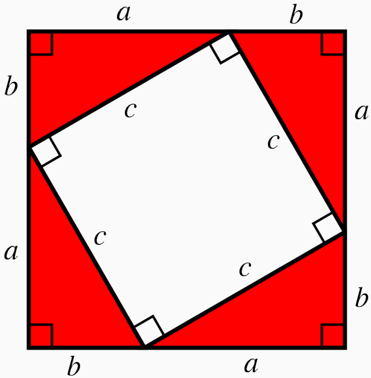


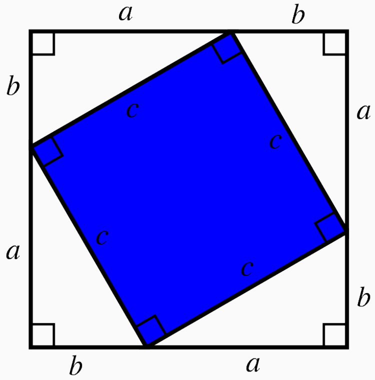
$A_{\text{big}\,\square}$

Proof: Pythagorean Theorem – ANIMATION DUMMY



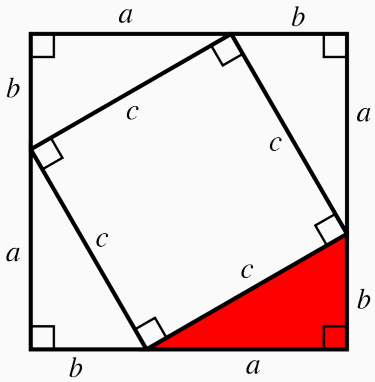
$$A_{\text{big}\,\square} = A_{\text{all}\,\triangle\text{s}} +$$

Proof: Pythagorean Theorem – ANIMATION DUMMY



$$A_{\text{big}\,\square} = A_{\text{all}\,\triangle\text{s}} + A_{\text{small}\,\square}$$

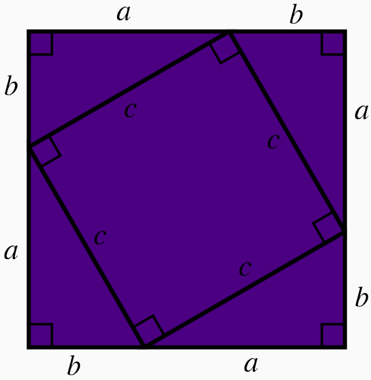
Proof: Pythagorean Theorem – ANIMATION DUMMY



$$A_{\text{big}\,\square} = A_{\text{all}\,\triangle\text{s}} + A_{\text{small}\,\square}$$

$$A_{\text{big}\,\square} = 4 \cdot A_{\triangle} + A_{\text{small}\,\square}$$

Proof: Pythagorean Theorem – ANIMATION DUMMY



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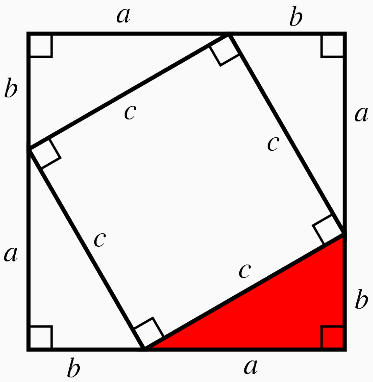
$$(a+b)^2 =$$

PROOF: PYTHAGOREAN THEOREM – ANIMATION DUMMY



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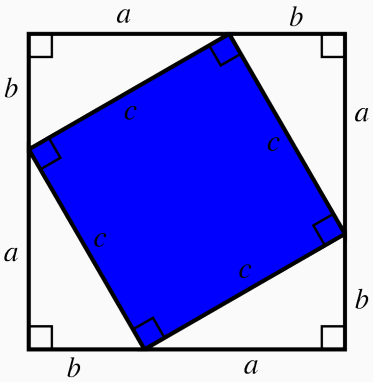
$$(a+b)^2 = 4 \cdot \left(\tfrac{1}{2} a \cdot b\right) +$$

Proof: Pythagorean Theorem – ANIMATION DUMMY



$$A_{\text{big}\,\square} = A_{\text{all}\,\triangle\text{s}} + A_{\text{small}\,\square}$$

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$$(a+b)^2 = 4 \cdot \left(\tfrac{1}{2} a \cdot b\right) + c^2$$

Take a breather, Tiger.

