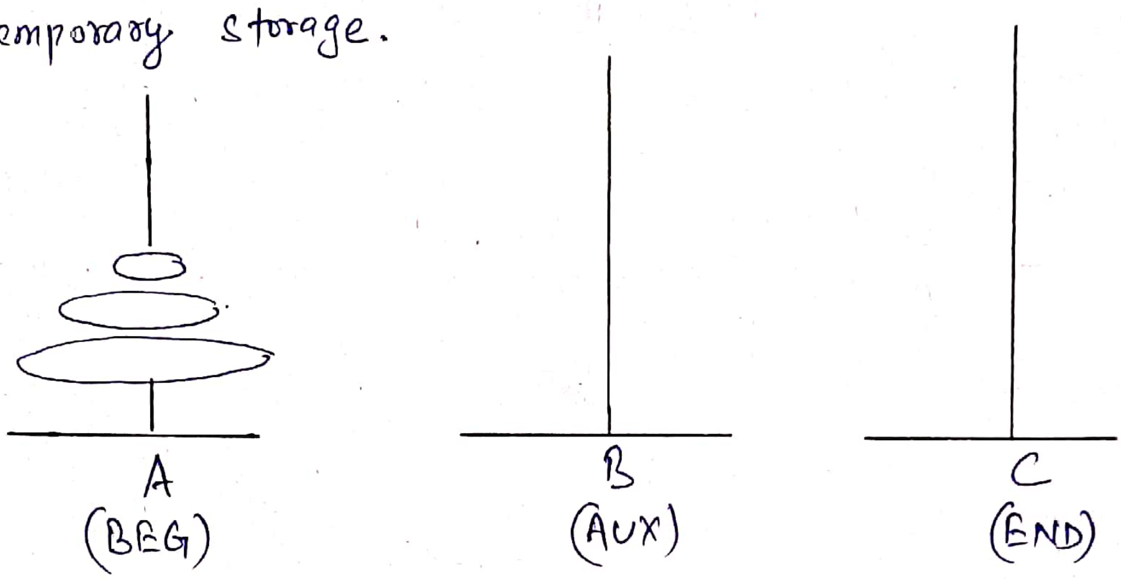
Towers of Hanoi

- There are n disks of different sizes.
- There are 3 towers or poles or pegs A, B, C.
- All the n disks are placed on tower A in such a way that a larger disk is always below a smaller disk.
- The other two towers are initially empty.
- The aim is to move n disks to tower C using tower B as temporary storage.



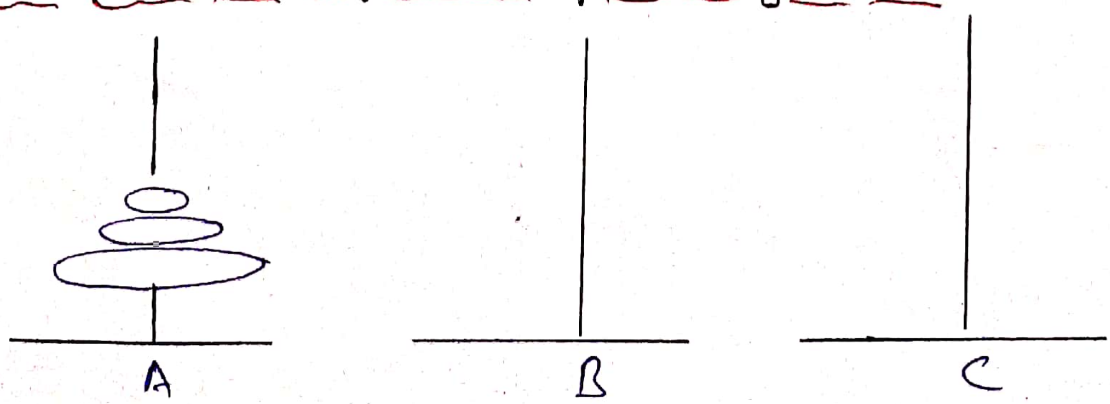
Rules:->

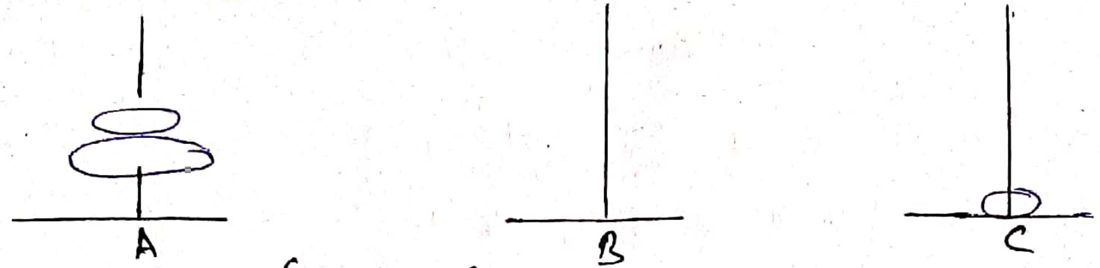
(1) Only one disc could be moved at a time.

