

Name: Chirag Rana
Class: SE Comps
Batch: C
UID: 2018130043

EXPERIMENT – 5

AIM: To perform experiments based on Knapsack Problem (C, B) using a greedy strategy.

THEORY:

The Greedy algorithm could be understood very well with a well-known problem referred to as Knapsack problem. Although the same problem could be solved by employing other algorithmic approaches, Greedy approach solves Fractional Knapsack problem reasonably in a good time. Let us discuss the Knapsack problem in detail.

Knapsack Problem

Given a set of items, each with a weight and a value, determine a subset of items to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.

The knapsack problem is in combinatorial optimization problem. It appears as a subproblem in many, more complex mathematical models of real-world problems. One general approach to difficult problems is to identify the most restrictive constraint, ignore the others, solve a knapsack problem, and somehow adjust the solution to satisfy the ignored constraints.

Applications

In many cases of resource allocation along with some constraint, the problem can be derived in a similar way of Knapsack problem. Following is a set of example.

- Finding the least wasteful way to cut raw materials
- portfolio optimization
- Cutting stock problems

Problem Scenario

A thief is robbing a store and can carry a maximal weight of W into his knapsack. There are n items available in the store and weight of i^{th} item is w_i and its profit is p_i . What items should the thief take?

In this context, the items should be selected in such a way that the thief will carry those items for which he will gain maximum profit. Hence, the objective of the thief is to maximize the profit.

Based on the nature of the items, Knapsack problems are categorized as

- Fractional Knapsack

- Knapsack

Fractional Knapsack

In this case, items can be broken into smaller pieces, hence the thief can select fractions of items.

According to the problem statement, □ There are n items

in the store

- Weight of i^{th} item $w_i > 0$.
- Profit for i^{th} item $p_i > 0$ and
- Capacity of the Knapsack is W

In this version of Knapsack problem, items can be broken into smaller pieces. So, the thief may take only a fraction x_i of i^{th} item.

$$0 \leq x_i \leq 1$$

The i^{th} item contributes the weight $x_i \cdot w_i$ to the total weight in the knapsack and profit $x_i \cdot p_i$ to the total profit. Hence, the objective of this algorithm is to

$$\text{maximize } \sum_{i=1}^n (x_i \cdot p_i)$$

subject to constraint,

$$\sum_{i=1}^n (x_i \cdot w_i) \leq W$$

It is clear that an optimal solution must fill the knapsack exactly, otherwise we could add a fraction of one of the remaining items and increase the overall profit.

Thus, an optimal solution can be obtained by:

$$\sum_{i=1}^n (x_i \cdot w_i) = W \quad \sum_{i=1}^n (x_i \cdot p_i) = \text{max}$$

In this context, first we need to sort those items according to the value of p_i/w_i , so that $p_{i+1}/w_{i+1} \leq p_i/w_i$.

ALGORITHM:

Fractional Knapsack (Array Wt, Array profit, int knapsack_capacity)

1. for $i \leftarrow 1$ to size (V)
2. calculate $\text{cost}[i] \leftarrow \text{profit}[i] / \text{Wt}[i]$
3. Sort-Descending (cost)
4. $i \leftarrow 0$
5. while ($i \leq \text{size}(\text{profit}) - 1$)
6. if $\text{Wt}[i] \leq \text{knapsack_capacity}$
7. $\text{knapsack_capacity} \leftarrow \text{knapsack_capacity} - \text{Wt}[i]$
8. $\text{total_profit} \leftarrow \text{total_profit} + \text{profit}[i];$

9. $i \leftarrow i+1$
10. else if $Wt[i] > knapsack_capacity$
11. $total_profittotal_ \leftarrow profit + knapsack_cap * cost[i];$
12. $i \leftarrow i+1$

TIME COMPLEXITY:

Time Complexity of Knapsack:

* The method uses 2 important steps sorting & finding the required elements.

1) Sorting the Array of Profit/wt \Rightarrow I have used Quick sort Algo but Heapsort is also a good preference.