This slide is just to say that we’re going to re-walk the previous part with blank space on the left. You’ll start talking again after the area equation finishes being written out with c^2.

You’ll trim all the dead air, natch.

Proof: Pythagorean Theorem — Area

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$A_{\text{big}\,\square}$

Proof: Pythagorean Theorem — Area

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$$(a+b)^2 =$$

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Proof: Pythagorean Theorem — Algebra

$$(a+b)^2 = 4 \cdot \left(\frac{1}{2}a \cdot b\right) + c^2$$

Proof: Pythagorean Theorem — Algebra

$$
\begin{aligned}
(a+b)^2 &= 4 \cdot \left(\tfrac{1}{2} a \cdot b\right) + c^2 \\
(a+b)(a+b) &= 2ab + c^2
\end{aligned}
$$

Proof: Pythagorean Theorem — Algebra

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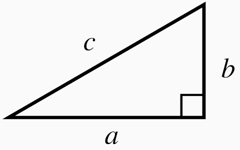

This logic would work for any right triangle.

Proof: Pythagorean Theorem — Algebra

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(a+b)^2 &= 4 \cdot \left(\tfrac{1}{2} a \cdot b\right) + c^2 \\
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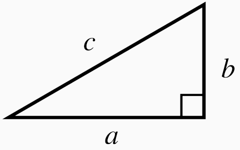

This logic would work for any right triangle. Therefore it must be the case that $a^2 + b^2 = c^2$ for all right triangles. $\square$

Take a breather, Tiger.

The next few slides are all animation ready slides, but not so useful for doing VO. As such, you should skip forward six slides until you reach

$$a^2 + b^2 > c^2$$

After that, take a breath, and get ready to start again. You'll be talking about how we can "go farther" with the **Pythagorean Converse**.

Pythagorean Converse and Inequalities

Consider a right triangle:

$$a^2 + b^2 = c^2$$

Pythagorean Converse and Inequalities

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What would happen if we "stretched" out c a bit?

Pythagorean Converse and Inequalities

What would happen if we "stretched" out c a bit?

$$a^2 + b^2 < c^2$$

Pythagorean Converse and Inequalities

$$a^2 + b^2 = c^2$$

Pythagorean Converse and Inequalities

$$a^2 + b^2 = c^2$$

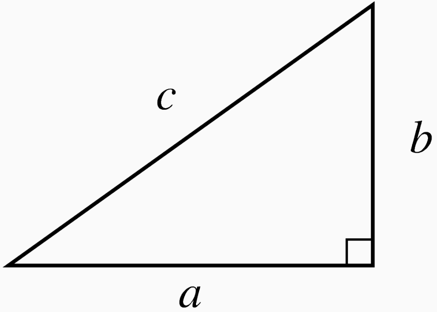
What would happen if "shrank" c down some?

Pythagorean Converse and Inequalities

What would happen if "shrank" c down some?

$$a^2 + b^2 > c^2$$

Pythag Conv. and Inequals – ANIMATION DUMMY



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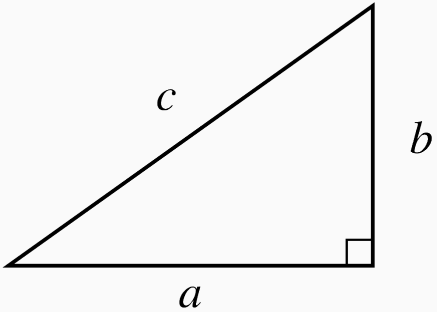

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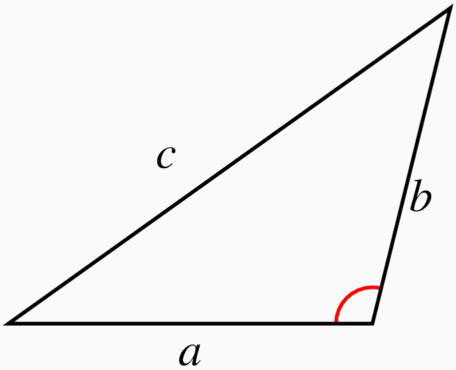

