PUX-321

GOVERNMENT OF KARNATAKA

KARNATAKA STATE PRE-UNIVERSITY EDUCATION EXAMINATION BOARD

II YEAR P.U.C. EXAMINATION OF

MARCH -2019

MAIN ANSWER BOOK

Answer Book Sl. No.
2068700

Register No. of the Candidate					
7	7	7	9	3	7

Please read the instructions overleaf before filling in

Subject Code: 35 Subject: MATHEMATICS

Sl. No. of Additional answer sheets used	No. of pages used in Main Answer book	No. of pages used in Addnl. answer book/s	Total No. of Pages used
1. 2518813	22	16	38+1 graph
2. 2518819			
3. 2518829			
4. 2518839			
5.			
6.			
7.			
8.			

Certified that the entries made above by the Candidate are found to be correct

Kendore 07-03-2019

Signature of the Invigilator with date

FOR THE USE OF EXAMINERS ONLY

Marks awarded

Part											Total Marks
	1	2	3	4	5	6	7	8	9	10	
	1	1	1	1	1	1	1	1	1	1	10
	11	12	13	14	15	16	17	18	19	20	
	2	2	2	2	2	2	2	1	-	2	16
	21	22	23	24	25	26	27	28	29	30	
	2	2	-	-	2	3	3	3	3	-	18
	31	32	33	34	35	36	37	38	39	40	
	3	3	3	0	03	3	-	-	5	5	25
	41	42	43	44	45	46	47	48	49	50	
	5	5	5	5	-	5	-	-	-	6/4	30
	51	52	53	54	55	56	57	58	59	60	
	61	62	63	64	65	66	67	68	69	70	
						99					
	71	72	73	74	75	76	77	78	79	80	
											99

Grand Total in Words	Ninty Nine only	Grand Total in Figures	99

INSTRUCTIONS TO CANDIDATES

1. Write your register number Correctly on the space provided on the Facing Sheet of the Answer book and the top left side of Additional answer sheets. Over writing should be attested by the Room Invigilator.
2. Write answers in both sides of the sheet using BLUE/BLACK ink or ball point pen.
3. Obtain Additional Sheets, Graph Sheets, Mathematical table from the Invigilator if required. Enter the serial numbers of all the Additional sheets used.
4. Intimate disorders if any, in the Main Answer book/ Additional sheets to the invigilator.
5. Indicate the Correct question number in the margin.
6. Obtain the permission of the Invigilator for change of PEN / INK.
7. All rough work should be made on a particular page with the heading ROUGH WORK and cross it.
8. Do not write in the margin and leave any page UNUSED except at the end of answers.
9. No Candidate is permitted to leave the examination hall within 30 minutes from the commencement of the examination. Any candidate who leaves after 30 minutes will not be allowed again to the examination hall.
10. If you want to make any request to the Room Invigilator, just stand up to attract his / her attention. Do not shout or leave your place. The Invigilator will come to you.
11. During the examination if the candidate wants to go out, for urination etc., same may be informed to the invigilator. While going out, the Answer paper, Question paper etc., should be handed over to the Room Invigilator for safe custody.
12. After completion, just stand up & inform the same to the Room Invigilator who in turn will collect the papers and gets your signature on the diary maintained by the Invigilator.
13. The following misdeeds will attract disciplinary actions and criminal prosecution.
    a) Breach of silence.
    b) Use of books, notes, manuscripts, etc., pertaining to the subject in the examination hall.
    c) Talking or signalling to other Candidates.
    d) Candidates copying from the answer books of the other candidates or from other source.
    e) Sending of answer books or additional sheets or question paper out of the examination hall.
    f) Impersonation.
    g) Taking the answer books or additional sheets received for writing the answers out of the examination hall during or after the examination.
    h) Tearing or insertion to the answer books and the additional sheets.
    i) Writing an appeal or request to the valuator in the answer book.
    j) Mobile Phones, Pagers are strictly prohibited in the Examination Hall.
    k) Simple calculators can be used, Scientific calculators allowed only for Statistics paper.
14. After completion of writing, Count the No. of pages used and fill the columns provided on the facing sheet of the main answer book.
15. Candidates suffering from infectious diseases are not allowed to sit in the examination hall.
16. Candidates should strike off the subject which is not applicable

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PART-A.

