## Birla Institute of Technology and Science, Pilani-K K Birla Goa Campus First Semester 2018-2019

## Numerical Analysis (MATH F313) Assignment - 1

**Note:** Solve all the problems by implementing the corresponding method using MATLAB software.

1. Mechanical engineers, as well as most engineers use thermodynamics extensively in their work. The following polynomial can be used to relate the zero-pressure specific heat of dry air  $c_p$  to temperature

$$c_p = 0.99403 + 1.671 \times 10^{-4} \, T + 9.7215 \times 10^{-8} \, T^2 - 9.5838 \times 10^{-11} \, T^3 + 1.9520 \times 10^{-14} \, T^4.$$

Determine the temperature that corresponds to a specific heat of 1.1 using secant method.

2. The upward velocity of a rocket can be computed by the following formula:

$$v = u \ln \frac{m_0}{m_0 - qt} - gt,$$

where v = upward velocity, u = the velocity at which fuel is expelled relative to the rocket,  $m_0 =$  the initial mass of the rocket at time t = 0, q = the fuel consumption rate, and g = the downward acceleration of gravity (assumed constant =  $9.81 \, m/s^2$ ). If  $u = 2000 \, m/s$ ,  $m_0 = 150000 \, kg$ , and  $q = 2700 \, kg/s$ . Compute the time at which  $v = 750 \, m/s$  using bisection method.

3. Water is flowing in a channel at a rate of  $Q = 20m^3/s$ . The critical depth y for such a channel must satisfy the equation

$$1 - \frac{Q^2}{g \, A_c^3} B = 0,$$

where  $g = 9.81 \, m/s^2$ ,  $A_c =$  the cross-sectional area  $(m^2)$ , and B = the width of the channel at the surface (m). For this case, the width and cross-sectional area can be related to depth y by

$$B = 3 + y$$
, and  $A_c = 3y + \frac{y^2}{2}$ .

Solve for the critical depth using

- The graphical method
- Bisection method and
- Method of false position.

Use initial interval [0.5, 2.5].

4. Use Newton-Raphson method to determine the mass m of the bungee jumper with a drag coefficient of  $c_d = 0.25 \, kg/m$  to have a velocity v of  $36 \, m/s$  after 4s of free fall. The acceleration of gravity is  $9.81 \, m/s^2$ . The governing model is

$$f(m) = \sqrt{\frac{g m}{c_d}} \tanh\left(\sqrt{\frac{g c_d}{m}}t\right) - v(t).$$

5. The volume of liquid V in a hollow horizontal cylinder of radius r and length L is related to the depth of the liquid h by

$$V = \left[r^2 \cos^{-1} \left(\frac{r-h}{r}\right) - (r-h)\sqrt{2rh - h^2}\right] L.$$

Determine h given  $r=2\,m, L=5\,m^3,$  and  $V=8.5\,m^3$  using regula-falsi method.

6. An oscillating current in an electric circuit is described by

$$I = 9e^{-t}\cos(2\pi t),$$

where t is in seconds. Determine all values of t such that I=3 using secant method.

7. Use the Newton-Raphson method to find the root of

$$f(x) = e^{-0.5x}(4 - x) - 2.$$

Take initial guesses of (i) 2, (ii) 6, and (iii) 8. Explain your results.

- Plotting graph of function
- Secant Method applied with different intervals.

```
clear all;
close all;
```

## Plotting graph of function

```
g = @(x) -1.1+0.99403+(1.671*10^-4)*x+(9.7215*10^-8)*x^2-(9.5838*10^-11)*x^3+(1.9520*10^-1
4)*x^4;

X = -1500:1000; Y = -1500:1000; n = 1;

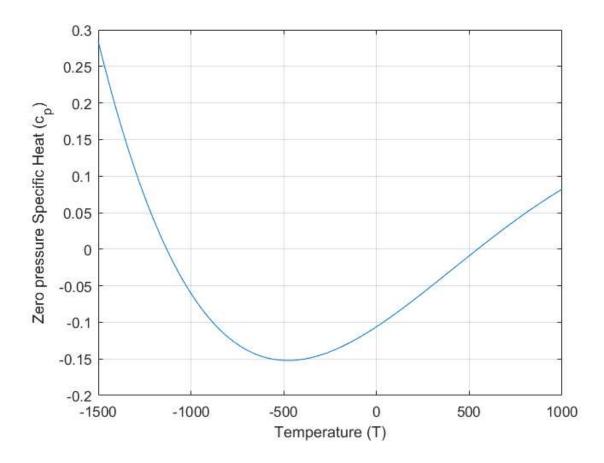
for x = -1500:1000

Y(n) = g(x);

n = n + 1;
end
plot(X,Y); grid on;
xlabel('Temperature (T)');
ylabel('Zero pressure Specific Heat (c_p)');

fprintf("Considering the interval [-1500, -1000]\n"); SecantMethod(-1500, -1000);
fprintf("Considering the interval [400, 600]\n"); SecantMethod(400,600);
```

Considering the interval [-1500, -1000]



## Secant Method applied with different intervals.

```
2
                       -1087.706301249115800
                  3
                       -1138.029509542062000
                       -1130.693760893146000
                  4
                  5
                       -1131.025602676162600
                       -1131.028101492426900
                       -1131.028100596541900
                       -1131.028100596544200
                  8
Considering the interval [400, 600]
                       544.867997840782210
                  2
                  3
                       544.081498954654420
                  4
                       544.087538233961940
                  5
                       544.087537655509440
                       544.087537655508980
```