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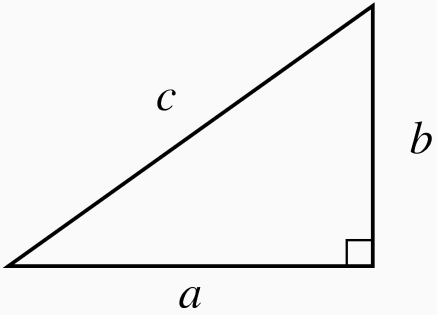

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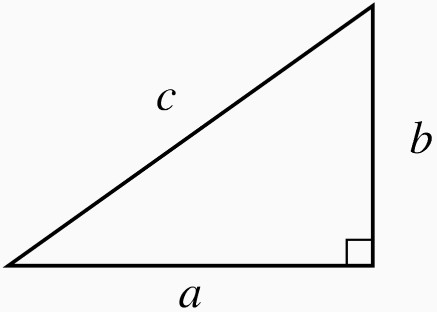
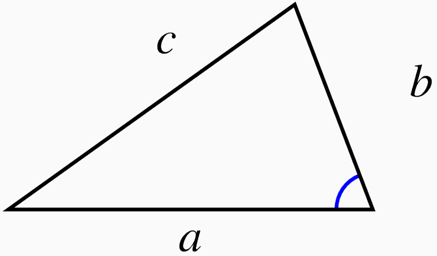
Pythag Conv. and Inequals – ANIMATION DUMMY



$$a^2 + b^2 = c^2$$

What would happen if "shrank" c down some?

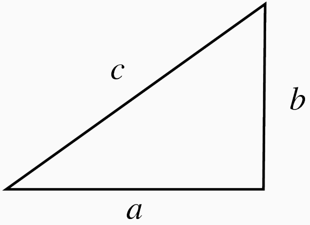
Pythag Conv. and Inequals – ANIMATION DUMMY



What would happen if "shrank" c down some?

$$a^2 + b^2 > c^2$$

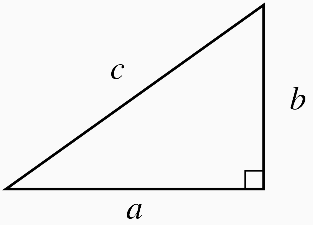
Triangle Test: Right, Obtuse, or Acute?



Pythagorean Converse with Inequalities

For any triangle that has side lengths of a, b, and c, where c is the longest side:

Triangle Test: Right, Obtuse, or Acute?

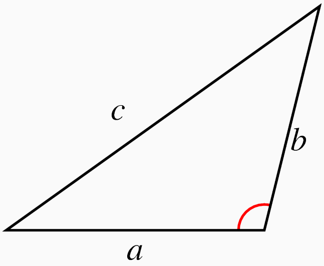


Pythagorean Converse with Inequalities

For any triangle that has side lengths of a, b, and c, where c is the longest side:

$$a^2 + b^2 = c^2 \iff \text{Right triangle}$$

Triangle Test: Right, Obtuse, or Acute?



Pythagorean Converse with Inequalities

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$$a^2 + b^2 < c^2 \iff \text{Obtuse triangle}$$

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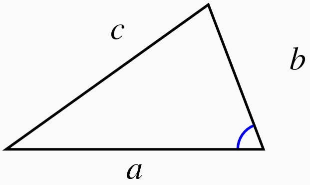

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$$a^2 + b^2 > c^2 \iff \text{Acute triangle}$$

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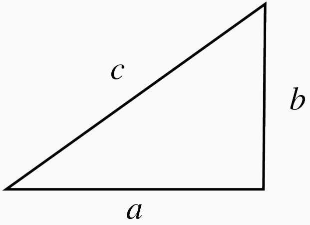

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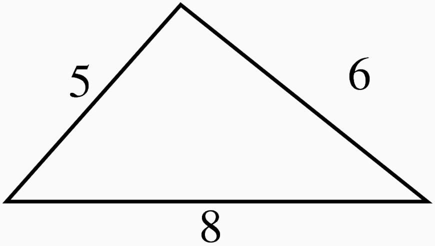
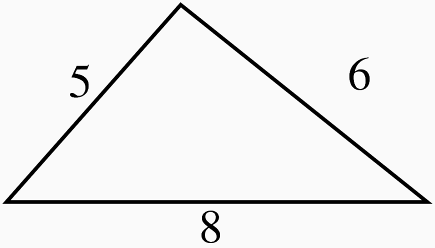
Pythagorean Converse with Inequalities — Example

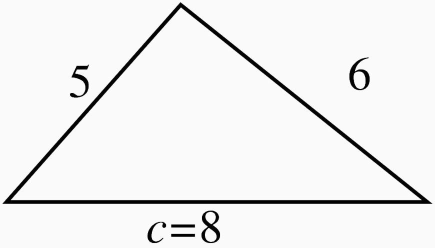


Pythagorean Converse with Inequalities — Example



Identify the longest side (c).

