Outline

- Requirements for a Quantum Computer
- Quantum Mechanics of Two State Systems
- Rotations
- Creating Gates with Rabi Oscillations
- Superconducting Qubits
  - How to make qubits with circuits
  - Superconductivity
  - Josephson junction and nonlinear LC circuits
  - SQUID: Tunable nonlinear inductance

Requirements for Quantum Computer

- Physical system with two uniquely addressable states
- A universal set of quantum gates
  - Ability to implement arbitrary rotations on the Bloch sphere
  - Ability to entangle two qubits
- Ability to measure the state of a cubit

Two State Systems in Quantum Mechanics

Schrodinger Equation

$$i\hbar\partial_t|\psi\rangle = H|\psi\rangle, \quad \partial_t \equiv \frac{\partial}{\partial t}$$

$$|\psi(t)\rangle = e^{-i\mathcal{E}t/\hbar}|\psi(0)\rangle$$

- *H* is an expression for the total energy of the system (kinetic + potential) and is called the *Hamiltonian*
- For two orthogonal states:

$$i\hbar\partial_t|\psi_0\rangle = H_0|\psi_0\rangle = E_0|\psi_0\rangle$$
$$i\hbar\partial_t|\psi_1\rangle = H_0|\psi_1\rangle = E_1|\psi_1\rangle$$

- What if we introduce a perturbation that weakly couples states 0 & 1?

$$i\hbar\partial_t|\psi_0\rangle = H_0|\psi_0\rangle + V_{01}|\psi_1\rangle$$
$$i\hbar\partial_t|\psi_1\rangle = H_0|\psi_1\rangle + V_{10}|\psi_0\rangle$$

Schrodinger Equation for 2x2 coupled system

$$i\hbar\partial_t|\psi_0\rangle = H_0|\psi_0\rangle + V_{01}|\psi_1\rangle$$
$$i\hbar\partial_t|\psi_1\rangle = H_0|\psi_1\rangle + V_{10}|\psi_0\rangle$$

- This can be written

$$i\hbar\partial_t\begin{bmatrix}|\psi_0\rangle\\|\psi_1\rangle\end{bmatrix} = \begin{bmatrix}E_0 & 0\\0 & E_1\end{bmatrix}\begin{bmatrix}|\psi_0\rangle\\|\psi_1\rangle\end{bmatrix} + \begin{bmatrix}0 & V_{01}\\V_{10} & 0\end{bmatrix}\begin{bmatrix}|\psi_0\rangle\\|\psi_1\rangle\end{bmatrix} = H\begin{bmatrix}|\psi_0\rangle\\|\psi_1\rangle\end{bmatrix}$$

$$H = \frac{1}{2}(E_0+E_1)\sigma_0 + \frac{1}{2}(E_0-E_1)\sigma_z + V\sigma_x,\quad \text{assuming } V_{01}=V_{10}$$

$$\sigma_0 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\ \sigma_z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix},\ \sigma_x = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$H = -\frac{1}{2}\hbar\omega_0\sigma_z + V\sigma_x,\quad \text{take zero reference energy} = \frac{1}{2}(E_0+E_1),\ E_1-E_0=\hbar\omega_0$$

Prototypical 2-state system: Spin in a Magnetic Field

$$\mathcal{H} = -\gamma_S \mathbf{S}\cdot\mathbf{B} = -\frac{q}{m_q}\mathbf{S}\cdot\mathbf{B},$$

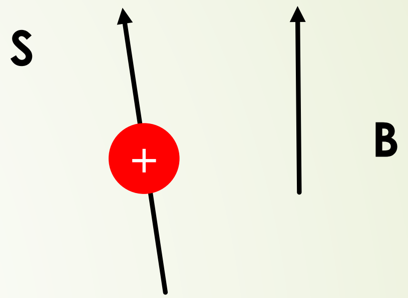
$$\mathbf{S} = \frac{\hbar}{2}\boldsymbol{\sigma},$$

$$\boldsymbol{\sigma} = \hat{\mathbf{x}}\sigma_x + \hat{\mathbf{y}}\sigma_y + \hat{\mathbf{z}}\sigma_z \qquad \mathcal{E} = \pm\frac{\hbar\omega_0}{2}$$

$$\mathcal{H}_0 = -\frac{1}{2}\hbar\omega_0\sigma_z,$$

$$|\psi(t)\rangle = e^{-i\mathcal{E}t/\hbar}\,|\psi(0)\rangle$$

$$\omega_0 = \frac{q}{m_q}B_0.$$

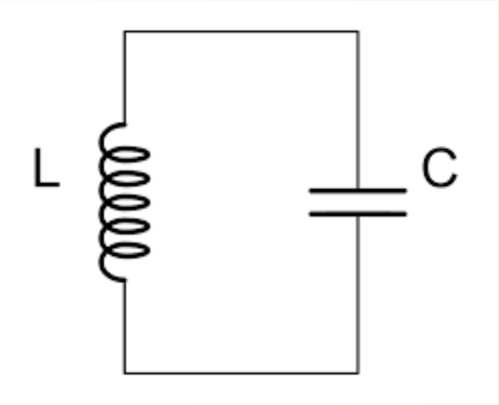


Superconducting Qubits

Qubit possibility: LC Resonant Circuits?

- In classical, linear circuit theory, the natural solution for the current is

$$i(t) = I_0 \cos \omega_0 t, \quad \omega_0 = 1/\sqrt{LC}$$



- The current can have any amplitude, independent of the frequency
- Energy is stored alternately in the electric field of the capacitor and the magnetic field of the inductor, and can have any value

$$U \propto I_0^2$$

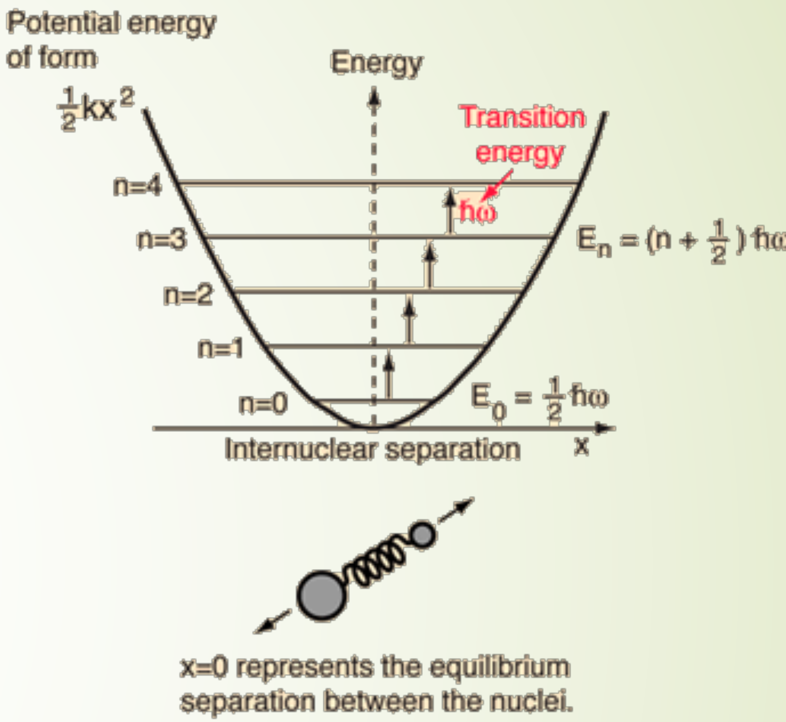
- In reality, the stored energy is quantized