1). Binary operation: A ~~operation~~ $*$ on set A is said to binary operation, which contains two variables defined as $a*a=ab$, which satisfies given condn. satisfies the given conditions is called binary operation For Ex. ~~$a*b$~~ $= a\bullet b$ $\forall a, b \in Z$ defined as $a \to a$

2). $\cos^{-1}(-\frac{1}{2}) = \pi - \cos^{-1}(\frac{1}{2})$

$= \pi - \frac{\pi}{3}$

$\cos^{-1}(-1/2) = \frac{2\pi}{3}$

3). Scalar matrix: A square matrix is said to be scalar matrix if all its diagonal elements are same and rest are zero.

Ex: For 2x2 sq. matrix $\rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

4). $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

$\therefore 3 - x^2 = 3 - 8$

$\therefore -x^2 = -5 - 3$. or $-x^2 = -8$ $\therefore x = \pm 2\sqrt{2}$.

4

5). $y = \sin(x^2+5)$

$\frac{dy}{dx} = \cos(x^2+5)\frac{d}{dx}(x^2+5)$

$\frac{dy}{dx} = 2x\cos(x^2+5)$

6). $\int (1-x)\sqrt{x}\, dx$

$\int \sqrt{x}\, dx - \int x\sqrt{x}\, dx.$

$\int x^{1/2} dx - \int x^{3/2} dx.$

$\frac{x^{1/2+1}}{1/2+1} - \frac{x^{3/2+1}}{3/2+1} + C.$

$\therefore \int (1-x)\sqrt{x}\, dx = \frac{2}{3}x^{3/2} - \frac{2x^{5/2}}{5} + C.$

7). $x(\hat{i}+\hat{j}+\hat{k}) = x\hat{i}+x\hat{j}+x\hat{k}$

Since, $\sqrt{x^2+x^2+x^2} = 1$ ... (given)

$\therefore \sqrt{3x^2} = 1$

or $\sqrt{3}\,x = 1$

$\therefore x = \frac{\pm 1}{\sqrt{3}}$

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8). Direction ratios $(2, -1, -2)$ — with a, b, c written above

Direction cosines, $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$

$= \frac{2}{\sqrt{2^2+1^2+2^2}}, \frac{-1}{\sqrt{2^2+1^2+2^2}}, \frac{-2}{\sqrt{2^2+1^2+2^2}}$

$= \left(\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}\right)$

9). Objective function: A linear function of involved variables, which we want to maximize or minimise the constraints in a linear programming problem is called objective function.

10). Given: $P(E) = 0.6$.

$P(F) = 0.3$

$P(F \cap E) = 0.2$.

To find: $P(F/E) = ?$

We have, $P(F/E) = \frac{P(F \cap E)}{P(E)}$

$= \frac{0.2}{0.6} = \frac{2}{6} = \frac{1}{3}$.

$\therefore \boxed{P(F/E) = 1/3}$

3

PART-B

12). To prove: $\sin^{-1}x + \cos^{-1}x = \pi/2$, $x \in [-1,1]$

put, $\sin^{-1}x = \theta$.

$\therefore x = \sin\theta$.

$\therefore x = \cos(\pi/2 - \theta)$ $\{\because \cos(\pi/2-\theta) = \sin\theta$

$\therefore \cos^{-1}x = \frac{\pi}{2} - \theta$.

$\therefore \theta + \cos^{-1}x = \frac{\pi}{2}$

$\therefore \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ $\{\because \theta = \sin^{-1}x$

Hence, proved.

13). $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, put $x = \sec\theta$ $\therefore \theta = \sec^{-1}x$.

$\therefore \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta - 1}}\right)$

$= \cot^{-1}\left(\frac{1}{\sqrt{\tan^2\theta}}\right)$ $\{\because 1+\tan^2\theta = \sec^2\theta$

$= \cot^{-1}\left(\frac{1}{\tan\theta}\right)$

$= \cot^{-1}(\cot\theta)$

$= \theta$ $\{\because \cot^{-1}(\cot\theta) = \theta\}$

H

ಪ್ರತಿ ಪುಟದ ಒಟ್ಟು ಅಂಕಗಳು $= \sec^{-1}x$ //.

14). Let $\overset{x_1\ y_1}{A(2,7)}$, $\overset{x_2\ y_2}{B(1,1)}$ & $\overset{x_3\ y_3}{C(10,8)}$ be the vertices of the triangle.

$$\therefore \text{Area of triangle} = \frac{1}{2} \bmod \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \bmod \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$A = \frac{1}{2} \bmod \left[2(1-8) - 7(1-10) + 1(8-10) \right]$$

2

$$= \frac{1}{2} \bmod (-14 + 63 - 2)$$

$$= \frac{47}{2} \text{ sq. units}$$

$\therefore$ Area of triangle is $\underline{\frac{47}{2}}$ sq. units

2

15). $y = (\log x)^{\cos x}$.