- Bisection Method for finding zeros of a function
- Function definition
- Plotting function f(t)
- Stopping criterium
- Main loop
- Absolute Error computation

## **Bisection Method for finding zeros of a function**

```
clear all;
close all;
```

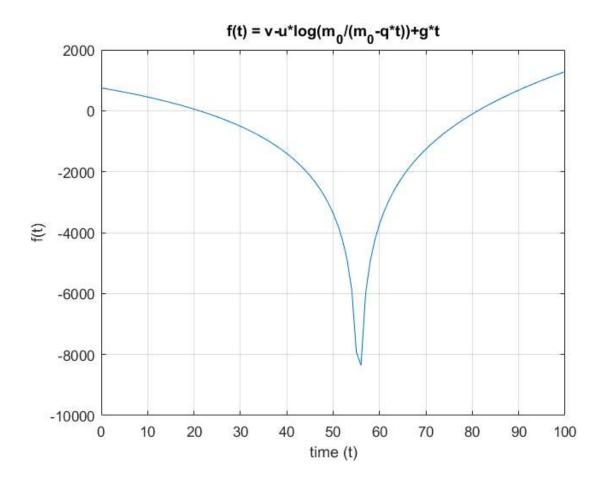
## **Function definition**

```
u = 2000; v = 750; m_0 = 150000;
g = 9.81; q = 2700;
f = @(t) v-u*log(m_0/(m_0-q*t))+g*t;
a=10; b=30;
```

## **Plotting function f(t)**

```
X = 0:100;
Y = 0:100;
n = 1;
for x = 0:100
Y(n) = f(x);
n = n + 1;
end
plot(X,Y); grid on;
xlabel("time (t)");
ylabel("f(t)");
title("f(t) = v-u*log(m_0/(m_0-q*t))+g*t");
```

```
Warning: Imaginary parts of complex X and/or Y arguments ignored
```



## Stopping criterium

```
TOL = 10^(-6);
Nmax = floor ( log((b-a)/TOL) / log(2) ) + 1
pvalues=zeros(Nmax,1);
```

Nmax = 25

## **Main loop**

```
for i = 1 : Nmax
    p = (a+b)/2;
        pvalues(i)=p;
    sfa = f(a);
    sfp = sign(f(p));
    fprintf( '\t\t %3d \t (%.6f,%.6f) \t %.10f \n', i, a, b, p)
        if ( (b-a)<2*TOL || sfp == 0 )
            break
    elseif ( sfa * sfp < 0 )
        b = p;
    else
        a = p;
        sfa = sfp;
    end
end</pre>
```

```
1
       (10.000000,30.000000)
                              20.0000000000
 2
       (20.000000,30.000000)
                              25.0000000000
       (20.000000,25.000000)
 3
                              22.5000000000
       (20.000000,22.500000) 21.2500000000
 4
 5
       (20.000000,21.250000) 20.6250000000
 6
       (20.625000,21.250000)
                              20.9375000000
       (20.937500,21.250000) 21.0937500000
 7
       (21.093750,21.250000) 21.1718750000
 8
 9
       (21.093750,21.171875) 21.1328125000
       (21.093750,21.132813)
                              21.1132812500
10
11
       (21.113281,21.132813) 21.1230468750
12
      (21.123047,21.132813) 21.1279296875
13
       (21.127930,21.132813)
                              21.1303710938
14
       (21.130371,21.132813) 21.1315917969
15
      (21.131592,21.132813) 21.1322021484
16
      (21.132202,21.132813) 21.1325073242
17
       (21.132202,21.132507)
                              21.1323547363
18
      (21.132355,21.132507) 21.1324310303
19
      (21.132355,21.132431) 21.1323928833
20
      (21.132393,21.132431)
                              21.1324119568
21
      (21.132412,21.132431) 21.1324214935
22
      (21.132412,21.132421) 21.1324167252
23
      (21.132412,21.132417) 21.1324143410
24
       (21.132414,21.132417)
                              21.1324155331
2.5
      (21.132414,21.132416) 21.1324149370
```

