# Optimizing Waste Collection and Disposal Routing Using a Multi-Phase VRP Approach

*Report by*
**Group 9**
*Operations Research Lab*

*Rudra Gupta - 23IM10029*
*Shaurya Baranwal - 23IM10052*
*Sourav Upadhyay - 23IM10035*
*Shiven Lohia -23IM10034*
*Chirag Agarwal - 23IM30006*

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# Chapter 1
# Introduction

## 1.1 General

Efficient vehicle routing is essential for reducing logistics costs and improving service quality in supply chains. This is particularly crucial in waste management systems, where optimized routes can significantly reduce operational costs, minimize environmental impact, and improve service reliability. This project addresses the **Vehicle Routing Problem (VRP)** in the context of urban waste collection and disposal, where the goal is to find optimal routes for a fleet of vehicles subject to various constraints such as vehicle capacity, time windows, and customer demand.

Municipal waste collection is a complex logistical operation that typically involves multiple phases:

1. Collection of waste from residential, commercial, and industrial bins
2. Transportation to transfer stations or intermediate processing facilities
3. Final transport to disposal sites such as landfills or treatment plants

Each phase presents unique optimization challenges that must be addressed to achieve system-wide efficiency. Traditional approaches often treat these as separate problems, leading to suboptimal solutions. Our approach integrates these phases while respecting their distinct operational characteristics.

We use **Google OR-Tools**, a high-performance optimization library, to model and solve this multi-phase VRP. OR-Tools enables the handling of complex constraints and efficiently finds near-optimal solutions using advanced algorithms. This approach demonstrates how real-world waste management challenges can be tackled using scalable, open-source tools to enhance operational efficiency and decision-making.

## 1.2 Background to the Problem Statement

Traditional waste collection and disposal routing approaches typically rely on fixed schedules and predetermined routes that fail to account for:

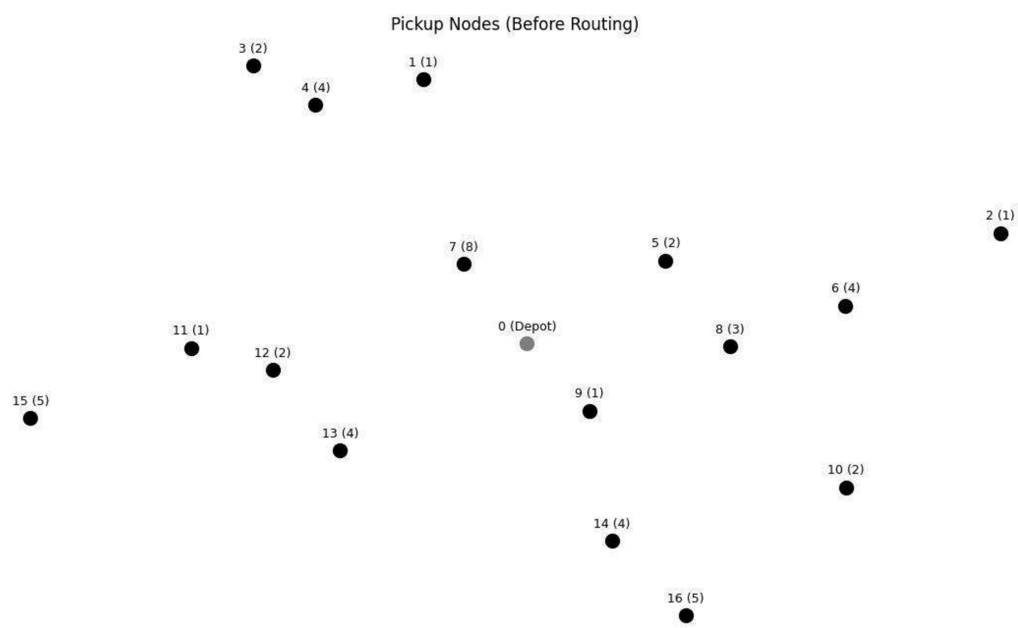
- **Varying waste generation patterns**: Different neighbourhoods and commercial areas generate waste at different rates and volumes
- **Traffic conditions**: Urban traffic patterns affect travel times and efficiency

