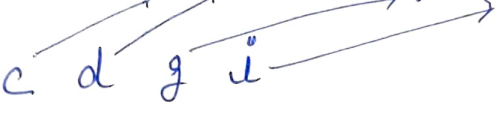
Longest Common Subsequence ⟶

Given two sequences, find the length of longest subsequence present in both of them. A subsequence is a sequence that appears in the same relative order but not necessarily continuous. For e.g. abc, abg, bdf, aeg, acefg are subsequences of abcdefg.

① String 1 : a b c d e f g h i j

String 2 : c d g i



c d g i is the longest subsequence

d g i] are subsequences but not the
g i] longest one.

② String 1 a b c d e f g h i j

String 2 e c d g i

① e g i

② c d g i] Longest subsequence.

③ String 1 : a b d a c e

String 2 b a b c e

① b a c e

② a b c e

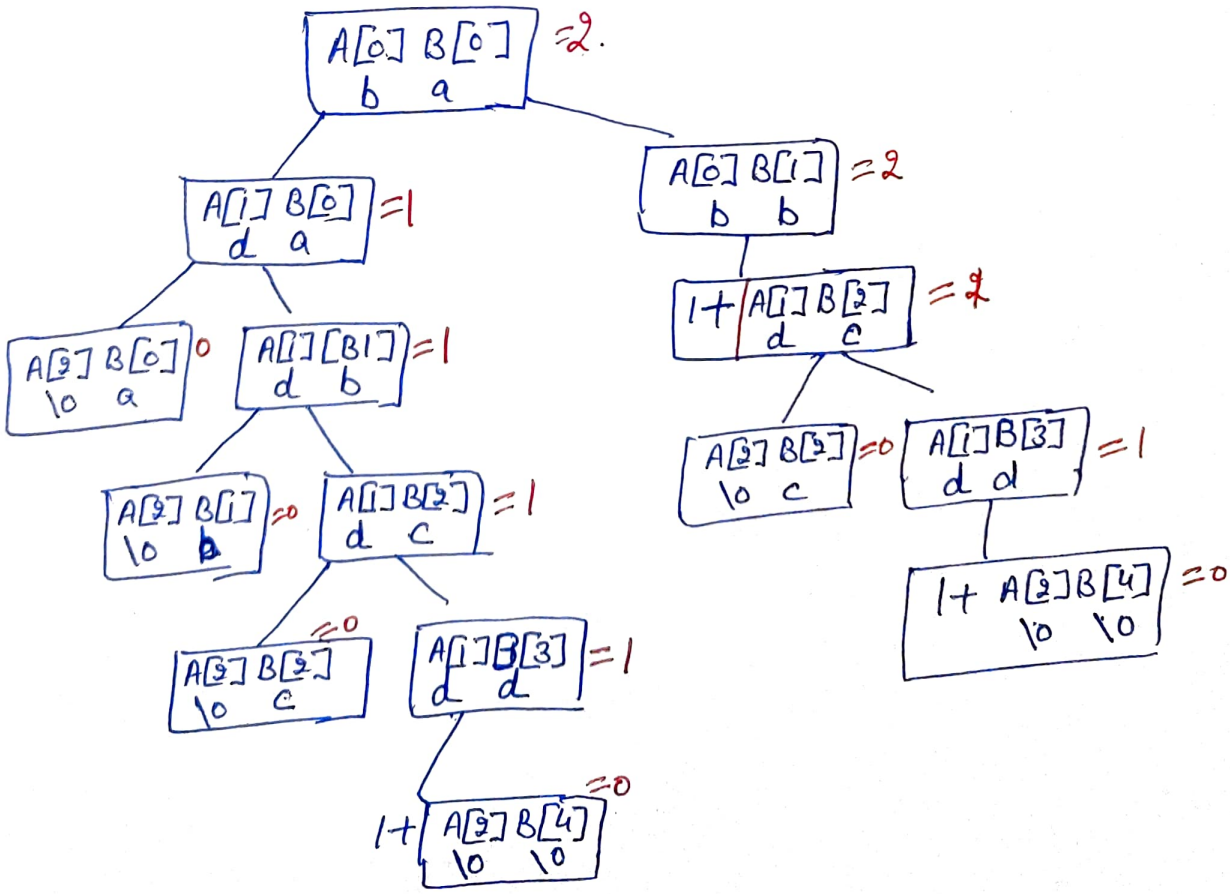
LCS Using Recursion ——

```
int LCS (m, n)
{
    if (A[m] == '\0' || B[n] == '\0')
        return 0;
    else if (A[m] == B[n])
        return (1 + LCS(m+1, n+1));
    else
        return (max(LCS(m+1, n), LCS(m, n+1)));
}
```

3

	0	1	2
A	b	d	\0

	0	1	2	3	4
B	a	b	c	d	\0



→ It is a exponential time taking ~~problem~~ algorithm.
→ It is a top down approach as it start from Ist element of string 1 and string 2
→ Overlapping problem exists in the above algo but it can be solved.