Take log on both sides,

$\log y = \cos x \log(\log x)$

Differenciate w.r.t 'x'.

$\therefore \frac{1}{y}\frac{dy}{dx} = \cos x \frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}\cos x$

$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{x\log x} + \log(\log x)(-\sin x)$

$\frac{dy}{dx} = y\left[\frac{\cos x}{x\log x} - \sin x \cdot \log(\log x)\right]$

or $\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x \cdot \log(\log x)\right]$

16) $ax + by^2 = \cos y$.

Differenciate wrt 'x'.

$a + b.2y\frac{dy}{dx} = -\sin y.\frac{dy}{dx}$.

$2by\frac{dy}{dx} + \sin y\frac{dy}{dx} = -a$.

$\therefore (2by + \sin y)\frac{dy}{dx} = -a$.

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$\therefore \frac{dy}{dx} = \frac{-a}{2by+\sin 2y}$.

✓

17). Vol. of cube is given by, $V = x^3$.

Differenciate wrt. 'x'.

$\frac{dV}{dx} = 3x^2$.

$\therefore dV = 3x^2 . dx$

or $\Delta V = 3x^2 . \Delta x$.

Given that, $\Delta x = 2\%$ of x

$= \frac{2}{100} \times x$.

$\therefore \Delta V = 3x^2 . \frac{2}{100} x$

2

$\Delta V = (0.06 x^3)\ m^3$

$\therefore$ The appropriate ~~change in~~ volume V of cube is $(0.06x^3)\ m^3$.

4

18). $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx$

~~put~~ $= \int \frac{\sec^2 x \, dx}{(1-\tan x)^2}$

put $1-\tan x = t$

$-\sec^2 x \, dx = dt$

or $\sec^2 x \, dx = -dt$

$\int \frac{-dt}{t^2} = -\int \frac{dt}{t^2}$

$-\int t^{-2} \cdot dt = -\frac{t^{-2+1}}{-2+1}$

$= -\frac{t^{-1}}{-1} = \frac{1}{t} + C$

$= \frac{1}{1-\tan x} + C$

1

20). Given differential eq^n: $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0.$

Order ~~Degree~~ = Two (2).

degree = Not defined (N.D).

Hence, order is two & degree is not defined.

2

2

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21). Given: $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 8$.

$|\vec{a}| = 8|\vec{b}|$.

To find: $|\vec{b}| = ?$

We know that,

$(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 8$.

$\therefore |\vec{a}|^2 - |\vec{b}|^2 = 8$.

$\therefore [8|\vec{b}|]^2 - |\vec{b}|^2 = 8$

$\therefore 64|\vec{b}|^2 - |\vec{b}|^2 = 8$

$63|\vec{b}|^2 = 8$

$|\vec{b}|^2 = \frac{8}{63}$

$\boxed{|\vec{b}| = \pm \frac{2\sqrt{2}}{\sqrt{63}}}$

To find: $|\vec{a}|$, put $|\vec{b}| = 2\sqrt{2}/\sqrt{63}$ in $|\vec{a}| = 8|\vec{b}|$.

$\therefore |\vec{a}| = 8 \cdot \frac{2\sqrt{2}}{\sqrt{63}} = \frac{16\sqrt{2}}{\sqrt{63}}$.

$\therefore |\vec{b}| = \frac{2\sqrt{2}}{\sqrt{63}}$

2

22). $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$.

$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

Projection of vector $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\therefore$ $\vec{a}\cdot\vec{b} = 2 + 6 + 2 = 10$ &

also, $|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$\therefore$ Projection of vector $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$= \frac{10}{\sqrt{6}}$

or. $\frac{10}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{10\sqrt{6}}{6}$

$= \frac{5\sqrt{6}}{3}$.

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II) 29) $f: N \rightarrow N$.

1) $f(x) = 2x$.

To prove:

i) $f(x)$ is one - one:

Let $x_1, x_2 \in N$, such that $f(x_1) = f(x_2)$

$\therefore 2x_1 = 2x_2$

$\therefore \boxed{x_1 = x_2} \; \forall \; x_1, x_2 \in N$.

Hence, $f(x)$ is one-one.

ii). $f(x)$ is not on-to.

Let $f(x) = y \; \forall \; y \in N$,

$\therefore 2x = y$

or $x = \frac{y}{2} \notin N$.

