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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

II Semester B. E. Examinations Oct-2023

(Common to EC, EE, EI, ET)

VECTOR CALCULUS, LAPLACE TRANSFORM AND NUMERICAL METHODS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of mathematics Handbook is permitted. Do not write anything on handbook.

PART-A

1	1.1	The region of convergence for the Laplace transform of e^{-4t} is ___________.	01
	1.2	Laplace Transform of the function $t^5\delta(t-3)$ is ___________.	01
	1.3	If $L[f(t)] = \frac{3}{s(s^2-5)}$, then $L\left[f\left(\frac{t}{2}\right)\right] =$ ___________.	01
	1.4	Find the Laplace transform of the function $f(t) = \frac{3}{\sqrt{t}}$.	01
	1.5	Relationship between line integral and surface integral is established in ___________.	01
	1.6	If $\overrightarrow{G}$ represents force acting on a particle, then $\int_C \overrightarrow{G}.d\vec{r}$ represents ___________.	01
	1.7	Find the gradient of the function $2yz + z^2$ at the point $(1,-1,3)$.	02
	1.8	Compute the divergence of $\vec{f} = (\cos\theta + \sin\theta)\widehat{e_r} + (\cos\theta - \sin\theta)\widehat{e_\theta} + \widehat{e_z}$ specified in the cylindrical coordinates (r, θ, z).	02
	1.9	$L^{-1}\left[\frac{1}{s(s-2)}\right]$ is ___________.	02
	1.10	If the signal in the frequency domain in $\frac{s}{(s+3)^2}$, then the function in the time domain is ___________.	02
	1.11	Evaluate $\int_C y^2dx - 2x^2dy$ along the parabola $y = x^2$ from (0,0) to (2,4).	02
	1.12	First approximate value of a real root of the equation $x^4 - x = 10$ near $x = 2$ by Newton –Raphson method is ___________.	02
	1.13	The approximate solution of $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 2$ at $x = 0.1$ using Taylor series up to second degree term is ___________.	02

PART-B

2	a	Show that the vector $\vec{F} = (y^2\cos x + z^3)\,\hat{\imath} + (2y\sin x - 4)\hat{\jmath} + 3xz^2\hat{k}$ is irrotational and find its scalar potential ϕ such that $\vec{F} = \nabla\phi$.	06
	b	Find the angle between the surface $x\log_e(z) = y^2 - 1$ and $x^2y = 2 - z$ at the point (1,1,1).	06
	c	If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = \lvert\vec{r}\rvert$, then show that $\frac{\vec{r}}{r^3}$ is solenoidal.	04

3	a	If $\vec{F} = (2x + y^2)\hat{\imath} + (3y - 4x)\hat{\jmath}$, evaluate $\oint_C \vec{F}.d\vec{r}$ around the triangle in the xy plane with vertices $(0,0), (2,0)$ and $(2,1)$.	08
	b	Evaluate $\iint_S \vec{F}.\hat{n}\, ds$ if $\vec{F} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.	08
		OR	
4	a	Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along the curve defined by $x^2 = 4y, 3x^3 = 8z$ from $x = 0$ to $x = 2$.	06
	b	Using Stoke's theorem, evaluate $\int_C (x + y)dx + (2x - z)dy + (y + z)dz$ where C is the boundary of the triangle with vertices $(2,0,0), (0,3,0)$ and $(0,0,6)$.	10
5	a	Find the Laplace transform of $t\cos(3t) + \int_0^t \frac{e^{-3t}\sin(2t)}{t} dt$.	08
	b	Show that the Laplace transform of the periodic function $f(t) = \begin{cases} 1 + t, & 0 \le t \le 1 \\ 3 - t, & 1 \le t \le 2 \end{cases}$ with $f(t + 2) = f(t)$ is $\frac{1}{s} + \frac{1}{s^2}\tanh\left(\frac{s}{2}\right)$. Also sketch the graph of the function.	08
		OR	
6	a	i) Evaluate $\int_0^\infty \frac{e^{-t}\sin(t)}{t} dt$ using Laplace transform. ii) Find $L[\sin 2t \cos 3t]$.	08
	b	Express the piecewise continuous function $f(t) = \begin{cases} 2t^2, & 0 \le t < 3 \\ t + 4, & 3 \le t < 5 \\ 9, & t \ge 5 \end{cases}$ in terms of the unit step function and then find its Laplace transform.	08
7	a	Find $L^{-1}\left[\log_e\left(\frac{s+2}{s+5}\right) + \frac{s+3}{s^2-10s+29}\right]$	08
	b	Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}, y(0) = 1, y'(0) = 0$ using Laplace transform.	08
		OR	
8	a	Using convolution theorem, obtain inverse Laplace transform of $\frac{s}{(s^2+1)(s^2+4)}$.	08
	b	Find the inverse Laplace transform of $\frac{s+2}{(s^2+4s+5)^2} + e^{-s}\left(\frac{1+\sqrt{s}}{s^3}\right)$	08
9	a	Compute the real root of the equation $x^3 - 4x + 1 = 0$ in the interval $[0,1]$ correct to three decimal places using Regula-Falsi method. Perform four iterations.	06
	b	Use the Runge-Kutta method of fourth order with $h = 0.1$ to find approximate value for the solution of the initial value problem $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$ at $x = 1.1$ and $x = 1.2$.	10
		OR	
10	a	Apply Milne's predictor-corrector method to find the solution of the differential equation $\frac{dy}{dx} = xy + y^2$ at $x = 0.4$ given that $y(0) = 1$, $y(0.1) = 0.1169, y(0.2) = 1.2773, y(0.3) = 1.5049$. Apply corrector formula twice.	10
	b	Using Newton –Raphson method, find the root of the equation $\sin x = 1 - x$ correct to four decimal places choosing the initial guess $x_0 = 0.6$.	06