#### **Absolute Error computation**

```
plast =p;
errors=pvalues-plast*ones(Nmax,1);
fprintf('Approximate value | Absolute Error\n\n')
fprintf(' %.10f | %.10f\n',[pvalues errors]');

figure; plot(errors); grid on;
xlabel("iteration n");
ylabel("|e_{n+1} - e_n|");
title('Absolute error e_n')
```

```
Approximate value | Absolute Error
20.0000000000
                  I = -1.1324149370
25.0000000000
                  1 3.8675850630
 22.5000000000
                    1.3675850630
 21.2500000000
                    0.1175850630
 20.6250000000
                    -0.5074149370
 20.9375000000
                     -0.1949149370
                  21.0937500000
                     -0.0386649370
 21.1718750000
                  0.0394600630
 21.1328125000
                  1 0.0003975630
                     -0.0191336870
 21.1132812500
                  21.1230468750
                    -0.0093680620
                  21.1279296875
                    -0.0044852495
                    -0.0020438433
 21.1303710938
                  21.1315917969
                     -0.0008231401
 21.1322021484
                  -0.0002127886
 21.1325073242
                  | 0.0000923872
 21.1323547363
                  | -0.0000602007
 21.1324310303
                  | 0.0000160933
```

```
      21.1323928833
      | -0.0000220537

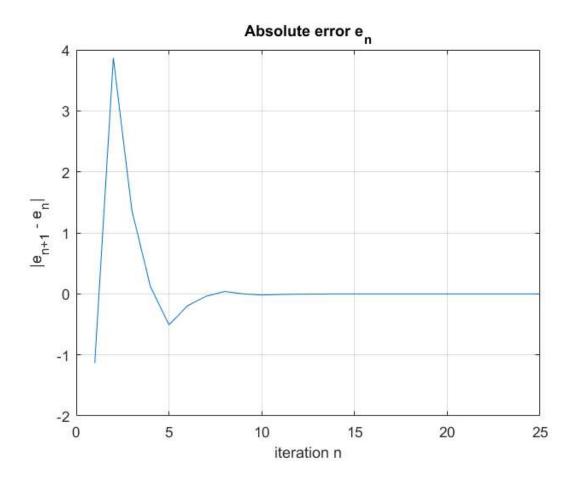
      21.1324119568
      | -0.0000029802

      21.1324214935
      | 0.0000065565

      21.1324167252
      | 0.0000017881

      21.1324143410
      | -0.0000005960

      21.1324149370
      | 0.0000000000
```



- Bisection Method for finding zeros of a function
- Function definition
- Plotting graph of f(y)
- Stopping criterium
- Iteration Scheme
- Absolute Error computation

## **Bisection Method for finding zeros of a function**

```
clear all;
close all;
```

## **Function definition**

```
Q = 20; g = 9.81;

A = @(y) 3*y + y^2/2;

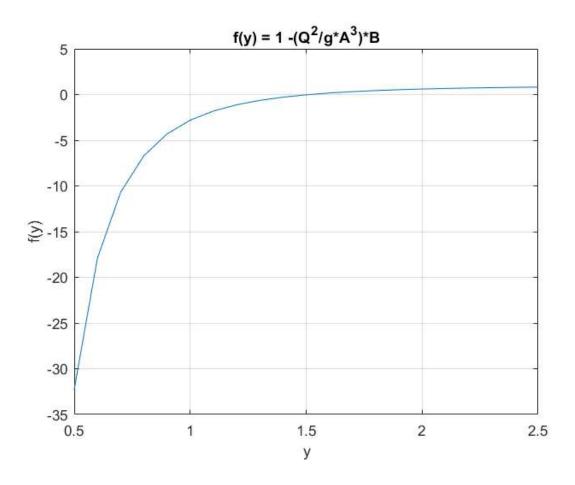
B = @(y) 3 + y;

f = @(y) 1 - (Q^2/(g^*(A(y)^3)))^*B(y);

a=0.5; b=2.5;
```

## Plotting graph of f(y)

```
X = a:0.1:b;
Y = a:0.1:b;
n = 1;
for x = a:0.1:b
Y(n) = f(x);
n = n + 1;
end
plot(X,Y); grid on;
xlabel("y");
ylabel("f(y)");
title("f(y) = 1 -({Q^2}/{g*A^3})*B")
```



## Stopping criterium

```
TOL = 10^(-6);

Nmax = floor ( log((b-a)/TOL) / log(2) ) + 1

pvalues=zeros(Nmax,1);
```

Nmax =

21

## **Iteration Scheme**

```
for i = 1 : Nmax
    p = (a+b)/2;
        pvalues(i)=p;
    sfa = f(a);
    sfp = sign(f(p));
    fprintf( '\t\ %3d \t (%.6f, %.6f) \t %.10f \n', i, a, b, p)
        if ( (b-a)<2*TOL || sfp == 0 )
            break
    elseif ( sfa * sfp < 0 )
        b = p;
    else
        a = p;
        sfa = sfp;
    end
end</pre>
```

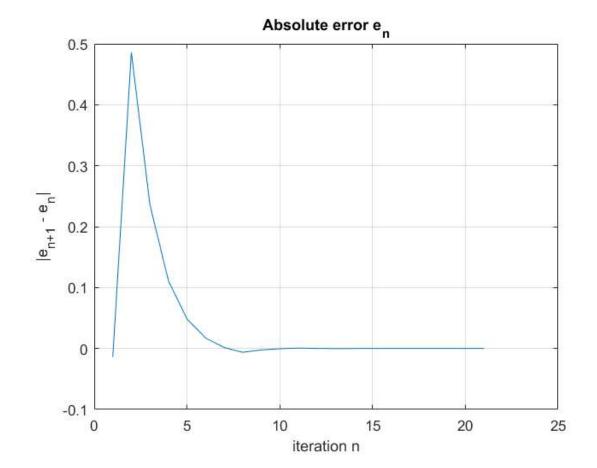
```
1
       (0.500000, 2.500000)
                                1.5000000000
 2
       (1.500000, 2.500000)
                                2.0000000000
       (1.500000, 2.000000)
 3
                                1.7500000000
       (1.500000, 1.750000)
 4
                                1.6250000000
 5
       (1.500000, 1.625000)
                                1.5625000000
 6
       (1.500000, 1.562500)
                                1.5312500000
 7
       (1.500000, 1.531250)
                                1.5156250000
 8
       (1.500000, 1.515625)
                                1.5078125000
 9
       (1.507813, 1.515625)
                                1.5117187500
10
       (1.511719, 1.515625)
                                1.5136718750
11
       (1.513672,1.515625)
                                1.5146484375
12
       (1.513672,1.514648)
                                1.5141601563
       (1.513672,1.514160)
13
                                1.5139160156
14
       (1.513916, 1.514160)
                                1.5140380859
15
      (1.514038, 1.514160)
                               1.5140991211
16
      (1.514038,1.514099)
                                1.5140686035
17
       (1.514038, 1.514069)
                                1.5140533447
       (1.514053,1.514069)
18
                                1.5140609741
19
       (1.514053, 1.514061)
                               1.5140571594
20
       (1.514053,1.514057)
                                1.5140552521
21
       (1.514053, 1.514055)
                               1.5140542984
```