Ex: if $y = 1$ then $x = 1/2 \notin N$.

~~Correct~~ or Range of $f \neq$ co-domain.

Hence, $f(x)$ is not on-to

Hence, proved.

PART-C

25). $R = \{(a,b) : a \leq b^3\}$

Reflexivity: As $a \leq a^3$ is not true or false.

Hence $(a,a) \notin R$.

Thus, R is not reflexive.

For Ex. $a = \frac{1}{2}$ $a^3 = \frac{1}{8}$, which is

Hence, $\frac{1}{2} \leq \frac{1}{8}$ is false.

Hence, R is not reflexive.

Symmetric: Let $(a,b) \in R$ $\forall a,b \in R$.

$a \leq b^3$ which does not implies $b \leq a^3$.

i.e. $a \leq b^3 \nRightarrow b \leq a^3$.

It means $(b,a) \notin R$.

Ex: $a = 1$, $b = 2$ $\therefore 1 \leq 8 \nRightarrow 2 \leq 1$.

Hence, R is not symmetric.

Transitive: Let (a,b) & $(b,c) \in R$ $\forall a,b,c \in R$

$\therefore a \leq b^3$ & $b \leq c^3$

$\Rightarrow a \leq c^3$

$\Rightarrow (a,c) \in R$

Ex: ~~a=2, b=3, c=4~~

~~2≤8 & 3≤8~~ $a = \frac{1}{2}$ ~~$b = \frac{1}{3}$~~ $c = \frac{1}{4}$

$\frac{1}{2} \leq \frac{1}{64}$

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Hence, R is neither reflexive nor symmetric but it is transitive...

28). <u>To prove:</u> $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$.

Let, $\cos^{-1}\frac{4}{5} = A$ | $\cos^{-1}\left(\frac{12}{13}\right) = B$

$\cos A = \frac{4}{5} = \frac{A}{H}$ | $\cos B = \frac{12}{13}$ $\frac{A}{H}$.

$\Rightarrow \sin A = \frac{3}{5}$ $\frac{O}{H}$. $\sin B = \frac{5}{13}$

We have

$\cos(A+B) = \cos A \cos B - \sin A \cdot \sin B$.

$\cos(A+B) = \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$

$= \frac{48}{65} - \frac{15}{65}$

$\cos(A+B) = \frac{33}{65}$.

$\therefore A + B = \cos^{-1}\left(\frac{33}{65}\right)$.

$\therefore \cos^{-1}(4/5) + \cos^{-1}(12/13) = \cos^{-1}(33/65)$.

Hence, proved.

27). Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

We have, $A = IA$.

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A.$$

$R_2 \leftrightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A.$$

$R_2 \leftrightarrow \frac{R_2}{-5}$.

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2/5 & -1/5 \end{bmatrix} A.$$

$R_1 \to$ ~~...~~ $R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/5. & 2/5. \\ 2/5 & -1/5 \end{bmatrix} A.$$

$$I = A^{-1} \quad A.$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix}$$

3

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28). $x = a(\theta + \sin\theta)$.

$y = a(1 - \cos\theta)$

Diff 'x' wrt θ

$$\frac{dx}{d\theta} = a(1 + \cos\theta)$$

Diff 'y' wrt 'θ'

$$\frac{dy}{d\theta} = a[\cancel{1}0 - (-\sin\theta)]$$

$$\frac{dy}{d\theta} = a\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cancel{a}\sin\theta}{\cancel{a}(1+\cos\theta)}$$

3

$$= \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{\cancel{2}\sin\theta/2\ \cancel{\cos}\theta/2}{\cancel{2}\cos^{\cancel{2}}\theta/2}$$

$$\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$$

3

29). $f(x) = x^2 + 2x - 8$. $x \in [-4, 2]$

Since, the given function being polynomial is continuous on $[-4, 2]$ and differenciable in $(-4, 2)$.

$f'(x) = 2x + 2$.

$\therefore f(-4) = (-4)^2 + 2(-4) - 8$

$f(-4) = 0$

$f(2) = 2^2 + 2(2) - 8$

$f(2) = 0$

$\therefore f(-4) = f(2)$.

Let, $f'(c) = 0$.

$2c + 2 = 0$

$\therefore 2c = -2$

$c = \frac{-2}{2}$

$\boxed{c = -1 \in [-4, 2]}$

Hence, Rolle's theorem is verified.