$$U = \hbar\omega_0\left(n + \frac{1}{2}\right)$$

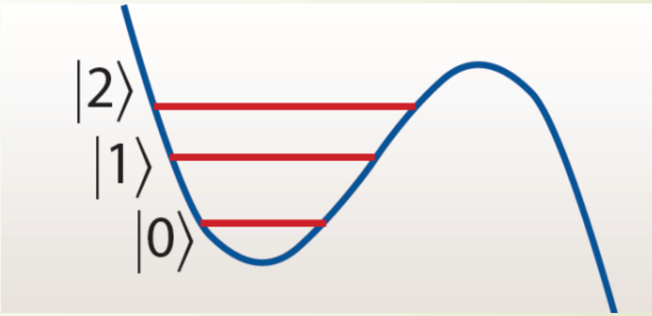
- Could we use two of these states for a qubit, say n=0 and n=1?

Nonlinear Resonant Circuits

- Problem: energy difference between 0,1 is the same as energy difference between all other states n,n+1
  - No way to address specific states
- Solution: if either L or C were nonlinear (i.e., their values depended on the magnitude of the current or voltage), then the energy levels would no longer be equally spaced!
  - If the energy difference between n=0 and n=1 is different from the energy difference from all other states, we can selectively address this particular transition by tuning the frequency of the applied excitation



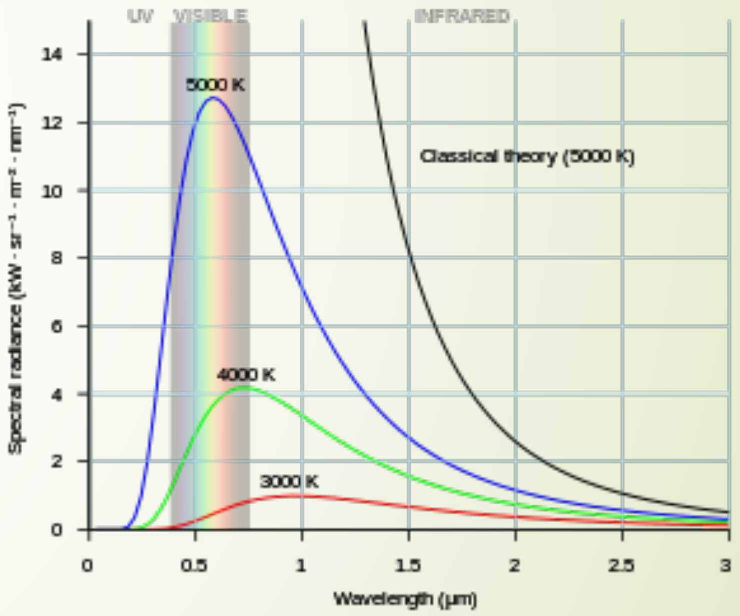
Graphic: http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/hosc.html



Graphic: Clarke & Wilhelm

RF Frequency

- The fact that we are talking about circuits suggests we are talking about RF rather than optical frequencies!
- Problem: any physical mass at finite temperature will emit electromagnetic radiation that depends on its temperature (Black body radiation)
  - We want the energy difference between qubit states to be large compared to thermal radiation
  - Highest frequency for widespread, economical instrumentation ~ 6 GHz (owing, e.g., to WIFI, etc.)
  - From kT=hν, the temperature corresponding to 6 GHz is 0.29K
  - Operating temperature must be much less than 0.3K!
- Solution: IBM Q systems operate at a temperature of about 15 mK using dilution refrigeration



Graphic: Wikipedia

Superconductors

A superconductor is a metal that allows a current to pass through it with no loss due to heat dissipation.

Typical values for the critical temperature range from mK to 100K

Using Superconductors we can preserve a wavefunction because the fact that the current wavefunction is not perturbed by its journey through the metal means that it will stay in a given state.

The current can be seen as a wavefunction, and is thus A probability distribution of different current values, this implies that clockwise and counter clockwise. It is this view of the current that enables us to create qubits from a simple loop of superconductor.

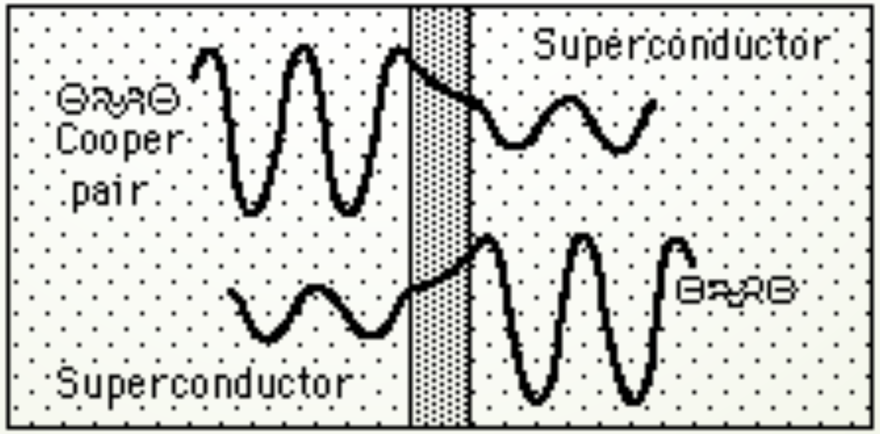
Metal	Critical T(K)
Aluminum	1.2K
Tin	3.7K
Mercury	4.2K
Niobium	9.3K
Niobium-Tin	17.9K
Tl-Ba-Cu-oxide	125K

Superconductivity

- At such low temperatures, metals such as Al and Nb become superconductors
  - At low temperatures, an attractive force between electrons appears
  - When this force gets sufficiently strong compared to thermal vibrations, electrons bind together into "Cooper pairs" with spin 1 and charge 2q
  - Cooper pairs form a macroscopic quantum state enabling charge to move without scattering or loss, resulting in superconductivity
- Makes it possible to realize extremely low-loss RF transmission lines
- Makes it possible to realize a nonlinear inductor using a Josephson junction

