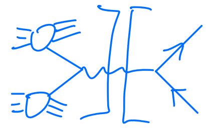
$$N_1(P) + N_2(l) \to l^+(P_1) + l^-(P_2) + X(P_X)$$

$$q = P_1 + P_2$$



$P_+ = n\cdot P$

$P_- = \bar{n}\cdot P$

$c \longrightarrow P_-$ dominent

$\bar{c} \longrightarrow P_+$ dominent

CM - frame

$$\frac{d\sigma}{d^4q} = \frac{1}{2S} \int d\Pi_2 \; \delta^{(4)}(q-P_1-P_2) \cdot \sum_X \left| \langle l^+l^- X | N_1 N_2 \rangle \right|^2 (2\pi)^4 \delta^{(4)}(P+l-P_X-q)$$

$$\to J^\mu = \sum_q e_q \bar{q}\gamma^\mu q$$

其中 $\langle l^+l^- X | N_1 N_2 \rangle = \frac{e^2}{q^2} \bar{u}(P_2)\gamma_\mu u(P_1) \langle X | J^\mu(0) | N_1 N_2 \rangle$

可以定义 Lepton tensor:

$$L_{\mu\nu} = \int d\Pi_2 \; \delta^{(4)}(q-P_1-P_2) \, tr[\not{P}_1 \gamma_\mu \not{P}_2 \gamma_\nu]$$

$$\Rightarrow L^{\mu\nu} = \int \frac{d^3P_1}{(2\pi)^3 2E_1} \frac{d^3P_2}{(2\pi)^3 2E_2} \delta^{(4)}(q-P_1-P_2) \, tr[\not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu]$$

记 $L^{\mu\nu} = q^\mu q^\nu L_1 + g^{\mu\nu} q^2 L_2$

$$g_{\mu\nu} L^{\mu\nu} = \int \frac{d^3P_1}{(2\pi)^3 2E_1} \frac{d^3P_2}{(2\pi)^3 2E_2} \delta^{(4)}(q-P_1-P_2) \, tr[\not{P}_1 \gamma^\mu \not{P}_2 \gamma_\mu]$$

$$= -8 \int \frac{d^3P_1}{(2\pi)^3 2E_1} \frac{d^3P_2}{(2\pi)^3 2E_2} \delta^{(3)}(q-P_1-P_2) \, \delta(|q|-|P_1|-|P_2|) \, (P_1\cdot P_2)$$

$$= -8 \times \frac{q^2}{2} \int d\Pi_2 \cdot \delta^{(4)}(q-P_1-P_2)$$

$$= -4q^2 \cdot \frac{1}{q} \cdot \int_0^q \frac{2\pi \, dP_1}{(2\pi)^6 \cdot 4}$$

$$= -\frac{1}{(2\pi)^4} \cdot \frac{1}{2\pi} q^2$$

$$= -8 \int \frac{d^3P_1}{(2\pi)^3 2|P_1|} \frac{1}{(2\pi)^3 2|q-P_1|} \delta(|q|-|P_1|-|q-P_1|) \left(P_1^0 \cdot P_2^0 - \vec{P}_1 \cdot (\vec{q}-\vec{P}_1)\right)$$

$$= -8 \int \frac{P_1^2 dP_1 \cdot \sin\theta \, \cancel{d\theta} \cdot \cancel{\delta(\cos\theta - 1)} \, d\varphi}{(2\pi)^3 2P_1 \cdot (2\pi)^3 \cdot 2 \cdot \cancel{\sqrt{q^2+P_1^2-2qP_1\cos\theta}}} \cdot \frac{\cancel{\sqrt{q^2+P_1^2-2qP_1\cos\theta}}}{\cancel{P_1 \cdot q}} \quad \cancel{P_1 \cdot q}$$

$$= -8 \cdot \frac{2\pi}{(2\pi)^6 \cdot 4} \int_0^q P_1 dP_1$$

$$= -\frac{1}{(2\pi)^4} \frac{1}{2\pi} q^2$$

$$q_\mu q_\nu L^{\mu\nu} = \int \frac{d^3P_1}{(2\pi)^3 2E_1} \frac{d^3P_2}{(2\pi)^3 2E_2} \delta^{(4)}(q-P_1-P_2) \cdot \left[8(P_1\cdot q)(P_2\cdot q) - 4q^2(P_1\cdot P_2) \right]$$