## **Absolute Error computation**

```
plast =p;
errors=pvalues-plast*ones(Nmax,1);
fprintf('Approximate value | Absolute Error\n\n')
fprintf('%.10f | %.10f\n',[pvalues errors]');

figure; plot(errors); grid on;
xlabel("iteration n");
ylabel("|e_{n+1} - e_n|");
title('Absolute error e_n')
```

```
Approximate value | Absolute Error
               -0.0140542984
1.5000000000
2.0000000000
               0.4859457016
               0.2359457016
1.7500000000
1.6250000000
               0.1109457016
1.5625000000
               | 0.0484457016
1.5312500000
               0.0171957016
               0.0015707016
1.5156250000
               -0.0062417984
1.5078125000
1.5117187500
              -0.0023355484
               -0.0003824234
1.5136718750
1.5146484375
               0.0005941391
               | 0.0001058578
1.5141601563
1.5139160156
               -0.0001382828
               -0.0000162125
1.5140380859
1.5140991211
               0.0000448227
1.5140686035
              0.0000143051
               -0.0000009537
1.5140533447
               0.0000066757
1.5140609741
              | 0.0000028610
1.5140571594
              | 0.0000009537
1.5140552521
              0.000000000
1.5140542984
```



- Function defination
- Plotting graph of f(y)
- Stopping criterium
- Iteration scheme
- Error plot "p\_n p\_{n-1}"
- Printing results

## **Function defination**

```
Q = 20; g = 9.81;

A = @(y) 3*y + y^2/2;

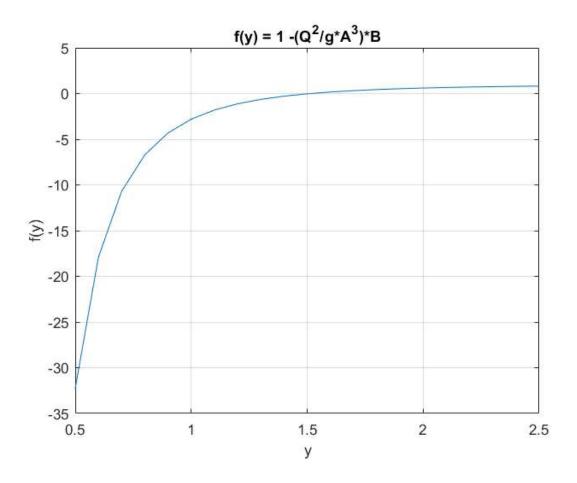
B = @(y) 3 + y;

f = @(y) 1 - (Q^2/(g^*(A(y)^3)))^*B(y);

a=0.5; b=2.5;
```

## Plotting graph of f(y)

```
X = a:0.1:b;
Y = a:0.1:b;
n = 1;
for x = a:0.1:b
Y(n) = f(x);
n = n + 1;
end
plot(X,Y); grid on;
xlabel("y");
ylabel("f(y)");
title("f(y) = 1 -({Q^2}/{g*A^3})*B")
```