Cooper Pairs and Superconductivity

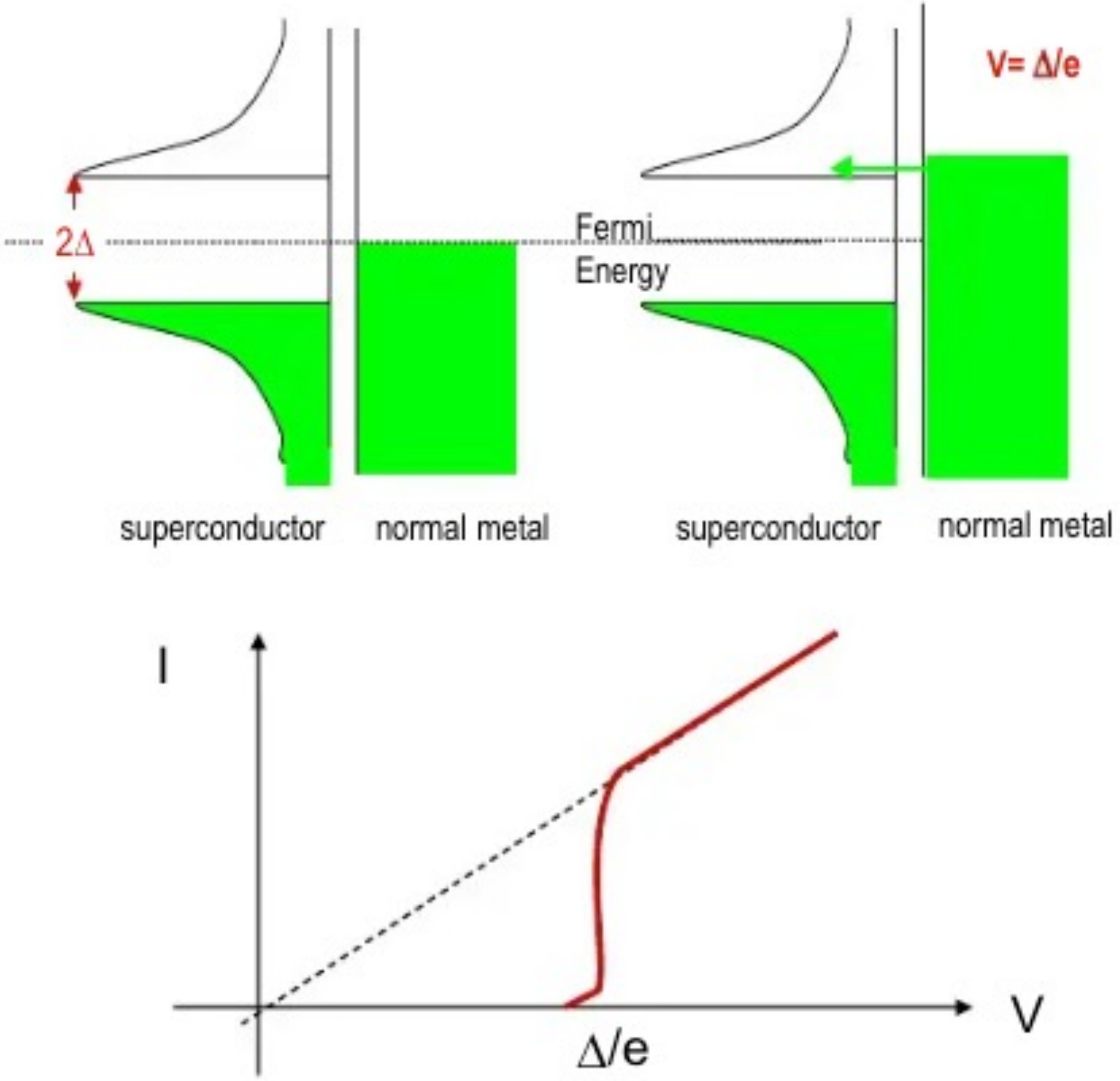
- Spin ½ particles are "Fermions"
  - Fermions obey the Pauli exclusion principle: no two can be in the same state
  - Electrons are Fermions
- Spin 1 particles are "Bosons"
  - Bosons do not obey the Pauli exclusion principle: you can have as many in a state as you want
  - Photons are Bosons
- In a superconductor, an effective attractive interaction between electrons causes them to be loosely bound together and act like a single spin 1 particle: "Cooper Pair"
- Since Cooper pairs are spin 1, they act like Bosons, and you can have multiple Cooper pairs in the same state
- All of the Cooper pairs in a macroscopic sample can be in the same coherent state



We have already noted tunneling effects between a superconductor and a normal metal (through an insulating barrier)

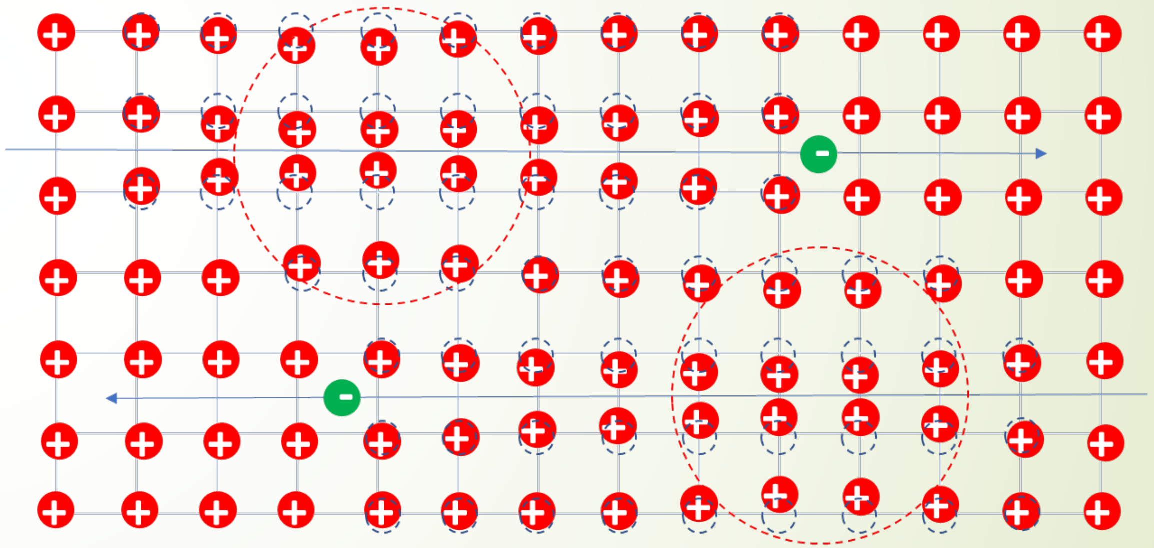
This provides evidence of a superconducting energy gap

Josephson demonstrated tunneling between two superconductors separated by an insulating barrier

