

## **Chapter 4: Knowledge Representation, Inferential reasoning**

# Knowledge Representation and Reasoning: Logic

## Logical Agents

### General features of a representation

#### *Representational adequacy*

The ability to represent all of the kinds of knowledge that are needed in a certain domain.

#### *Inferential adequacy*

The ability to represent all of the kinds of inferential procedures (procedures that manipulate the representational structures in such a way as to derive new structures corresponding to new knowledge inferred from old).

#### *Inferential efficiency*

The ability to represent efficient inference procedures (for instance, by incorporating into the knowledge structure additional information that can be used to focus the attention of the inference mechanisms in the most promising directions).

#### *Acquisitional efficiency*

The ability to acquire new information easily.

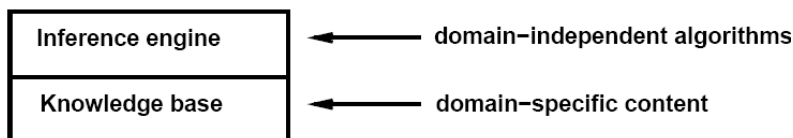
## Logic

Logic (Knowledge-Based) agents combine general knowledge with current percepts to infer hidden aspects of current state prior to selecting actions

- Crucial in partially observable environments

### Knowledge Base

**Knowledge Base** : set of sentences represented in a knowledge representation language and represents assertions about the world.



Then it can ask itself what to do-answers should follow from the KB

Agents can be viewed at the knowledge level

i.e., what they know, regardless of how implemented

Or at the implementation level

i.e., data structures in KB and algorithms that manipulate them

### **Abilities KB agent**

**Agent must be able to:**

- Represent states and actions,
- Incorporate new percepts
- Update internal representation of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

### **Logic**

Logics are formal languages for representing information such that conclusions can be drawn: It has syntax and semantics:

- Syntax defines the sentences in the language
- Semantics define the meaning of sentences. that defines truth of a sentence in a world
- 

E.g., the language of arithmetic

$x + 2 \geq y$  is a sentence;  $x2 + y >$  is not a sentence

$x + 2 \geq y$  is true iff the number  $x + 2$  is no less than the number  $y$

$x + 2 \geq y$  is true in a world where  $x=7$ ;  $y=1$

$x + 2 \geq y$  is false in a world where  $x=0$ ;  $y=6$

### **E n t a i l m e n t**

Entailment means that one thing follows from another:

$$KB \models \alpha$$

Knowledge base KB entails sentence  $\alpha$  if and only if  $\alpha$  is true in all worlds where KB is true

E.g.,  $x + y = 4$  entails  $4 = x + y$

Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

## M o d e l s

Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated.

$m$  is a model of a sentence  $\alpha$  if  $\alpha$  is true in  $m$ .

$M(\alpha)$  is the set of all models of  $\alpha$

## L o g i c a l i n f e r e n c e

If an inference algorithm  $i$  can derive  $\alpha$  from  $KB$ , we write  $KB \vdash_i \alpha$  which is pronounced as “ $i$  derives  $\alpha$  from  $KB$ ”

If an algorithm only derives entailed sentences it is called *sound* or *truth preserving*.

– Otherwise it just makes things up.

*$i$  is sound if whenever  $KB \vdash_i \alpha$  it is also true that  $KB \models \alpha$*

Completeness : the algorithm can derive any sentence that is entailed.

*$i$  is complete if whenever  $KB \models \alpha$  it is also true that  $KB \vdash_i \alpha$*

## Propositional Logic:Syntax

Propositional logic is the simplest logic. We use the symbols like  $P_1, P_2$  to represent sentences. Propositional logic is defined as:

If  $S$  is a sentence,  $\neg S$  is a sentence (negation)

If  $S_1$  and  $S_2$  are sentences,  $S_1 \wedge S_2$  is a sentence (conjunction)

If  $S_1$  and  $S_2$  are sentences,  $S_1 \vee S_2$  is a sentence (disjunction)

If  $S_1$  and  $S_2$  are sentences,  $S_1 \Rightarrow S_2$  is a sentence (implication)

If  $S_1$  and  $S_2$  are sentences,  $S_1 \Leftrightarrow S_2$  is a sentence (biconditional)

Formal grammar for propositional logic can be given as below:

Sentence	$\rightarrow$ AtomicSentence   ComplexSentence
AtomicSentence	$\rightarrow$ True   False   Symbol
Symbol	$\rightarrow$ P   Q   R .....
ComplexSentence	$\rightarrow$ $\neg$ Sentence   (Sentence $\wedge$ Sentence)   (Sentence $\vee$ Sentence)   (Sentence $\Rightarrow$ Sentence)   (Sentence $\Leftrightarrow$ Sentence)

## Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

Rules for evaluating truth with respect to a model:

$\neg S$  is true if,  $S$  is false

$S1 \wedge S2$  is true if,  $S1$  is true and  $S2$  is true

$S1 \vee S2$  is true if,  $S1$  is true or  $S2$  is true

$S1 \Rightarrow S2$  is true if,  $S1$  is false or  $S2$  is true

$S1 \Leftrightarrow S2$  is true if,  $S1 \Rightarrow S2$  is true and  $S2 \Rightarrow S1$  is true

Truth Table showing the evaluation of semantics of complex sentences:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true

## Logical equivalence

Two sentences  $\alpha$  and  $\beta$  are *logically equivalent* ( $\alpha \equiv \beta$ ) iff true they are true in same set of models

or Two sentences  $\alpha$  and  $\beta$  are *logically equivalent* ( $\alpha \equiv \beta$ ) iff  $\alpha \models \beta$  and  $\beta \models \alpha$ .

$$\begin{aligned}(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\ \neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\ \neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{de Morgan} \\ \neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{de Morgan} \\(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge\end{aligned}$$

## Validity

A sentence is *valid* if it is true in all models,

e.g., *True*,  $A \vee \neg A$ ,  $A \Rightarrow A$ ,  $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Valid sentences are also known as tautologies

Every valid sentence is logically equivalent to True

## Satisfiability

A sentence is *satisfiable* if it is true in *some* model

- e.g.,  $A \vee B, C$

A sentence is *unsatisfiable* if it is true in *no* models

- e.g.,  $A \neg \wedge A$

Validity and satisfiability are related concepts

- $\alpha$  is valid iff  $\neg\alpha$  is unsatisfiable
- $\alpha$  is satisfiable iff  $\neg\alpha$  is not valid

Satisfiability is connected to inference via the following:

- $KB \models \alpha$  if and only if  $(KB \wedge \neg\alpha)$  is unsatisfiable

## Inference rules in PL

Modus Ponens

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

And-elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

**Monotonicity:** the set of entailed sentences can only increase as information is added to the knowledge base.

For any sentence  $\alpha$  and  $\beta$  if  $KB \models \alpha$  then  $KB \wedge \beta \models \alpha$ .

## Resolution

**Unit resolution rule:**

Unit resolution rule takes a clause – a disjunction of literals – and a literal and produces a new clause. Single literal is also called unit clause.

$$\frac{\ell_1 \vee \dots \vee \ell_k, \quad m}{\ell_1 \vee \dots \vee \ell_{i-1} \vee \ell_{i+1} \vee \dots \vee \ell_k}$$

Where  $\ell_i$  and  $m$  are complementary literals

**Generalized resolution rule:**

Generalized resolution rule takes two clauses of any length and produces a new clause as below.

$$\frac{\ell_1 \vee \dots \vee \ell_k, \quad m_1 \vee \dots \vee m_n}{\ell_1 \vee \dots \vee \ell_{i-1} \vee \ell_{i+1} \vee \dots \vee \ell_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

For example

$$\frac{\ell_1 \vee \ell_2, \quad \neg \ell_2 \vee \ell_3}{\ell_1 \vee \ell_3}$$

Resolution Uses CNF (Conjunctive normal form)

- Conjunction of disjunctions of literals (clauses)

The resolution rule is sound:

- Only entailed sentences are derived

Resolution is complete in the sense that it can always be used to either confirm or refute a sentence (it can not be used to enumerate true sentences.)

**Conversion to CNF**

A sentence that is expressed as a conjunction of disjunctions of literals is said to be in conjunctive normal form (CNF).

A sentence in CNF that contains only k literals per clause is said to be in k-CNF.