## Stopping criterium

```
TOL=10^(-6);
format long;
old = b;
fa = feval(f,a);
fb = feval(f,b);
Nmax = 100;
```

#### Iteration scheme

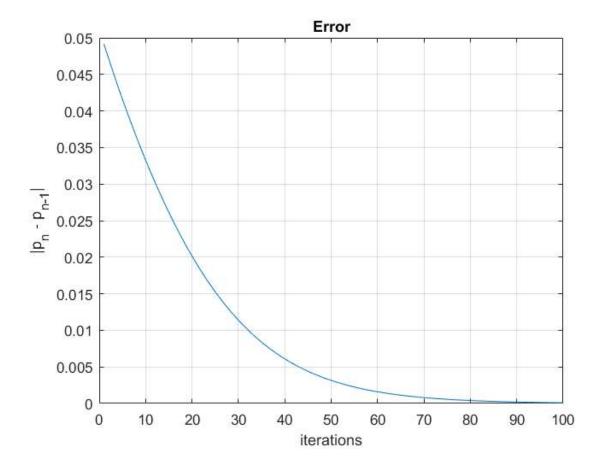
```
pvalues=[]; flag =0;
for i = 1 : Nmax
        new = b - fb * (b - a) / (fb - fa);
        fnew = feval(f,new);
        fprintf ( '\t\t %3d \t (%.10f, %.10f) \t %.10f \n', i, a, b, new )
        if ( abs(new-old) < TOL )</pre>
        flag=1;
           break
        elseif ( fa * fnew < 0 )
           b = new;
           fb = fnew;
        else
           a = new;
           fa = fnew;
    end
    error(i) = abs(new-old);
        old = new;
        pvalues = [pvalues;old];
end
```

```
1
       (0.5000000000, 2.5000000000)
                                           2.4508314769
 2
        (0.5000000000, 2.4508314769)
                                           2.4036291706
 3
       (0.50000000000, 2.4036291706)
                                           2.3583419190
        (0.5000000000, 2.3583419190)
 4
                                           2.3149191727
 5
        (0.5000000000, 2.3149191727)
                                           2.2733109175
 6
        (0.50000000000, 2.2733109175)
                                           2.2334676049
 7
        (0.5000000000, 2.2334676049)
                                           2.1953400929
 8
        (0.50000000000, 2.1953400929)
                                           2.1588795962
 9
        (0.50000000000, 2.1588795962)
                                           2.1240376472
10
       (0.50000000000, 2.1240376472)
                                           2 0907660677
11
        (0.5000000000, 2.0907660677)
                                           2.0590169517
12
        (0.50000000000, 2.0590169517)
                                           2.0287426587
13
        (0.50000000000, 2.0287426587)
                                           1.9998958179
14
       (0.5000000000, 1.9998958179)
                                           1.9724293424
15
        (0.50000000000, 1.9724293424)
                                           1.9462964529
       (0.50000000000, 1.9462964529)
16
                                           1.9214507099
17
       (0.5000000000, 1.9214507099)
                                           1.8978460542
18
       (0.5000000000, 1.8978460542)
                                           1.8754368537
19
       (0.50000000000, 1.8754368537)
                                           1.8541779556
20
        (0.5000000000, 1.8541779556)
                                           1.8340247435
21
       (0.50000000000, 1.8340247435)
                                           1.8149331968
22
        (0.5000000000, 1.8149331968)
                                           1.7968599526
23
       (0.5000000000, 1.7968599526)
                                           1.7797623671
24
       (0.50000000000, 1.7797623671)
                                           1.7635985779
25
       (0.5000000000, 1.7635985779)
                                           1.7483275630
26
       (0.50000000000, 1.7483275630)
                                           1.7339091977
27
        (0.5000000000, 1.7339091977)
                                           1.7203043082
28
       (0.50000000000, 1.7203043082)
                                           1.7074747200
29
        (0.5000000000, 1.7074747200)
                                           1.6953833021
       (0.50000000000, 1.6953833021)
30
                                           1.6839940055
31
        (0.5000000000, 1.6839940055)
                                           1.6732718953
32
       (0.5000000000, 1.6732718953)
                                           1.6631831780
33
        (0.5000000000, 1.6631831780)
                                           1.6536952215
34
        (0.50000000000, 1.6536952215)
                                           1.6447765697
35
       (0.50000000000, 1.6447765697)
                                           1.6363969518
36
        (0.5000000000, 1.6363969518)
                                           1.6285272841
        (0.5000000000, 1.6285272841)
37
                                           1.6211396688
38
        (0.5000000000, 1.6211396688)
                                           1.6142073856
39
       (0.5000000000, 1.6142073856)
                                           1.6077048800
40
        (0.5000000000, 1.6077048800)
                                           1.6016077471
41
        (0.5000000000, 1.6016077471)
                                           1.5958927111
42
       (0.50000000000, 1.5958927111)
                                           1.5905376021
43
        (0.50000000000, 1.5905376021)
                                           1.5855213296
44
        (0.50000000000, 1.5855213296)
                                           1.5808238536
45
        (0.50000000000, 1.5808238536)
                                           1.5764261538
46
       (0.50000000000, 1.5764261538)
                                           1.5723101972
47
        (0.5000000000, 1.5723101972)
                                           1.5684589036
       (0.5000000000, 1.5684589036)
48
                                           1.5648561116
49
       (0.5000000000, 1.5648561116)
                                           1.5614865425
50
       (0.5000000000, 1.5614865425)
                                           1.5583357652
51
        (0.50000000000, 1.5583357652)
                                           1.5553901595
       (0.50000000000, 1.5553901595)
52
                                           1.5526368813
                                           1.5500638265
53
       (0.5000000000, 1.5526368813)
54
       (0.5000000000, 1.5500638265)
                                           1.5476595967
55
       (0.50000000000, 1.5476595967)
                                           1.5454134645
56
        (0.5000000000, 1.5454134645)
                                           1.5433153408
57
        (0.5000000000, 1.5433153408)
                                           1.5413557415
        (0.5000000000, 1.5413557415)
58
                                           1.5395257565
```

```
1.5378170192
 59
         (0.5000000000, 1.5395257565)
 60
         (0.5000000000, 1.5378170192)
                                           1.5362216769
 61
        (0.5000000000, 1.5362216769)
                                           1.5347323625
 62
        (0.5000000000, 1.5347323625)
                                           1,5333421676
 63
         (0.5000000000, 1.5333421676)
                                           1.5320446161
 64
        (0.5000000000, 1.5320446161)
                                           1.5308336399
         (0.5000000000, 1.5308336399)
                                           1.5297035546
 65
        (0.5000000000, 1.5297035546)
 66
                                           1.5286490375
        (0.5000000000, 1.5286490375)
 67
                                           1.5276651054
 68
        (0.50000000000, 1.5276651054)
                                           1.5267470952
 69
        (0.50000000000, 1.5267470952)
                                           1.5258906436
        (0.50000000000, 1.5258906436)
 70
                                           1.5250916693
 71
        (0.5000000000, 1.5250916693)
                                           1.5243463557
 72
        (0.5000000000, 1.5243463557)
                                           1.5236511342
 73
        (0.5000000000, 1.5236511342)
                                           1.5230026691
 74
        (0.5000000000, 1.5230026691)
                                           1.5223978427
 75
        (0.5000000000, 1.5223978427)
                                           1.5218337418
 76
        (0.5000000000, 1.5218337418)
                                           1.5213076446
 77
        (0.50000000000, 1.5213076446)
                                           1.5208170085
 78
        (0.5000000000, 1.5208170085)
                                           1.5203594589
 79
        (0.5000000000, 1.5203594589)
                                           1.5199327779
 80
        (0.5000000000, 1.5199327779)
                                           1.5195348948
 81
        (0.50000000000, 1.5195348948)
                                           1.5191638761
 82
        (0.5000000000, 1.5191638761)
                                           1.5188179169
 83
        (0.5000000000, 1.5188179169)
                                           1.5184953323
 84
        (0.5000000000, 1.5184953323)
                                           1.5181945498
 85
        (0.5000000000, 1.5181945498)
                                           1.5179141017
 86
        (0.50000000000, 1.5179141017)
                                           1.5176526184
 87
        (0.50000000000, 1.5176526184)
                                           1.5174088219
 88
        (0.5000000000, 1.5174088219)
                                           1.5171815196
 89
        (0.5000000000, 1.5171815196)
                                           1.5169695990
 90
        (0.5000000000, 1.5169695990)
                                           1.5167720222
        (0.5000000000, 1.5167720222)
 91
                                           1.5165878207
 92
        (0.50000000000, 1.5165878207)
                                           1.5164160914
 93
        (0.5000000000, 1.5164160914)
                                           1.5162559917
 94
        (0.5000000000, 1.5162559917)
                                           1.5161067357
 95
        (0.50000000000, 1.5161067357)
                                           1.5159675903
 96
        (0.5000000000, 1.5159675903)
                                           1.5158378720
 97
        (0.50000000000, 1.5158378720)
                                           1.5157169431
        (0.5000000000, 1.5157169431)
 98
                                           1.5156042091
 99
        (0.5000000000, 1.5156042091)
                                           1.5154991154
        (0.5000000000, 1.5154991154)
100
                                           1.5154011449
```