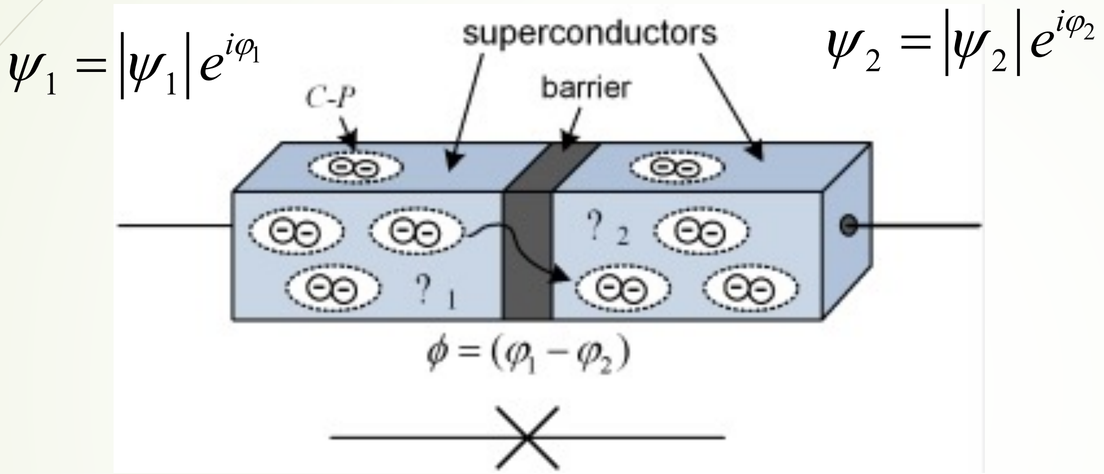


Cooper Pairs are the result of the Electron-Phonon interaction in the theory of Bardeen, Cooper, and Schreifer (BCS Theory)

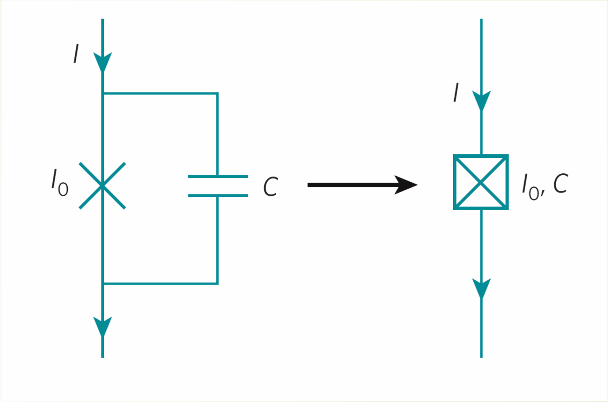
- Electrons normally repel one another, but are attracted to ions in the crystal lattice
- If the ions are pulled slightly toward an electron, from a distance it can appear as though there is a net positive charge, attracting another electron



Josephson tunnel junction



Circuit Symbols



Graphic: Clarke & Wilhelm

- Two superconductors separated by a thin insulating layer
- Wave functions for superconducting Cooper pairs decay exponentially in the insulating layer
- If the layer is thin enough to allow appreciable tunneling, then phases are no longer independent but are related to each other through the size of the tunneling current

Graphic: Ph.D. thesis: Vratislav Michal

Josephson Junction as nonlinear inductor

$$\varphi = \varphi_2 - \varphi_1$$
$$I = I_c \sin\varphi$$
$$V = \frac{\Phi_0}{2\pi}\frac{d\varphi}{dt}, \quad \Phi_0 = \frac{h}{2q} \text{ is the flux quantum}$$
$$\frac{dI}{dt} = I_c \cos\varphi \frac{d\varphi}{dt}$$
$$= I_c \cos\varphi \frac{2\pi V}{\Phi_0}$$
$$\Rightarrow$$
$$V = \frac{\Phi_0}{2\pi I_c \cos\varphi}\frac{dI}{dt}$$
$$= \frac{\Phi_0}{2\pi I_c \sqrt{1-\sin^2\varphi}}\frac{dI}{dt}$$
$$= \frac{\Phi_0}{2\pi I_c \sqrt{1-(I/I_c)^2}}\frac{dI}{dt} = L_{eff}(I)\frac{dI}{dt}$$

- Effective inductance depends on the current
- Looks like a non-linear inductor: origin of *anharmonicity*: spacing between energy levels is not the same
  - *Enables the individual addressing of a single pair of states*
  - *In contrast, in a linear circuit, all states are equally spaced*

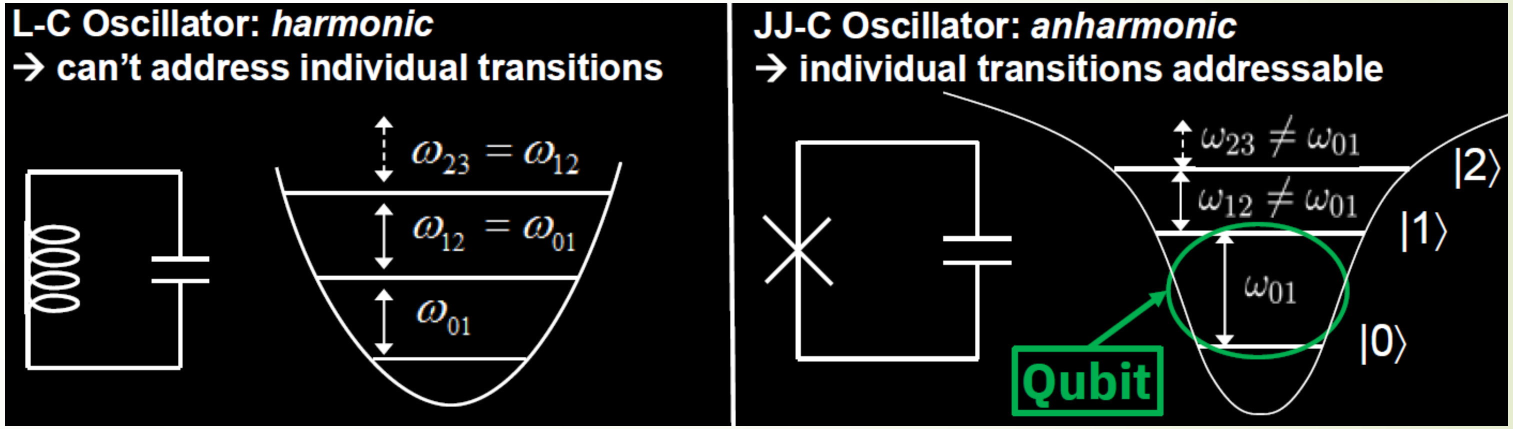


Image credit: Doug McClure, IBM

Superconducting Quantum Interference Device (SQUID)