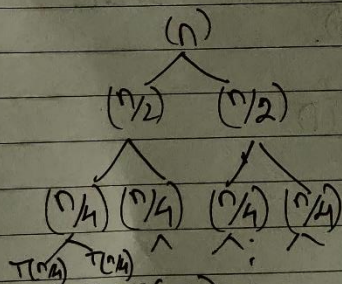
$$\therefore T(n) = O(n \log n) \Rightarrow \text{Best Case / Avg case.}$$

$$= O(n^2) \Rightarrow \text{Worst Case.}$$

2) Find the right Amount of profit i.e. Total profit i.e. $O(n)$.

$$\text{i.e. } \sum_{i=1}^n (1 \times C) \Rightarrow Cn = O(n).$$

Quick Sort Time Complexity:-



$$\text{i.e. } T(n) = 2T(n/2) + cn$$

$$\therefore T(n) = 2[2T(n/4) + cn/2] + cn$$

$$\Rightarrow 4T(n/4) + 2cn + cn$$

$$= 8T(n/8) + 3cn$$

\vdots

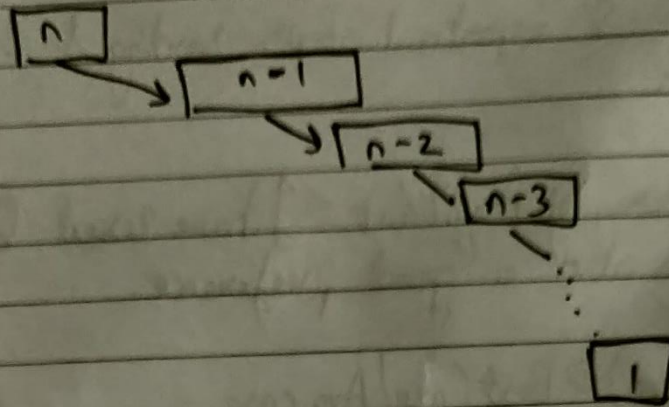
$$\Rightarrow 2^k T(n/2^k) + kcn$$

i.e. At level where $n = 2^k \Rightarrow k = \log(n)$

$$\therefore T(n) = n \cdot T(1) + cn \cdot \log(n)$$

$$\text{i.e. } T(n) = O(n \log n) \Rightarrow \text{for Avg / Best Case.}$$

for Worst Case (A.P.A \Rightarrow Sorted Array)



$$\therefore T(n) = T(n-1) + Cn$$

$$\therefore T(n-1) = T(n-2) + C(n-1)$$

\vdots

$$T(2) = T(1) + C(2) \dots$$

$$\therefore T(n) = Cn + C(n-1) + C(n-2) + C(n-3) \dots + C(2) + T(1)$$

$$\Rightarrow C[n + (n-1) + (n-2) + (n-3) \dots 3+2+1]$$

$$\Rightarrow C \sum_{i=1}^n (i) + T(1)$$

$$\Rightarrow C \cdot \frac{n(n+1)}{2} + T(1)$$

$$\text{i.e. } O(n^2)$$

DESIGN AND IMPLEMENTATION:

Design & Implementation of Knapsack:-

1) $n \rightarrow 7$ (number of objects)

(Profit, weight) $\Rightarrow [(10, 2), (5, 3), (15, 5), (7, 7), (6, 1), (18, 4), (3, 1)]$

\therefore Profit/weight $\Rightarrow [5, 1.67, 3, 1, 6, 4.5, 3]$

\therefore After sorting the array:-

Profit $\Rightarrow [6, 10, 18, 3, 15, 5, 7]$

weight $\Rightarrow [1, 2, 4, 1, 5, 3, 7]$

Profit/weight $\Rightarrow [6, 5, 4.5, 3, 3, 1.67, 1]$

for Knapsack:- Profit $\Rightarrow [2 \times 1.67 + 3 \times 1 + 3 \times 5 + 4.5 \times 4 + 5 \times 2 + 6 \times 1]$
 $\Rightarrow [55.33]$