$$\begin{cases} 8(P_1\cdot q)(P_2\cdot q) = 8(\cancel{P_1^2}+P_1\cdot P_2)(\cancel{P_2^2}+P_1\cdot P_2) \\ \qquad = 2q^4 \\ -4q^2(P_1\cdot P_2) = -2q^4 \end{cases} \Rightarrow q_\mu q_\nu L^{\mu\nu} = 0$$

$$\Rightarrow q_\mu q_\nu L^{\mu\nu} = q^4 (L_1 + L_2) = 0$$

$$g_{\mu\nu} L^{\mu\nu} = q^2 (L_1 + 4L_2) = -\frac{1}{(2\pi)^4} \frac{1}{2\pi} q^2 .$$

$$\Rightarrow L_2 = -\frac{1}{(2\pi)^4} \frac{1}{6\pi}$$

$$\Rightarrow L^{\mu\nu} = \frac{1}{(2\pi)^4} \frac{1}{6\pi} (q^\mu q^\nu - g^{\mu\nu} q^2)$$

有 Ward identity: $q^\mu L_{\mu\nu} = q^\nu L_{\mu\nu} = 0$

$$\alpha = \frac{e^2}{4\pi} \Rightarrow e^4 = 16\pi^2\alpha^2$$

$$\Rightarrow \frac{d\sigma}{d^4q} = \frac{1}{2s}\frac{e^4}{q^4} L_{\mu\nu} W^{\mu\nu} = \frac{1}{2s}\frac{16\pi^2\alpha^2}{q^4}\cdot\frac{1}{(2\pi)^4}\frac{1}{6\pi}(q^\mu q^\nu - g^{\mu\nu}q^2) W_{\mu\nu}$$

$$= \frac{4\pi\alpha^2}{3sq^2}\frac{1}{(2\pi)^4}\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right) W_{\mu\nu}$$

其中 $W_{\mu\nu} = \sum_X \langle N_1 N_2 | J_\mu^\dagger(0) | X\rangle\langle X | J_\nu(0) | N_1 N_2\rangle (2\pi)^4 \delta^{(4)}(P+l-q-P_X)$

$$\left(= \int d^4x \cdot e^{i(P+l-q-P_X)\cdot x}\right)$$

$$\langle N_1 N_2 | J_\mu^\dagger(0) | X\rangle = \langle N_1 N_2 | e^{-i\hat{p}\cdot x} J_\mu^\dagger(x) e^{i\hat{p}\cdot x} | X\rangle$$

$$= e^{-i(P+l-P_X)} \langle N_1 N_2 | J_\mu^\dagger(x) | X\rangle$$

$$= \sum_X \int d^4x\, e^{-iq\cdot x} \langle N_1 N_2 | J_\mu^\dagger(x) | X\rangle\langle X | J_\nu(0) | N_1 N_2\rangle$$

$$= \int d^4x\, e^{-iq\cdot x}\langle N_1 N_2 | J_\mu^\dagger(x) J_\nu(0) | N_1 N_2\rangle = \int d^4x\, e^{-iq\cdot x}\langle N_1 N_2 | T[J_\mu^\dagger(x) J_\nu(0)] | N_1 N_2\rangle$$

$q^0 > 0$ → $T[\cdots]$; $q^0 \in \mathbb{R}$

In SCET:

$$J^\mu(x) = \int dr\,dt\; C_V(r,t)\; \bar{\chi}_{\bar{c}}^{\alpha\bar{a}}(x+rn)\left[S_{\bar{n}}^\dagger(x) S_n(x)\right]^{\bar{a}\bar{b}} (\gamma_\perp^\mu)^{\alpha\beta} \chi_c^{\beta\bar{b}}(x+t\bar{n})$$

Fierz identity: $\bar{u}_1\Gamma_1 u_2\, \bar{u}_3\Gamma_3 u_4 = \sum_{A,B} C_{AB}\, \bar{u}_1\Gamma_A u_4\, \bar{u}_3\Gamma_B u_2$ (Sum over all the rep.)