- Parallel Josephson Junctions
- Magnetic field induces circulating current
- Simple analysis: neglect inductance of loop, assume both JJs are identical

$$I_T = I_1 + I_2 = I_c \sin\varphi_1 + I_c \sin\varphi_2$$

$$\varphi_2 = \varphi_1 + 2\pi\Phi / \Phi_0$$

$$\Phi = BA$$

- The second equation comes from integrating the canonical momentum around the loop (See e.g., Van Duzer & Turner)
- If the total current is zero:

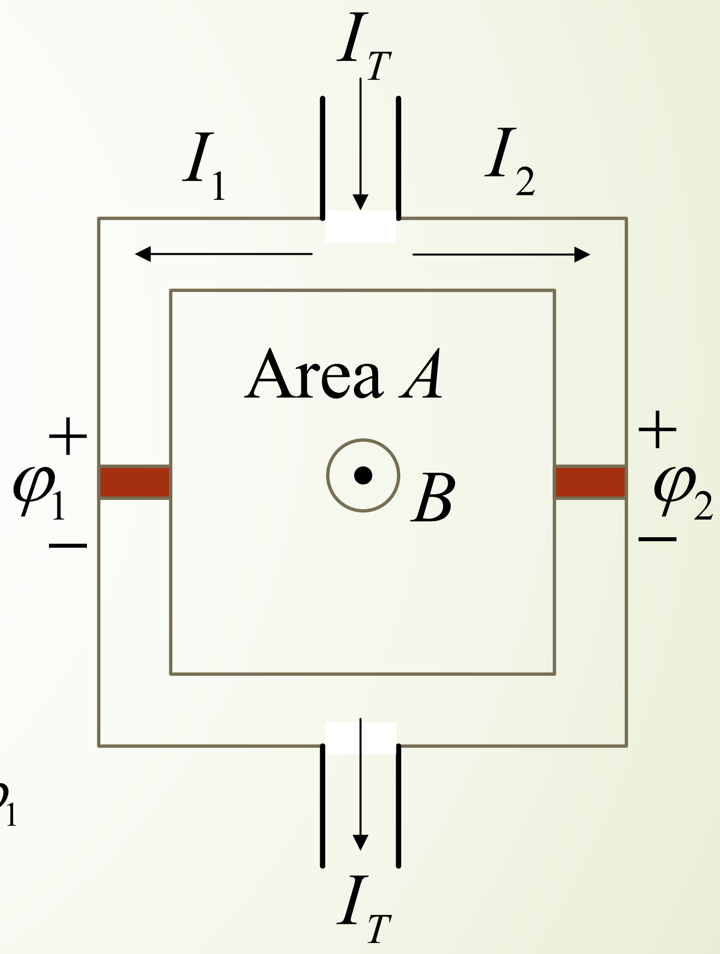
$$\sin\varphi_2 = -\sin\varphi_1$$

$$\sin\left(\varphi_1 + 2\pi\Phi / \Phi_0\right) = -\sin\varphi_1$$

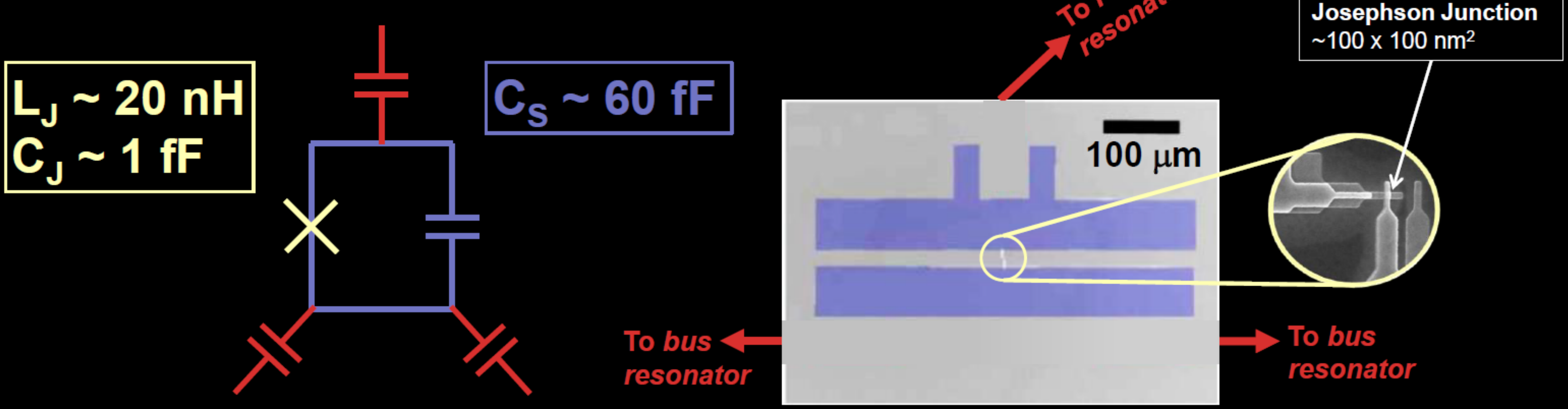
$$\varphi_1 + 2\pi\Phi / \Phi_0 \approx -\varphi_1$$

$$\varphi_1 = -\pi\Phi / \Phi_0 = -\varphi_2$$

- Thus applying a magnetic field will induce a current, and consequently tune the inductance