Let's illustrate the conversion to CNF by using an example.

$$B \Leftrightarrow (A \vee C)$$

- Eliminate  $\Leftrightarrow$ , replacing  $\alpha \Leftrightarrow \beta$  with  $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$ .
  - $(B \Rightarrow (A \vee C)) \wedge ((A \vee C) \Rightarrow B)$
- Eliminate  $\Rightarrow$ , replacing  $\alpha \Rightarrow \beta$  with  $\neg \alpha \vee \beta$ .
  - $(\neg B \vee A \vee C) \wedge (\neg(A \vee C) \vee B)$
- Move  $\neg$  inwards using de Morgan's rules and double-negation:
  - $(\neg B \vee A \vee C) \wedge ((\neg A \wedge \neg C) \vee B)$
- Apply distributivity law ( $\wedge$  over  $\vee$ ) and flatten:
  - $(\neg B \vee A \vee C) \wedge (\neg A \vee B) \wedge (\neg C \vee B)$

## Resolution algorithm

Convert KB into CNF

Add negation of sentence to be entailed into KB i.e.  $(KB \wedge \neg\alpha)$

Then apply resolution rule to resulting clauses.

The process continues until:

- There are no new clauses that can be added
  - Hence  **$KB$  does not** entail  $\alpha$
- Two clauses resolve to entail the empty clause.
  - Hence  **$KB$  does** entail  $\alpha$

## Resolution example

Consider the knowledge base given as below:

$$KB = (B \Leftrightarrow (A \vee C)) \wedge \neg B$$

Prove that  $\neg A$  can be inferred from above KB by using resolution.

Convert KB into CNF

$$B \Rightarrow (A \vee C) \wedge ((A \vee C) \Rightarrow B) \wedge \neg B$$

$$(\neg B \vee A \vee C) \wedge (\neg(A \vee C) \vee B) \wedge \neg B$$

$$(\neg B \vee A \vee C) \wedge ((\neg A \wedge \neg C) \vee B) \wedge \neg B$$

$$(\neg B \vee A \vee C) \wedge (\neg A \vee B) \wedge (\neg C \vee B) \wedge \neg B$$

Add negation of sentence to be inferred from KB into KB

Now KB contains following sentences all in CNF

$$(\neg B \vee A \vee C)$$

$$(\neg A \vee B)$$

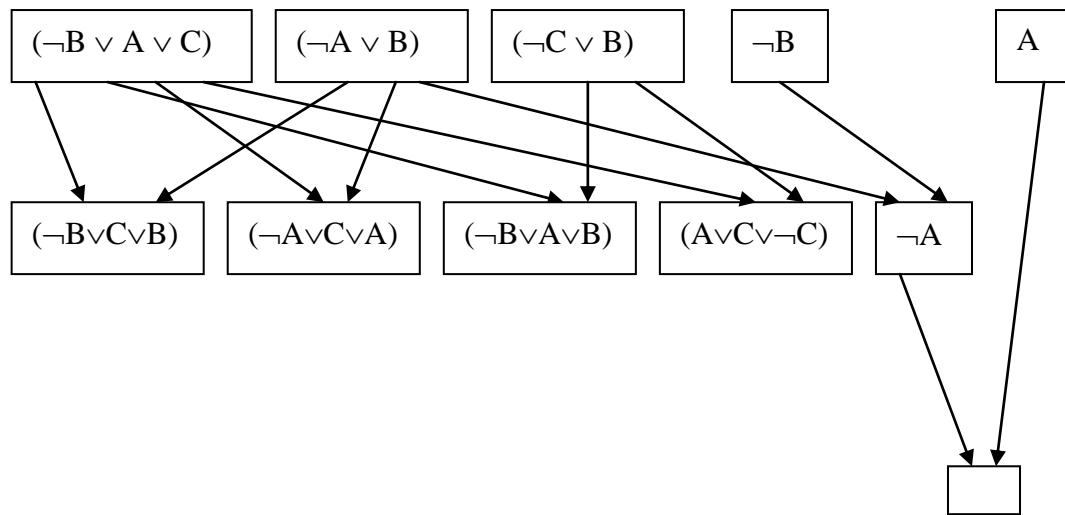
$$(\neg C \vee B)$$

$$\neg B$$

$$A$$

Now use Resolution algorithm





## Forward and backward chaining

The completeness of resolution makes it a very important inference model. But in many practical situations full power of resolution is not needed. Real-world knowledge bases often contain only clauses of restricted kind called **Horn Clause**. A Horn clause is disjunction of literals with at most one positive literal

Three important properties of Horn clause are:

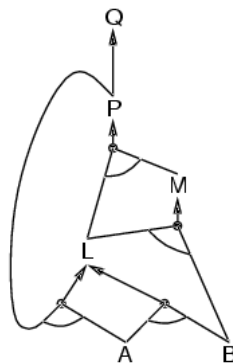
- ✓ Can be written as an implication
- ✓ Inference through forward chaining and backward chaining.
- ✓ Deciding entailment can be done in a time linear size of the knowledge base.