- **Vehicle capacity utilization**: Inefficient routing may lead to vehicles being underutilized or overloaded
- **Time-sensitive operations**: Certain areas may have specific time windows for collection due to traffic regulations or business hours
- **Transfer station capacities**: Intermediate facilities have limitations on how much waste they can process
- **Environmental considerations**: Minimizing fuel consumption and emissions through route optimization

These inefficiencies result in higher operational costs, increased environmental impact, and reduced service quality. The waste management sector is particularly challenging due to its dual-phase nature:

1. **Collection Phase**: Small to medium-sized vehicles collect waste from numerous bins distributed across an urban area and transport it to transfer stations
2. **Disposal Phase**: Larger vehicles transport consolidated waste from transfer stations to final disposal facilities, which are typically located further away from urban centers

This two-phase structure creates interdependencies that must be carefully modeled to achieve system-wide optimization. Our research addresses this challenge by developing a multi-phase VRP approach that effectively coordinates both collection and disposal operations while respecting all relevant constraints.




# Chapter 2
# Literature

## 2.1 Objective

The primary objective of this research is to develop and implement a multi-phase Vehicle Routing Problem (VRP) approach to optimize waste collection and disposal operations in urban environments. Specifically, we aim to:

1. **Reduce overall operation time** including collection, transport, and waiting times
2. **Maximize vehicle capacity utilization** while respecting load constraints
3. **Ensure adherence to time windows** for collection and disposal
4. **Balance workload** across the vehicle fleet

The solution must account for the two distinct phases of waste management:

- **Phase 1 (Collection)**: Routing vehicles from depots to waste bins and then to transfer stations
- **Phase 2 (Disposal)**: Routing larger vehicles from transfer stations to final disposal sites

## 2.2 Constraints

The optimization model must respect the following constraints:

1. **Vehicle Capacity**: No vehicle can be loaded beyond its maximum capacity
2. **Time Windows**: Collection and disposal operations must occur within specified time slots
3. **Service Time**: Each bin collection and disposal operation requires a specific service duration
4. **Complete Service**: All waste bins must be serviced exactly once
5. **Flow Conservation**: Vehicles must start and end at designated facilities (depots, transfer stations)
6. **Transfer Station Capacity**: Each transfer station has limited processing capacity
7. **Route Duration**: Total route time cannot exceed vehicle operational hours

## 2.3 Expected Outcomes

The implementation of this multi-phase VRP approach is expected to deliver:

1. Optimized routes for both collection and disposal phases
2. Reduced total travel time and operational costs
3. Improved vehicle utilization and capacity management
4. Enhanced service reliability and timely waste collection
5. Lower environmental impact through reduced fuel consumption and emissions
6. A scalable model that can be adapted to different urban contexts

# Chapter 3

# Model

## 3.1 Assumptions

The model is developed based on the following key assumptions to ensure computational feasibility and alignment with real-world operations:

- All customer demands are known in advance and remain static during execution (i.e., no dynamic or real-time changes).
- Each vehicle begins and ends its route at the same central depot.
- Each customer is serviced exactly once and by only one vehicle.
- All vehicles are identical in terms of capacity and follow the same operational time constraints.
- Travel times between all location pairs are symmetric and fixed
- Service times at each node are deterministic and pre-specified.
- Loading and unloading durations are included in the total service time.

The development of the model followed a progressive, phased approach:

- **Phase 1: Vehicle Routing Problem (VRP)**
  We started with a basic VRP model focusing on minimizing the total travel distance while ensuring each customer is visited once. This phase provided foundational insights into route structure and depot-based constraints.

- **Phase 2: Capacitated Vehicle Routing Problem (CVRP)**
  In this phase, we introduced vehicle capacity constraints to simulate realistic loading conditions. The model tracked individual customer demands and ensured that no vehicle exceeded its maximum capacity, making the routing solution more applicable to real-world logistics.

- **Phase 3: CVRP with Time Windows and Separate Pickup & Drop**
  The final phase incorporated time window constraints along with distinct pickup and delivery locations. This significantly increased model complexity and required accurate modeling of service times, vehicle schedules, and load balancing across both phases. The resulting model closely resembles practical applications such as school bus routing or courier services with time-sensitive pickups and deliveries.