IBM qubits: single-junction transmons



Patterned superconducting metal (**niobium** + **aluminum**) on silicon

– Total capacitance dominated by **shunting capacitance C_S**

Interactions mediated by **capacitively coupled co-planar waveguide resonators**

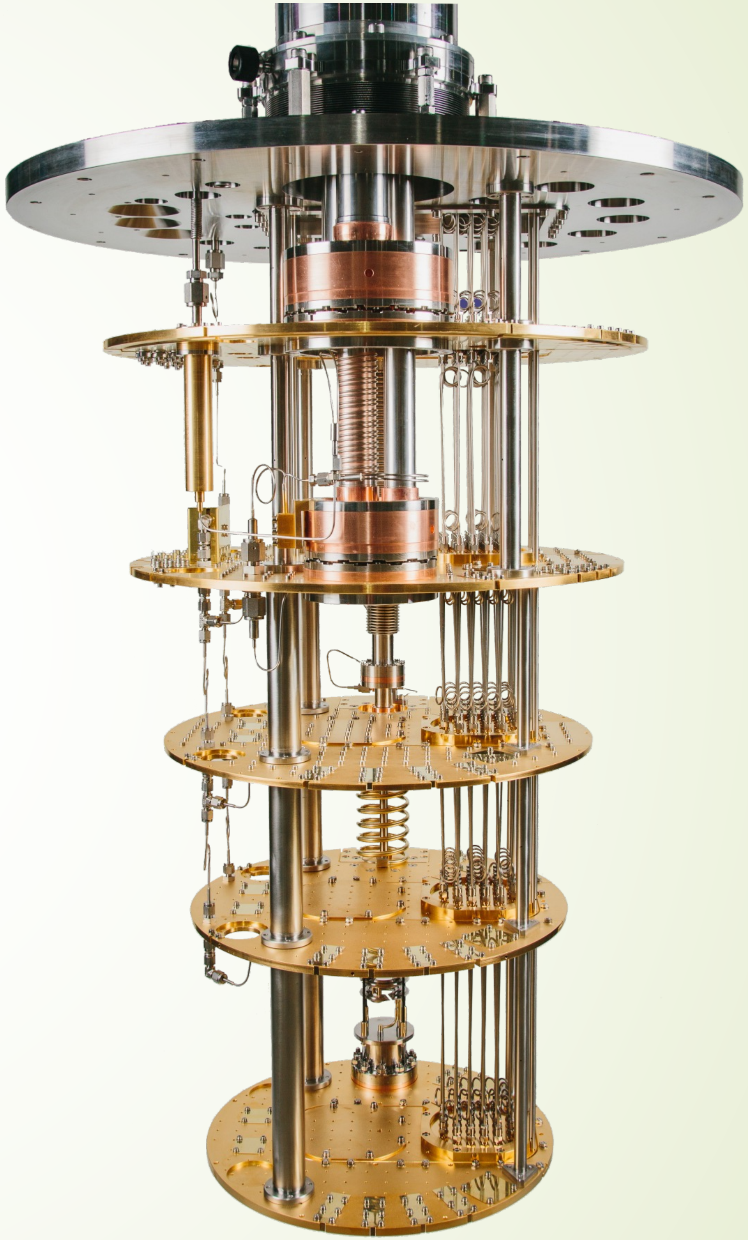
– *Bus resonators* provide controlled coupling to adjacent qubits

– *Readout resonators* couple to outside world; resonant frequency indicates qubit state

Slide credit: Doug McClure, IBM

Dilution Refrigerator (Oxford)

Base temperature	< 10 mK
Cooling power at 20 mK	> 12 μW
Cooling power at 100 mK	> 450 μW
Sample space diameter	360 mm plate
Line-of-sight-access	1 × Secondary Insert (117 mm × 252 mm), 2 × KF40, 2 × KF25
PTR options	1.35 W, 1.50 W or 1.80 W
Temperature control range	10 mK to 30 K with magnet at full field
Magnet options	Solenoid: up to 14 T Vector rotate: up to 9.1.1 T Field cancellation: < 10 mT



ANGSTROM ENGINEERING

Quantum Series | J-J Fabrication System

Superconducting Circuits

- Strong coupling to environment – short coherence times
- Strong qubit-qubit coupling – fast gates

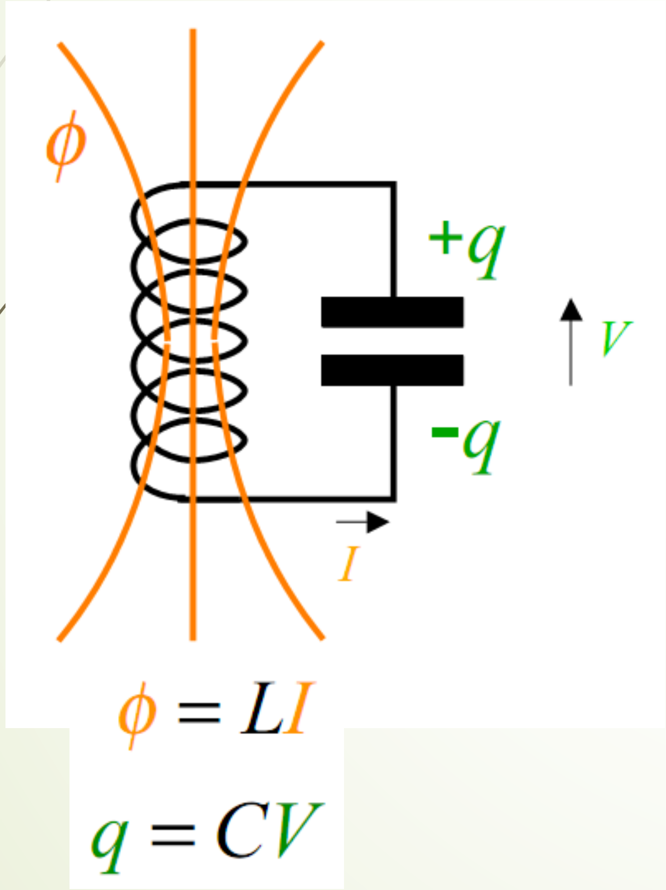
Superconducting Circuits

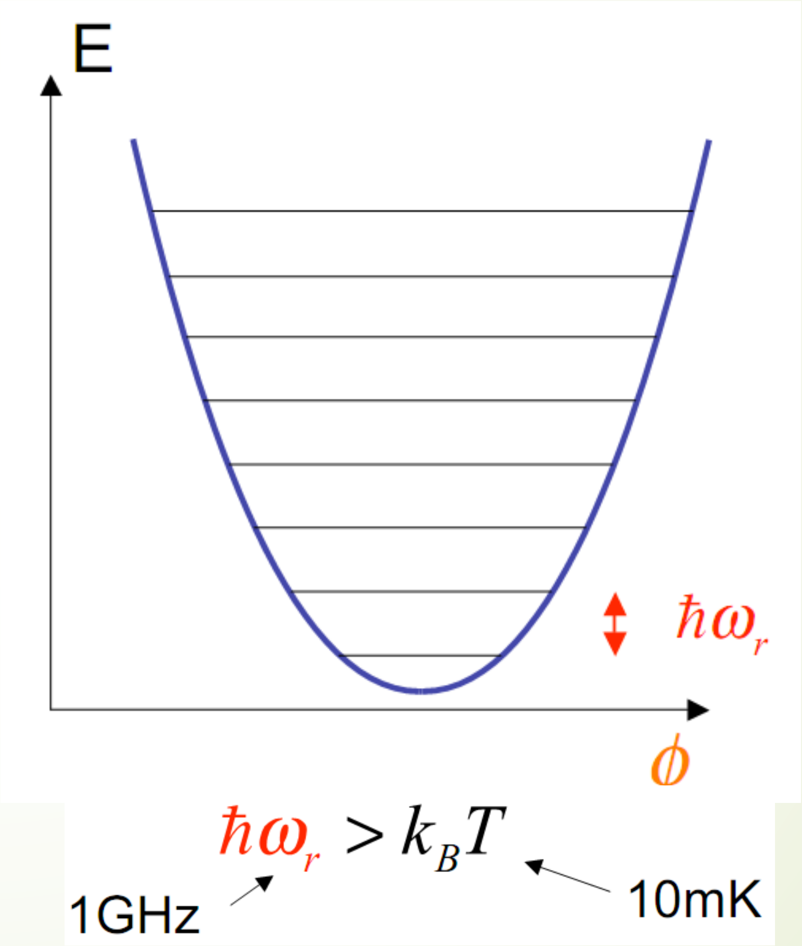
- Easy electrical access
- Easily engineered with capacitors, inductors, Josephson junctions
- Easy to fabricate and integrate