完备集: $\mathbb{1}$, γ^μ, γ^5, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]$, $\gamma^\mu\gamma^5$

1, 4, 1, 6, 4

fermion loop, no chirality

故 $$\gamma_\mu \otimes \gamma^\mu \longrightarrow -\frac{1}{2}\gamma_\mu\otimes\gamma^\mu \underline{-\frac{1}{2}\gamma^\mu\gamma^5\otimes\gamma_\mu\gamma^5} + \underline{\mathbb{1}\otimes\mathbb{1}} \underline{- \gamma^5\otimes\gamma^5} + 0\,\cancel{\sigma^{\mu\nu}\otimes\sigma_{\mu\nu}}$$

由 $\chi_c = W_c^\dagger \xi_c = \frac{\not{n}\not{\bar{n}}}{4} W_c^\dagger \psi_c$, $\chi_{\bar{c}} = W_{\bar{c}}^\dagger \xi_{\bar{c}} = \frac{\not{\bar{n}}\not{n}}{4} W_{\bar{c}}^\dagger \psi_{\bar{c}}$ $\Rightarrow$ $\not{n}\chi_c = 0$, $\not{\bar{n}}\chi_{\bar{c}} = 0$

($\not{n}\not{n} = n_\mu n_\nu \gamma^\mu\gamma^\nu = (n)^2 = (n)^2 = 0$)

$\gamma^\mu = \gamma_\perp^\mu + \cancel{\not{n}\frac{\bar{n}^\mu}{2}} + \cancel{\not{\bar{n}}\frac{n^\mu}{2}}$

only contribution.

$J^\mu J^\nu$

$$\bar{\chi}_c\gamma^\mu\chi_c = \bar{\chi}_c\gamma_\perp^\mu\chi_c + \cancel{\bar{n}^\mu\bar{\chi}_c\frac{\not{n}}{2}\chi_c} + n^\mu\bar{\chi}_c\frac{\not{\bar{n}}}{2}\chi_c = n^\mu\bar{\chi}_c\frac{\not{\bar{n}}}{2}\chi_c$$

$$\bar{\chi}_{\bar{c}}\gamma^\mu\chi_{\bar{c}} = \bar{\chi}_{\bar{c}}\gamma_\perp^\mu\chi_{\bar{c}} + \bar{n}^\mu\bar{\chi}_{\bar{c}}\frac{\not{n}}{2}\chi_{\bar{c}} + \cancel{n^\mu\bar{\chi}_{\bar{c}}\frac{\not{\bar{n}}}{2}\chi_{\bar{c}}} = \bar{n}^\mu\bar{\chi}_{\bar{c}}\frac{\not{n}}{2}\chi_{\bar{c}}$$

$$\bar{\chi}_c\chi_c = \left(\frac{\not{n}\not{\bar{n}}}{4}\chi_c\right)^\dagger\chi_c = \chi_c^\dagger\frac{\not{\bar{n}}\not{n}}{4}\chi_c = 0$$

$$\bar\chi_c \gamma_{\perp\mu} \chi_{\bar c}\, \bar\chi_{\bar c}\gamma_\perp^\mu \chi_c$$

$$= \bar\chi_c\left(\gamma_\mu - \frac{n}{2}\bar n_\mu - \cancel{\frac{\bar n}{2} n_\mu}\right)\chi_{\bar c}\,\bar\chi_{\bar c}\left(\gamma^\mu - \cancel{\frac{n}{2}\bar n^\mu} - \frac{\bar n}{2} n^\mu\right)\chi_c$$

$$= \bar\chi_c \gamma_\mu \chi_{\bar c}\bar\chi_{\bar c}\gamma^\mu\chi_c - \bar\chi_c \not n\, \chi_{\bar c}\bar\chi_{\bar c}\not{\bar n}\,\chi_c + n\cdot\bar n\, \bar\chi_c \frac{\not n}{2}\chi_{\bar c}\bar\chi_{\bar c}\frac{\not{\bar n}}{2}\chi_c$$

exchange

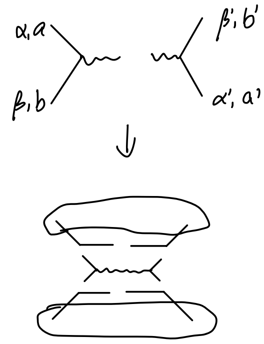
$$\xrightarrow{\text{Fierz}} -\frac{(-1)^3}{2}\left(\bar\chi_c\gamma_\mu\chi_c\, \bar\chi_{\bar c}\gamma^\mu\chi_{\bar c} - \cancel{2\bar\chi_c\frac{\not n}{2}\chi_c\,\bar\chi_{\bar c}\frac{\not{\bar n}}{2}\chi_{\bar c}}\right)$$