3

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31) $I = \int x \log x \, dx. = \int \log x \cdot x \, dx.$

Integrate,

$I = \log x \int x\,dx - \int \frac{d}{dx} \log x \cdot \int x\,dx \; dx + C$

$I = \frac{x^2}{2} \log x - \int \frac{1}{x}\left(\frac{x^2}{2}\right) dx + C$

$I = \frac{x^2}{2} \log x - \int \frac{x}{2} \, dx + C$

$I = \frac{x^2}{2} \log x - \frac{1}{2} \int x \, dx + C$

$I = \frac{x^2}{2} \log x - \frac{1}{2} \cdot \frac{x^2}{2} + C$

3 $I = \frac{x^2}{2} \log x - \frac{x^2}{4} + C$

$\therefore \int x \log x \, dx = \frac{x^2}{2} \log x - \frac{x^2}{4} + C$

$\boxed{\int u v \, dx = u \int v \, dx - \int \frac{du}{dx} \int v\,dx \; dx + C.}$

3

32) $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

put $\cos x = t$

$-\sin x\,dx = dt$

or $\sin x\,dx = -dt$

$\displaystyle\int_0^{\pi/2} \frac{\sin x\,dx}{1+\cos^2 x} = \int_0^{\pi/2} \frac{-dt}{1+t^2}$

$\displaystyle = -\int_0^{\pi/2} \frac{dt}{1+t^2}$

Since, $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

$\displaystyle = -\frac{1}{1}\tan^{-1}\left(\frac{t}{1}\right)\Big]_0^{\pi/2}$

$\displaystyle = -\tan^{-1}(\cos x)\Big]_0^{\pi/2}$

3

$= -\left[\tan^{-1}\cos(\pi/2) - \tan^{-1}\cos(0)\right]$

$= -\left[\tan^{-1} 0 - \tan^{-1}(1)\right]$

$\displaystyle = -0 + \frac{\pi}{4}$

$\displaystyle\int_0^{\pi/2} \frac{\sin x\ \cancel{x}}{1+\cos^2 x} = \frac{\pi}{4}$

3

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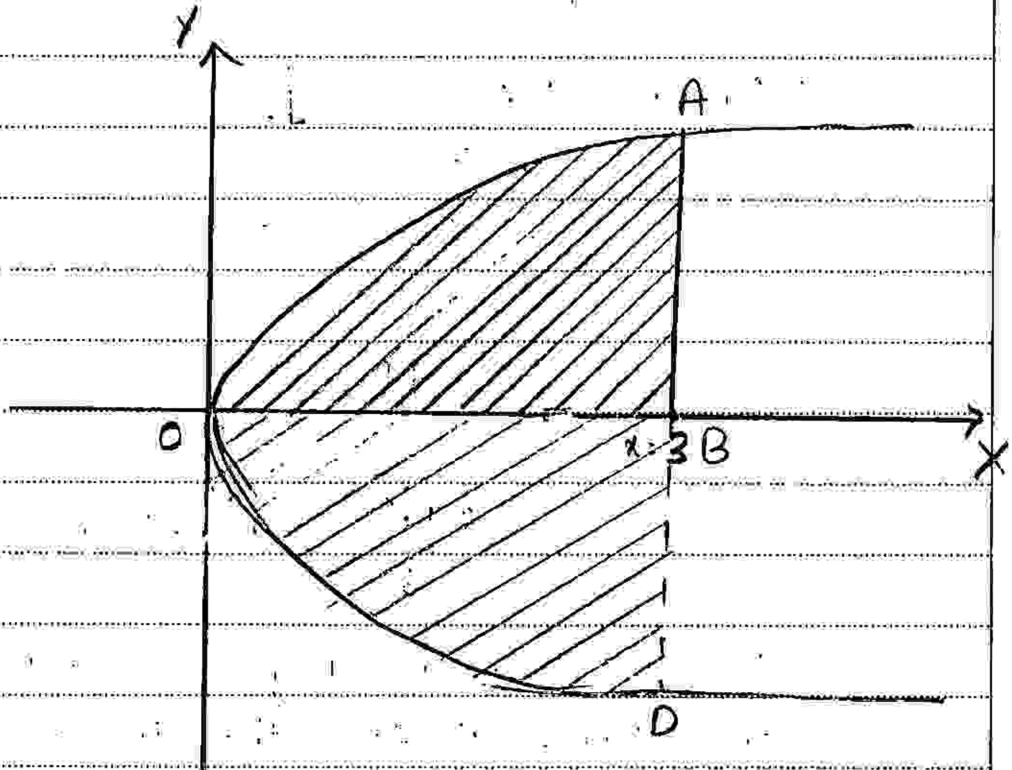
33)