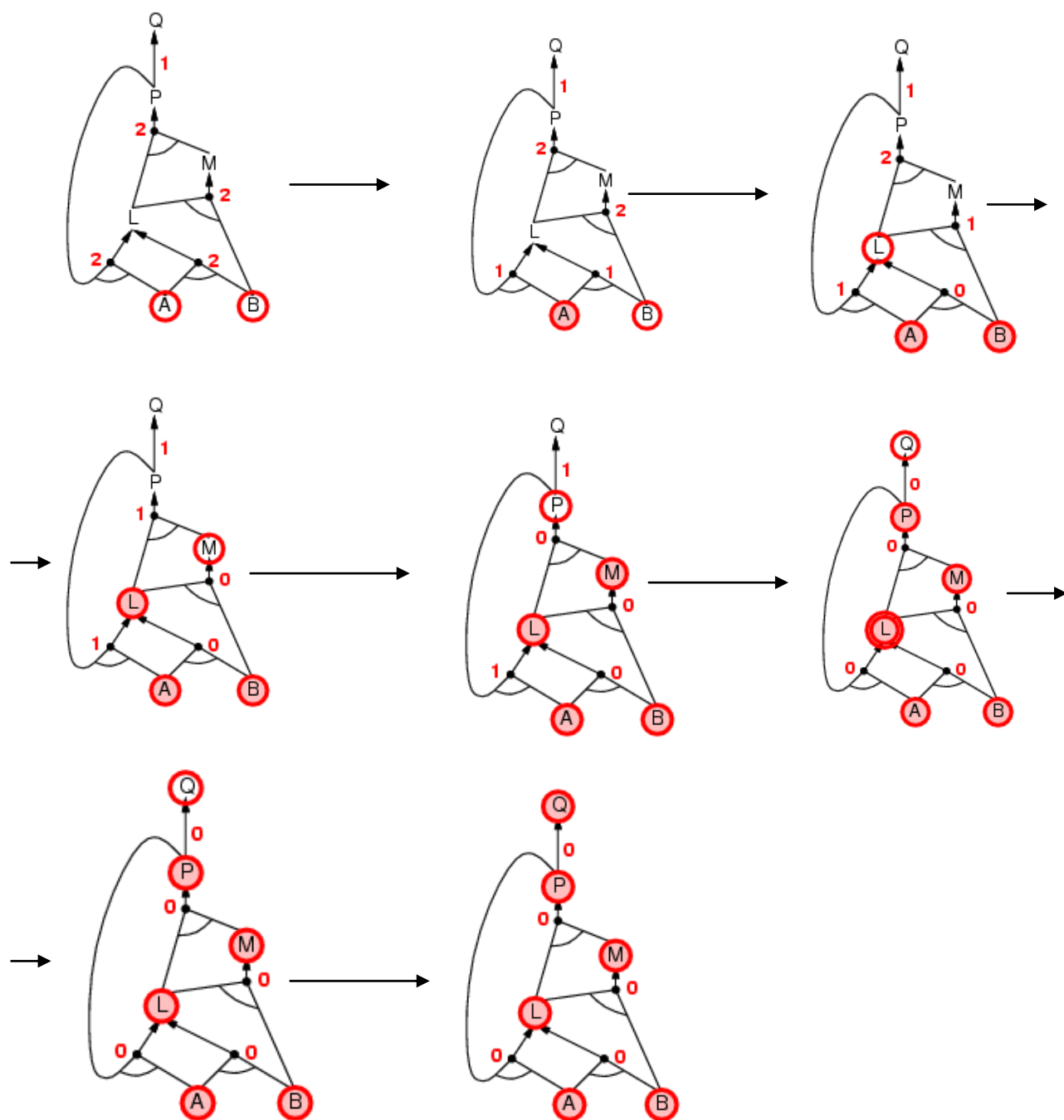
## Forward chaining

Idea: fire any rule whose premises are satisfied in the *KB*,

- add its conclusion to the *KB*, until query is found

$P \Rightarrow Q$   
 $L \wedge M \Rightarrow P$   
 $B \wedge L \Rightarrow M$   
 $A \wedge P \Rightarrow L$   
 $A \wedge B \Rightarrow L$   
 $A$   
 $B$