#### **Error plot "p\_n - p\_{n-1}"**

```
figure; plot([1:i], error); grid on;
xlabel("iterations"); ylabel("|p_n - p_{n-1}|");
title("Error")
```



# **Printing results**

```
fprintf('The approximate root is %.10f',new)
if flag == 0
    disp(' Maximum number of iterations exceeded')
end
```

The approximate root is 1.5154011449 Maximum number of iterations exceeded

- Function Defination
- Plotting Graph
- Stopping criterium
- Iteration Scheme

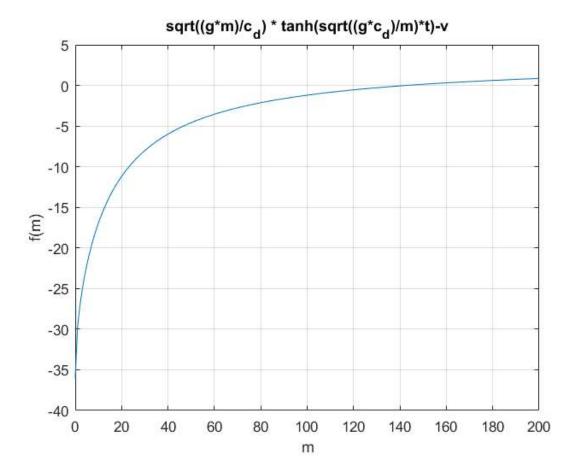
```
clear all;
close all;
```

#### **Function Defination**

```
syms m;
c_d = 0.25; v = 36; t = 4; g = 9.81;
f = @(m) sqrt((g*m)/c_d) *tanh(sqrt((g*c_d)/m)*t)-v;
fprime = eval(['@(m)' char(diff(f(m)))]);
```

## **Plotting Graph**

```
M = 0:200;
f_M = 0:200;
n = 1;
for m = 0:200;
    f_M(n) = f(m);
    n = n+1;
end
plot(M, f_M); grid on;
xlabel("m"); ylabel("f(m)");
title("sqrt((g*m)/c_d) * tanh(sqrt((g*c_d)/m)*t)-v");
```



## Stopping criterium

```
fprintf("(i) Taking the initial guess as x0 = 50 \n");
  x0 = 50; % initial approximation to location of root
  TOL = 10^{(-5)}; % absolute error convergence tolerance
  Nmax = 100; % maximum number of iterations to be performed
```

(i) Taking the initial guess as x0 = 50

#### **Iteration Scheme**

```
flag=0;
for i = 1 : Nmax
    fold=f(x0);
    fprimeold=fprime(x0);
        dx = fold / fprimeold;
    x0 = x0 - dx;
        fprintf ( '\t\t %3d \t %.10f \n', i, x0 );

    if ( abs(dx) < TOL )
        flag=1;
        break
    end
end</pre>
```

```
1 88.3992785546
```

<sup>2 124.0896500820</sup> 

<sup>3 140.5416962708</sup> 

```
4 142.7071837659
5 142.7376272539
6 142.7376331084
```

```
if flag == 0
    disp('Maximum number of iterations exceeded.')
end
```

```
fprintf('\n The approximate solution is %f \n', x0)
```

