Birla Institute of Technology and Science, Pilani-K K Birla Goa Campus

First Semester 2018-2019

Numerical Analysis
(MATH F313)
Assignment - 1

Note: Solve all the problems by implementing the corresponding method using MATLAB software.

1. Mechanical engineers, as well as most engineers use thermodynamics extensively in their work. The following polynomial can be used to relate the zero-pressure specific heat of dry air c_p to temperature

   $$c_p = 0.99403 + 1.671 \times 10^{-4}\, T + 9.7215 \times 10^{-8}\, T^2 - 9.5838 \times 10^{-11}\, T^3 + 1.9520 \times 10^{-14}\, T^4.$$

   Determine the temperature that corresponds to a specific heat of 1.1 using secant method.

2. The upward velocity of a rocket can be computed by the following formula:

   $$v = u \ln \frac{m_0}{m_0 - qt} - gt,$$

   where v = upward velocity, u = the velocity at which fuel is expelled relative to the rocket, m_0 = the initial mass of the rocket at time $t = 0$, q = the fuel consumption rate, and g = the downward acceleration of gravity (assumed constant = $9.81\, m/s^2$). If $u = 2000\, m/s, m_0 = 150000\, kg$, and $q = 2700\, kg/s$. Compute the time at which $v = 750\, m/s$ using bisection method.

3. Water is flowing in a channel at a rate of $Q = 20 m^3/s$. The critical depth y for such a channel must satisfy the equation

   $$1 - \frac{Q^2}{g\, A_c^3} B = 0,$$

   where $g = 9.81\, m/s^2$, A_c = the cross-sectional area (m^2), and B = the width of the channel at the surface (m). For this case, the width and cross-sectional area can be related to depth y by

   $$B = 3 + y, \quad \text{and} \quad A_c = 3y + \frac{y^2}{2}.$$

   Solve for the critical depth using

   - The graphical method
   - Bisection method and
   - Method of false position.

   Use initial interval $[0.5, 2.5]$.

4. Use Newton-Raphson method to determine the mass m of the bungee jumper with a drag coefficient of $c_d = 0.25\, kg/m$ to have a velocity v of $36\, m/s$ after $4s$ of free fall. The acceleration of gravity is $9.81\, m/s^2$. The governing model is

   $$f(m) = \sqrt{\frac{g\, m}{c_d}} \tanh\left(\sqrt{\frac{g\, c_d}{m}} t\right) - v(t).$$

5. The volume of liquid V in a hollow horizontal cylinder of radius r and length L is related to the depth of the liquid h by

$$V = \left[r^2 \cos^{-1}\left(\frac{r-h}{r}\right) - (r-h)\sqrt{2rh-h^2} \right] L.$$

Determine h given $r = 2\,m, L = 5\,m^3$, and $V = 8.5\,m^3$ using regula-falsi method.

6. An oscillating current in an electric circuit is described by

$$I = 9e^{-t}\cos(2\pi t),$$

where t is in seconds. Determine all values of t such that $I = 3$ using secant method.

7. Use the Newton-Raphson method to find the root of

$$f(x) = e^{-0.5x}(4-x) - 2.$$

Take initial guesses of (i) 2, (ii) 6, and (iii) 8. Explain your results.