Pythagorean Converse with Inequalities — Example



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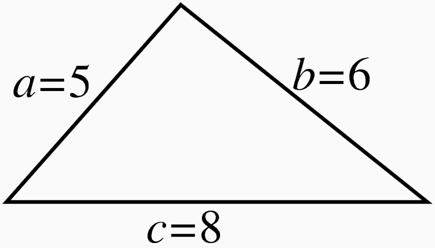
Pythagorean Converse with Inequalities — Example



Identify the longest side (c).

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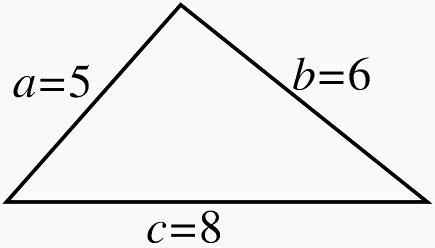

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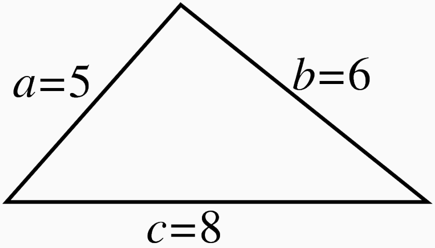

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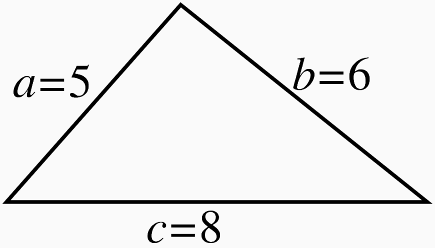

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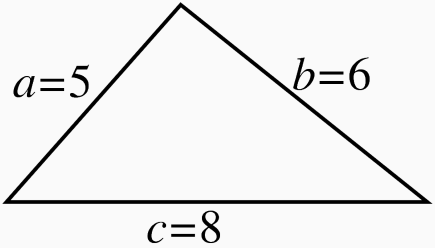

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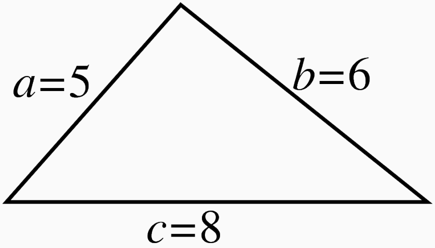

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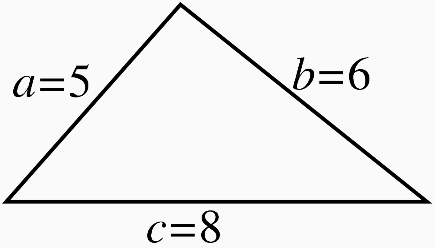

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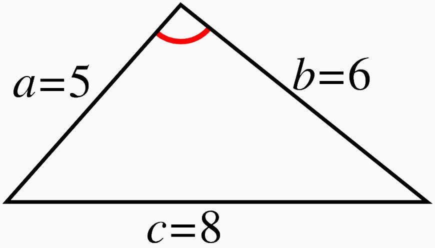

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Thus the triangle is obtuse.

Proof of Pythagorean Converse

The converse of the Pythagorean theorem *seems* reasonable, but in math we want **proof**!

Proof of Pythagorean Converse

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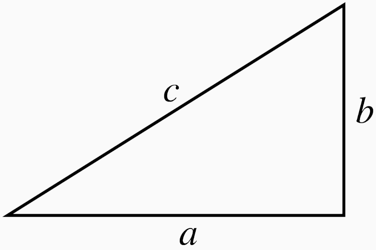
Let's prove it!