$$= \bar\chi_c\frac{\not{\bar n}}{2}\chi_c\,\bar\chi_{\bar c}\frac{\not n}{2}\chi_{\bar c}$$

$$\Rightarrow g_{\mu\nu}J^{\mu\dagger}J^\nu = \bar\chi_c\gamma_\perp^\mu S_n^\dagger S_{\bar n}\chi_{\bar c}\,\bar\chi_{\bar c} S_{\bar n}^\dagger S_n \gamma_\perp^\nu \chi_c$$

$$= (\bar\chi_c)^{\alpha,a}\left(\frac{\not{\bar n}}{2}\right)^{\alpha\beta}(S_n^\dagger S_{\bar n})^{ba}(\chi_c)^{\beta,b'}(\bar\chi_{\bar c})^{\alpha' a'}(S_{\bar n}^\dagger S_n)^{a'b'}\left(\frac{\not n}{2}\right)^{\alpha'\beta'}(\chi_{\bar c})^{\beta',b}$$

$$= \left(\bar\chi_c\frac{\not{\bar n}}{2}\chi_c\right)^{a,b'}\left(\bar\chi_{\bar c}\frac{\not n}{2}\chi_{\bar c}\right)^{a',b}(S_n^\dagger S_{\bar n})^{ba}(S_{\bar n}^\dagger S_n)^{a'b'}$$



每条闭合线上色守恒 → color singlet.

引入色平均 $\frac{1}{N_c}\delta^{a,b'}$

$$\Rightarrow g_{\mu\nu}J^{\mu\dagger}(x)J^\nu(0) = \frac{1}{N_c}\left(\bar\chi_c\frac{\not{\bar n}}{2}\chi_c\right)^{a,a}\frac{1}{N_c}\left(\bar\chi_{\bar c}\frac{\not n}{2}\chi_{\bar c}\right)^{b,b}\underbrace{(S_n^\dagger S_{\bar n})^{ba}(x)(S_{\bar n}^\dagger S_n)^{ab}(0)}_{\mathrm{tr}\left[S_n^\dagger(x)S_{\bar n}(x)\, S_{\bar n}^\dagger(0)S_n(0)\right]}$$

$$= \frac{1}{N_c^2}\int dr\,dt\,dr'\,dt'\, C_V^*(r',t')C_V(r,t)\bar\chi_c(x+t\bar n)\frac{\not{\bar n}}{2}\chi_c(t\bar n)\cdot\bar\chi_{\bar c}(sn)\frac{\not n}{2}\chi_{\bar c}(x+s'n)$$
$$\times\,\mathrm{tr}\left[S_n^\dagger(x)S_{\bar n}(x)\cdot S_{\bar n}^\dagger(0)S_n(0)\right]$$

$$\Rightarrow g_{\mu\nu}W^{\mu\nu} = \int dr\,dt\,dr'\,dt'\; C_V(r,t)\,C_V^*(r',t')\int d^4x\, e^{-iq\cdot x}\times$$
$$\frac{1}{N_c}\langle N_1 N_2|\,\bar\chi_c(x+t\bar n)\frac{\not{\bar n}}{2}\chi_c(t\bar n)\cdot\bar\chi_{\bar c}(sn)\frac{\not n}{2}\chi_{\bar c}(x+s'n)\,\mathrm{tr}\left[\bar T S_n^\dagger(x)S_{\bar n}(x)\,T S_{\bar n}^\dagger(0)S_n(0)\right]|N_1 N_2\rangle$$

⇒ 定义 Soft function (S_{DY}) / Soft Wilson Line (W_{DY}):

$$\hat S_{DY}(x) = \frac{1}{N_c}\mathrm{tr}\langle 0|\bar T\left(S_n^\dagger(x)S_{\bar n}(x)\right)T\left(S_{\bar n}^\dagger(0)S_n(0)\right)|0\rangle$$

真空平均值是由于初态质子 p^μ 均为(反)共线的，不含软贡献.

$$\Rightarrow -g_{\mu\nu}W^{\mu\nu} = -\frac{1}{N_c}\int d^4x\, e^{-iq\cdot x}\int dr\,dt\,dr'\,dt'\, C_V(r,t)C_V^*(r',t')\,\hat S_{DY}(x)\,\langle N_1(p)|\bar\chi_c(x+t\bar n)\frac{\not{\bar n}}{2}\chi_c(t\bar n)|N_1(p)\rangle$$
$$\times\langle N_2(l)|\bar\chi_{\bar c}(rn)\frac{\not n}{2}\chi_{\bar c}(x+r'n)|N_2(l)\rangle$$

