Data Science, 2022 Tut 5: Evaluation and Measurement- Hypothesis Testing

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BE COMPS

Make Assumptions about values when it is necessary in a consistent manner. Refer to the necessary table from the following link when necessary.

https://www.sheffield.ac.uk/polopoly_fs/1.43999!/file/tutorial-10-reading-tables.pdf Testing a Proportion of small samples

1. H_0: $p = p_0$

2. One of the alternatives H\: $p < p_{n,}$ $p > p_{o,}$ or $p \neq p_o$

3. Choose a level of significance equal to a.

4. Test statistic: Binomial variable X with $p = p_0$.

5. Computations: Find x, the number of successes, and compute the appropriate P-value.

6. Decision: Draw appropriate conclusions based on the P-value

Ex. 1

A builder claims that air-conditions are installed in 70% of all homes being constructed today in the city of Mumbai. Would you agree with this claim if a random survey of new homes in this city shows that 8 out of 15 had air-conditions installed? Use a 0.10 level of significance

Tutorial - 5

1) $H_0 : p = 0.7$ $\qquad$ $H_1 : p \neq 0.7$

Significance level $= \alpha = 0.1$

Test statistic :-

Binomial variable x with $p = 0.7$ & $n = 15$

$x = 8$ & $np_0 = 15 \times 0.7 = 10.5$

$\therefore P = 2P(X \leq 8 \text{ when } p = 0.7)$

$= 2 \sum_{x=0}^{8} b(x, 15, 0.7)$

$= 2 \times 0.1311$ (value taken from binomial prob. table)

$= \underline{\underline{0.2622}}$

$\therefore P > 0.1$ i.e $P > \alpha$

Therefore, we do not reject H_0.
Since, there is insufficient reason to doubt the builder's claim.

Ex.2

A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.

2) $H_0: p = 0.6$
$H_1: p > 0.6$

Significance level $= \alpha = 0.05$

$\because x = 70,\ n = 100,\ P_0 = 0.6$

$$\therefore Z = \frac{x - nP_0}{\sqrt{nP_0q_0}}$$

$$= \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}}$$

$$= \frac{10}{\sqrt{24}}$$

$$= \underline{\underline{2.04}}$$

$P = P(Z > 2.04)$

$P = 0.0207$ (from the table)

$\therefore P < \alpha$

We reject H_0 and conclude that new drug is superior.

Ex.3

A vote is to be taken among the residents of Mumbai and the surrounding area to determine whether a proposed Nuclear plant should be constructed. The construction site is within the Mumbai limits, and for this reason many voters in the surrounding area feel that the proposal will pass because of the large proportion of Mumbai voters who favor the construction. To determine if there is a significant difference in the proportion of Mumbai voters and surrounding area voters favoring the proposal, a poll is taken. If 120 of 200 Mumbai voters favor the proposal and 240 of 500 surrounding area residents favor it, would you agree that the proportion of Mumbai voters favoring the proposal is higher than the proportion of surrounding area voters? Use an a = 0.05 level of significance.

3) Let:-

P_1 – proportion of Mumbai voters.
P_2 – surrounding area residentials proportion.

$\alpha = 5\% = 0.05$

$\hat{P}_1 = \frac{120}{200} = 0.6$

$\hat{P}_2 = \frac{240}{300} = 0.48$

$\hat{P}_P = \frac{120+240}{200+300} = 0.514$

Hypothesis

$H_0: P_1 \le P_0$

$H_1: P_1 > P_2$

$$z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_P(1-\hat{P}_P)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.6-0.48}{\sqrt{0.514 \times 0.486 \times \left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$\therefore z = 2.869$

$P = P(z > 2.869) = 0.0044$

$\because P < \alpha$

Rejecting H_0 and concluding that proportion of Mumbai voters favouring proposal is higher.

Ex.4

State the null and alternative hypotheses to be used in testing the following claims, and determine generally where the critical region is located:

(a) At most, 20% of next year's wheat crop will be exported to Russia..