### **Backward chaining**

Idea: work backwards from the query  $q$ : to prove  $q$  by BC,

Check if  $q$  is known already, or

Prove by BC all premises of some rule concluding  $q$

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal

1. has already been proved true, or
2. has already failed

For example for above KB

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

$A$

$B$

Prove that  $Q$  can be inferred from above KB

We know  $P \Rightarrow Q$ , try to prove  $P$

$L \wedge M \Rightarrow P$

Try to prove  $L$  and  $M$

$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

Try to prove  $B$ ,  $L$  and  $A$  and  $P$

$A$  and  $B$  is already known, since  $A \wedge B \Rightarrow L$ ,  $L$  is also known

Since,  $B \wedge L \Rightarrow M$ ,  $M$  is also known

Since,  $L \wedge M \Rightarrow P$ ,  $P$  is known, hence the proof

# First-Order Logic

## Pros and cons of propositional logic

Propositional logic is declarative

Propositional logic allows partial/disjunctive/negated information

- (unlike most data structures and databases)

Propositional logic is compositional:

- meaning of  $B \wedge P$  is derived from meaning of  $B$  and of  $P$

Meaning in propositional logic is context-independent

- (unlike natural language, where meaning depends on context)

Propositional logic has very limited expressive power

- (unlike natural language)
- E.g., cannot say "pits cause breezes in adjacent squares"
  - except by writing one sentence for each square

## First-order logic

Whereas propositional logic assumes the world contains facts,

first-order logic (like natural language) assumes the world contains

- Objects: people, houses, numbers, colors, baseball games, wars, ...
- Relations: red, round, prime, brother of, bigger than, part of, comes between, ...
- Functions: father of, best friend, one more than, plus, ...

## Logics in General

The primary difference between PL and FOPL is their ontological commitment:

Ontological Commitment: What exists in the world — TRUTH

- PL: facts hold or do not hold.
- FL : objects with relations between them that hold or do not hold

Another difference is:

Epistemological Commitment: What an agent believes about facts — BELIEF

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	degree of truth $\in [0, 1]$	known interval value

## Representing knowledge in first-order logic

The objects from the real world are represented by constant symbols (a,b,c,...). For instance, the symbol “Tom” may represent a certain individual called Tom.

Properties of objects may be represented by predicates applied to those objects (P(a), ...): e.g "male(Tom)" represents that Tom is a male.

Relationships between objects are represented by predicates with more arguments: "father(Tom, Bob)" represents the fact that Tom is the father of Bob.

The value of a predicate is one of the boolean constants T (i.e. true) or F (i.e. false). "father(Tom, Bob) = T" means that the sentence "Tom is the father of Bob" is true. "father(Tom, Bob) = F" means that the sentence "Tom is the father of Bob" is false.

Besides constants, the arguments of the predicates may be functions (f,g,...) or variables (x,y,...).

Function symbols denote mappings from elements of a domain (or tuples of elements of domains) to elements of a domain. For instance, weight is a function that maps objects to their weight: weight (Tom) = 150. Therefore the predicate greater-than (weight (Bob), 100) means that the weight of Bob is greater than 100. The arguments of a function may themselves be functions.

Variable symbols represent potentially any element of a domain and allow the formulation of general statements about the elements of the domain.

The quantifier's  $\forall$  and  $\exists$  are used to build new formulas from old ones.

" $\exists x P(x)$ " expresses that there is at least one element of the domain that makes  $P(x)$  true.

" $\exists x \text{ mother}(x, \text{Bob})$ " means that there is  $x$  such that  $x$  is mother of Bob or, otherwise stated, Bob has a mother.

" $\forall x P(x)$ " expresses that for all elements of the domain  $P(x)$  is true.