Reminder of Pythagorean Converse

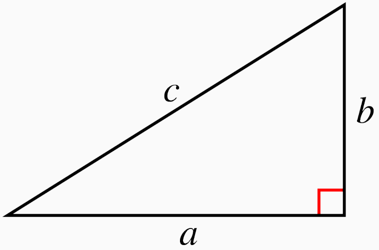
Converse of Pythagorean Theorem



If we have a triangle where

$$a^2 + b^2 = c^2,$$

Converse of Pythagorean Theorem

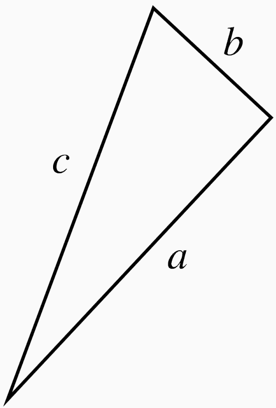


If we have a triangle where

$$a^2 + b^2 = c^2,$$

Then it's a right triangle.

Proof of Pythagorean Converse



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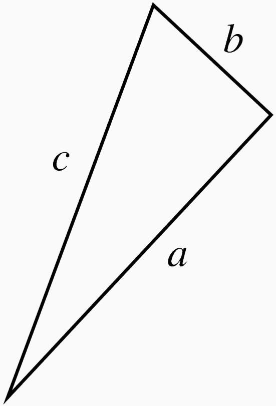
Given: $a^2 + b^2 = c^2$



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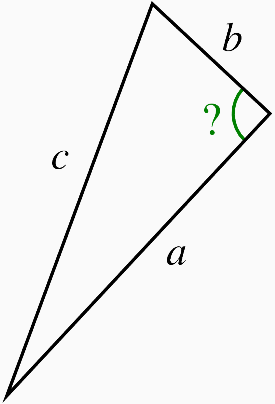

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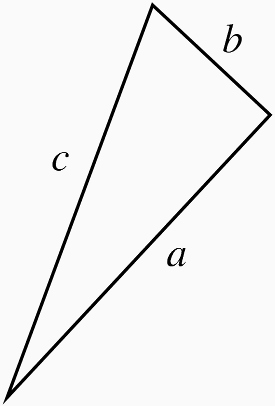

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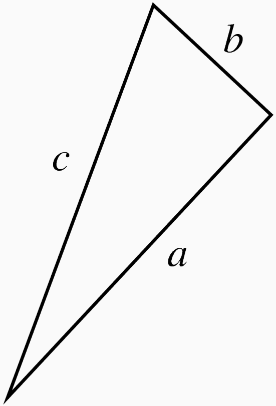

Construct another triangle:

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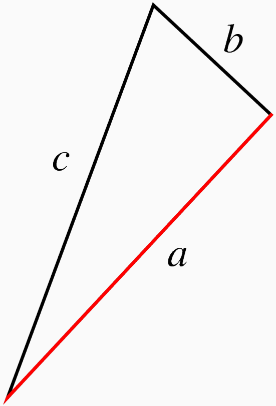

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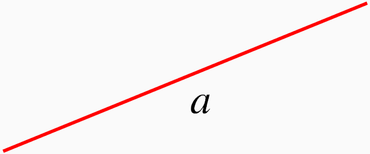

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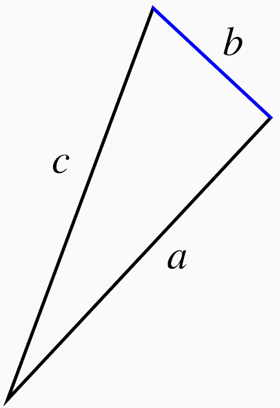

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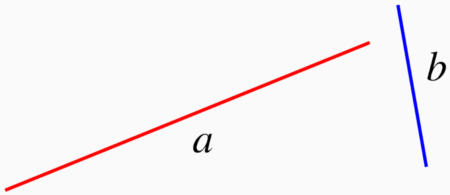

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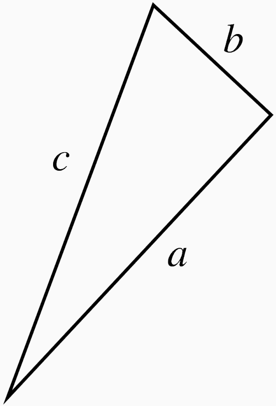