The area of the region bounded by the curve $y^2 = 4x$ & the line $x = 3$ is given by

Area OABDO = 2 Area (OAB)

$$= 2 \int_0^3 y \, dx$$

$$= 2 \int_0^3 2\sqrt{x} \, dx$$

$$= 2 \times 2 \left[\frac{x^{1/2+1}}{1/2+1} \right]_0^3$$

= 4

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$\therefore$ Area $= 4 \times \frac{2}{3} x^{3/2} \Big]_0^3$

$= \frac{8}{3} \left(3^{3/2} - 0^{3/2} \right)$

$= \frac{8}{3} \left(3\sqrt{3} - 0 \right)$

$= \frac{24\sqrt{3}}{3}$ sq. units.

3

$\therefore$ The area of the region bounded by the curve $y^2 = 4x$ & the line $x = 3$ is $\frac{24\sqrt{3}}{3}$ sq. units.

39). $y = ae^{3x} + be^{-2x}$.

Diff wrt 'x'.

$\frac{dy}{dx} = a e^{3x}(3) + be^{-2x} \cdot (-2)$

$\frac{dy}{dx} = 3ae^{3x} - 2be^{-2x}$.

$\frac{d^2y}{dx^2} = 3ae^{3x}(3) - 2be^{-2x}(-2)$

0

$= 9ae^{3x} + 4be^{-2x}$.

$= 3(3ae^{3x} - 2be^{-2x}) + 10be^{-2x}$

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$$\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 10be^{-2x}$$

$$= 3\frac{dy}{dx}$$

35). $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 3k.$

$\therefore \vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$

$\cancel{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})} = \frac{(\vec{a}+\vec{b})(\vec{a}-\vec{b})}{\cancel{\sqrt{1^2+1^2+1^2}}\sqrt{1^2+2^2+3^2}}$

$= \frac{-3-8}{\cancel{\sqrt{3}\sqrt{14}}}$

$= \frac{-11}{\cancel{\sqrt{42}}}$

$\cancel{(\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})}$

$$[(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})] = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$$

$= \hat{i}(-6+4) - \hat{j}(-4+0) + \hat{k}(-2+0)$

$= -2\hat{i} + 4\hat{j} - 2\hat{k}$

0

0

36). Let $OA = 9\hat{i} + 8\hat{j} + 12\hat{k}$

$OB = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$OC = 3\hat{i} + 5\hat{j} + 4\hat{k}$

$OD = 5\hat{i} + 8\hat{j} + 5k$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -2\hat{i} - 4\hat{j} - 6\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -\hat{i} - 3\hat{j} - 8\hat{k}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \hat{i} - 7\hat{k}$

consider, $[\overrightarrow{AB}\ \overrightarrow{AC}\ AD]$

$$\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix}$$

$= -2(21-0) + 4(7+8) - 6(0+3)$

$= -42 + 60 - 18$

$= -60 + 60$

3

$= 0 \quad \therefore [AB\ AC\ AD] = 0$

Hence, the given points are collinear.

3

ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರು ಕಡ್ಡಾಯವಾಗಿ ಭರ್ತಿ ಮಾಡಿ ಸಹಿ ಮಾಡುವುದು.

ವಿದ್ಯಾರ್ಥಿ ನೋಂದಣಿ ಸಂಖ್ಯೆ | 7 | 7 | 7 | 9 | 3 | 7 |

ವಿದ್ಯಾರ್ಥಿ/ನಿ ಪಡೆದಿರುವ ಒಟ್ಟು ಪುಟಗಳ ಸಂಖ್ಯೆ	ಮುಖ್ಯ ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಬರೆದಿರುವ ಒಟ್ಟು ಪುಟಗಳ ಸಂಖ್ಯೆ	ಹೆಚ್ಚುವರಿ ಹಾಳೆಯಲ್ಲಿ ಬರೆದಿರುವ ಒಟ್ಟು ಪುಟಗಳ ಸಂಖ್ಯೆ	ಬರೆದಿರುವ ಒಟ್ಟು ಪುಟಗಳ ಸಂಖ್ಯೆ	ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರ ಸಹಿ
22	22	16	38+1 graph	Kundaraj

Answer Sheet No. 2518813

Reg. No. 777937

Main Answer Book Sl. No. 2068700.

Kundaragi 07-03-2019
Signature of the Invigilator with date

PART - D

39). $f : N \to Y$. $f(x) = 4x+3$.

$Y = \{y \in N : y = 4x+3, x \in N\}$.