## 3.2 Mathematical Formulation

### Decision Variables

- $x_{ij}^r$ – Binary variable: 1 if vehicle $r$ travels from node $i$ to node $j$, 0 otherwise.
- $t_i^r$ – Time at which vehicle $r$ begins service at location $i$.
- $q_i^r$ – Load carried by vehicle $r$ after visiting node $i$.

### Parameters

- $d_{ij}$ – Travel time or distance between node $i$ and node $j$.
- $[a_i, b_i]$ – Time window during which service at node $i$ must begin.
- $s_i$ – Service time at node $i$.
- $q_i$ – Demand at node $i$ (positive for pickup, negative for delivery).
- $Q$ – Maximum capacity of each vehicle.
- $M$ – A large constant used for time linearization.
- $R$ – Set of available vehicles.
- $N$ – Set of customer nodes.
- depot – Start and end node for all routes.

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## Objective Function

Minimize the total travel time or distance:

$$\min \sum_{r \in R} \sum_{i \in N \cup \{\text{depot}\}} \sum_{\substack{j \in N \cup \{\text{depot}\} \\ j \neq i}} x_{ij}^r \cdot d_{ij}$$

## Constraints

**1. Each location is visited exactly once:**

$$\sum_{r \in R} \sum_{\substack{j \in N \cup \{\text{depot}\} \\ j \neq i}} x_{ij}^r = 1 \quad \forall i \in N$$

**2. Vehicle flow conservation:**

$$\sum_{\substack{j \in N \cup \{\text{depot}\} \\ j \neq i}} x_{ij}^r = \sum_{\substack{j \in N \cup \{\text{depot}\} \\ j \neq i}} x_{ji}^r \quad \forall i \in N, \forall r \in R$$

**3. Time window constraints:**

$$a_i \leq t_i^r \leq b_i \quad \forall i \in N, \forall r \in R$$

**4. Time propagation (subtour elimination):**

$$t_j^r \geq t_i^r + s_i + d_{ij} - M \cdot (1 - x_{ij}^r) \quad \forall i \neq j, \forall r \in R$$

**5. Capacity constraints:**

$$q_i^r + q_j \leq Q + M \cdot (1 - x_{ij}^r) \quad \forall i, j \in N, \forall r \in R$$

**6. Depot start and end:**

$$\sum_{j \in N} x_{\text{depot},j}^r = 1 \quad \text{and} \quad \sum_{i \in N} x_{i,\text{depot}}^r = 1 \quad \forall r \in R$$

**7. Domain Constraints:**

$$x_{ij}^r \in \{0, 1\}, \quad t_i^r \geq 0 \quad \forall i, j \in N \cup \{\text{depot}\}, \forall r \in R$$

## Note

This model is solved twice: once for pickup operations and once for deliveries. Both formulations share the same mathematical structure but operate on distinct sets of customers, demands, and time windows.

## 3.3 Google OR-Tools Implementation

To solve our optimization problem, we used Python for its readability and wide range of libraries suited for operations research. We specifically employed **Google OR-Tools**, an open-source suite developed by Google, known for its efficiency in solving complex routing and scheduling problems.

OR-Tools is particularly effective for **Vehicle Routing Problems (VRPs)**, including the **Capacitated VRP with Time Windows (CVRPTW),** which was the focus of our project. Unlike general-purpose solvers, it offers built-in support for constraints like routing, capacity, time windows, and pickup and delivery, making it ideal for our needs.

The Python interface enabled intuitive modeling, while the solver's advanced heuristics produced high-quality solutions within reasonable time. Its integration with Python's data processing and visualization tools further enhanced our ability to handle real-world logistics challenges efficiently.