①

$$\begin{cases} P_c \sim Q(\lambda^2, 1, \lambda) \\ P_{\bar c} \sim Q(1, \lambda^2, \lambda) \\ P_s \sim Q(\lambda, \lambda, \lambda) \end{cases} \Rightarrow P_c + P_{\bar c} + P_s \sim Q(1, 1, \lambda)$$

$$\Rightarrow x^\mu \sim \frac{1}{Q}\left(1, 1, \frac{1}{\lambda}\right)$$

对于(反)共线场: $P_c \sim Q(\lambda^2, 1, \lambda)$

$$\chi_c(x) = \chi_c(x_+, x_\perp) + \overset{O(1)}{x_-}\,\underbrace{\overset{O(\lambda^2)}{\partial_+}\chi_c(x_+, x_\perp)}_{\text{小量}} + O(x_-^2)$$

$x_+ \cdot \partial_-^c \sim O(1)$

$x_- \cdot \partial_+^{\bar{c}} \sim O(1)$

$x_\perp \cdot \partial_\perp \sim O(1)$

$$\chi_{\bar{c}}(x) = \chi_{\bar{c}}(x_-, x_\perp) + x_+ \partial_- \chi_{\bar{c}}(x_-, x_\perp) + O(x_+^2)$$

对于软场: $P_s \sim Q(\lambda, \lambda, \lambda)$

$$S(x) = S(x_\perp) + \overset{O(1)}{x_-}\,\overset{O(\lambda)}{\partial_+} S(x_\perp) + x_+ \partial_- S(x_\perp) + O(x_+^2, x_-^2, x_+ x_-)$$

若 $\lambda \ll 1 \Leftrightarrow \frac{P_\perp}{Q} \ll 1 \Leftrightarrow \frac{P_\perp}{M} \ll 1 \Rightarrow$ 从 $\frac{P_\perp}{M}$ 展开:

$$\xrightarrow{LO} \langle N_1(p)|\bar{\chi}_c(x+t\bar{n})\frac{\bar{n}\!\!\!/}{2}\chi_c(t\bar{n})|N_1(p)\rangle = \langle N_1(p)|\bar{\chi}_c(x_+ + x_\perp + t'\bar{n})\frac{\bar{n}\!\!\!/}{2}\chi_c(t\bar{n})|N_1(p)\rangle$$

$$\equiv \bar{n}\cdot p \int_{-1}^{1} d\zeta_1 \underbrace{f_{\bar{q}/N_1}(\zeta_1, x_\perp, \mu)}_{\text{PDF}}\, e^{i\zeta_1(x_+ + t'\bar{n} - t\bar{n})p}$$

$*$ $\quad f_{\bar{q}/N_1}(z) = f^*_{\bar{q}/N_1}(z) = -f_{q/N_1}(-z)$

$$\int_0^1 dz\, f_{\bar{q}/N_1}(z) = -\int_0^1 dz\, f_{q/N_1}(-z) = \int_0^{-1} dz\, f_{q/N_1}(z) = -\int_{-1}^{0} dz\, f_{q/N_1}(z)$$

$$f_{q/N_1}(z, x_\perp, \mu) = \frac{1}{2\pi}\int dt\, e^{-izt(n\cdot p)}\langle N_1(p)|\bar{\chi}_c(t\bar{n}+x_\perp)\frac{\bar{n}\!\!\!/}{2}\chi_c(0)|N_1(p)\rangle$$

—— Schwartz (32.117)

$$\Rightarrow -g_{\mu\nu}W^{\mu\nu} \overset{LO}{=} -\frac{1}{N_c}\int d^4x\, e^{-iq\cdot x}\int dr\,dt\,dr'\,dt'\, C_V(r,t)\,C_V^*(r',t')\,\hat{S}_{DY}(x_\perp,\mu)$$
$$\times\, \bar{n}\cdot p\int_{-1}^{1} d\zeta_1\, f_{\bar{q}/N_1}(\zeta_1, x_\perp, \mu)\, e^{i\zeta_1(x_+ + t'\bar{n} - t\bar{n})p}$$
$$\times\, n\cdot l\int_{-1}^{1} d\zeta_2\, f_{q/N_2}(\zeta_2, x_\perp, \mu)\, e^{i\zeta_2(x_- + r'n - rn)l}$$