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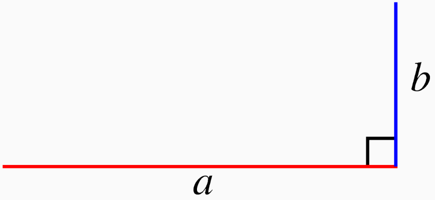

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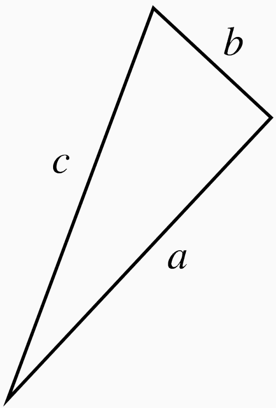

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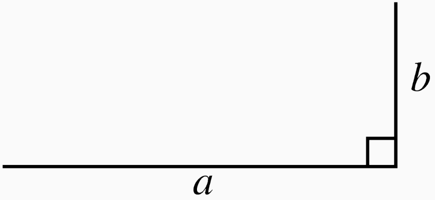

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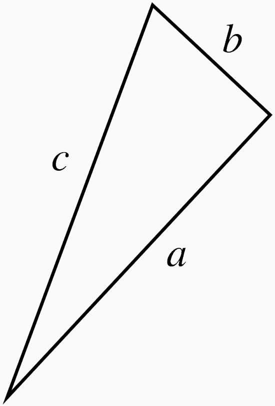

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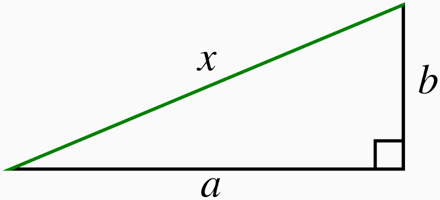

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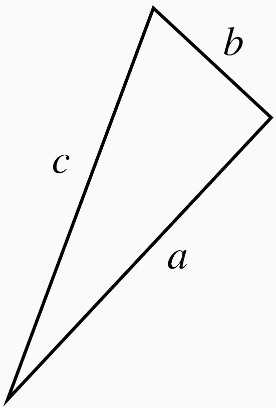

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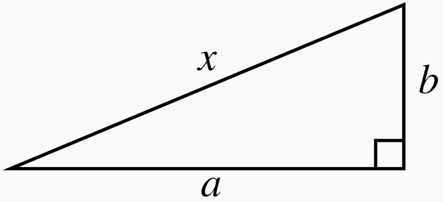

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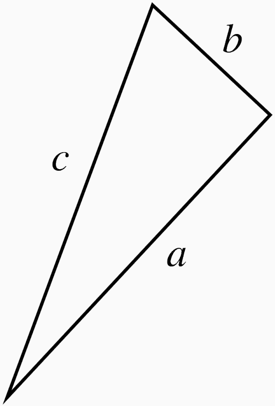

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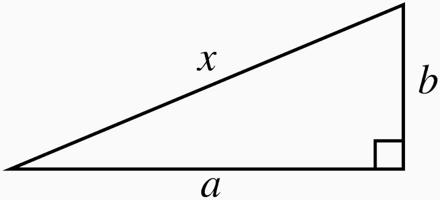

Pythagorean Theorem on new triangle:

$$a^2 + b^2 = x^2$$

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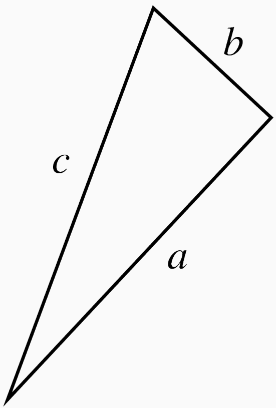

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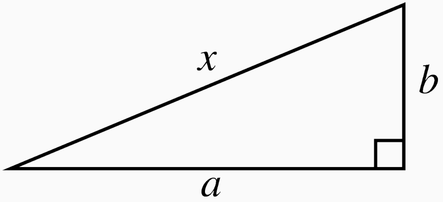

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Use substitution:

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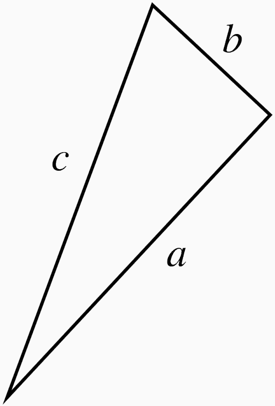

