Types of functions —

$O(1)$ — constant

$O(\log n)$ = Logarithmic

$O(n)$ = Linear

$O(n^2)$ — Quadratic

$O(n^3)$ — Cubic

$O(2^n)$ — exponential

$O(3^n)$ —

$O(n^n)$ —

$f(n) = 2 \rightarrow O(1)$

$f(n) = 5$

$f(n) = 5000$

$f(n) = 2n + 3$ — $O(n)$

$f(n) = 5000n + 700$

$f(n) = \frac{n}{5000} + 6$

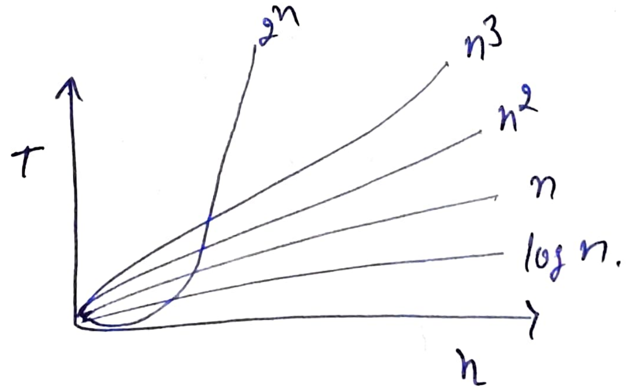
~~$1 < \log n < n \log n$~~

$1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 < \dots < 2^n < 3^n \dots < n^n.$

$\log n$	n	n^2	2^n
0	1	1	1
$\log_2 2 = 1$	2	4	4
2	4	16	16
3	8	64	256
3.1	9	81	512

$n^{100} < 2^n.$

$n^k < 2^n.$



Asymptotic Notations →

O - big-oh upper bound
Ω - big Omega Lower bound
Θ - big theta Average bound.

1. Big-oh Notation →

The function $f(n) = O(g(n))$ iff
$\exists$ +ive constant c and n_0
such that $f(n) \leq c * g(n) \ \forall n \geq n_0$

e.g. $f(n) = 2n+3$

$2n+3 \leq 10n$
or
$2n+3 \leq 7n$
or

Make all terms as multiple of n ← $2n+3 \leq 2n+3n \quad n \geq 1$

$f(n) = O(n)$

$2n+3 \leq 2n^2+3n^2$
$2n+3 \leq 5n^2$

$f(n) = O(n^2)$

Always Use the tightest Bound

2 Omega Notation →

The function $f(n) = \Omega g(n)$ iff $\exists$ +ive constants c and n_0 such that

$f(n) \geq c * g(n) \ \forall n \geq n_0$

e.g.

$f(n) = 2n+3$

$2n+3 \geq 1 * n \quad \forall n \geq 1$
$\uparrow$ $f(n)$ $\quad \uparrow c \quad \uparrow g(n)$

$f(n) = \Omega(n)$

3) Theta Notation →

The function $f(n) = \Theta(g(n))$ iff $\exists$ +ive constants c and n_0 such that

$$c_1 * g(n) \le f(n) \le c_2 * g_2(n)$$

eg

$f(n) = 2n+3$

$\underset{c_1\ g(n)}{1 \times n} \le 2n+3 \le \underset{c_2\ g(n)}{5 \times n}$ $\therefore f(n) = \Theta(n)$

$f(n) = n$
$g(n) = n+1$

1) $n \le c_1(n+10)$ $n_0 \geq 1$ $n_0 = 10$

2 $n \geq c_2(n+10)$ $n_0 \geq 10$ $c_1 = 1$

$n = \Theta(n+10)$ $c_2 = \frac{1}{2}$

examples

① $f(n) = 2n^2+3n+4$

$2n^2+3n+4 \le 2n^2+3n^2+4n^2$

$2n^2+3n+4 \le 9 n^2$ ⇒ so $f(n) = O(n^2)$, $n \geq 1$

$2n^2+3n+4 \geq 1 \times n^2$ $f(n) = \Omega(n^2)$

$1n^2 < 2n^2+3n+4 \le 9n^2$ $f(n) = \Theta(n^2)$

$2n^2+3n+4 \le 2n^2+3n+\ldots$
$\le 2n^2+4n$

$2n^2+4n \le 2n^2+n^2 \le 3\ldots$

$c = 3$
$g(n) = n^2$
$n_0 = 4$

② $f(n) = n^2 \log n + n$

$1 \times n^2 \log n \le n^2 \log n + n \le 10 n^2 \log n.$

$O(n^2 \log n)$

$\Omega(n^2 \log n)$

$\Theta(n^2 \log n)$

eg 2 $f(n) = n! = n \times n-1 \times n-2 \times \dots \times 3 \times 2 \times 1$

