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Huffman Coding Algorithm →

An extended binary tree or 2 tree is a binary tree T in which each node has either 0 or 2 children. The nodes with 0 children are called external nodes and nodes with 2 childrens are called internal nodes. Internal nodes are represented by a circle and external nodes are represented by square.

In any 2 tree no of external nodes N_E is 1 more than the no of internal nodes

i.e $N_E = N_I + 1$

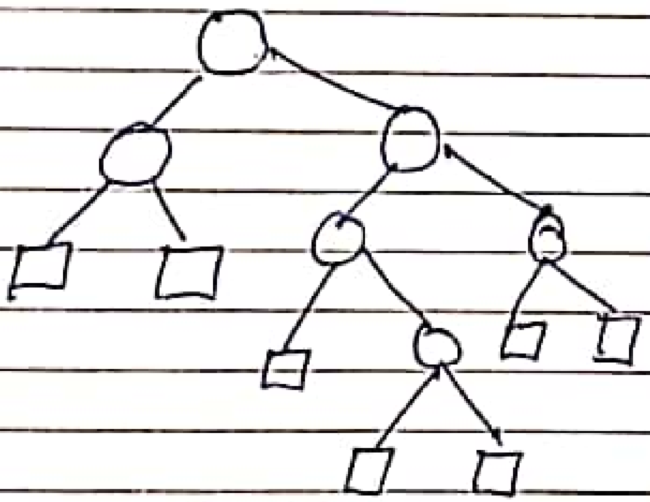


Fig 2 Tree

External Path length (L_E) :

The external path length L_E of a 2 tree T to be the sum of all path length summed over each path from the root R of T to an external node

$$L_E = 2+2+3+4+4+3+3 = 21$$

Internal Path Length (L_I) —

$$L_I = 0+1+1+2+3+2 = 9$$

Observe that

$$\boxed{L_I + 2n = L_E}$$

Internal ~~external~~

where n is no of nodes.

Suppose T is a 2 tree with n external nodes and suppose each of external node is assigned a non negative weight. The weighted Path length P of the tree T is defined to be the sum of weighted path length

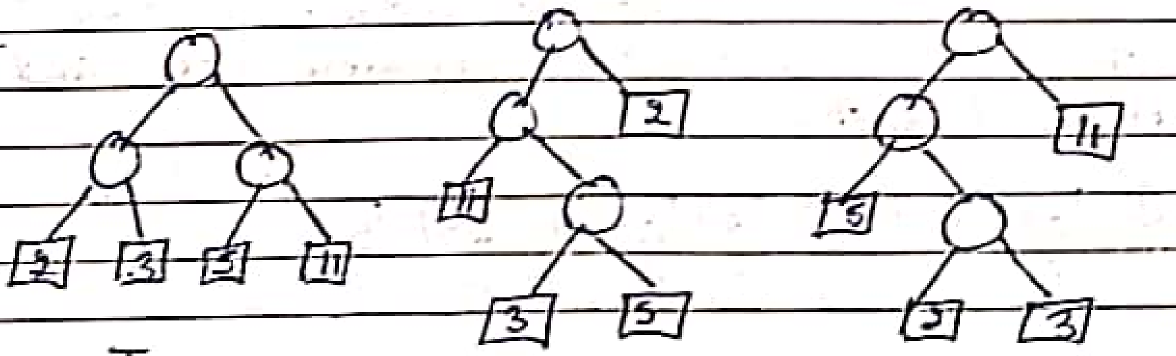
i.e $\quad P = \omega_1 L_1 + \omega_2 L_2 + \cdots + \omega_n L_n$.

where $\omega \rightarrow$ weight

$L \rightarrow$ length of external Node N_i

Consider now the collection of all 2 trees with n external nodes. Clearly the complete tree among them will have a minimal external path length L_E. On the other hand suppose each tree is given the same n weight for its external nodes. Then it is not clear which tree will give a minimal weighted path length P.

e.g



T_1 $\quad$ T_2 $\quad$ T_3

$P_1 = 2\times2 + 3\times2 + 5\times2 + 11\times2 = 42$

$P_2 = 2\times1 + 3\times3 + 5\times3 + 11\times2 = 48$

$P_3 = 2\times3 + 3\times3 + 5\times2 + 11\times1 = 36$

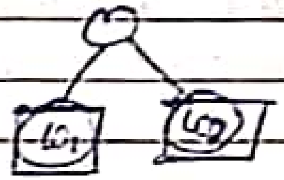
Huffman Algorithm :

Alg : Suppose w_1 and w_2 are two minimum weights among the n given weight $w_1, w_2, \ldots, w_n$. Find a tree T' which gives a solution for the n-1 weights.

$w_1 + w_2, w_3, w_4, \ldots, w_n$.

