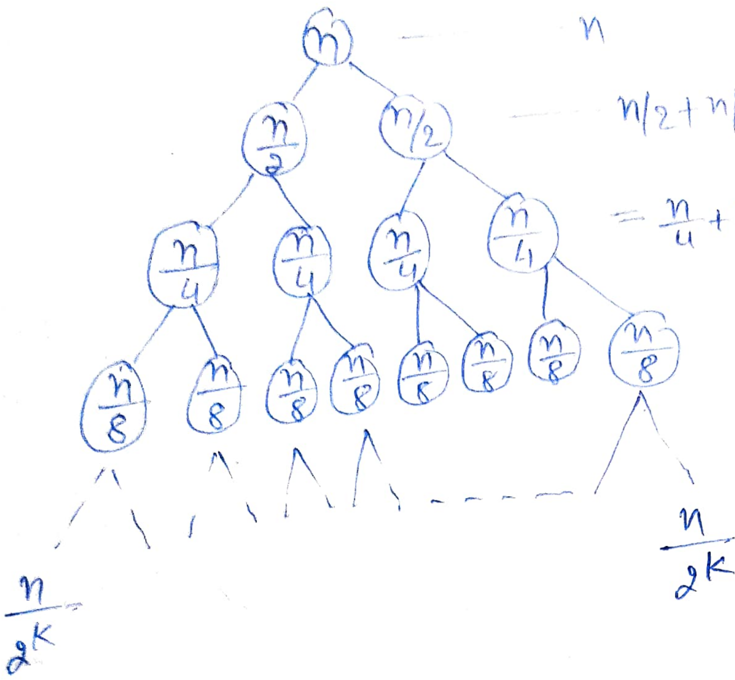
Examples
Recursive Tree Method →

e.g. 1 $T(n) = T(n/2) + T(n/2) + n$



— n

— $n/2 + n/2 = n$

$= \frac{n}{4} + \frac{n}{4} + \frac{n}{4} + \frac{n}{4} = n.$

$= n.k$ times

$\boxed{= n.\log n \text{ times}}$ Ans

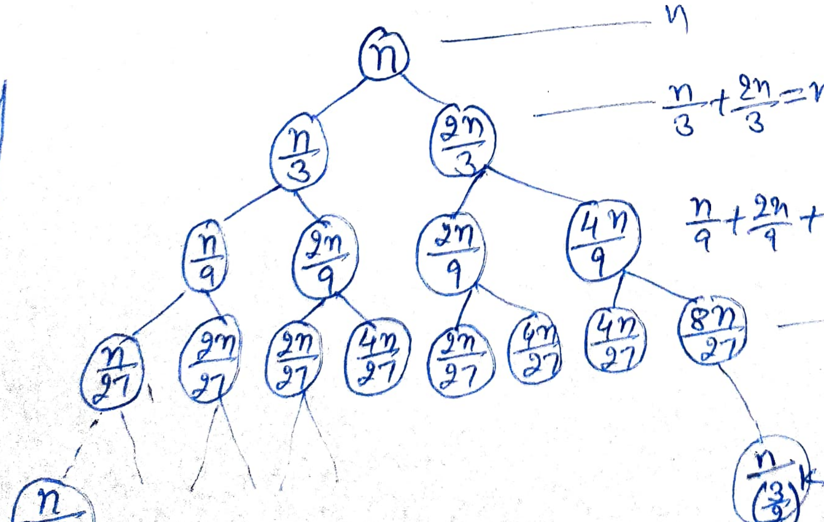
Let $n/2^k = 1$

$n = 2^k$

$k = \log n.$

$\Rightarrow \Theta(n \log n)$

e.g. 2 $T(n) = T(n/3) + T(2n/3) + n$



$\log_3 n.$

— n

— $\frac{n}{3} + \frac{2n}{3} = n.$