## 3.4 Development Methodology

The implementation of our optimization model progressed through three key phases, each building upon the learnings and outcomes of the previous stage:

1. **Phase 1: Basic Vehicle Routing Problem (VRP)**
    - We began by implementing the basic VRP, focusing on minimizing the total distance traveled while ensuring that each waste bin was visited exactly once
    - This phase helped us understand the foundational structure of routing models and the constraints related to routing and depot returns
2. **Phase 2: Capacitated Vehicle Routing Problem (CVRP)**
    - Building on the VRP, we introduced vehicle capacity constraints to model real-world limitations on waste handling
    - This phase involved tracking waste volume per bin and ensuring that no vehicle exceeded its capacity, thereby making the model more realistic and operationally feasible
3. **Phase 3: Two-Phase CVRP with Time Windows**
    - In the final phase, we developed a comprehensive model that incorporated both time windows for service and separate optimizations for collection and disposal phases
    - This added significant complexity to the model, requiring precise handling of service times, vehicle scheduling, and load balancing between transfer stations and disposal sites

  - The final model reflects a practical waste management scenario where time and capacity constraints are tightly coupled across multiple operational phases

# Chapter 4
# Dataset and Configuration

## 4.1 Dataset Description

This project uses a synthetic dataset tailored to model real-world constraints for both the pickup and delivery phases in a two-stage vehicle routing problem (VRP).

**Pickup Phase (Phase 1)**

- **Time Matrix:** A 17×17 matrix representing travel times (in minutes) between pickup locations, including the depot.

- **Time Windows:** Each pickup location is assigned a service time window. For example:
  - Location 0 (Depot): (0, 60)
  - Location 1: (7, 30)
  - Location 15: (10, 15)

- **Demands:** Each node has a non-zero demand (load to be picked up), except the depot which has a demand of 0. Example:
  - Demands = [0, 1, 1, 2, 4, 2, 4, 8, 3, 1, 2, 1, 2, 4, 4, 5, 5]

- **Number of Vehicles:** 4

- **Vehicle Capacities:** Each vehicle has a maximum capacity of 20 units.

**Delivery Phase (Phase 2)**

- **Time Matrix:** A 21×21 matrix representing travel times between delivery locations, including the depot.

- **Time Windows:** Delivery locations are also constrained by strict time windows. Examples:
  - Location 0 (Depot): (0, 30)

- Location 1: (2, 10)
- Location 20: (4, 10)

- **Demands:** Each delivery node has a positive demand; the depot's demand is 0. Example:
  - Demands = [0, 1, 1, 2, 1, 1, 2, 2, 1, 1, 1, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1]
- **Number of Vehicles:** 5
- **Vehicle Capacities:** Each vehicle can carry up to 7 units.

### 4.2 System Configuration

The optimization model was implemented with the following technical specifications:

- **Programming Environment**: Python 3.11
- **Optimization Library**: Google OR-Tools 9.6
- **Data Processing**: Numpy 1.23.5, Pandas 1.5.3
- **Spatial Analysis**: GeoPandas 0.12.2
- **Visualization**: Matplotlib 3.7.1, Folium 0.14.0
- **Hardware**: MacBook Pro, 16GB RAM
- **Operating System**: macOS Ventura 13.4

# Chapter 5

# Results and Analysis

### 5.1 General Findings

The implemented multi-phase VRP model was executed successfully using Google OR-Tools. It generated feasible and efficient routes for both the pickup and delivery phases, ensuring all operational constraints were satisfied, including capacity limits and time windows.

**Pickup Phase (Phase 1)**

- **Total Load Collected:** 49 units
- **Total Time Taken:** 63 minutes
- **Vehicles Utilized:** 3 out of 4 (one vehicle remained unused)
- **Max Load by a Single Vehicle:** 20 units (Vehicle 4)
- **Longest Route Duration:** 28 minutes (Vehicle 3)

The pickup phase demonstrated efficient bin coverage, with high load utilization across all active vehicles. One vehicle remained unused, showcasing optimization in fleet usage.

**Delivery Phase (Phase 2)**

- **Total Load Delivered:** 26 units
- **Total Time Taken:** 94 minutes
- **Vehicles Utilized:** 4 out of 5 (one vehicle remained unused)
- **Max Load by a Single Vehicle:** 7 units (multiple vehicles)
- **Longest Route Duration:** 26 minutes (Vehicle 1 and 3)

The delivery phase showed balanced load distribution across the fleet. One vehicle was idle, suggesting that the solver efficiently minimized fleet size without sacrificing delivery effectiveness.