(b) On the average, Indian homemakers drink 3 cups of tea per day.

(c) The proportion of graduates in engineering this year majoring in the computer sciences is at least. 0.15.

(d) The average donation to the Indian Autism Association is no more than 500 INR.

(e) Residents in suburban Mumbai commute, on the average, 15 kilometers to their place of employment.

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4)

a) Null Hypothesis

$H_0: p = 0.20$

Alternative Hypothesis

$H_1: p > 0.20$

The critical region is in right tail.

b) Null Hypothesis

$H_0: \mu = 3$

Alternative Hypothesis

$H_1: \mu \neq 3$

The critical region is in both tails.

c) Null Hypothesis

$H_0: p = 0.15$

Alternative Hypothesis

$H_1: p < 0.15$

The critical region is in the left tail.

d) Null Hypothesis

$H_0: \mu = 500$

Alternative Hypothesis

$H_1: \mu > 500$

The critical region is in right tail.

e) Null Hypothesis

$H_0: \mu = 15$

Alternative Hypothesis

$H_1: \mu \neq 15$

The critical region is in both tails.

Ex.5

In a study conducted by the Department of computer Engineering and analyzed by the Statistics Consulting Center at SPIT the laptops supplied by two different companies were compared. Ten sample laptops were made out of the Intel chips supplied by each company and the "robustness" was studied. The data are as follows:

Company A: 9.3 8.8 6.8, 8.7 8.5 6.7 8.0 6.5 9.2 7.0

Company B: 11.0 9.8 9.9 10.2,10.1 9.7 11.0 11.1 10.2 9.6

Can you conclude that there is virtually no difference in means between the laptops supplied by the two companies? Use a P-value to reach your conclusion. Should variances be pooled here?

5) Let μ_1 & μ_2 be population mean robustness of laptops supplied by company A & company B respectively.

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Significance level = $\alpha = 0.05$

$$\bar{x}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1i}$$

$$= \frac{9.3+8.8+6.8+8.7+8.5+6.7+8+6.5+9.2+7}{10}$$

$$= \underline{\underline{7.95}}$$

$$\bar{x}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} x_{2i}$$

$$= \frac{11+9.8+9.9+10.2+10.1+9.7+11+11.1+10.2+9.6}{10}$$

$$\therefore \bar{x}_2 = \underline{\underline{10.26}}$$

$$S_1^2 = \frac{1}{n_1-1}\left[\sum_{i=1}^{n_1} x_{1i}^2 - n_1 \bar{x}_1^2\right] = \frac{10.65}{9} = \underline{\underline{1.207}}$$

Similarly, $S_2^2 = \underline{\underline{0.325}}$

Since sample variances come different we cannot assume the population variances are equal.
Hence, we will use unpooled t-test.

The degrees of freedom for this test are calculated as:-

$$V = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{S_2^2}{n_2}\right)^2}$$

$$= 10.3$$

The test statistic used to test the hypothesis is

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

this under the null hypothesis follows approximate t-distribution with $V=10$ degrees of freedom

Also, under null hypothesis $\mu_1 - \mu_2 = 0$

$$\therefore \; T = \frac{7.93 - 10.26}{\sqrt{\frac{12.0\cancel{7}}{10} + \frac{0.325}{10}}} = -5.9$$

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$\because$ this test is two sided, the value of test is doubled area under the density curve of t-distribution with 10 degrees of freedom, right of the absolute value to test statistic.

$$|t| = |-5.90| = \underline{\underline{5.9}}$$

$$P = 2 \cdot P(T \geq |t|) = 2 \cdot P(T \geq 5.9)$$

$$t_{0.0005}(10) = 4.587$$

$$\therefore |t| = 5.9 > P(T \geq 5.9) < 0.0005. \text{ So}$$

$$\underline{\underline{P < 0.001}}$$

$\therefore P < \alpha$

Null Hypothesis rejected

We conclude that mean robustness of laptops is not the same for both companies.