$1 \times 1 \times 1 \dots 1 \le 1 \times 2 \times 3 \dots \times n \le n \times n \times n \times \dots n$

$1 \le n! \le n^n$

$\Omega(1)$ Lower bound

$O(n^n)$ upper bound

we can not find Θ for this

eg 3. $f(n) = \log n!$

$\log(1 \times 1 \times \dots 1) \le \log(1 \times 2 \times 3 \dots n) \le \log(n \times n \times n \dots n)$

$1 \le \log n! \le \log n^n$

$\le n \log n$

$\Omega(1)$ $O(n \log n)$

Properties of Asymptotic Notations →

General Property

① If $f(n)$ is $O(g(n))$ then $a * f(n)$ is $O(g(n))$

e.g. $f(n) = 2n^2 + 5$ is $O(n^2)$

$7f(n) = 14n^2 + 35$ is $O(n^2)$

a is some constant

② If $f(n)$ is $\Theta(g(n))$ then $a * f(n)$ is $\Theta(g(n))$

③ If $f(n)$ is $\Omega(g(n))$ then $a * f(n)$ is $\Omega(g(n))$

Reflexive Properties →

① If $f(n)$ is given then $f(n)$ is $O(f(n))$

i.e. function is upper bound of itself

e.g. $f(n) = n^2$ $O(n^2)$

② $f(n) = \Omega(f(n))$

$\therefore f(n) = \Theta(f(n))$

Transitive Property :-

If $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$

then $f(n) = O(h(n))$

e.g. $f(n) = n$ $g(n) = n^2$ $h(n) = n^3$

n is $O(n^2)$ and n^2 is $O(n^3)$

then n is $O(n^3)$

** This Property is true for all 3 Notations

Symmetric Property → (True only for Theta Notation)

If $f(n)$ is $\Theta(g(n))$ then $g(n)$ is $\Theta(f(n))$

$f(n) = O(g(n))$
then $g(n) \neq O(f(n))$
$n^2 = O(n^3)$
$\Rightarrow n^3 = O(n^2)$ ✗
i.e $g(n) \neq O(f(n))$

e.g $f(n) = n^2$ $g(n) = n^2$

$f(n) = \Theta(n^2)$

$g(n) = \Theta(n^2)$

$f(n) = \Omega(g(n))$
then $g(n) \neq \Omega f(n)$
$n^3 = \Omega(n)$
$n = \Omega(n^3)$ ✗
i.e $g(n) \neq \Omega f(n)$

Transpose Symmetric →

If $f(n) = O(g(n))$ then $g(n)$ is $\Omega(f(n))$

eg $f(n) = n$ $g(n) = n^2$

then n is $O(n^2)$ and

n^2 is $\Omega(n)$

Note

If $f(n) = O(g(n))$

and $f(n) = \Omega(g(n))$

$g(n) \leq f(n) \leq g(n)$

$f(n) = \Theta(g(n))$

Question

If $f(n) = O(g(n))$
and $d(n) = O(e(n))$
then $f(n) + d(n) =$

$O(\max(g(n), e(n)))$

$\Rightarrow$ Let $f(n) = n = O(n)$

$d(n) = n^2 = O(n^2)$

$f(n) + d(n) = n + n^2 = O(n^2)$

$$\boxed{\begin{array}{l}\text{If } f(n) = O(g(n)) \\ \text{and } d(n) = O(e(n)) \\ \text{then } f(n) \times d(n) = O(g(n) \times e(n))\end{array}}$$

Comparision of Two Function →

①

n	$n^2 < n^3$	
2	$2^2=4$	$2^3=8$
3	$3^2=9$	$3^3=27$
4	$4^2=16$	$4^3=64$

② Apply log on both side

$n^2 \qquad n^3$

$\log n^2 \qquad \log n^3$

$2 \log n < 3 \log n.$

$$\boxed{\begin{array}{l}\log a.b = \log a + \log b \\ \log a/b = \log a - \log b. \\ \log a^b = b \log a \\ a^{\log_c b} = b^{\log_c a} \\ a^b = n \text{ then } b = \log_a n\end{array}}$$

e.g. $f(n) = n^2 \log n \qquad g(n) = n (\log n)^{10}$

Apply log

$\log (n^2 \log n) \qquad \log (n (\log n)^{10})$

$\log n^2 + \log \log n \qquad \log n + \log (\log n)^{10}$

$2 \log n + \log \log n \qquad \log n + 10 \log \log n$

$>$

2 $f(n) = 3n^{\sqrt{n}}$ $g(n) = 2^{\sqrt{n}\log n}$

$3n^{\sqrt{n}}$ $2^{\log_2 n^{\sqrt{n}}}$

$3n^{\sqrt{n}}$ $\left(n^{\sqrt{n}}\right)^{\log_2 2}$

$3n^{\sqrt{n}}$ $n^{\sqrt{n}}$