Let $x_1, x_2 \in N$ such that $f(x_1) = f(x_2)$

$\therefore 4x_1 + \cancel{3} = 4x_2 + \cancel{3}$.

$\therefore 4x_1 = 4x_2$

$\therefore x_1 = x_2 \quad \forall x_1, x_2 \in N$.

$\therefore f$ is one - one

Let $f(x) = y$ ~~such~~ that $y \in Y$.

then, $4x+3 = y$.

$4x = y-3$

$x = \dfrac{y-3}{4} \in N \quad \forall y \in Y$.

corresponding to every $y \in Y$ there exists $\dfrac{y-3}{4} \in X$, such that $f\left(\dfrac{y-3}{4}\right) = y$.

$\therefore f$ is on-to

Hence, it is a bijective function, and is invertible.

Thus, f^{-1} exists.

<u>To find f^{-1}:</u>

We have,

$$f(x) = y$$

$$x = f^{-1}(y)$$

$$\therefore f^{-1}(y) = \frac{y-3}{4} \text{ or }$$

$$f^{-1}(x) = \frac{x-3}{4}.$$

40). $A =$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}.$$

<u>To prove:</u> $A^3 - 23A - 40I = O$.

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 19+12+32 & 38-8+16 & 57+4+8 \\ 1+36+32 & 2-24+16 & 3+12+8 \\ 14+18+60 & 28-12+30 & 42+6+15 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$\text{Now, } 23A = 23 \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix}$$

$$40I = 40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Consider, LHS: $A^3 - 23A - 40I$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46-0 & 69-69+0 \\ 69-69-0 & -6+46-40 & 23-23-0 \\ 92-92-0 & 46-46-0 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= O$

$=$ RHS

$\therefore$ LHS $=$ RHS

i.e. $A^3 - 23A - 40I = O$.

41). The given system of linear equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & +2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$|A| = 3(2-3) + 2(4+4) + 3(-6-4)$$
$$= -3 + 16 - 30$$
$$= -17 \neq 0.$$

System of eq^n has unique sol^n.

We have, $A^{-1} = \dfrac{\text{adj} A}{|A|}$.

To find adj A.

$A_{11} = +(2-3) = -1$

$A_{12} = -(4+4) = -8$

$A_{13} = +(-6-4) = -10$

$A_{21} = -(-4+9) = -5$

$A_{22} = +(6-12) = -6$

$A_{23} = -(-9+8) = 1$

$A_{31} = +(2-3) = -1$

$A_{32} = -(-3-6) = 9$

$A_{33} = +(3+4) = 7$

$$\therefore \operatorname{adj} A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

We have, $AX = B$

$$\therefore X = A^{-1}B.$$

$$X = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= -\frac{1}{17}\begin{bmatrix} -8-5-4 \\ -64-6+36 \\ -80+1+28 \end{bmatrix}$$

$$= -\frac{1}{17}\begin{bmatrix} -17 \\ -34 \\ -54 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \underline{x = 1}, \ \underline{y = 2}, \ \underline{z = 3}$$

5

5

42) $y = \sin^{-1} x$.

Differenciate wrt 'x'.

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = 1.$$

Squaring on both sides,

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 1.$$

Differenciate again wrt 'x'.

$$\therefore (1-x^2)\, 2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 (-2x) = 0.$$

$$2(1-x^2)\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx}\right) - 2\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)x = 0$$

~~take $2\frac{dy}{dx}$ to both sides~~

$$2\frac{dy}{dx}\left[(1-x^2)\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)\right] = 0.$$

5

5

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right) = 0$$

Hence, proved.

93) Let x be the length of rectangle &
y be width of rectangle.

Given: $\frac{dx}{dt} = -3$ cm/min.

$\frac{dy}{dt} = 2$ cm/min.

$x = 10$ cm, $y = 6$ cm.

i). Rate of change of perimeter = ?

$$P = 2(x+y)$$

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$= 2(-3+2)$$

$$= 2(-1)$$

$$= -2 \text{ cm/min}$$

4

ii) Rate of change of area = ?

$$A = xy$$

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$= 10(2) + 6(-3)$$

$$= 20 - 18$$

$$= 2 \text{ cm}^2/\text{min}$$

4

$\therefore$ Rate of change of perimeter is -2 cm/min &
area is 2 cm^2/min.