## Quantifiers

Allows us to express properties of collections of objects instead of enumerating objects by name. Two quantifiers are:

Universal: “for all”  $\forall$

Existential: “there exists”  $\exists$

### Universal quantification:

$\forall \langle \text{Variables} \rangle \langle \text{sentence} \rangle$

Everyone at VUB is smart:

$$\forall x \text{ At}(x, \text{VUB}) \Rightarrow \text{Smart}(x)$$

$\forall x P$  is true in a model  $m$  iff  $P$  is true for all  $x$  in the model

Roughly speaking, equivalent to the conjunction of instantiations of  $P$

$\text{At}(\text{KingJohn}, \text{VUB}) \Rightarrow \text{Smart}(\text{KingJohn}) \wedge \text{At}(\text{Richard}, \text{VUB}) \Rightarrow \text{Smart}(\text{Richard})$   
 $\wedge \text{At}(\text{VUB}, \text{VUB}) \Rightarrow \text{Smart}(\text{VUB}) \wedge \dots$

Typically,  $\Rightarrow$  is the main connective with  $\forall$

- A universally quantifier is also equivalent to a set of implications over all objects

Common mistake: using  $\wedge$  as the main connective with  $\forall$ :

$\forall x \text{ At}(x, \text{VUB}) \wedge \text{Smart}(x)$

means “Everyone is at VUB and everyone is smart”

### Existential quantification

$\exists \langle \text{variables} \rangle \langle \text{sentence} \rangle$

Someone at VUB is smart:

$\exists x \text{ At}(x, \text{VUB}) \wedge \text{Smart}(x)$

$\exists x P$  is true in a model  $m$  iff  $P$  is true for at least one  $x$  in the model

Roughly speaking, equivalent to the disjunction of instantiations of  $P$

$\text{At}(\text{KingJohn}, \text{VUB}) \wedge \text{Smart}(\text{KingJohn}) \vee \text{At}(\text{Richard}, \text{VUB}) \wedge \text{Smart}(\text{Richard})$   
 $\vee \text{At}(\text{VUB}, \text{VUB}) \wedge \text{Smart}(\text{VUB}) \vee \dots$

Typically,  $\wedge$  is the main connective with  $\exists$

Common mistake: using  $\Rightarrow$  as the main connective with  $\exists$ :

$\exists x \text{ At}(x, \text{VUB}) \Rightarrow \text{Smart}(x)$

is true even if there is anyone who is not at VUB!

### Properties of quantifiers

$\forall x \forall y$  is the same as  $\forall y \forall x$

$\exists x \exists y$  is the same as  $\exists y \exists x$

$\exists x \forall y$  is not the same as  $\forall y \exists x$

$\exists x \forall y \text{ Loves}(x, y)$

- “There is a person who loves everyone in the world”

$\forall y \exists x \text{ Loves}(x, y)$

- “Everyone in the world is loved by at least one person”

Quantifier duality: each can be expressed using the other

$\forall x \text{ Likes}(x, \text{IceCream}) \equiv \exists \neg x \neg \text{Likes}(x, \text{IceCream})$

$\exists x \text{ Likes}(x, \text{Broccoli}) \equiv \neg \forall x \neg \text{Likes}(x, \text{Broccoli})$

### Equality

$term1 = term2$  is true under a given interpretation if and only if  $term1$  and  $term2$  refer to the same object

e.g.

Father (John) = Henry

Here Objects referred by father of John and Henry is same.

### **Inference in first-order logic**

#### **First Order Logic to Predicate Logic**

First order inference can be done by converting the knowledge base to PL and using propositional inference.

- How to convert universal quantifiers?
  - Replace variable by ground term.
- How to convert existential quantifiers?
  - Skolemization.

#### **Universal instantiation (UI)**

Substitute ground term (term without variables) for the variables.

For example consider the following KB

$\forall x \text{ King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$

King (John)

Greedy (John)

Brother (Richard, John)

It's UI is:

King (John)  $\wedge$  Greedy (John)  $\Rightarrow$  Evil(John)

King (Richard)  $\wedge$  Greedy (Richard)  $\Rightarrow$  Evil(Richard)

King (John)

Greedy (John)

Brother (Richard, John)

Note: Remove universally quantified sentences after universal instantiation.

#### **Existential instantiation (EI)**

For any sentence  $\alpha$  and variable  $v$  in that, introduce a constant that is not in the KB (called skolem constant) and substitute that constant for  $v$ .

e.g.