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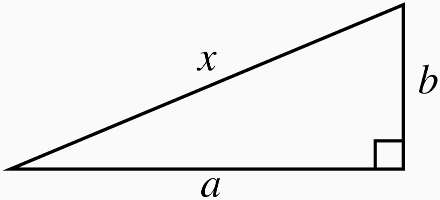

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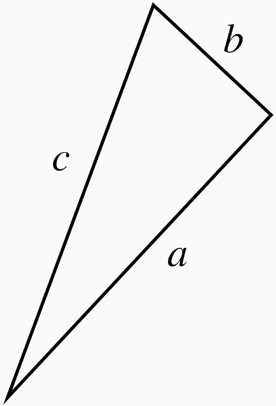

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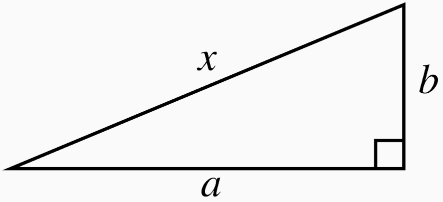

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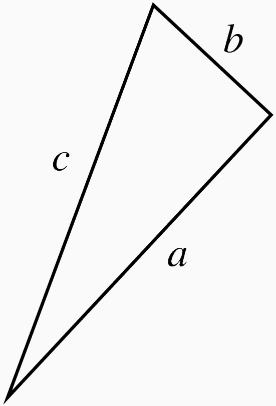

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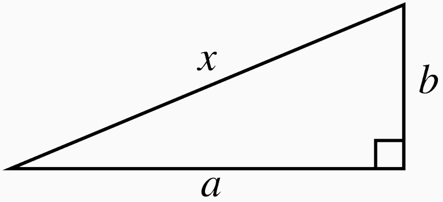

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Use substitution: $c^2 = x^2 \implies c = x$

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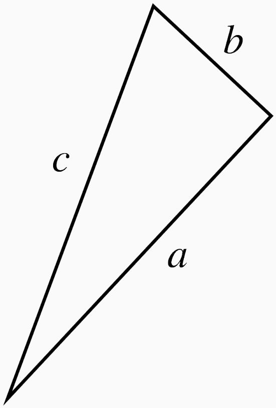

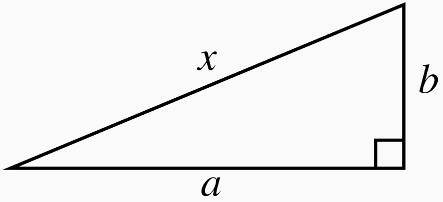

We now know that $c = x$.

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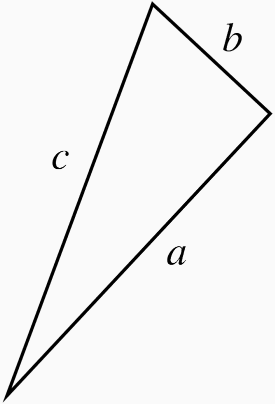

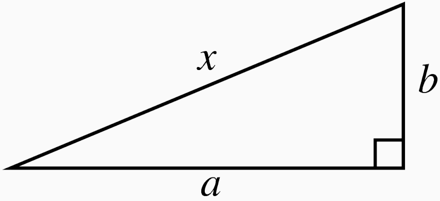

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Thus, by *SSS Congruence*, the triangles must be congruent to each other.

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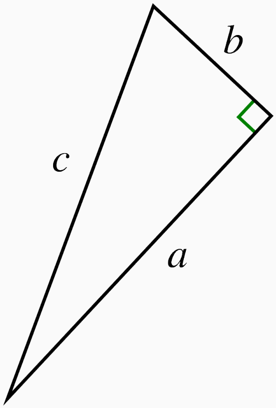

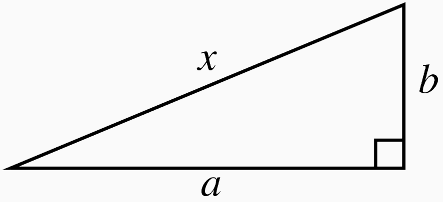

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Thus, by *SSS Congruence*, the triangles must be congruent to each other.

Therefore the original triangle must have a right angle as well. $\square$

Thanks for Watching!

Watch the rest of the videos on this topic!

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