94) $\int \frac{1}{x^2-a^2}\,dx = ?$

Consider, $\frac{1}{x^2-a^2} = \frac{1}{(x+a)(x-a)}$

$= \frac{1}{2a} \frac{2a}{(x+a)(x-a)}$

$= \frac{1}{2a} \frac{a+a+x-x}{(x+a)(x-a)}$

$= \frac{1}{2a}\left[\frac{(x+a)-(x-a)}{(x+a)(x-a)}\right]$

$= \frac{1}{2a}\left[\frac{\cancel{x+a}}{(\cancel{x+a})(x-a)} - \frac{(\cancel{x-a})}{(x+a)(\cancel{x-a})}\right]$

$= \frac{1}{2a}\left[\frac{1}{x-a} - \frac{1}{x+a}\right]$

Now, $\int \frac{1}{x^2-a^2}\,dx = \int \frac{1}{2a}\left[\frac{1}{x-a} - \frac{1}{x+a}\right] dx$

P.T.O

$$= \frac{1}{2a} \int \frac{1}{x-a} dx - \int \frac{1}{x+a} dx.$$

$$= \frac{1}{2a} \left[\log |x-a| - \log |x+a| \right] + C$$

$$= \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$\therefore \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

i) $\int \frac{1}{x^2-16} dx = \int \frac{1}{(x)^2-(4)^2} dx$

$$= \frac{1}{2(4)} \log \left| \frac{x-4}{x+4} \right| + C$$

$$= \frac{1}{8} \log \left| \frac{x-4}{x+4} \right| + C$$

5

4

46). $\frac{dy}{dx} + (\sec x)y = \tan x$.

Here, $P = \sec x$ $Q = \tan x$.

Integrating factor, $IF = e^{\int p\,dx}$

$= e^{\int \sec x\,dx}$

$= e^{\log|\sec x + \tan x|}$

$= \sec x + \tan x$.

Now, General sol^n of D.E is -

$y\,IF = \int Q\,IF\,dx + C$.

$y(\sec x + \tan x) = \int \tan x(\sec x + \tan x)\,dx + C$

$= \int(\sec x \tan x + \tan^2 x)\,dx + C$

$= \int \sec x \tan x\,dx + \int \tan^2 x\,dx + C$

$= \sec x + \int(\sec^2 x - 1)\,dx + C$

$y(\sec x + \tan x) = \sec x + \tan x - x + C$.

$\therefore y(\sec x + \tan x) = \sec x + \tan x - x + C$

5

50).

b).

$$f(x) = \begin{cases} kx+1 & \text{if } x \le 5 \\ 3x-5. & \text{if } x > 5. \end{cases}$$

Since, the given function is continuous at $x=5$.

$\therefore$ LHL = RHL = f(5).

$f(5) = 5k+1$.

$$\lim_{x\to 5^-}(3x-5) = \lim_{x\to 5^+}(5k+1).$$

$\Rightarrow \quad 15-5 = 5k+1$

$5k+1 = 10$

$5k = 10-1$

$5k = 9$

$$\boxed{k = \frac{9}{5}}$$

$\therefore$ The value of k is $\frac{9}{5}$//.

4

50). a). $Z = 5x + 10y$.

$x + 2y \leq 120$ —— (1).

$x + y \geq 60$ —— (2).

$x - 2y \geq 0$ —— (3).

From (1), $x + 2y = 120$.

put, $x = 0$. $y = 60$ A(0,60)

$x = 120$ $y = 0$ B(120,0)

Test pt at (0,0)

$0 + 0 \leq 120$.

Turue, sol^n. region contains origin.

From (2), $x + y = 60$

put $x = 0$ $y = 60$ C(0,60)

$x = 60$ $y = 0$ D(60,0)

Test pt. at 0,0.

$0 \geq 60$

False, It doesn't contain origin.

35) $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$

$$\left[(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})\right] = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = \hat{i}(-6+4) - \hat{j}(-4 \ldots + \hat{k}(-2 \ldots$$

$$= -2\hat{i} + 4\hat{j} - \ldots$$

$$|(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})| = \sqrt{2^2 + 4^2 + 2^2}$$

$$= \sqrt{24}.$$

$$\widehat{(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})} = \frac{(\vec{a}+\vec{b})(\vec{a}-\vec{b})}{\sqrt{24}}$$

$$= \frac{-2\hat{i} + 4\hat{j} - 2\hat{k}}{\sqrt{24}}.$$

3

March, 2019

Subject Name : **MATHEMATICS** *Subject Code :* **35 (NS)**

Register No. of the candidate 777937.

Invigilator's Signature 04-03-2019

Scale: x-axis 2 cm = 20 units
y axis 2 cm = 20 units