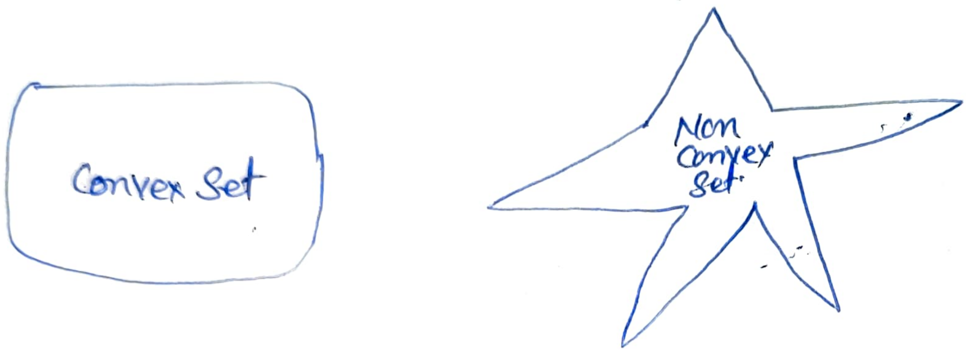
Convex Hull Problem →

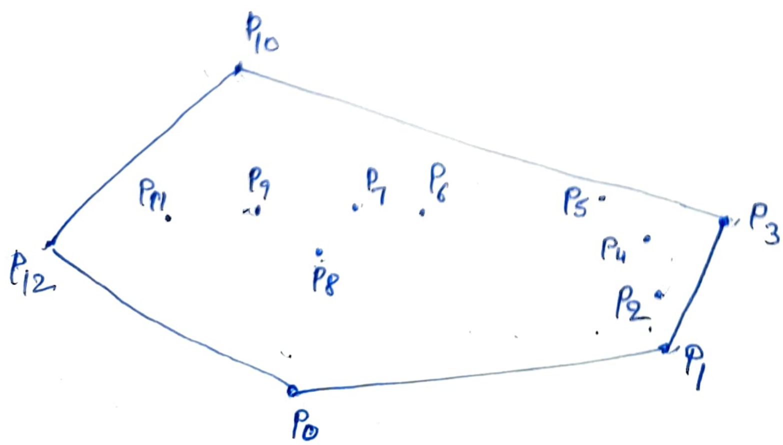


Convex : A Set of points in the plane is called convex if for any two points P and Q in the set, the entire line segment with endpoints P and Q belong to the Set




Convex Hull : The convex hull of a Set S of points is the smallest convex set containing S.

Convex Polygon — Convex hull of any set of $n > 2$ points (not all on the same line) is a convex polygon with vertices at some of the points of S. (If all the points do lie on the same line, the polygon degenerates to a line segment).



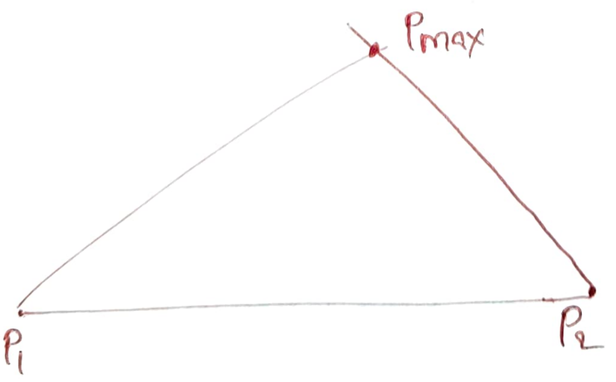
Convex Hull Problem → Problem of constructing convex hull for given set S of n points.

Brute Force Approach → If there are n points, for those n points, there are $\binom{n}{3}$ triangles, enlist all such triangles. For each Δ, whatever point lies inside it, remove those points. The points which do not lie inside are extreme points.

Any Point which lies inside a Δ will never lies on the convex hull

$^{n}C_3$ $O(n^3)$ $\frac{n(n-1)(n-2)}{6}$

$\underline{O(1)}$ $O(n)$



QuickHull (S)



{

// Find convex hull from the set S of n points.

convex hull = { }

1. Find left and right most points, say A and B and add A and B to convex hull

2 Segment AB divides the remaining (n-2) points into two groups $S_1$ and $S_2$

$S_1$ ⇒ Points in S that are on right side of oriented line from A to B

$S_2$ ⇒ Points in S that are on right side of the oriented line from B to A

3 Find_Hull ($S_1$, A, B)

Find-Hull($S_2$, B, A)

Find-hull ($S_K$, P, Q)

{

Find points on convex hull from the set $S_K$ of points that are on the right side of the oriented line from P to Q

If $S_K$ has no Point,

then return

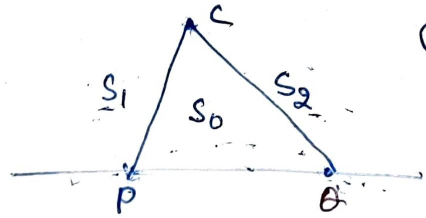
From given set of Points in $S_K$, find farthest Point, say C, from segment PQ. Add Point C to convex hull at the location between P and Q. three Points P and Q and C Partitions the remaining points of $S_K$ into 3 subsets:

$S_0$ are Points inside the Δ

$S_1$ are points ~~right~~ Left side of the oriented line from P to C

$S_2$ are points on the ~~right~~ Left side of oriented line from C to Q

Findhull (S1, P, C)
Findhull (S2, C, Q.




3.

## Time Complexity

**QuickHull**

(P-Q partition the set S in size $\ell$ and $n-\ell$)

(Complexity to find the distance of all Point frome line PQ and finding max

$$T(n) = T(\ell) + T(n-\ell) + O(n)$$

$$T(n) = 2T(n/2) + O(n)$$

$$T(n) = O(n \log n)$$ Avarage

Worst

$$T(n) = O(n^2)$$

## Applications of ~~cer~~ Convex Hull —

→ Geographic Information System
→ Image Processing
→ Pattern Recognition
→ Game theory.