To the Treasure

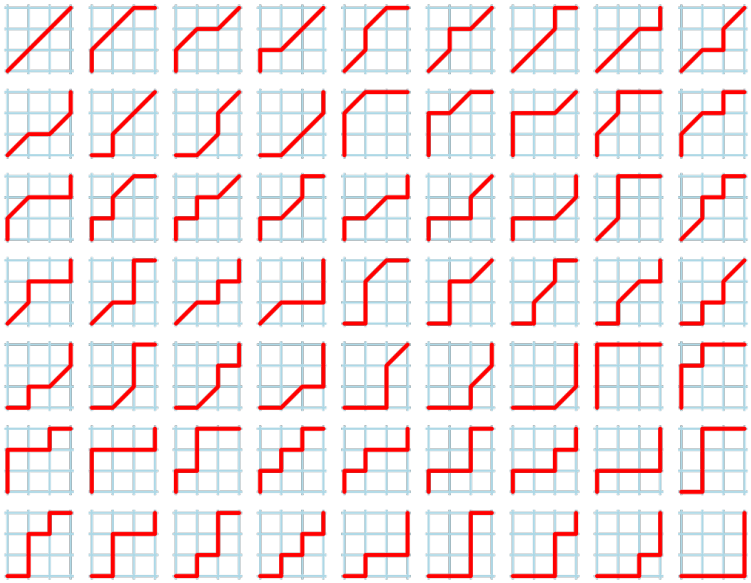
DiPS CodeJam 22

Prompt

Pranav and Prithvi are on an adventure. They find themselves at the southwest corner of an $n \times n$ grid, and they must get to the northeast corner. They can only move in one of these three ways:

- Directly north,
- Directly east, or
- Directly north-east.

For example, if we take $n = 3$, there are 63 paths they can take:



Can you tell them how many different paths there are to their destination?

Input Format

The first and only line of input contains a single integer n.

Output Format

The first and only line of your output must contain the number of different paths.

Constraints

$0 \leq n \leq 100$

Sample Input/Output

Input	Output
2	63

Solution

The number of paths from the southwest corner $(0, 0)$ of a rectangular grid to the northeast corner (m, n), using only single steps north, northeast, or east is called a Delannoy Number $D(m, n)$. To find the answer, we must calculate $D(n, n)$.

Solving the Problem

The recurrence relation for Delanoy Numbers where $m, n \neq 0$ is

$$D(m, n) = D(m - 1, n) + D(m - 1, n - 1) + D(m, n - 1)$$

As m and n are equal, we calculate $D(n, n) = D(n - 1, n) + D(n - 1, n - 1) + D(n, n - 1)$.

Sample Program

```
n = int(input())

def delannoy(m, n):
     if m==0 or n==0:
          return 1

     return delannoy(m-1, n) + delannoy(m-1, n-1) + delannoy(m, n-1)

print(delannoy(n, n))
```