$S = (p+l)^2$

$= \underset{0}{p^2} + \underset{0}{l^2} + 2p\cdot l$

$= 2(\bar{n}\cdot p)(n\cdot l)\frac{\bar{n}\cdot n}{4}$

$= (\bar{n}\cdot p)(n\cdot l)$

$\hat{S} = \zeta_1\zeta_2 S$

$$= -\frac{S}{N_c}\int_0^1 d\zeta_1\, d\zeta_2 \int dr\,dt\, C_V(r,t)\, e^{-i\zeta_1 t\bar{n}\cdot p - i\zeta_2 r n\cdot l}\int dr'\,dt'\, C_V^*(r',t')\, e^{i\zeta_1 t'\bar{n}\cdot p + i\zeta_2 r' n\cdot l}$$
$$\times \int \frac{dx^+ dx^-}{2}\, d^2x_\perp\, \hat{S}_{DY}(x_\perp,\mu)\, e^{i\zeta_1\frac{1}{2}x_+ P_- + i\zeta_2\frac{1}{2}x_- l_+ - i\frac{q_+}{2}x_- - i\frac{q_-}{2}x_+}\, e^{-iq_\perp\cdot x_\perp}$$
$$\times \left[f_{q/N_1}(\zeta_1, x_\perp, \mu)\, f_{\bar{q}/N_2}(\zeta_2, x_\perp, \mu) + (q \leftrightarrow \bar{q})\right]$$

def: $\quad \tilde{C}_V(-\hat{s},\mu) = \int dr\,dt\, C_V(r,t)\, e^{-i\zeta_1 t\bar{n}\cdot p - i\zeta_2 r n\cdot l}$

$\quad \tilde{C}_V^*(-\hat{s},\mu) = \quad \cdots$

$$\int dx^+\, e^{i(\zeta_1 P_- - q_-)\frac{x^+}{2}} = 2(2\pi)\,\delta(\zeta_1 P_- - q_-) = \frac{2(2\pi)}{P_-}\,\delta\!\left(\zeta_1 - \frac{q_-}{P_-}\right)$$

$$\int dx^-\, e^{i(\zeta_2 l_+ - q_+)\frac{x^-}{2}} = 2(2\pi)\,\delta(\zeta_2 l_+ - q_+) = \frac{2(2\pi)}{l_+}\,\delta\!\left(\zeta_2 - \frac{q_+}{l_+}\right)$$

$$\Rightarrow -g_{\mu\nu}W^{\mu\nu} = -\frac{s}{N_c}\int_0^1 dz_1 dz_2 \left|\tilde{C}_V(-\hat{s},\mu)\right|^2 2(2\pi)^2 \frac{1}{s}\,\delta\!\left(z_1-\frac{q_-}{P_-}\right)\delta\!\left(z_2-\frac{q_+}{l_+}\right)$$

$$\times\int d^2x_\perp e^{-iq_\perp\cdot x_\perp}\left[f_{q/N_1}(z_1,x_\perp,\mu)f_{\bar q/N_2}(z_2,x_\perp,\mu)+(q\leftrightarrow\bar q)\right]\hat{S}_{DY}(x_\perp,\mu)$$

$$= -\frac{1}{N_c}\left|\tilde{C}_V(-\hat{s},\mu)\right|^2 2(2\pi)^2\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\left[f_{q/N_1}(z_1,x_\perp,\mu)f_{\bar q/N_2}(z_2,x_\perp,\mu)+(q\leftrightarrow\bar q)\right]\hat{S}_{DY}(x_\perp,\mu)$$

$$z_1=\frac{q_-}{P_-}\;;\;z_2=\frac{q_+}{l_+}$$

$$\Rightarrow d\sigma = \frac{d^4q}{(2\pi)^4}\left|\tilde{C}_V(-\hat{s},\mu)\right|^2\boxed{\frac{1}{2s}\frac{e^4}{q^4}\frac{q^2}{(2\pi)^4 6\pi}\frac{1}{N_c}\times 2(2\pi)^2}\sum_q Q_q^2\int d^2x_\perp e^{-iq_\perp\cdot x_\perp}\left[f_{q/N_1}f_{\bar q/N_2}+(q\leftrightarrow\bar q)\right]\hat{S}_{DY}(x_\perp,\mu)$$