(2) A larger disc must never be stacked above a smaller one.

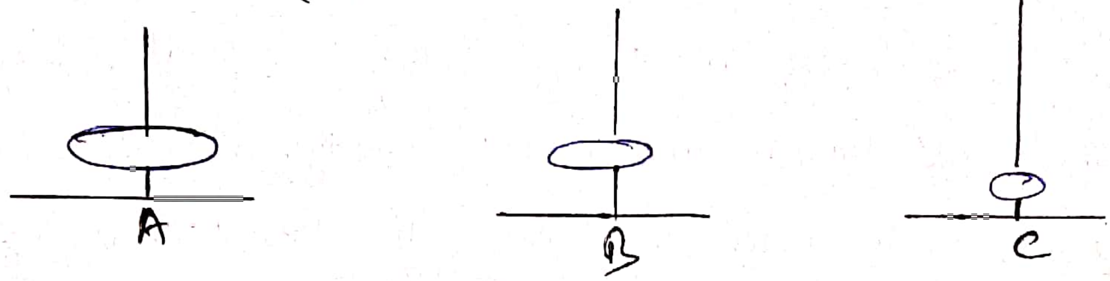
(3) One and only one extra tower could be used for immediate storage of disks.

Solution to Towers of Hanoi problem for n = 3

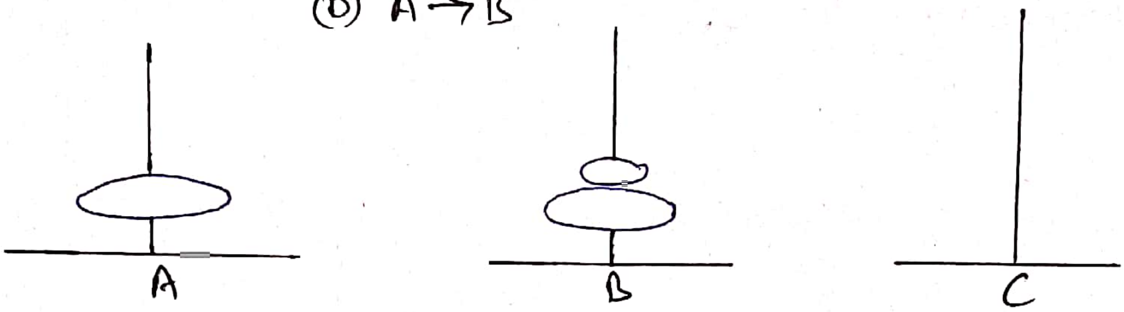




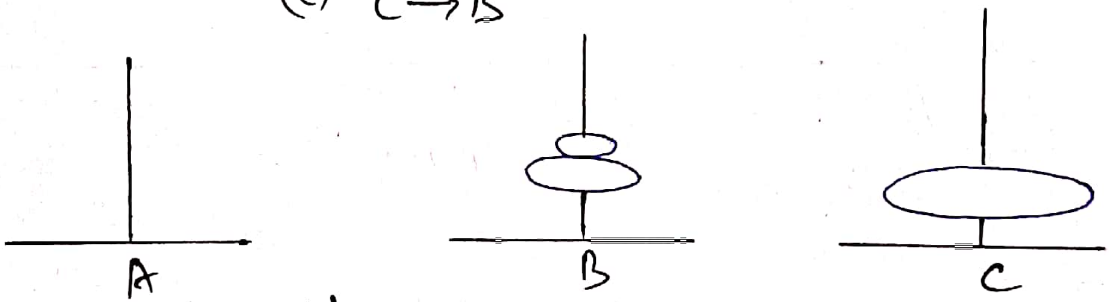
(a) $A \rightarrow C$



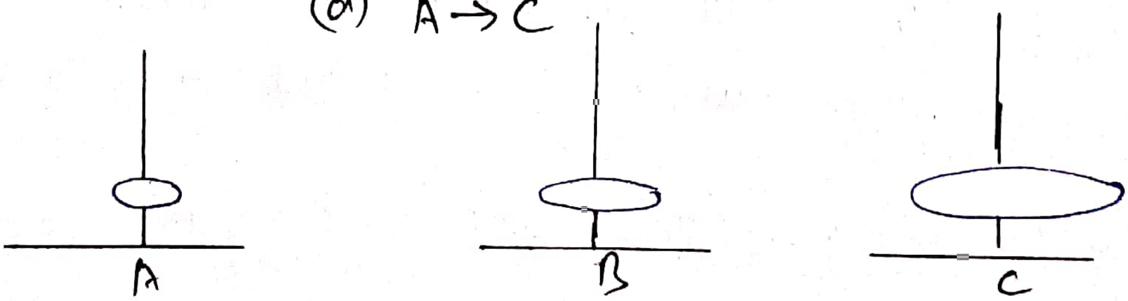
(b) $A \rightarrow B$



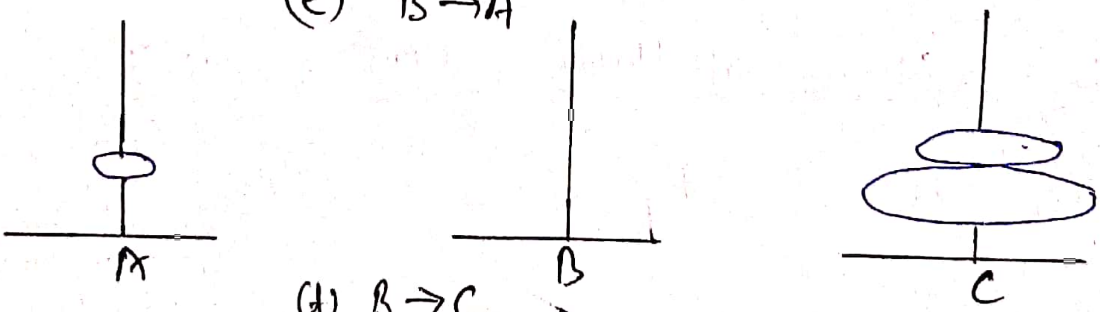
(c) $C \rightarrow B$



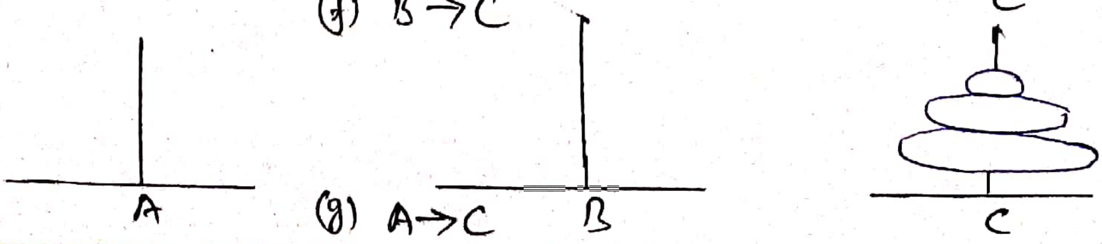
(d) $A \rightarrow C$



(e) $B \rightarrow A$



(f) $B \rightarrow C$



(g) $A \rightarrow C$

For $n=3$, steps are

1. $A \to C$
2. $A \to B$
3. $C \to B$
4. $A \to C$
5. $B \to A$
6. $B \to C$
7. $A \to C$

Number of disks (n)	Number of Moves
1	$2^1-1 = 1$
2	$2^2-1 = 4-1 = 3$
3	$2^3-1 = 8-1 = 7$
4	$2^4-1 = 16-1 = 15$
5	$2^5-1 = 32-1 = 31$
6	$2^6-1 = 64-1 = 63$

So, the formula for finding the number of steps it takes to transfer n disks from A to C is 2^n-1

The Solution to Towers of Hanoi problem for $n > 1$ disks may be reduced to the following subproblems:

1. Move top $n-1$ disks from A to B.
2. Move top disk from A to C ie $A \to C$.
3. Move top $n-1$ disks from B to C.

For $n=4$ disks, 15 moves are:→

$A \to B$, $A \to C$, $B \to C$, $A \to B$, $C \to A$, $C \to B$, $A \to B$, $A \to C$, $B \to C$, $B \to A$, $C \to A$, $B \to C$, $A \to B$, $A \to C$, $B \to C$.