Consider the sentence.

$\exists x \text{ crown}(x) \wedge \text{OnHead}(x, \text{John})$

After EI

$\text{crown}(C1) \wedge \text{OnHead}(C1, \text{John})$  where C1 is Skolem Constant.

### **EI versus UI**

UI can be applied several times to *add* new sentences; the new KB is logically equivalent to the old.

EI can be applied once to replace the existential sentence; the new KB is not equivalent to the old but is satisfiable if the old KB was satisfiable.

Note: After propositionalization, we can use any algorithm fool PI to derive the conclusion.

### **Unification and Lifting**

Instead of translating the knowledge base to PL, we can redefine the inference rules into FOL.

- Lifting: they only make those substitutions that are required to allow particular inferences to proceed.
- E.g. *generalized Modus Ponens*
  - To introduce substitutions different logical expressions have to be look identical
  - To identify the substitution we need unification algorithm.

#### **Unification:**

We can get the inference immediately if we can find a substitution  $\alpha$  such that  $\text{King}(x)$  and  $\text{Greedy}(x)$  match  $\text{King}(\text{John})$  and  $\text{Greedy}(y)$

$\alpha = \{x/\text{John}, y/\text{John}\}$  works

$\text{Unify}(\alpha, \beta) = \theta$  if  $\alpha\theta = \beta\theta$

p	q	$\theta$
$\text{Knows}(\text{John}, x)$	$\text{Knows}(\text{John}, \text{Jane})$	$\{x/\text{Jane}\}$
$\text{Knows}(\text{John}, x)$	$\text{Knows}(y, \text{OJ})$	$\{x/\text{OJ}, y/\text{John}\}$
$\text{Knows}(\text{John}, x)$	$\text{Knows}(y, \text{Mother}(y))$	$\{y/\text{John}, x/\text{Mother}(\text{John})\}$
$\text{Knows}(\text{John}, x)$	$\text{Knows}(x, \text{OJ})$	$\{\text{fail}\}$

Last unification is failed due to overlap of variables. x can not take the values of John and OJ at the same time.

We can avoid this problem by renaming to avoid the name clashes (standardizing apart)

e.g.

$\text{Unify}\{\text{Knows}(\text{John}, x) \quad \text{Knows}(z, \text{OJ})\} = \{x/\text{OJ}, z/\text{John}\}$



*Another complication:*

To unify  $Knows(John, x)$  and  $Knows(y, z)$ ,

Unification of  $Knows(John, x)$  and  $Knows(y, z)$  gives  $\alpha = \{y/John, x/z\}$  or  $\alpha = \{y/John, x/John, z/John\}$

First unifier gives the result  $Knows(John, z)$  and second unifier gives the result  $Knows(John, John)$ . Second can be achieved from first by substituting john in place of z. The first unifier is more general than the second.

There is a single most general unifier (MGU) that is unique up to renaming of variables.

MGU =  $\{y/John, x/z\}$

### **Generalized Modus Ponens (GMP)**

$p_1', p_2', \dots, p_n', (p_1 \wedge p_2 \wedge \dots \wedge p_n \Rightarrow q)$

$q\theta$

where  $p_i'\theta = p_i\theta$  for all i

$p_1'$  is King(John)

$p_1$  is King(x)

$p_2'$  is Greedy(y)

$p_2$  is Greedy(x)

$\theta$  is  $\{x/John, y/John\}$

$q$  is Evil(x)

$q\theta$  is Evil(John)

GMP used with KB of definite clauses (exactly one positive literal).

### **Forward Chaining:**

We can use FC for FOPL also:

Consider the following KB:

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Prove that Col. West is a criminal

Soln:

it is a crime for an American to sell weapons to hostile nations:

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Nono ... has some missiles, i.e.,

$\exists x Owns(Nono, x) \wedge Missile(x)$ :

Use Existential Instantiation:

$Owns(Nono, M1) \text{ and } Missile(M1)$

... all of its missiles were sold to it by Colonel West  
 $Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

