Chirag
102103335
3COE-12

DATE: / 20
PAGE No

Q1.

To find Maximum Likelihood Estimates of the parameters θ_1 (mean) and θ_2 (variance) for a normal distribution, we will use likelihood function and then maximize it.

Given that $X_1, X_2, ..., X_n$ is a random sample from a normal distribution with mean θ_1 and variance θ_2 the likelihood function is:

$$L(\theta_1, \theta_2 | x_1, x_2, ..., x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Taking log on both sides:

$$\ln L(\theta_1, \theta_2 | x_1, x_2, ..., x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

To find MLE, we'll differentiate the log-likelihood with respect to θ_1 and θ_2, set derivative equal to zero

(i) For θ_1:

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2 | x_1, x_2, ..., x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

Setting this equal to zero:

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0 \Rightarrow \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0$$

$$\therefore \hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

So, the MLE for θ_1 is the sample mean.

(ii) For θ_2:

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 | x_1, x_2, \ldots, x_n) = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Setting this equal to zero:

$$-\frac{n}{2\hat{\theta}_2} + \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2 = 0$$

$$\Rightarrow \frac{n}{2\hat{\theta}_2} - \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

$$\Rightarrow \hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

So, MLE for θ_2 is the sample variance.

DATE: / 20
PAGE No

Q2

To find the MLE of θ for a random sample $X_1, X_2, \ldots X_n$ from a Bernoulli distribution with parameter θ and a known m. The likelihood for this scenario is:

$$L(\theta | x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(X_i = x_i | \theta)$$

Since X_i follows a Bernoulli distribution, $P(X_i = x_i | \theta)$
$= \theta^{x_i}(1-\theta)^{m-x_i}$ for each i.

Taking the log on both sides:

$$\ln L(\theta | x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \ln(\theta^{x_i}(1-\theta)^{m-x_i})$$

$$= \sum_{i=1}^{n} (x_i \ln\theta + (m-x_i)\ln(1-\theta))$$

Now differentiate with respect to θ and set to zero

$$\frac{d}{d\theta}(\ln L(\theta | x_1, x_2, \ldots, x_n)) = 0$$

$$\sum_{i=1}^{n}\left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} = \frac{n \cdot m - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\therefore \theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}$$

So, max likelihood estimate for θ is:

$$\hat{\theta}_{MLE} = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}$$