Quantum Characteristics

- How can a macroscopic device exhibit quantum properties?
- LC oscillator circuit is like a quantum harmonic oscillator
- $L=3nH$, $C=10pF \rightarrow f=1GHz$

Quantum Characteristics





Types of Superconducting Qubits

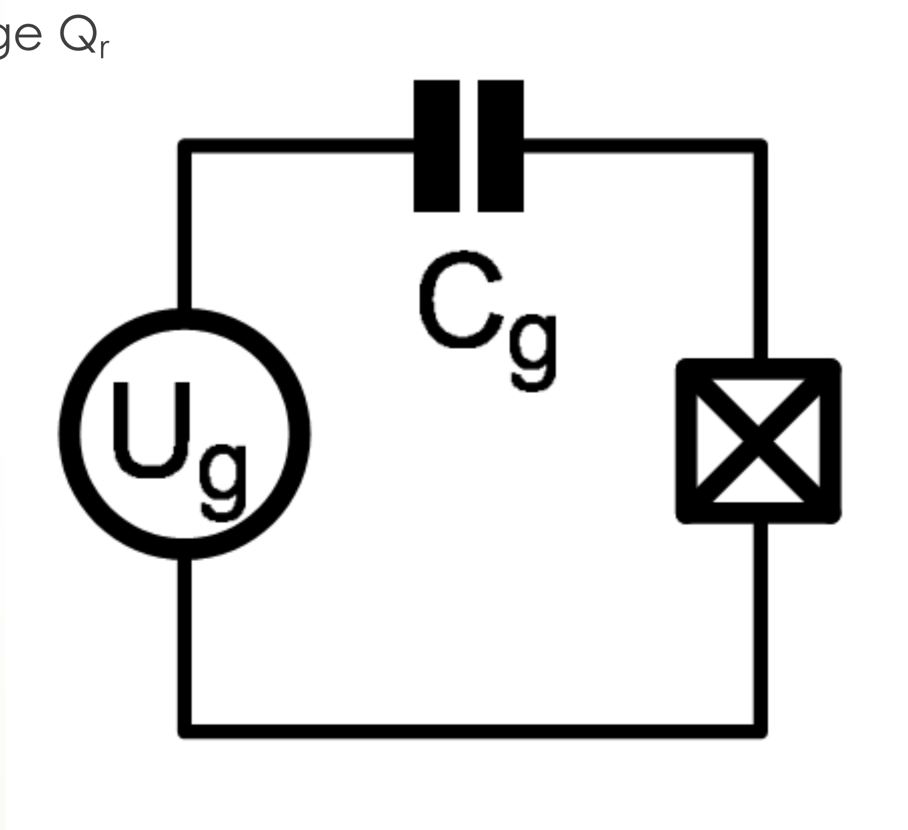
- Charge Qubit – Cooper Pair Box
- Flux Qubit – RF-SQUID
- Phase Qubit – Current Biased Junction

Readout

- Switch reading ON and OFF
- Controls Coupling
- Doesn't Contribute Noise (ON or OFF)
- Strong read and repeat rather than weak continuous measurements

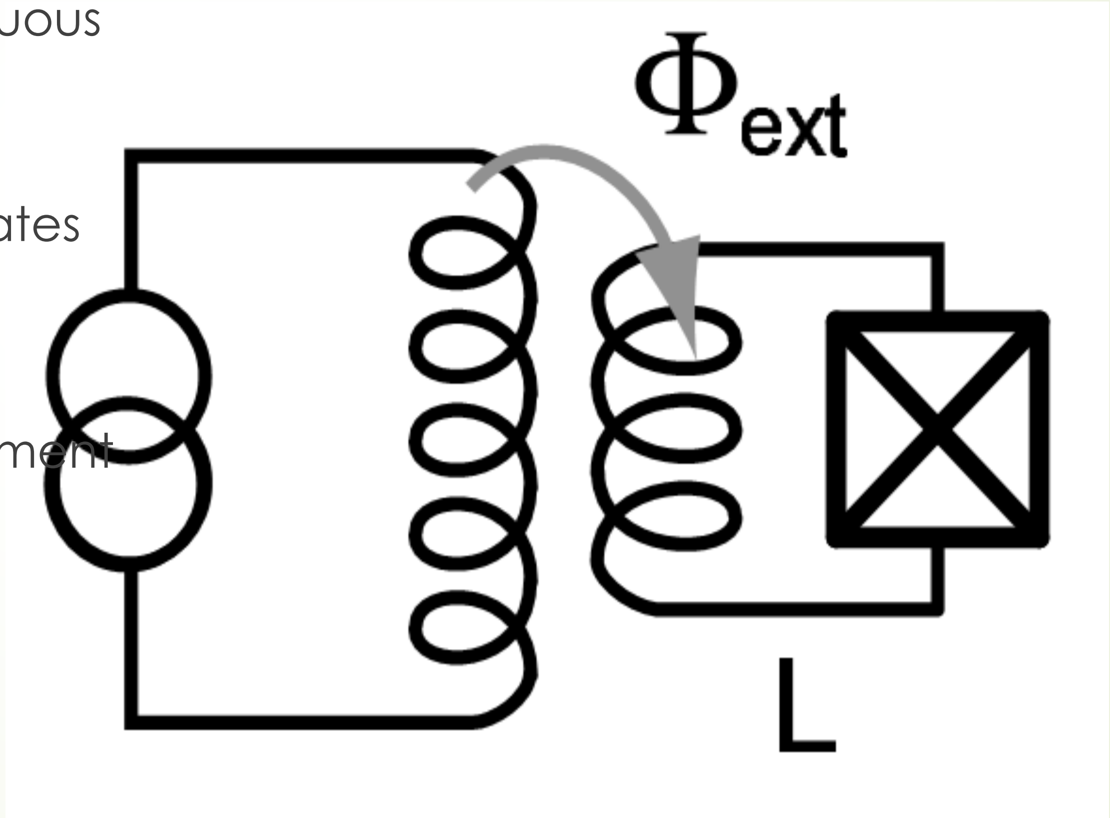
Charge Qubit – Cooper Pair Box

- Biased to combat continuous charge Q_r
- Cooper pairs are trapped in box between capacitor and Josephson junction
- Charge in box correlates to energy states