Then in the tree, replace the external node

$\boxed{w_1 + w_2}$ by subtree



The new 2-tree T is the desired solution

Q.1 Suppose A, B, C, D, ~~and~~ E, F, G and H are 8 data items and suppose they are assigned weights as follows.

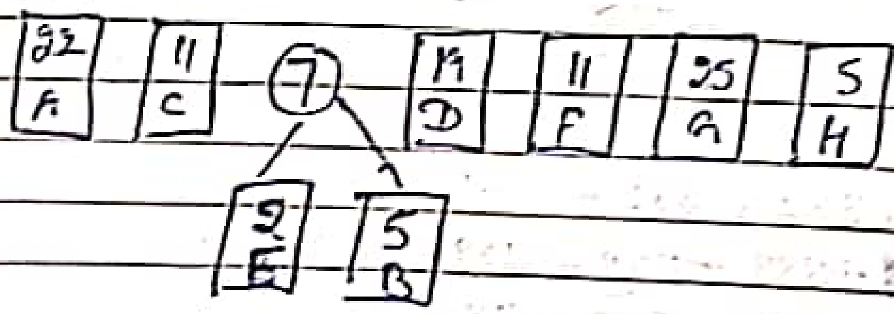
Data	A	B	C	D	E	F	G	H
Weight	22	5	11	19	2	11	25	5

Show how to construct the tree T with minimum weighted Path using above data and Huffman's Algorithm.