Tower (N , BEG , AUX , END)

where BEG is Beginning tower

AUX is Auxiliary tower

END is End / Final tower

Tower (N , BEG , AUX , END)

1. If N = 1 , then write BEG → END and exit .
2. Call Tower (N-1 , BEG , END , AUX)
3. Write BEG → END
4. Call Tower (N-1 , AUX , BEG , END)
5. Exit .

Que Write the recursive solution for N = 3 disks with 3 towers A , B , C .

Tower (3 , A , B , C)

here, N = 3

BEG = A

AUX = B

END = C

∴ Steps are

A → C

A → B

C → B

A → C

B → A

B → C

A → C

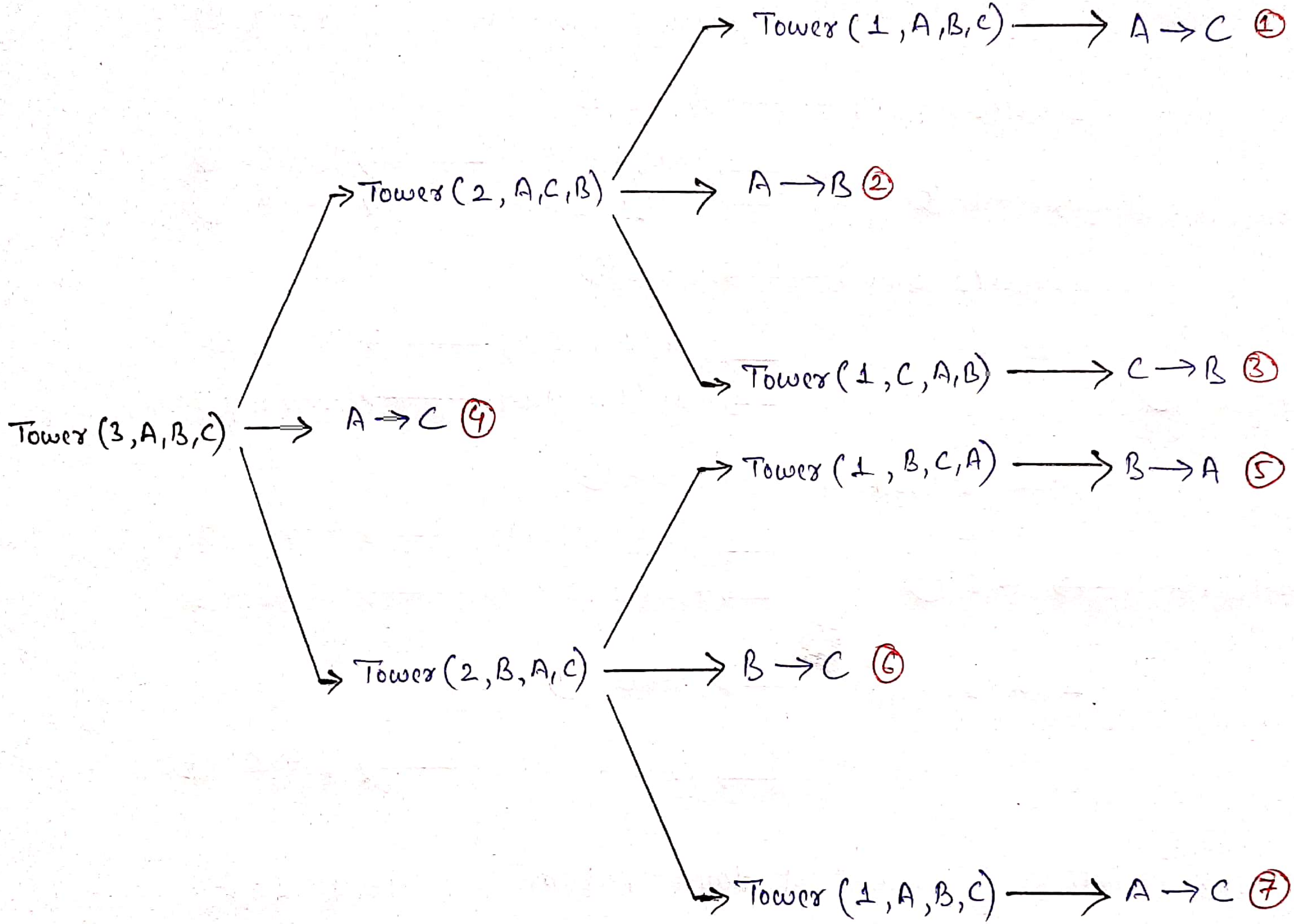


fig. Recursive tree Solution for Tower of Hanoi Problem for N=3 disks.