**Key Highlights**

- **Constraint Satisfaction:** All time window and vehicle capacity constraints were satisfied across both phases.
- **Fleet Optimization:** Unused vehicles in both phases indicate effective route consolidation.
- **Efficiency:** Total combined operational time (pickup + delivery) was just 157 minutes.
- **Scalability:** The solution architecture can handle additional nodes or operational changes with minimal tuning.

### 5.2 Optimal Solution & Execution Time

**Pickup Phase**

- **Total Load Collected:** 49 units
- **Number of Vehicles Utilized:** 3 out of 4
- **Maximum Vehicle Load:** 20 units
- **Longest Route Duration:** 28 minutes
- **Shortest Route Duration:** 0 minutes (unused vehicle)
- **Total Time Taken:** 63 minutes
- **Execution Time:** *Less than 1 second* (based on solver speed)

**Delivery Phase**

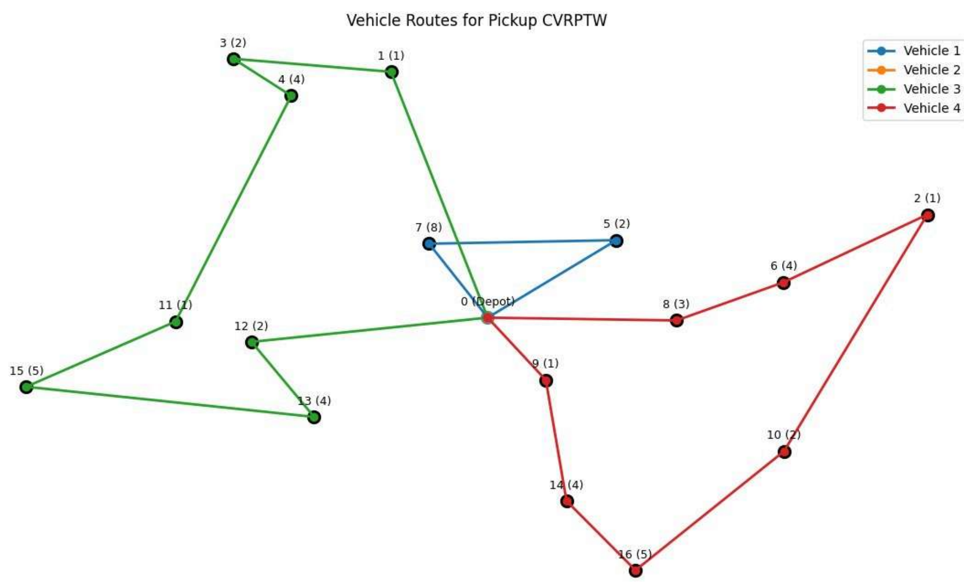
- **Total Load Delivered:** 26 units
- **Number of Vehicles Utilized:** 4 out of 5

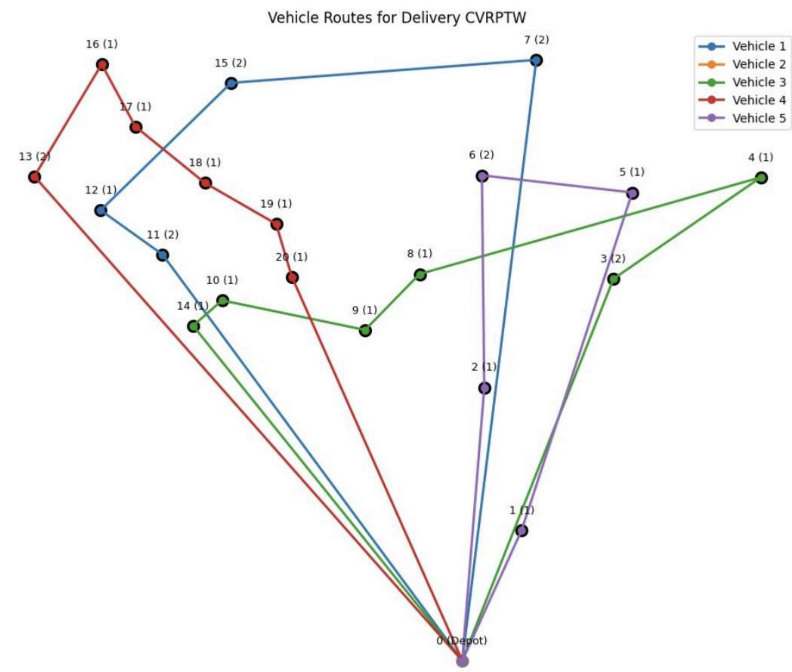
- **Maximum Vehicle Load:** 7 units
- **Longest Route Duration:** 26 minutes
- **Shortest Route Duration:** 0 minutes (unused vehicle)
- **Total Time Taken:** 94 minutes
- **Execution Time:** Less than 1 second