The approximate solution is 142.737633

- Function defination
- Plotting graph of f(y)
- Stopping criterium
- Iteration scheme
- Error plot "p\_n p\_{n-1}"
- Printing results

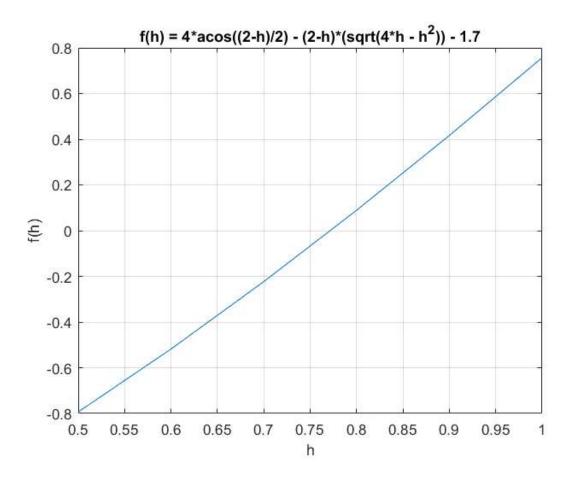
## **Function defination**

```
f = @(h) 4*acos((2-h)/2) - (2-h)*(sqrt(4*h - h^2)) - 1.7;

a=0.5; b=1;
```

## Plotting graph of f(y)

```
X = a:0.1:b;
Y = a:0.1:b;
n = 1;
for x = a:0.1:b
Y(n) = f(x);
n = n + 1;
end
plot(X,Y); grid on;
xlabel("h");
ylabel("f(h)");
title("f(h) = 4*acos((2-h)/2) - (2-h)*(sqrt(4*h - h^2)) - 1.7")
```



## Stopping criterium

```
TOL=10^(-6);
format long;
old = b;
fa = feval(f,a);
fb = feval(f,b);
Nmax = 100;
```

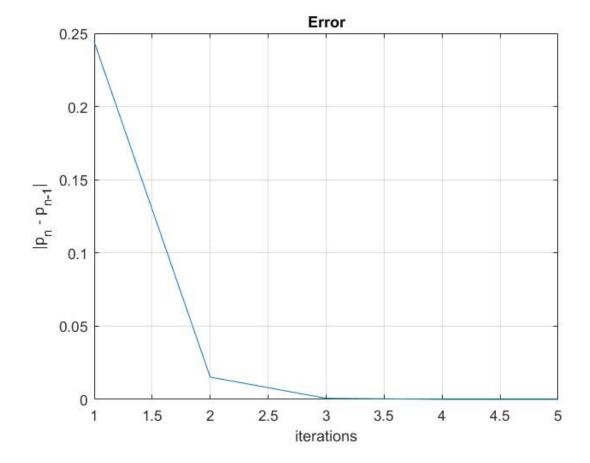
### **Iteration scheme**

```
pvalues=[]; flag =0;
for i = 1 : Nmax
        new = b - fb * (b - a) / (fb - fa);
        fnew = feval(f,new);
        fprintf ( '\t\t %3d \t (%.10f, %.10f) \t %.10f \n', i, a, b, new )
        if ( abs(new-old) < TOL )</pre>
        flag=1;
           break
        elseif ( fa * fnew < 0 )
           b = new;
           fb = fnew;
        else
           a = new;
           fa = fnew;
    end
    error(i) = abs(new-old);
        old = new;
        pvalues = [pvalues;old];
end
```

```
1
      (0.5000000000, 1.0000000000)
                                       0.7559087671
2
      (0.7559087671,1.0000000000)
                                       0.7711574965
3
      (0.7711574965,1.0000000000)
                                       0.7719060141
      (0.7719060141,1.0000000000)
                                       0.7719423707
      (0.7719423707,1.0000000000)
5
                                       0.7719441357
      (0.7719441357,1.0000000000)
                                       0.7719442214
```

## Error plot "p\_n - p\_{n-1}"

```
figure; plot([1:5], error); grid on;
xlabel("iterations"); ylabel("|p_n - p_{n-1}|");
title("Error")
```



## **Printing results**

```
fprintf('The approximate root is %.10f',new)
if flag == 0
    disp(' Maximum number of iterations exceeded')
end
```

The approximate root is 0.7719442214

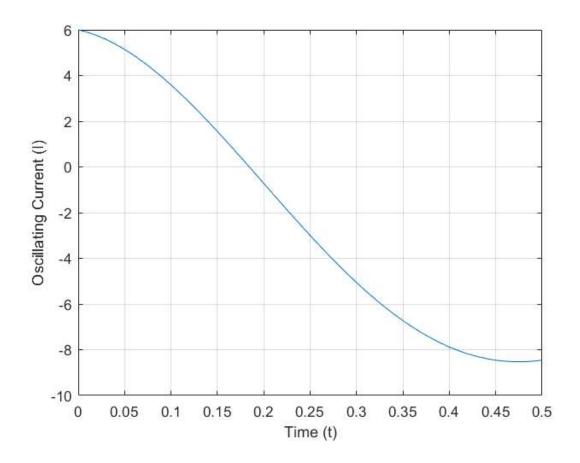
- Plotting graph of function
- Secant Method applied with different intervals.

```
clear all;
close all;
```