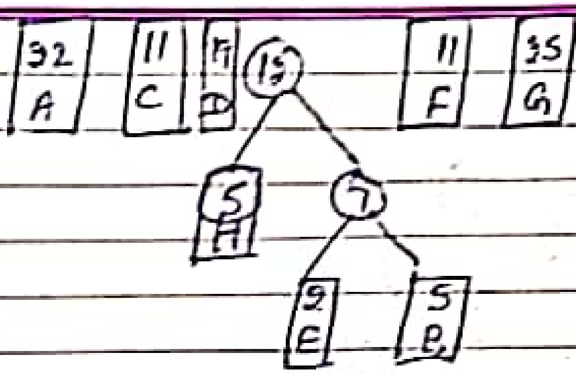
Sol^n ①

22	5	11	19	2	11	25	5
A	B	C	D	E	F	G	H

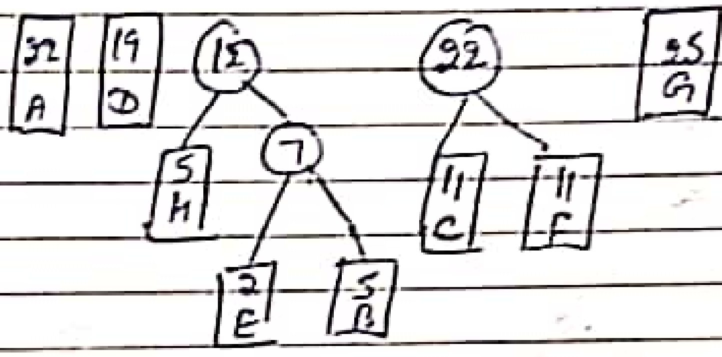
②



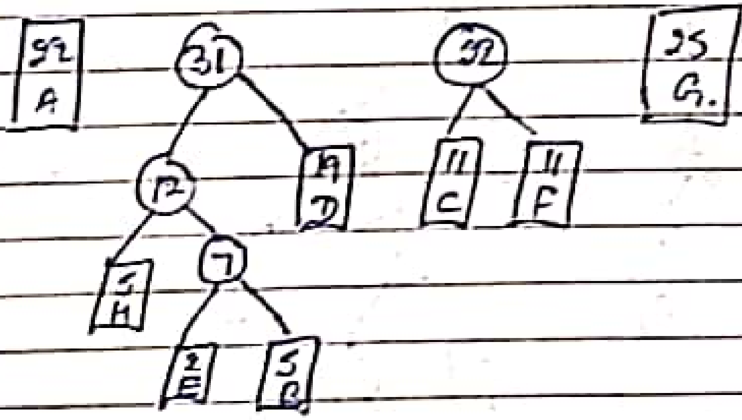
3


32
A
11
C
19
D
12
5
H
7
2
E
5
B
11
F
25
G

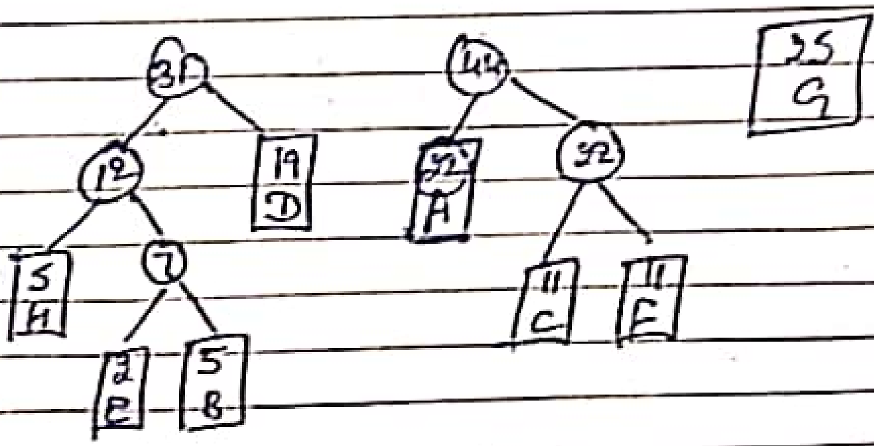
4


32
A
19
D
12
5
H
7
2
E
5
B
22
11
C
11
F
25
G

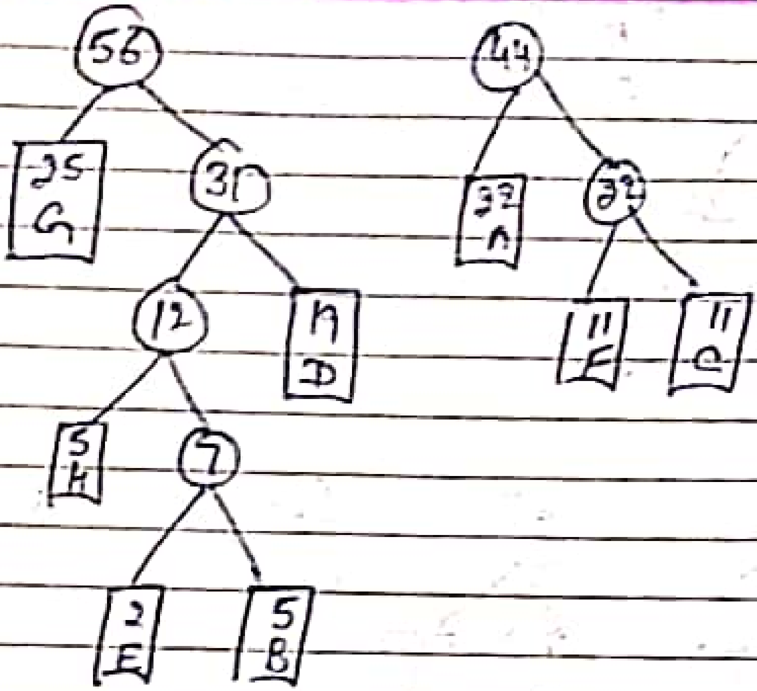
5


32
A
31
12
19
D
5
H
7
2
E
5
B
22
11
C
11
F
25
G.

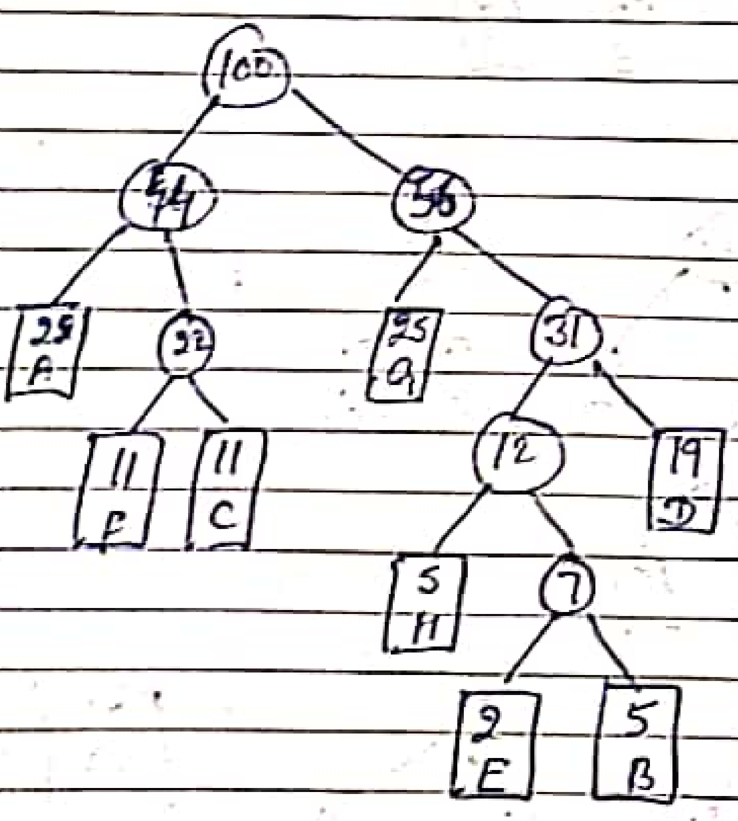
6


31
12
19
D
5
H
7
2
E
5
B
44
32
A
22
11
C
11
F
25
G

1

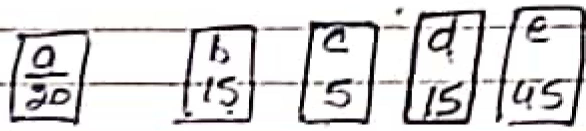

56
25
G
31
12
19
D
5
H
7
2
E
5
B
44
22
A
22
11
F
11
C

2

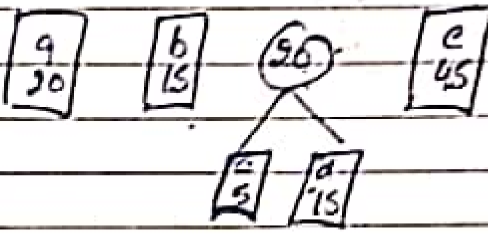

100
44
56
25
A
22
25
G
31
11
F
11
C
12
19
D
5
H
7
2
E
5
B

Ans

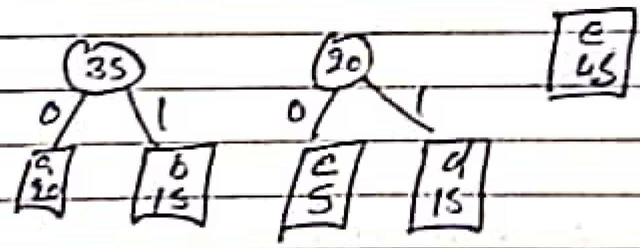
Q. let $A = \{a/20, b/15, c/5, d/15, e/45\}$