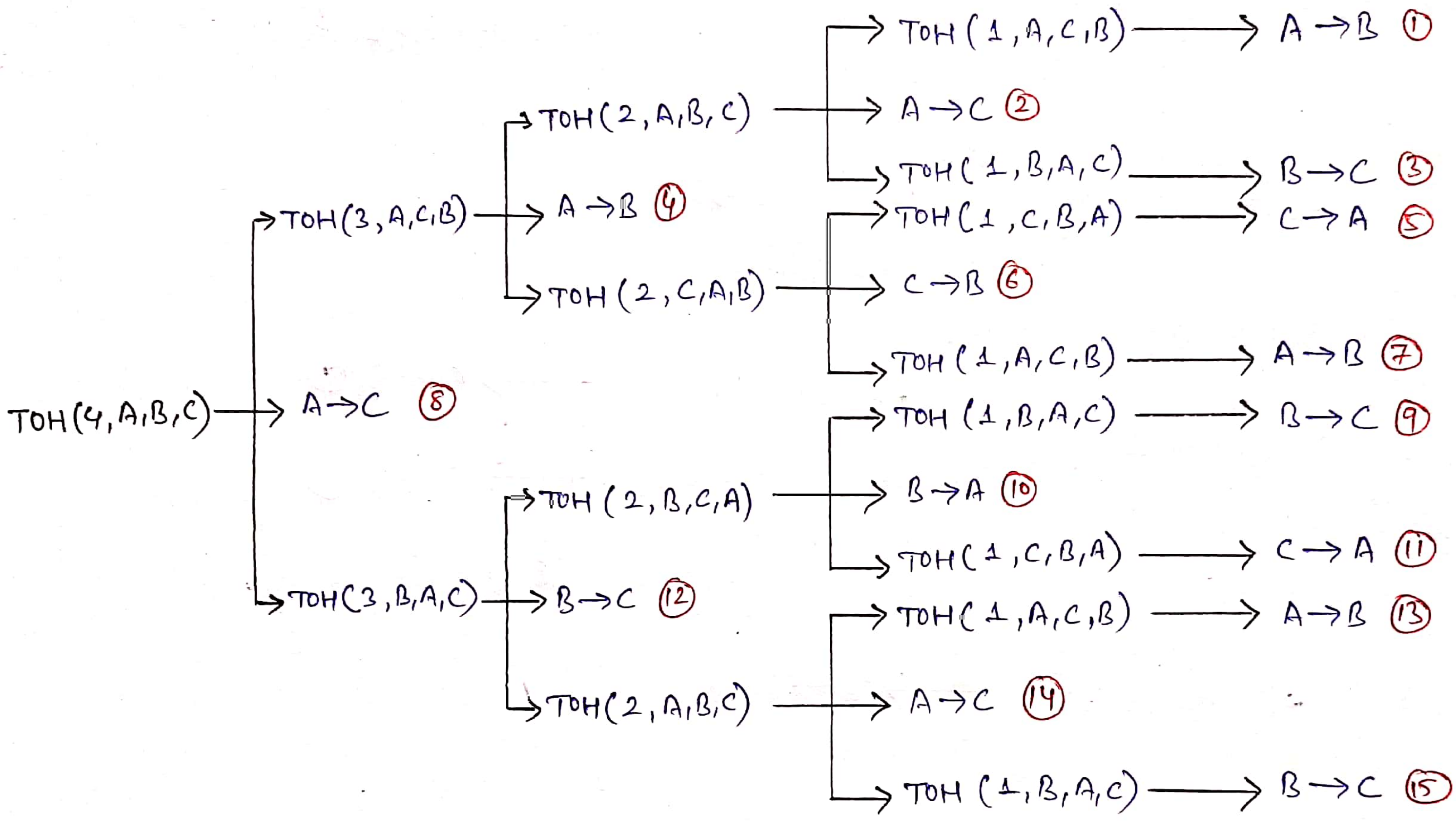


fig. Recursive Tree Solution for Tower of Hanoi problem for N=4 disks.