## Plotting graph of function

```
g = @(x) 9*exp(-x)*cos(2*pi*x) - 3;
X = 0:0.01:0.5; Y = -100:4:100;
n = 1;
for x = 0:0.01:0.5
Y(n) = g(x);
n = n + 1;
end
plot(X,Y); grid on;
xlabel('Time (t)');
ylabel('Oscillating Current (I)');
fprintf("Considering the interval [0,0.5]\n"); SecantMethod(0,0.5);
```

Considering the interval [0,0.5]



## Secant Method applied with different intervals.

```
2 0.207486443733952
3 0.165147507080017
4 0.184233316589298
5 0.184364220463863
6 0.184363081087201
7 0.184363081134278
```

- Plotting Graph
- Analyzing results for different initial guess
- The Newton Raphson Iteration Scheme
- Function Defination
- Stopping criterium
- Iteration Scheme
- Results

```
clear all;
close all;
```

## **Plotting Graph**

```
f = @(x) [exp(-0.5*x)]*(4-x)-2;

x = -2:0.1:2;

y = -2:0.1:2;

n = 1;

for x = -2:0.1:2;

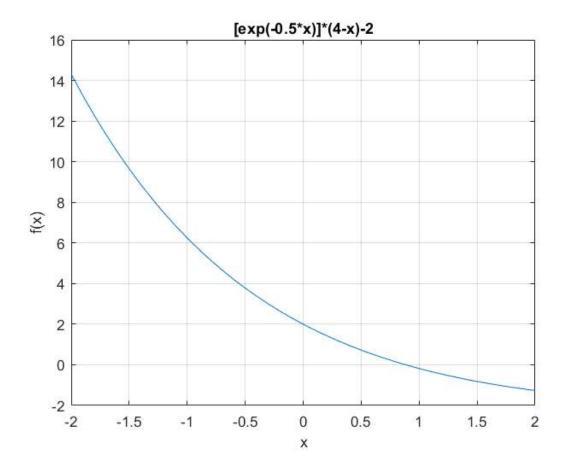
Y(n) = f(x);

n = n + 1;
end

plot(X,Y); grid on;

xlabel("x"); ylabel("f(x)");

title("[exp(-0.5*x)]*(4-x)-2");
```



## Analyzing results for different initial guess

```
NewtonRaphson(2);
NewtonRaphson(6);
NewtonRaphson(8);
```

## **The Newton Raphson Iteration Scheme**

```
function y = NewtonRaphson(x0)
```

#### **Function Defination**

```
syms x;

f = @(x) [exp(-0.5*x)]*(4-x)-2;

fprime = eval(['@(x)' char(diff(f(x)))]);
```

## Stopping criterium

x0 = 2; % initial approximation to location of root

```
TOL = 10^(-5); % absolute error convergence tolerance
Nmax =50; % maximum number of iterations to be performed
```

## **Iteration Scheme**

```
flag=0;
for i = 1 : Nmax
```

```
fold=f(x0);
  fprimeold=fprime(x0);
  dx = fold / fprimeold;
  x0 = x0 - dx;
  fprintf ( '\t\t %3d \t %.10f \n', i, x0 );

if ( abs(dx) < TOL )
     flag=1;
     break
end
end</pre>
```

```
1 0.2817181715
 2
     0.7768868450
     0.8817078789
 3
 4
     0.8857032412
 5
      0.8857088020
 1
      Inf
 2
      NaN
 3
      NaN
      NaN
 4
 5
      NaN
 6
      NaN
 7
      NaN
      NaN
 8
 9
      NaN
10
      NaN
11
      NaN
12
      NaN
13
      NaN
14
      NaN
15
      NaN
16
      NaN
17
      NaN
18
      NaN
      NaN
19
20
      NaN
21
      NaN
22
      NaN
      NaN
23
24
      NaN
25
      NaN
      NaN
26
27
      NaN
28
      NaN
29
      NaN
30
      NaN
31
      NaN
32
      NaN
33
      NaN
34
      NaN
35
      NaN
36
      NaN
37
      NaN
38
      NaN
39
      NaN
40
      NaN
```

```
42
       NaN
43
       NaN
44
       NaN
45
       NaN
46
       NaN
47
       NaN
48
       NaN
49
       NaN
50
       NaN
 1
       121.1963000663
 2
       7212131452880262800000000.0000000000
 3
       Inf
 4
       NaN
 5
       NaN
 6
       NaN
 7
       NaN
 8
       NaN
 9
       NaN
10
       NaN
11
       NaN
12
       NaN
13
       NaN
14
       NaN
15
       NaN
16
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17
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18
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19
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20
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21
22
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29
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30
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31
       NaN
32
       NaN
33
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34
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35
       NaN
36
       NaN
37
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38
       NaN
       NaN
39
40
       NaN
41
       NaN
42
       NaN
       NaN
43
44
       NaN
45
       NaN
       NaN
46
47
       NaN
```

41

48

NaN

NaN

```
49 NaN50 NaN
```

#### Results

```
fprintf("# Taking the initial guess as x_0 = %.1f \n", x0);
if flag == 0
    disp('Maximum number of iterations exceeded.')
end
fprintf('\n The approximate solution is %f \n', x0)
```

```
# Taking the initial guess as x_0 = 0.9
The approximate solution is 0.885709
# Taking the initial guess as x_0 = NaN
Maximum number of iterations exceeded.
The approximate solution is NaN
# Taking the initial guess as x_0 = NaN
Maximum number of iterations exceeded.
The approximate solution is NaN
```

end