inv Mass

$$\int d^4q = \int dM^2\int d^4q\,\delta(q^2-M^2)\theta(q^0)$$

$$=\int dM^2\int\frac{d^3q}{2q^0} = \frac{1}{2}\int dM^2 d^2q_T\,dy$$

$(q^2=\hat{s})$

$$\boxed{} = \frac{4\pi\alpha^2}{3sq^2}\frac{1}{N_c}\frac{2}{(2\pi)^2}$$

$$\Rightarrow \frac{d\sigma}{dM^2d^2q_T\,dy} = \frac{4\pi\alpha^2}{3sM^2}\frac{1}{N_c}|\tilde{C}_V|\sum_q Q_q^2\int\frac{d^2x_\perp}{(2\pi)^2}e^{-iq_\perp\cdot x_\perp}\left[f_{q/N_1}(z_1,x_\perp,\mu)f_{\bar q/N_2}(z_2,x_\perp,\mu)+(q\leftrightarrow\bar q)\right]\hat{S}_{DY}(x_\perp,\mu)+\mathcal{O}\!\left(\frac{q_T^2}{M^2}\right)$$

$$=\sum_q Q_q^2 H(M^2,\mu)\int\frac{d^2x_\perp}{(2\pi)^2}e^{-iq_\perp\cdot x_\perp}\left[f_{q/N_1}f_{\bar q/N_2}+(q\leftrightarrow\bar q)\right]\hat{S}_{DY}(x_\perp,\mu)$$

其中 $$H(M^2,\mu)=\frac{4\pi\alpha^2}{3sM^2N_c}\left|C_V(-M^2,\mu)\right|^2$$

$$\hat{S}(x_\perp,\mu)=\langle 0|\bar{T}[S_{\bar n}^\dagger S_n(x_\perp)]\,T[S_n^\dagger S_{\bar n}(0)]|0\rangle$$

② Threshold 极限下，质心系能量几乎全部用来产生末态目标产物，而不会产生任何共线的末态部分子。

故末态共线部分子均来源于入射质子，其 $|P_{X_T}| \sim \Lambda_{QCD} \ll |q|$

$$\Rightarrow \langle N_1(p)|\bar\chi_c(x+t\bar n)\frac{\bar{\not n}}{2}\chi_c(t\bar n)|N_1(p)\rangle$$

$$= \langle N_1(p)|\bar\chi_c(x_+ + t'\bar n)\frac{\bar{\not n}}{2}\chi_c(t\bar n)|N_1(p)\rangle$$

$$\equiv \bar n\cdot p\int_{-1}^{1} dz_1\, f_{q/N_1}(z_1,\mu)\, e^{iz_1(x_+ + t'\bar n - t\bar n)\cdot p}$$

其中 $f_{q/N_1}(z)=\int\frac{dt}{2\pi}\, e^{-izt\bar n\cdot p}\langle N_1(p)|\bar\chi_c(t\bar n)\frac{\bar{\not n}}{2}\chi_c(0)|N_1(p)\rangle$ ⟵ $\chi_c = W^\dagger\frac{\not n\bar{\not n}}{4}\psi$, $\bar\chi_c=\bar\psi\frac{\bar{\not n}\not n}{4}W$

$$=\langle N_1(p)|\bar\psi(t\bar n)\frac{\bar{\not n}\not n}{4}W\frac{\bar{\not n}}{2}W^\dagger\frac{\not n\bar{\not n}}{4}\psi(0)|N_1(p)\rangle$$

$$=\langle N_1(p)|\bar\psi(t\bar n)\frac{\bar{\not n}}{2}[t\bar n,0]\psi(0)|N_1(p)\rangle$$ ⟵ Schwartz

$$\Rightarrow (-g_{\mu\nu}W^{\mu\nu}) = \frac{s}{N_c}\int_0^1 dz_1dz_2\,|\tilde C_V(-\hat s,\mu)|^2\int d^4x\,\hat S_{DY}(x,\mu)\,e^{ix\cdot(z_1P_- + z_2 l_+ - q)}\left[f_{q/N_1}(z_1,\mu)f_{\bar q/N_2}(z_2,\mu)+(q\leftrightarrow\bar q)\right]$$

$$\Rightarrow d\sigma = \frac{d^4q}{(2\pi)^4}\frac{4\pi\alpha^2}{3q^2N_c}|\tilde C_V(-\hat s,\mu)|^2\int_0^1dz_1dz_2\sum_q Q_q^2\left[f_{q/N_1}f_{\bar q/N_2}+(q\leftrightarrow\bar q)\right]\int d^4x\,\hat S_{DY}(x,\mu)\,e^{ix\cdot(z_1P_-+z_2l_+-q)}$$