LCS using memorization. —

Storing the result of recursion is called memorization.

To improve the recursion we take help of memorization and reduce its time.

Memorization reduces the no of function calls.

size of table is $m \times n$ $\quad A \to m$ size

Time = $O(m \times n)$ $\quad B \to n$ size.

		a	b	c	d	\0
		0	1	2	3	4
a	0	2	2			
b	1	1	1	1	1	
\0	2	0	0	0		0

LCS using Dynamic Programming →

(1)

	1	2
A	b	d

	1	2	3	4
B	a	b	c	d

If (A[m] == B[n])

LCS[m,n] = 1 + LCS[m-1, n-1]

else

LCS[m,n] = max(LCS[m-1,n], LCS[m,n-1])

		a	b	c	d
	0	1	2	3	4
0	0	0	0	0	0
b 1	0	0	1	1	1
d 2	0	0	1	1	2

b d

size of largest common subsequence is 2

Time is O(m×n)

e.g (2)

string 1 stone

string 2 longest

			L	O	N	G	E	S	T
		0	1	2	3	4	5	6	7
$	0	0	0	0	0	0	0	0	0
S	1	0	0	0	0	0	0	1	1
T	2	0	0	0	0	0	0	1	2
O	3	0	0	1	1	1	1	1	2
N	4	0	0	1	2	2	2	2	2
E	5	0	0	1	2	2	3	3	3

