USN

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

II Semester B. E. Regular / Supplementary Examinations Aug-2024

VECTOR CALCULUS, LAPLACE TRANSFORM AND NUMERICAL METHODS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of mathematics Handbook is permitted. Do not write anything on handbook.

PART-A

			M	BT	CO
1	1.1	If vector $\vec{f} = 2ax\hat{\imath} - ay\hat{\jmath} + 4z\hat{k}$ is solenoidal, then the value of the constant 'a' is ________.	01	1	1
	1.2	The normal vector to the surface $y^2 - 4x^2 + 3z = 3$ is ________.	01	1	1
	1.3	The region of convergence for the Laplace transform of $sin(2t)$ is ________.	01	1	1
	1.4	If $L[f(t)] = \frac{5}{s(s^2+7),}$ then $L[f(2t)] =$ ________.	01	1	1
	1.5	$L^{-1}\left[\frac{1}{(s+2)^5}\right] =$ ________.	02	1	2
	1.6	$L[\sqrt{t}\, e^{-\frac{t}{2}}]=$ ________.	02	2	1
	1.7	Inverse Laplace transform of $\frac{s-3}{s^2-6s+2}$ is ________.	02	2	1
	1.8	Given $\frac{dy}{dx} = x + y^2, y(0) = 1, h = 0.1, k_1 = 0.1, k_2 = 0.1152, k_3 = 0.1168, then$ the solution of the differential equation at $x = 0.1$ using Runge kutta method of fourth order is ________.	02	2	2
	1.9	First approximate value of a real root of the equation $2x - \log_{10} x = 7$ in the interval [3.5,4] by false position method is ________.	02	1	1
	1.10	If $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and V is the region bounded by cube $x = 0, y = 0, z = 0, x = 1, y = 2, z = 3$, then $\iiint_v \nabla.\vec{F}$ is ________.	02	2	2
	1.11	If $\vec{F} = (x^2 + 5y^2)\hat{\imath} + (3y^2 - 4x)\hat{\jmath}$, then evaluate $\int_c \vec{F}.d\vec{r}$ along the straight line from (0,0) to (0,2).	02	2	2
	1.12	Compute gradient of the scalar field $\phi = r + zcos\theta$, specified in the cylindrical coordinates (r,θ, z).	02	2	2

PART-B

2	a	Find the directional derivative of $\phi(x,y,z) = \frac{1}{x^2+y^2+z^2}$ at the point $(1,2,-3)$ in the direction of $\hat{\imath} + 3\hat{\jmath} + \hat{k}$.	06	2	2
	b	Show that the vector $\vec{F} = (y^2\cos(x) + z^3)\hat{\imath} + (2ysin(x) - 4)\hat{\jmath} + 3xz^2\hat{k}$ is irrotational and find its scalar potential such that $\vec{F} = \nabla\phi$	06	3	3
	c	If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}\ and\ r = \|\vec{r}\|$, then show that $\nabla r^n = nr^{n-2}\vec{r}$	04	2	2
3	a	Using divergence theorem, evaluate $\iint_S [(x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}].\hat{n}ds$, over the surface of the rectangular parallelepiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$	08	2	3
	b	Using Green's theorem in the plane evaluate $\int_c \{(3x^2 - y^2)dx + (x^2 + 5y^2)dy\}, C$ is the boundary of the region bounded by $x = 0, y = 0, x + y = 2$.	08	2	3

		OR			
4	a	Using Stokes theorem evaluate $\int_c (x+y)dx+(2x-z)dy+(y+z)dz \; where \; C$ is the boundary of the triangle with vertices $(2,0,0),(0,3,0)$ and $(0,0,6)$.	10	3	3
	b	Evaluate $\int_c \vec{F}.d\vec{r} \;\; where \; \vec{F}=(xy)\hat{\imath}+(x^2+y^2)\hat{\jmath}$ along i) The path of the straight line from (0,0) to (1,0) and then to (1,1). ii) The straight line joining the origin and (1,2).	06	2	2
5	a	Find the Laplace transform of $\int_0^t \frac{sint}{t}\, dt + te^{-t}\cos^2 2t$	08	2	2
	b	Express the piecewise continuous function $f(t)=\begin{cases} 1, & 0<t\le 1 \\ t, & 1<t\le 2 \\ t^2, & t>2 \end{cases}$ in terms of the unit step function and also find its Laplace transform.	08	2	3
		OR			
6	a	i) Evaluate $\int_0^\infty e^{-6t} t\sin(t)\, dt \; using$ Laplace transform. ii) Find the Laplace transform of the function $(e^{-at}-\cos(bt))/t$	08	2	2
	b	Show that the Laplace transform of the periodic function $f(t)$ is $\frac{a}{s}\tanh\left(\frac{as}{2}\right), where$ f(t)=$\begin{cases} k, 0\le t\le a \\ -k, a\le t<2a \end{cases}$ $f(t+2a)=f(t)$. Also sketch the waveform for $f(t)$.	08	2	2
7	a	Find the inverse Laplace transform of $\frac{4s+5}{(s+1)^2(s+2)}$	08	2	2
	b	Using convolution theorem, obtain the inverse Laplace transform of $\frac{1}{(s+1)(s^2+4)}$	08	3	3
		OR			
8	a	Solve $\frac{d^2y}{dt^2}+\frac{dy}{dt}=t^2+2t, y(0)=4, y'(0)=-2$ using Laplace transform.	08	3	4
	b	Find i) $L^{-1}\left[\cot^{-1}\left(\frac{2}{s+1}\right)\right]$ ii) $L^{-1}\left[e^{-4s}\left(\frac{s+3}{s^2+4s+13}\right)\right]$	08	2	2
9	a	Use the Runge Kutta method of fourth order with $h=0.1$ to find approximate value of the solution of the initial value problem $\frac{dy}{dx}=2x^2+3y^2-2, y(2)=1 \; at \; x=2.1.$	08	2	4
	b	Employ Taylor series method to obtain the approximate value of y at $x=0.1$ for the differential equation $\frac{dy}{dx}=e^x-y^2; y(0)=1$ considering the terms upto fourth degree.	08	2	2
		OR			
10	a	Apply Milne's predictor-corrector method to find the solution of the differential equation $\frac{dy}{dx}+x^3=y$ at $x=0.4$ given that $y(0)=1$, $y(0.1)=1.1051$, $y(0.2)=1.2210$, $y(0.3)=1.3477$	10	2	2
	b	Using Newton Raphson method, find the root of the equation $xcosx=x^2$ correct to four decimal places choosing the initial guess $x_0=1$.	06	2	2