由于在 threshold 极限处，且 $q^2=\hat s+O(\lambda^2)$

故：$q^0=\sqrt{\hat s}+O(\lambda^2)$ $\quad |\vec q|\sim\lambda^2$

$$\sigma=\int dM^2\int\frac{d^3q}{2q_0}\cdots$$

把 d^4q 的 $(2\pi)^4$ 给它

$$\overset{LO}{=}\int dM^2\frac{1}{2\sqrt{\hat s}}\cdots\int d^4x\,\hat S_{DY}(x,\mu)\int d^3q\, e^{ix\cdot(z_1P_-+z_2l_+-q)}$$

$$=\int\frac{dM^2}{2\sqrt{\hat s}}\cdots\int d^4x\,\hat S_{DY}(x,\mu)\,e^{-ix^0q^0}e^{ix\cdot(z_1P_-+z_2l_+)}\,\delta^{(3)}(\vec x)\,(2\pi)^3$$

$$=\int\frac{dM^2}{2\sqrt{\hat s}}\cdots\int dx^0\,\hat S_{DY}(x^0,\vec x=0,\mu)\,e^{ix^0(z_1P^0+z_2l^0-q^0)}$$

def: $z\equiv\frac{M^2}{\hat s}$; 此时有 $1-z=\frac{\hat s-M^2}{\hat s}\sim O(\lambda^2)$

$$\hat s=(q+P_X)^2=q^2+2q\cdot P_X+P_X^2$$

$$=M^2+2(q^0P_X^0-\vec q\cdot\vec P_X)$$

$$=M^2+2P_X^0\sqrt{M^2+(P_X^0)^2}+2(P_X^0)^2$$

$\begin{cases} q^2=M^2 \\ P_X^2=0\end{cases}$ & $\vec q=-\vec P_X$

$\Rightarrow \vec q\cdot\vec P_X=-|\vec P_X|^2=-(P_X^0)^2$

$q^0=\sqrt{M^2+\vec q^2}=\sqrt{M^2+(P_X^0)^2}$

代入z: $1-z=2\frac{P_X^0}{\sqrt{\hat s}}\sqrt{z+\left(\frac{P_X^0}{\sqrt{\hat s}}\right)^2}+2\left(\frac{P_X^0}{\sqrt{\hat s}}\right)^2$

令 $\frac{P_X^0}{\sqrt{\hat s}}=x \Rightarrow 2x^2+2x\sqrt{z+x^2}-(1-z)=0$

$$\Rightarrow x=\frac{1-z}{2\sqrt{z^2-z+1}}=\frac{1-z}{2\sqrt{(z-1)z+1}}\simeq\frac{1-z}{2}+O(\lambda^4)$$

$$\Rightarrow P_X^0=\sqrt{\hat s}\,\frac{1-z}{2}=M\frac{1-z}{2\sqrt z}\equiv\frac{\omega}{2}$$

$$\Rightarrow (z_1P_-+z_2l_+-q)^{(0)}=P_X^0=\frac{\sqrt{\hat s}}{2}(z-1)+O(\lambda^4)$$

$$\Rightarrow \frac{d\sigma}{dM^2}=H(M^2,\mu)\int_0^1dz_1dz_2\sum_q Q_q^2\left[f_{q/N_1}(z_1,\mu)f_{\bar q/N_2}(z_2,\mu)+(q\longleftrightarrow\bar q)\right]\frac{1}{\sqrt{\hat s}}\tilde S_{DY}(\sqrt{\hat s}(1-z),\mu)$$

其中 $$\tilde{S}_{DY}(\omega,\mu) = \int \frac{dx^0}{2(2\pi)} \hat{S}_{DY}(x,\mu)\, e^{i x^0 \frac{\omega}{2}}$$

$$H(M^2,\mu) = \frac{4\pi\alpha^2}{3M^2 N_c} \left| \tilde{C}_V(-M^2,\mu) \right|^2$$