CODE:

```
#include<stdio.h>
#include<stdlib.h>
void swap(float* a, float* b)
{
    float t = *a;
    *a = *b;
    *b = t;
}

int partition (float arr[], float arrwe[], float arrpr[], int low, int high)
{
    float pivot = arr[high];
    int i = (low - 1);
```

```

for (int j = low; j <= high- 1; j++)
{
    if (arr[j] < pivot)
    {
        //printf("The pivot: %f",pivot);
        i++;
        swap(&arr[i], &arr[j]);
        swap(&arrwe[i], &arrwe[j]);
        swap(&arrpr[i], &arrpr[j]);
    }
}
swap(&arr[i + 1], &arr[high]);
swap(&arrwe[i + 1], &arrwe[high]);
swap(&arrpr[i + 1], &arrpr[high]);

return (i + 1);
}

void quickSort(float arr[],float arrwe[],float arrpr[], int low, int high)
{
    if (low < high)
    {
        int pi = partition(arr,arrwe,arrpr, low, high);
        quickSort(arr,arrwe,arrpr, low, pi - 1);
        quickSort(arr,arrwe,arrpr, pi + 1, high);
    }
}

void printArray(float arr[], int size)
{
    for (int i=0; i < size; i++)
        printf(" /%.3f/ ", arr[i]);
    printf("\n\n");
}

void submitsolution(float arrra[],float arrwe[],float arrpr[],int sum,int length){
    int temp = 0;

```

```

float finalval = 0;
float* arr = (float*)malloc(length*sizeof(float));
for (int i = length-1;i>=0; i --){
    if(temp < sum){
        if((sum-temp) >= arrwe[i]){
            temp += arrwe[i];
            arr[i] = arrwe[i];
            finalval += arr[i]*arrra[i];
        }else{
            arr[i] = sum-temp;
            finalval += arr[i]*arrra[i];
            temp = sum;
            break;
        }
    }
}
printf("Weight selected array: ");
printArray(arr,length);
printf("The final Answer is: %.3f",finalval);
}
int main()
{
    int n,sum;

    float *arrpr,*arrwe;
    float *arrra;
    printf("Enter the array length:");
    scanf("%d",&n);

    arrpr = (float*)malloc(n*sizeof(float));
    arrwe = (float*)malloc(n*sizeof(float));
    arrra = (float*)malloc(n*sizeof(float));

    printf("Enter the profit and weight value:");

    for (int i = 0; i< n;i++){
        scanf("%f",&arrpr[i]);
        scanf("%f",&arrwe[i]);
    }
}

```

```

        arrra[i] = arrpr[i]/arrwe[i];
    }
    for (int i = 0; i < n; i++) {
        printf("%f ", arrra[i]);
    }
    printf("\nEnter the maximum value of Sum: ");
    scanf("%d", &sum);

    quickSort(arrra, arrwe, arrpr, 0, n-1);
    printf("Sorted profit/wt array: \n");
    printArray(arrra, n);
    printf("Sorted weight array: \n");
    printArray(arrwe, n);
    printf("Sorted profit array: \n");
    printArray(arrpr, n);
    submitsolution(arrra, arrwe, arrpr, sum, n);
    return 0;
}
// 10 2 5 3 15 5 7 7 6 1 18 4 3 1

```

OUTPUT:

```

D:\Submission\DAA\KNapsack\knapsack.exe
Enter the array length:7
Enter the profit and weight value:
10 2
5 3
15 5
7 7
6 1
18 4
3 1
5.000000 1.666667 3.000000 1.000000 6.000000 4.500000 3.000000
Enter the maximum value of Sum: 15
Sorted profit/wt array:
/1.000/ /1.667/ /3.000/ /3.000/ /4.500/ /5.000/ /6.000/

Sorted weight array:
/7.000/ /3.000/ /1.000/ /5.000/ /4.000/ /2.000/ /1.000/

Sorted profit array:
/7.000/ /5.000/ /3.000/ /15.000/ /18.000/ /10.000/ /6.000/

Weight selected array: /0.000/ /2.000/ /1.000/ /5.000/ /4.000/ /2.000/ /1.000/

The final Answer is: 55.333
Process returned 0 (0x0) execution time : 40.622 s
Press any key to continue.

```


CONCLUSION:

1. I implemented Fractional Knapsack Problem with the Quicksort algorithm.
2. I understood how to sort three array parallelly.