# Data Science, 2022 Tut 6: Machine Learning 1

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**BE COMPS**

1. [Probability] Assume that the probability of obtaining heads when tossing a coin is $\lambda$.

a. What is the probability of obtaining the first head at the $(k + 1)$-th toss?

b. What is the expected number of tosses needed to get the first head?

# Tutorial - 6

1) a) $P(H) = \lambda$

$P(T) = 1 - \lambda$

$P(H \text{ at } k{+}1\text{th toss}) = P(T \text{ at } k \text{ toss and } H \text{ at } k{+}1\text{th toss})$

$$= (1-\lambda)^k \lambda$$

b) Let $n$ be the no. of tosses required to get the first head & $S = E[n]$

As tosses are independent and expectation is additive

$$S = \lambda \times 1 + (1-\lambda)(S+1)$$

$$= \lambda + S + 1 - \lambda S - \lambda$$

$$\therefore S\lambda = 1$$

$$\therefore S = \frac{1}{\lambda}$$

2. [Probability] Assume X is a random variable.

a. We define the variance of X as: $Var(X) = E[(X - E[X])^2]$. Prove that $Var(X) = E[X^2] - E[X]^2$.

b. If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X? If $Y = a + bX$, what is the variance of Y ?

2) $X \rightarrow$ random variable

a) Variance of $X$ : $var(X) = E[(X - E[X])^2]$

TPT :- $Var(X) = E[X^2] - E[X]^2$

Given :- $var(X) = E[(X - E[X])^2]$

$$= E[(x^2 - 2E[x]x + E[x]^2)]$$

$$= E[x^2] - 2E[E[x]x] + E[x]^2$$

$$= E[x^2] - 2E[x]^2 + E[x]^2$$

$$= E[x^2] - E[x]^2$$

– Hence Proved.

b) $E[x] = 0$ & $E[x^2] = 1$

To find :- ① $var(x)$
② if $y = a + bx$, $var(y)$

① $var(x) = E[x^2] - E[x]^2$

$$= 1 - 0^2 = \underline{\underline{1}}$$

② $E[Y] = E[ax + b] = E[x]b + a = 0b + a = \underline{\underline{a}}$
$E[Y^2] = E[a^2x^2 + b^2 + 2axb] = \underline{\underline{a^2 + b^2}}$

$\therefore var(y) = E[Y^2] - E[Y]^2 = a^2 + b^2 - a^2 = \underline{\underline{b^2}}$

3. [Probability] Your friend Aku is a great predictor about winning a horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99.
2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999.
3) With probability $10^{-5}$, Aku predicts that a black beauty is a winning horse. This also means with probability $1 - 10^{-5}$, Aku predicts that a black beauty will not win. a. Given a horse, what is the probability that it wins?

b. What is the probability that Aku correctly predicts a black beauty is winning ?

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3) Let A be the event that "Aku predicts that the given horse is winning horse".

¬A vice versa

Let B be the event that given horse wins & ¬B vice versa.

a) $P(B) = P(B, A) + P(B, \neg A)$

$= P(B|A) P(A) + P(B|\neg A) P(\neg A)$

$= 0.99 \times 10^{-5} + (1 - 0.99999) \times (1 - 10^{-5})$

$\therefore P(B) \approx 1.99 \times 10^{-5}$

b) $P(A|B) = \frac{P(A, B)}{P(B)} = \frac{P(A|B)(P(A)}{P(B)}$

$= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$

$= 0.497$