$\frac{n}{9} + \frac{2n}{9} + \frac{2n}{9} + \frac{4n}{9} = n.$

— n

$\log_{3/2} n$

$\Rightarrow \boxed{n \log_{3/2} n}$ Ans

Let $\frac{n}{3^k} = 1$

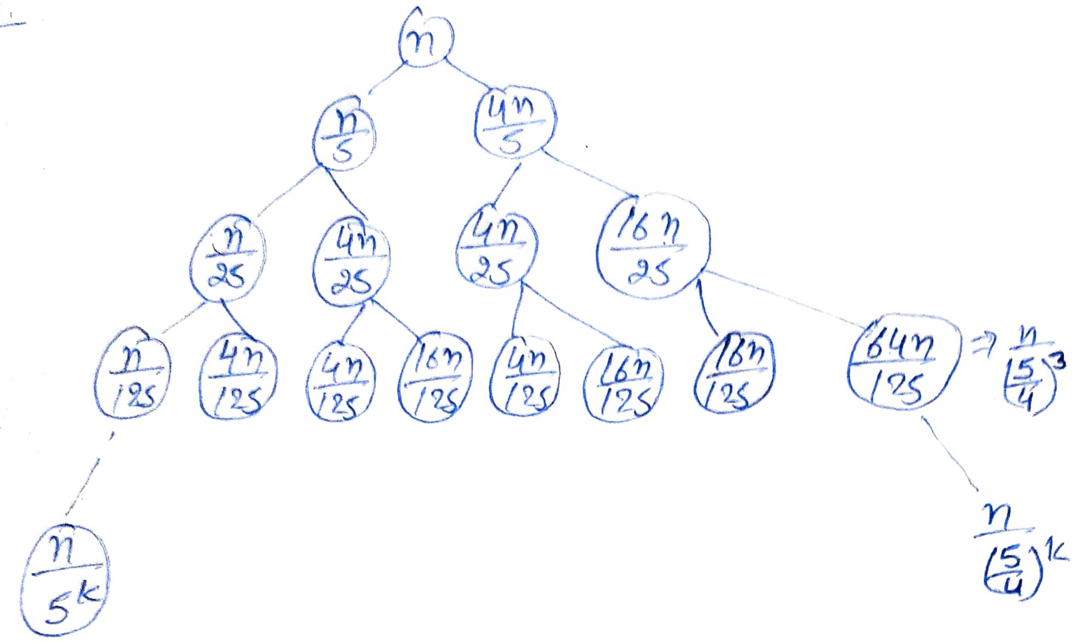
$\log_3 n = k$

$\frac{n}{(3/2)^k} = 1 \Rightarrow n = \left(\frac{3}{2}\right)^k$

$= \log_{\frac{3}{2}} n = k$

Q.3 $T(n) = T(n/5) + T(4n/5) + n.$

Sol.^{n}



$\frac{4n}{5} \Rightarrow \frac{n}{(\frac{5}{4})^1}$ height of Left subtree $=$ Let $\frac{n}{5^k} = 1$