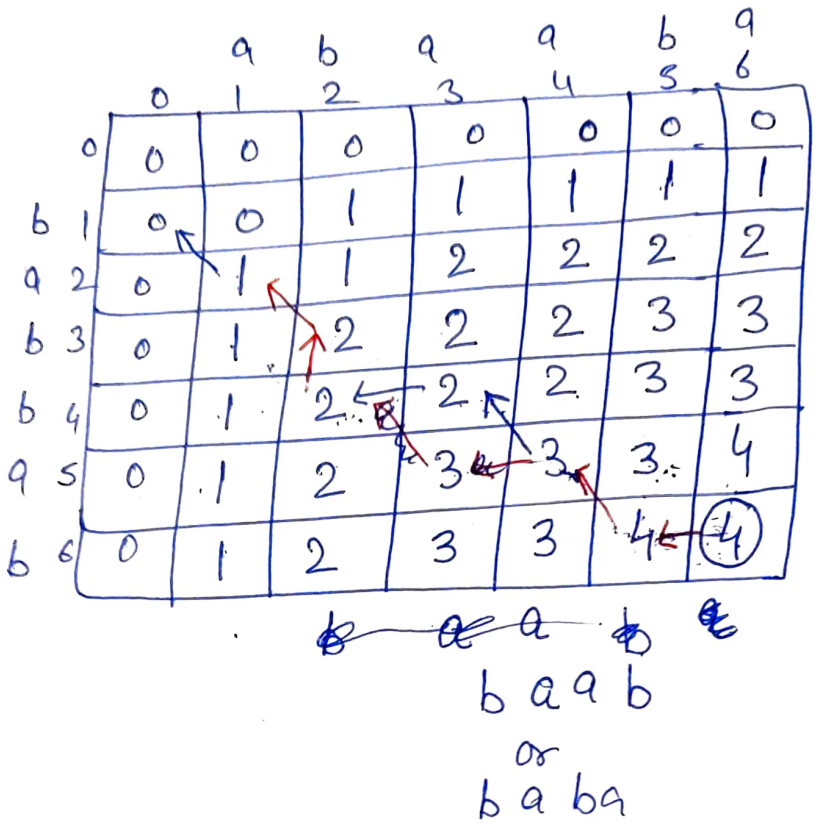
O N e

Ans = one

largest common subsequence is one of size 3

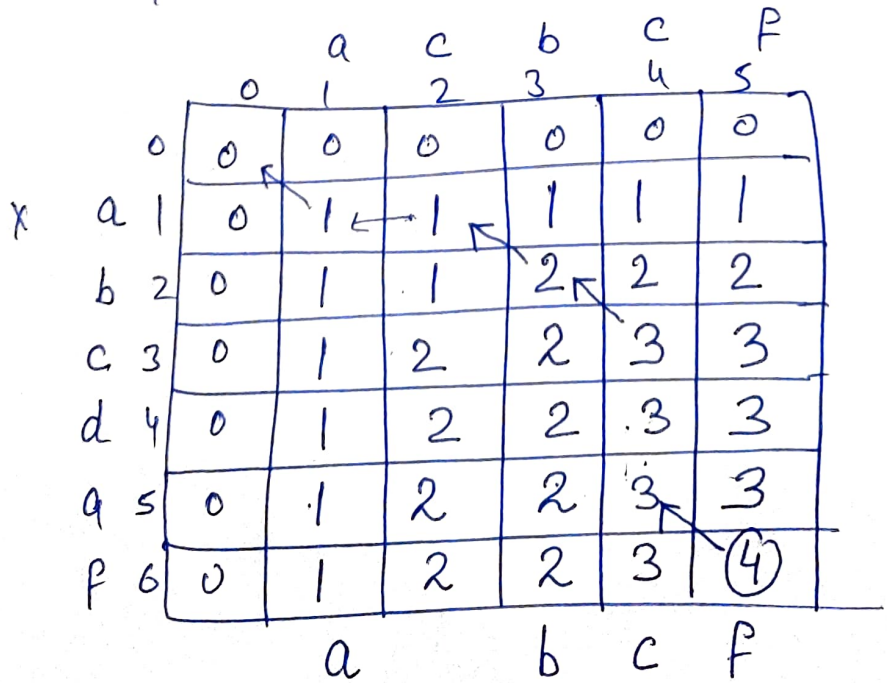
eg 3 $X = a\ b\ a\ a\ b\ a$

$Y = b\ a\ b\ b\ a\ b$



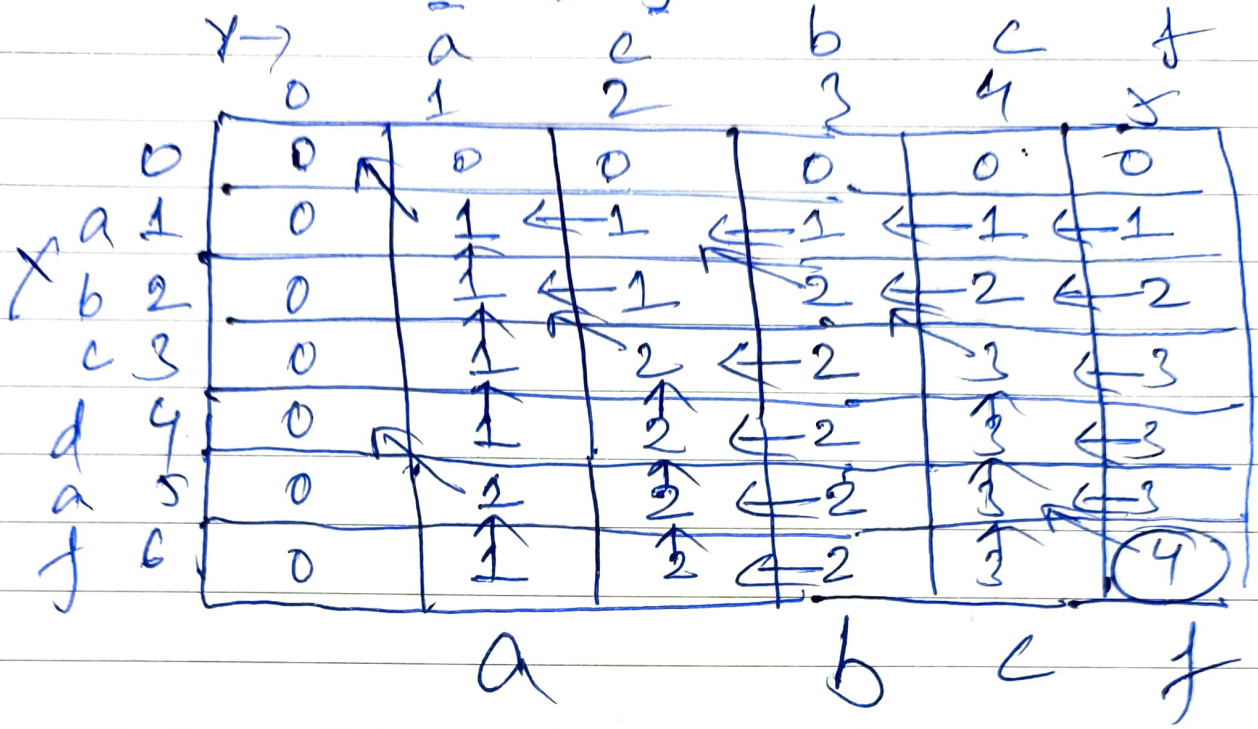
eg 4 $X = a\ b\ c\ d\ a\ f$

$Y = a\ c\ b\ c\ f$



Q2 $X = a\ b\ c\ d\ a\ f$

$Y = a\ c\ b\ c\ f$



a b c f

a b c f