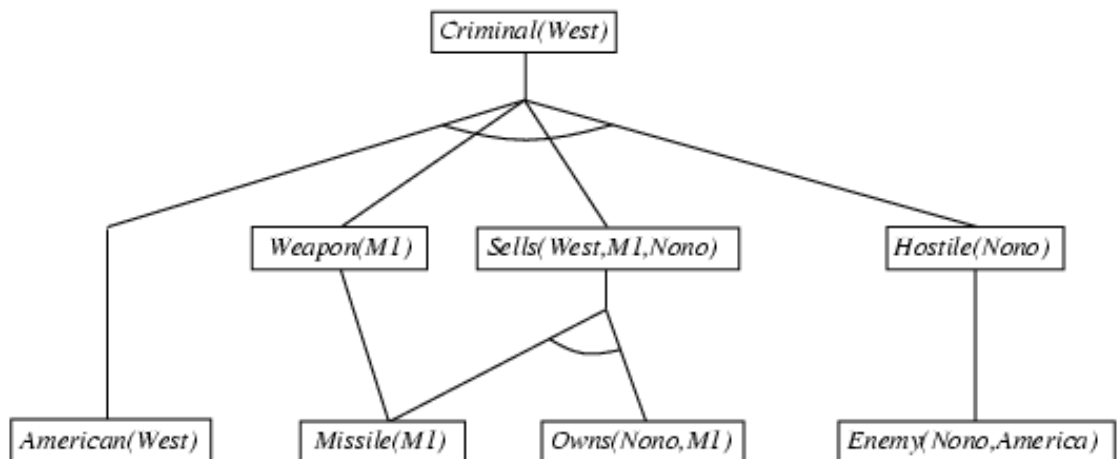
$Enemy(x,America) \Rightarrow Hostile(x)$

West, who is American ...

$American(West)$

The country Nono, an enemy of America ...

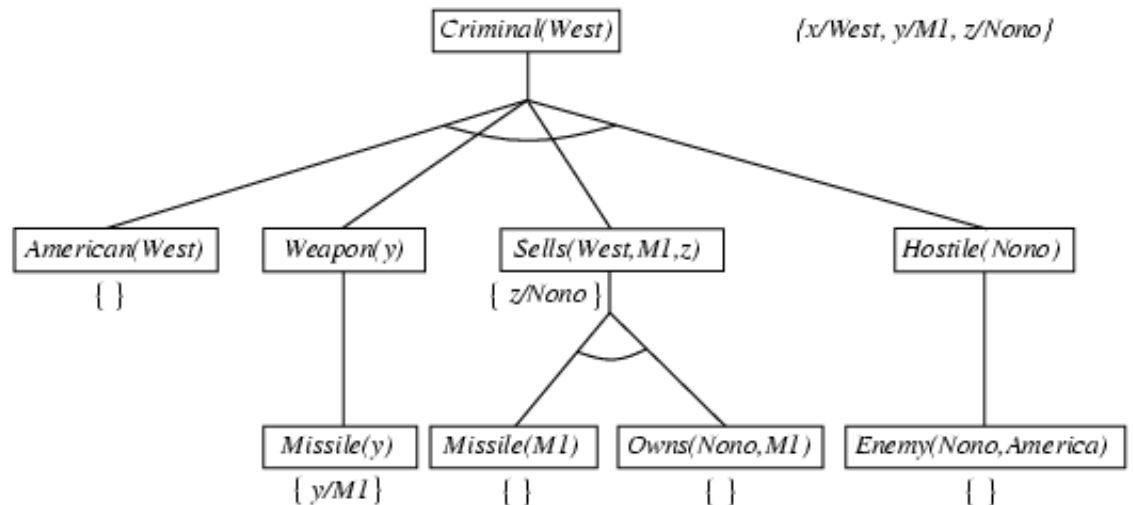
$Enemy(Nono,America)$



### **Backward Chaining:**

We can also use BC algorithm for FOPL.

Eg. For above KB we can use BC al below:



**Exercise:**

- Robert is an American. All Americans who sells weapons are criminals. All missiles are weapons. Robert sells weapons. Prove that Robert is criminal.
- SRK is a superstar. All superstars are rich. Rich mans have fast cars. Fast cars consume a lot of petrol. Prove that SRK's car consumes a lot of petrol.
- All greedy leaders are autocrat. Autocrats are evils. Prashant is a greedy leader. Ram is a honest leader. Prove that Prashant is evil.
- All students of this class takes a course in BSC IT. BSC IT students are good in AI. Ritu is student of this class. Prove that Ritu is good in AI.