Find huffman tree.



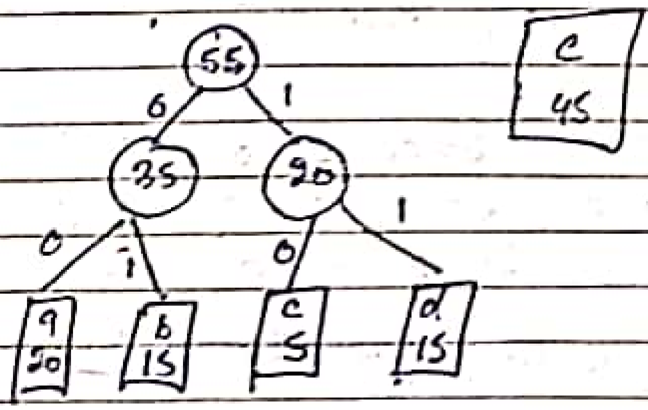
① merge c and d.



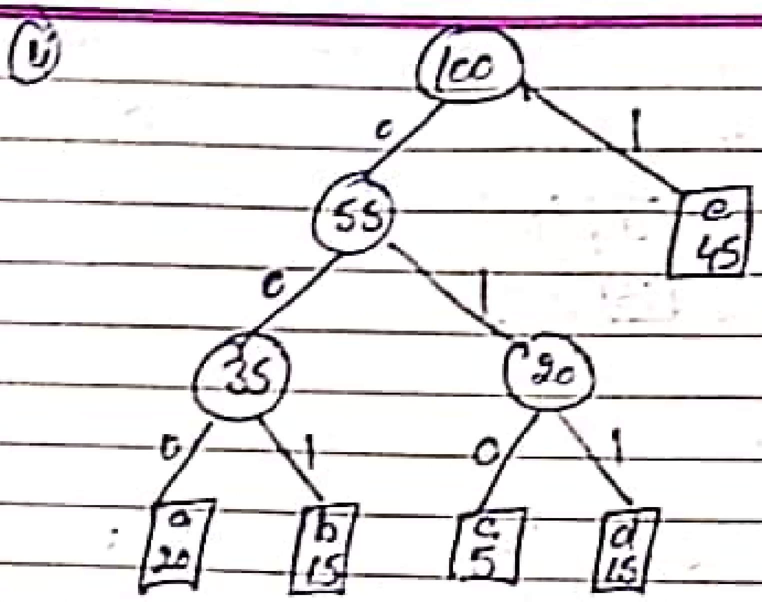
② Merge a and b



③



(ii)



a = 000
b = 001
c = 010
d = 011
e = 1

Ans

e.g. (1) a = 0, b = 110 c = 10 d = 111

Prefix free code

(2) C_1 = { a = 00 b = 01 c = 10 d = 11
C_2 = { a = 0 b = 110 c = 10 d = 111
C_3 = { a = 1 b = 110 c = 10 d = 111

In C_1 010011 is uniquely decodable as bad

C_2 1100111 is uniquely decodable as bad

C_3 1101111 is not uniquely decodable
It could be bad or good

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	a	b	c	d	e	f
Freq in 000	45	13	12	16	9	5
a fixed length	000	001	010	011	100	101
Variable length	0	101	100	111	1101	1100

Fixed length code requires 300 000
Variable length code uses 224000 bits