# **Steps to Convert to CNF (Conjunctive Normal Form)**

Every sentence in Propositional Logic is logically equivalent to a conjunction of disjunctions of literals. A sentence expressed as a conjunction of disjunctions of literals is said to be in Conjunctive normal Form or CNF.

1. Eliminate implication ' $\rightarrow$ '

2. Eliminate Existential Quantifier '∃'

To eliminate an independent Existential Quantifier, replace the variable by a Skolem constant. This process is called as Skolemization.

**Example**:  $\exists y$ : President (y)

Here 'y' is an independent quantifier so we can replace 'y' by any name (say – George Bush).

So,  $\exists y$ : President (y) becomes President (George Bush).

To eliminate a dependent Existential Quantifier we replace its variable by Skolem Function that accepts the value of 'x' and returns the corresponding value of 'y.'

**Example:**  $\forall x : \exists y : father\_of(x, y)$ 

Here 'y' is dependent on 'x', so we replace 'y' by S(x).

So,  $\forall x : \exists y : father_of(x, y)$  becomes  $\forall x : \exists y : father_of(x, S(x))$ .

3. Eliminate Universal Quantifier '∀'

To eliminate the Universal Quantifier, drop the prefix in PRENEX NORMAL FORM i.e. just drop  $\forall$  and the sentence then becomes in PRENEX NORMAL FORM.

4. Eliminate AND '^'

a  $^b$  splits the entire clause into two separate clauses i.e. a and b  $(a \ v \ b) ^c$  c splits the entire clause into two separate clauses a  $v \ b$  and c  $(a ^b) \ v \ c$  splits the clause into two clauses i.e. a  $v \ c$  and b  $v \ c$ 

To eliminate '^' break the clause into two, if you cannot break the clause, distribute the OR 'v' and then break the clause.

### **EXAMPLE**

Now let us see an example which uses resolution.

#### **Problem Statement:**

- 1. Ravi likes all kind of food.
- 2. Apples and chicken are food
- 3. Anything anyone eats and is not killed is food
- 4. Ajay eats peanuts and is still alive
- 5. Rita eats everything that Ajay eats

Prove by resolution that Ravi likes peanuts using resolution.

#### **Solution:**

Step 1: Converting the given statements into Predicate/Propositional Logic

- i.  $\forall x : food(x) \rightarrow likes(Ravi, x)$
- ii. food (Apple) ^ food (chicken)
- iii.  $\forall a : \forall b : \text{eats } (a, b) \land \text{killed } (a) \rightarrow \text{food } (b)$
- iv. eats (Ajay, Peanuts) ^ alive (Ajay)
- v.  $\forall c : eats (Ajay, c) \rightarrow eats (Rita, c)$
- vi.  $\forall d$ : alive(d)  $\rightarrow \sim$  killed (d)
- vii.  $\forall e: \sim killed(e) \rightarrow alive(e)$

**Conclusion**: likes (Ravi, Peanuts)

## Step 2: Convert into CNF

- i.  $\sim$ food(x) v likes (Ravi, x)
- ii. Food (apple)
- iii. Food (chicken)
- iv.  $\sim$  eats (a, b) v killed (a) v food (b)

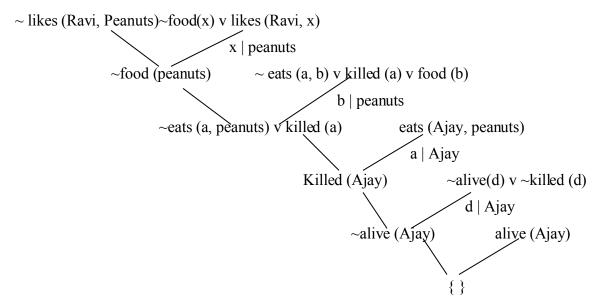
- v. Eats (Ajay, Peanuts)
- vi. Alive (Ajay)
- vii. ~eats (Ajay, c) V eats (Rita, c)
- viii.  $\sim$ alive (d)  $v \sim$  killed (d)
  - ix. Killed (e) v alive (e)

Conclusion: likes (Ravi, Peanuts)

Step 3: Negate the conclusion

~ likes (Ravi, Peanuts)

Step 4: Resolve using a resolution tree



Hence we see that the negation of the conclusion has been proved as a complete contradiction with the given set of facts.

Hence the negation is completely invalid or false or the assertion is completely valid or true.

## **Hence Proved**