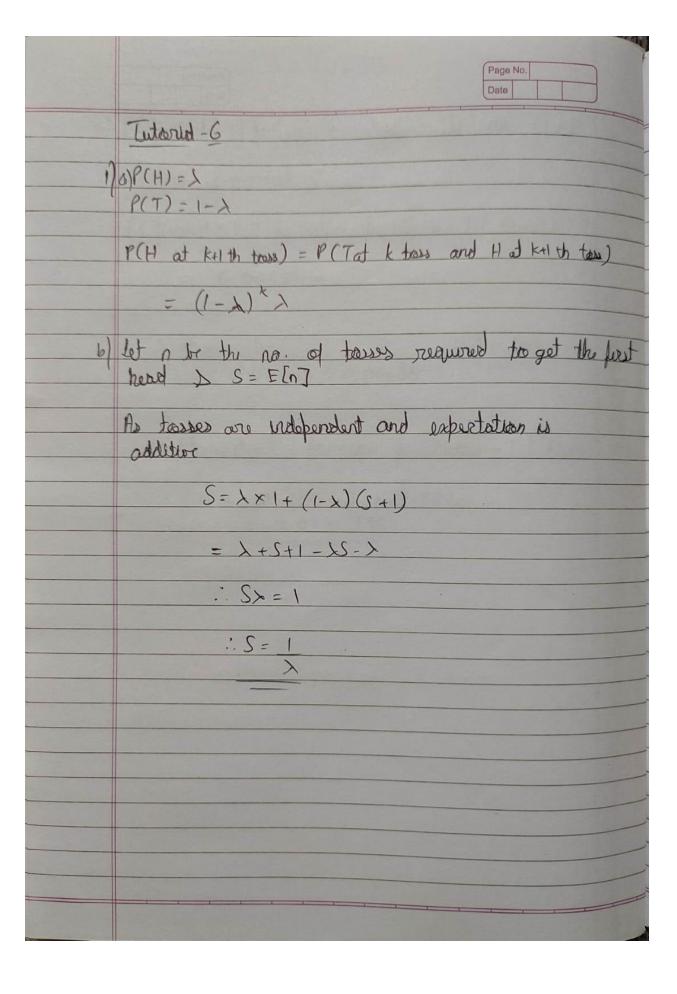
Data Science, 2022 Tut 6: Machine Learning 1

Name: Chirag Rana UID: 2018130043 BE COMPS

- 1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ .
- a. What is the probability of obtaining the first head at the (k + 1)-th toss?
- b. What is the expected number of tosses needed to get the first head?



- 2. [Probability] Assume X is a random variable.
- a. We define the variance of X as: $Var(X) = E[(X E[X])^2]$. Prove that $Var(X) = E[X^2] E[X]^2$.
- b. If E[X] = 0 and $E[X^2] = 1$, what is the variance of X? If Y = a + bX, what is the variance of Y?

2) X -> random variable a) laruanze of x: wor (x) = E[(x-E[x])2] TPT: - Uan (x) = E[x2] - E[x]2 fron: - (DUT (X) = E[(X-E[7])] $= E[(x^2 - 2EDxJx + EDxJ^2)]$ = E[x2] - 2 E[E[x]x] + E[x]2 $= E[x^2] - 2E[x]^2 + E[x]^2$ = E[x2] - ELGZ b Ebi] = 0 & Ebz2] = 1 To find: 1 var(x)

(2) If y=a+bx, var(y) 1) var(x) = E(x2) - E(x]2 $= 1 - 0^2 = 1$ (2) E[Y] = E[ax+b] = E[x]b+a = 0b+a = a $E[Y^2] = E[a^2x^2+b^2+2axb] = a^2+b^2$ $var(y) = E[y^2] - E[y]^2 = o^2 + b^2 - a^2 = b^2$

- 3. [Probability] Your friend Aku is a great predictor about winning a horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99.
- 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999.
- 3) With probability 10^{-5} , Aku predicts that a black beauty is a winning horse. This also means with probability $1-10^{-5}$, Aku predicts that a black beauty will not win. a. Given a horse, what is the probability that it wins?
- b. What is the probability that Aku correctly predicts a black beauty is winning?

	Page No. Date
3)	Let A be the swort that "Aku predicts that the
	nA oure versa
	Let B In the event that goin house wins & MB vice vorsa.
al	$f(B) = p(B,A) + p(B, \neg A)$
	= P(BIA) P(A) + P(BI-A) P(-A)
	$= 0.99 \times 10^{-5} + (1 - 0.99999) \times (1 - 10^{-5})$
	:,PB) = 1.99×10-5
	- Managail , should be seen a see
b/	P(AIB) = P(AB) (P(A)
	P(B) P(B)
	- 0.99 × 10-5 1.99 × 10-5
	1.14 × 10-3
	= 0.497
	0 = 01-d0 = p+d[A] = Fd+yb+q = Obite = c
	"4+ 90 Cdx 05 (3- 1/3)] - [5/7]