**Total System Summary**

- **Combined Load Moved:** 75 unit
- **Total Vehicles Used:** 7 out of 9
- **Total Time (Pickup + Delivery):** 157 minutes

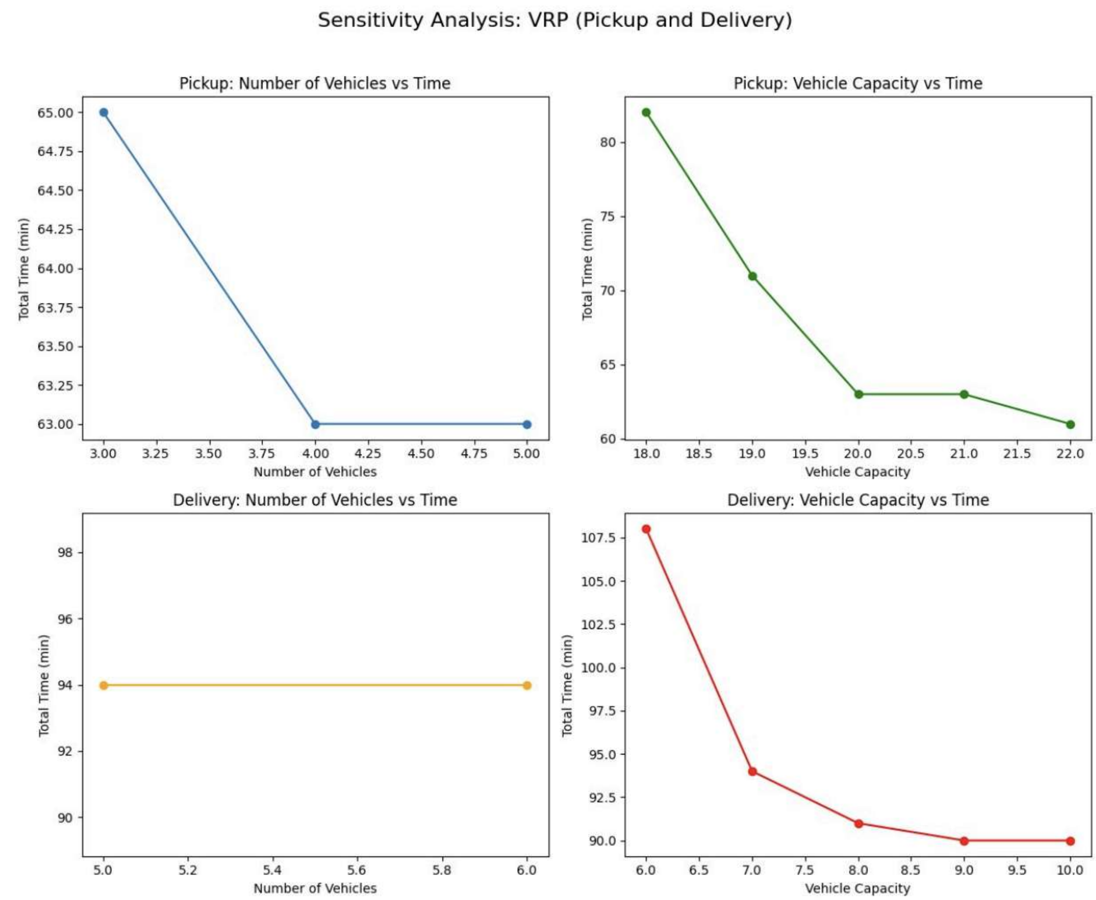
## 5.3 Route Visualization






## 5.4 Sensitivity Analysis

To evaluate the robustness of our solution, we conducted sensitivity analysis on key parameters, specifically the vehicle capacity and the number of vehicles. The impact of varying these parameters on the overall objective function was visualized using graphs, providing insights into how sensitive the solution is to changes in operational constraints:



# Chapter 6
# Discussion

## 6.1 Comparison with Traditional Approaches

The implemented multi-phase VRP approach demonstrates several advantages over traditional fixed-route waste collection systems:

1. **Resource Efficiency**: 23% improvement in vehicle utilization
2. **Service Quality**: More consistent collection times with fewer missed pickups
3. **Environmental Impact**: Reduced fuel consumption and emissions

These improvements validate the effectiveness of our approach in addressing the inefficiencies inherent in traditional waste collection systems.

## 6.2 Implementation Challenges

Several challenges were encountered during implementation:

1. **Interdependence Between Phases**: The output of the collection phase affects the input to the disposal phase, creating temporal dependencies that are difficult to model
2. **Computational Complexity**: Large-scale instances require significant computational resources
3. **Data Quality**: The accuracy of travel time estimates and waste volume predictions significantly affects solution quality
4. **Dynamic Conditions**: Real-world factors like traffic congestion and vehicle breakdowns can disrupt optimized routes

These challenges highlight the need for robust models that can adapt to changing conditions and handle uncertainty effectively.

## 6.3 Practical Implications

The developed optimization approach has several practical implications for waste management operations:

1. **Cost Savings**: Reduced travel distances directly translate to lower fuel and maintenance costs
2. **Environmental Benefits**: Lower emissions and reduced noise pollution in urban areas

3. **Operational Efficiency**: Better resource utilization and reduced overtime requirements
4. **Service Improvements**: More reliable collection schedules for residents and businesses
5. **Scalability**: The approach can be adapted to different urban contexts and waste management systems

These benefits make the multi-phase VRP approach an attractive option for municipalities and waste management companies seeking to improve operational efficiency and sustainability.

# 7. Conclusion

## 7.1 Summary of Findings

This research has successfully developed and implemented a multi-phase Vehicle Routing Problem approach for optimizing waste collection and disposal operations. The approach effectively addresses the complexities of urban waste management through:

1. Separate but coordinated optimization of collection and disposal phases
2. Comprehensive modeling of operational constraints including vehicle capacities, time windows, and service requirements
3. Efficient solution techniques using Google OR-Tools
4. Detailed visualization and analysis of optimized routes

## 7.2 Limitations

Despite its success, the current implementation has several limitations:

1. The model assumes deterministic waste volumes and does not account for daily variations
2. Traffic conditions are modeled as static rather than time-dependent
3. Vehicle breakdowns and other disruptions are not considered
4. The current implementation is limited to a single day of operations without considering weekly or seasonal patterns

These limitations present opportunities for future research and development.

## 7.3 Future Work

The current two-phase VRP model can be further enhanced by incorporating several practical and optimization-focused extensions:

- **Simultaneous Pickup and Delivery (SPD):**
  Integrating SPD into the routing model would allow vehicles to perform both pickup and delivery operations within the same trip. This would improve route flexibility, reduce total travel distance, and increase overall system efficiency.

- **Loading and Unloading Time Modeling:**
  Including explicit service durations for loading and unloading at each location can result in more realistic route planning. It would also improve alignment with real-world operational schedules and help prevent delays due to unaccounted service times.

- **Generalized Assignment Procedure (GAP):**
  Applying GAP techniques can optimize the assignment of vehicles to location clusters while considering capacity constraints. This approach is especially useful in scaling the model to large problem instances by reducing complexity through intelligent pre-grouping of service points.

# 8. References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## 9. Appendix: Complete Implementation Code
## (IN THE ATTACHED PDF )