$$\boxed{k = \log_5 n}$$

height of right subtree Let $\frac{n}{(\frac{5}{4})^k} = 1$

$$n = \left(\frac{5}{4}\right)^k$$

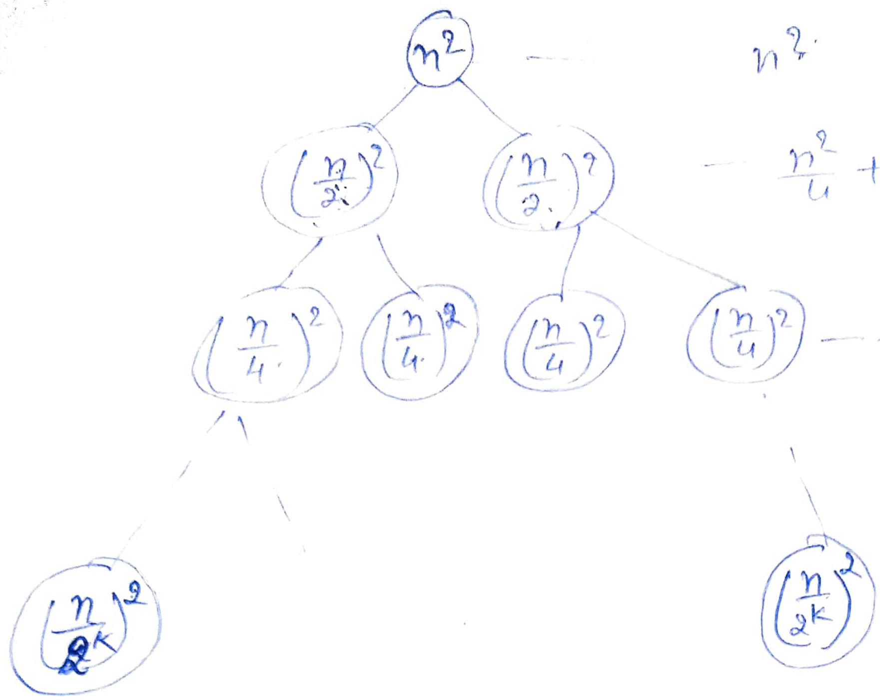
$$\boxed{\log_{\frac{5}{4}} n = k}$$

$$\boxed{n \log_5 n} \le f(n) \le \boxed{T(n) \Rightarrow n \log_{5/4} n}$$

4 $T(n) = 2T(n/2) + n^2$

$= T(n/2) + T(n/2) + n^2.$

P.T.O



n^2

$\frac{n^2}{4} + \frac{n^2}{4} = \frac{2n^2}{4} = \frac{n^2}{2}$

$\frac{n^2}{16} + \frac{n^2}{16} + \frac{n^2}{16} + \frac{n^2}{16} = \frac{4n^2}{16} = \frac{n^2}{4}$

height of Left subtree $\rightarrow$ $\frac{n}{2^k} = 1$

$k = \log_2 n$

Total Time $= n^2 + \frac{n^2}{2} + \frac{n^2}{4} + \frac{n^2}{8} + \ldots$

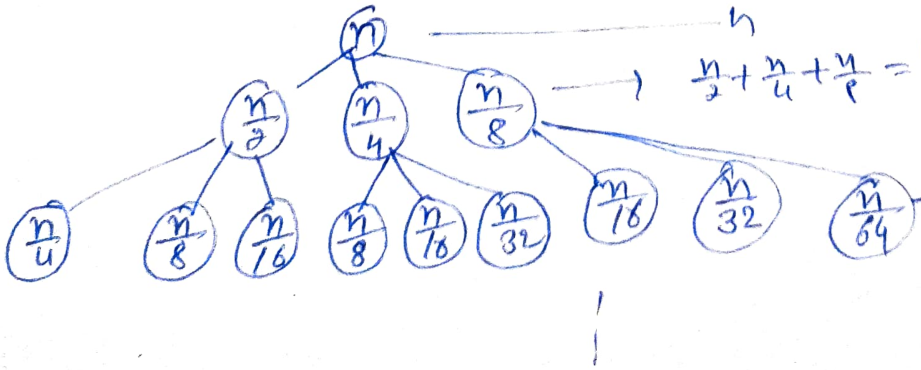
$n^2\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right)$ $\quad$ $\because$ G.P Series. $\left(\frac{a}{a-r}\right)$

$= n^2\left(\frac{1}{1-\frac{1}{2}}\right)$

$= O(n^2)$

e.g 5 $\quad$ ~~$T(n) = 2T(\frac{n}{2}$~~

$T(n) = T(n/2) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + n.$



n

$\frac{n}{2} + \frac{n}{4} + \frac{n}{8} = \frac{4n+2n+n}{8} = \frac{7}{8}n$

$\frac{n}{4} + 2\left(\frac{n}{8}\right) + 2\left(\frac{n}{16}\right) + 2\left(\frac{n}{32}\right) + \frac{n}{64}$

$= \frac{49}{64}n = \left(\frac{7}{8}\right)^2 n$

$= \left(\frac{7}{8}\right)^3 n$

$\left(\frac{7}{8}\right)^k n.$

Now to find k, $\frac{n}{2^k} = 1$

$k = \log n$

$$T(n) = n\left(1 + \left(\frac{7}{8}\right)^1 + \left(\frac{7}{8}\right)^2 + \left(\frac{7}{8}\right)^3 + \cdots \left(\frac{7}{8}\right)^{\log n}\right)$$

$= 8n$ (decreasing G.P $= 1$) $\frac{1}{1-\frac{7}{8}}$

$T(n) = O(n)$