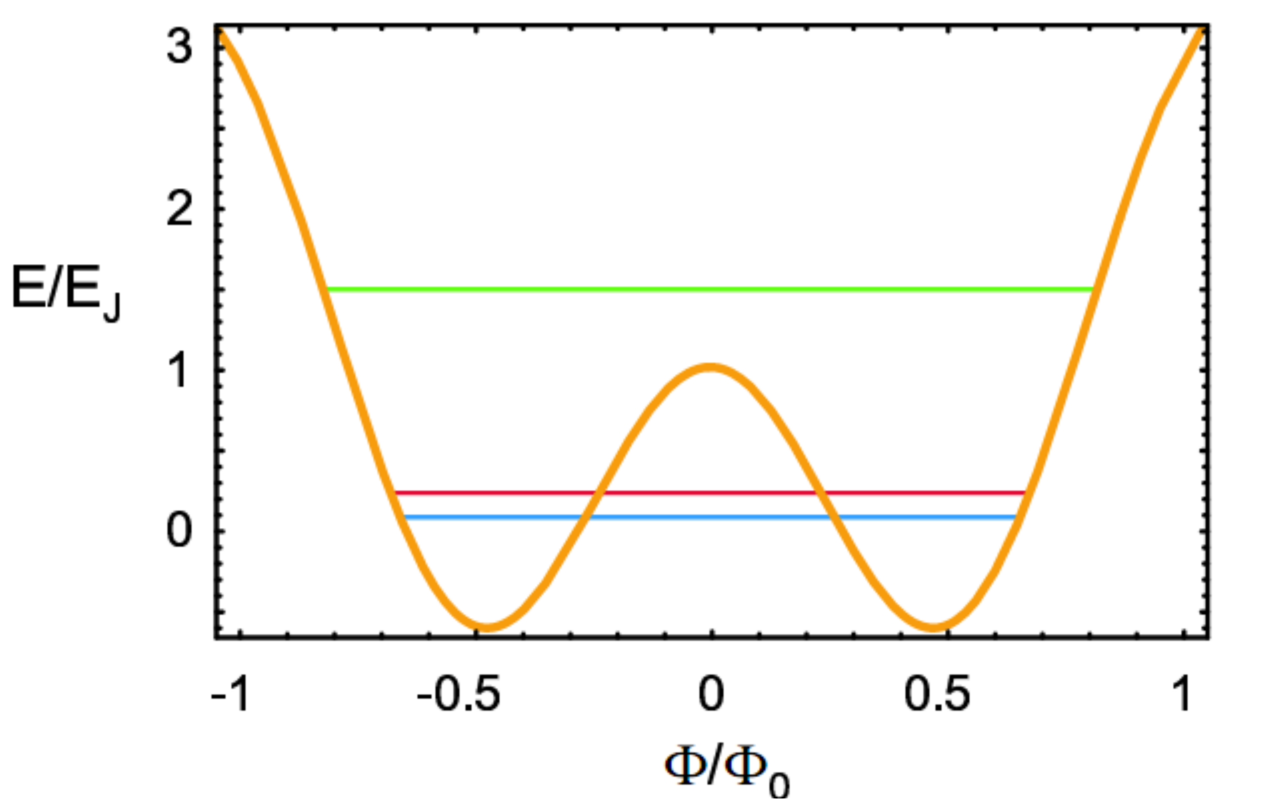


Flux Qubit – RF-SQUID

- Shunted to combat continuous charge Q_r
- Current in right loop correlates to energy states
- Can use RF pulses to implement gates



Flux Qubit – RF-SQUID



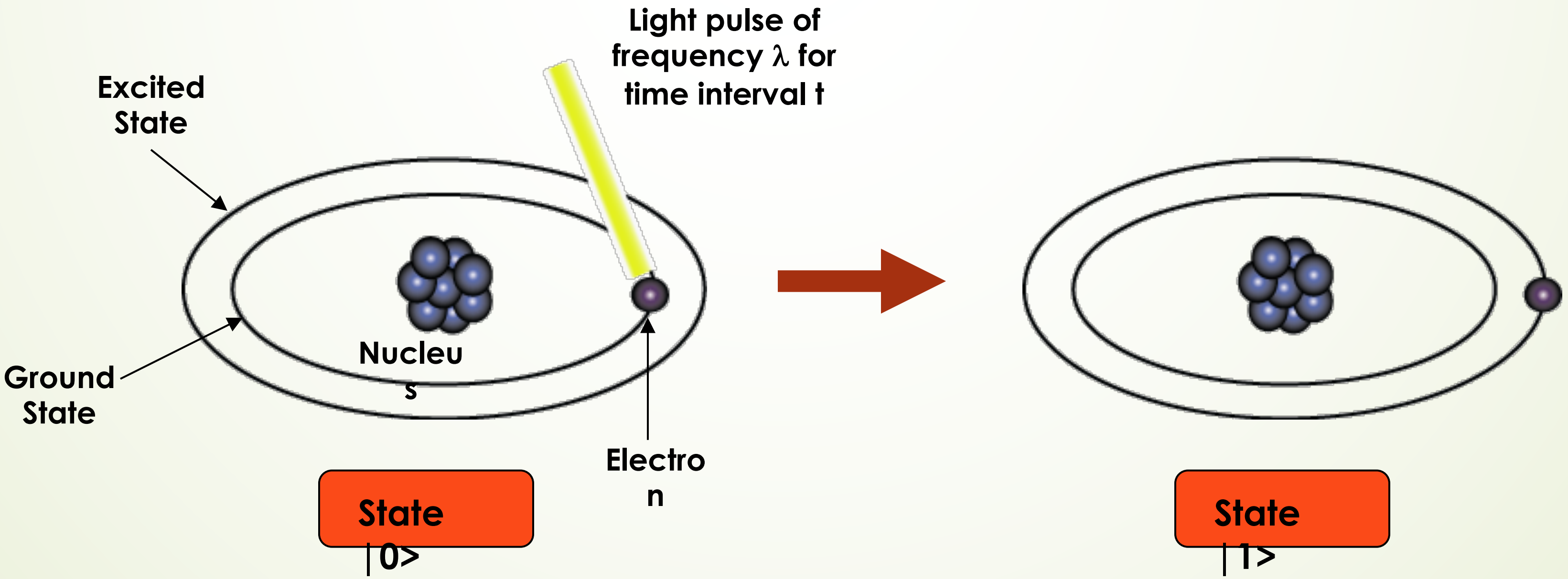
$$H = \frac{q^2}{2C_J} + \frac{\phi^2}{2L} - E_J \cos\left[\frac{2e}{\hbar}\left(\phi - \Phi_{\text{ext}}\right)\right]$$

Data Representation

Representation of Data - Qubits

A bit of data is represented by a single atom that is in one of two states denoted by **|0>** and **|1>**. A single bit of this form is known as a ***qubit***

A physical implementation of a qubit could use the two energy levels of an atom. An excited state representing |1> and a ground state representing |0>.



Representation of Data - **Superposition**

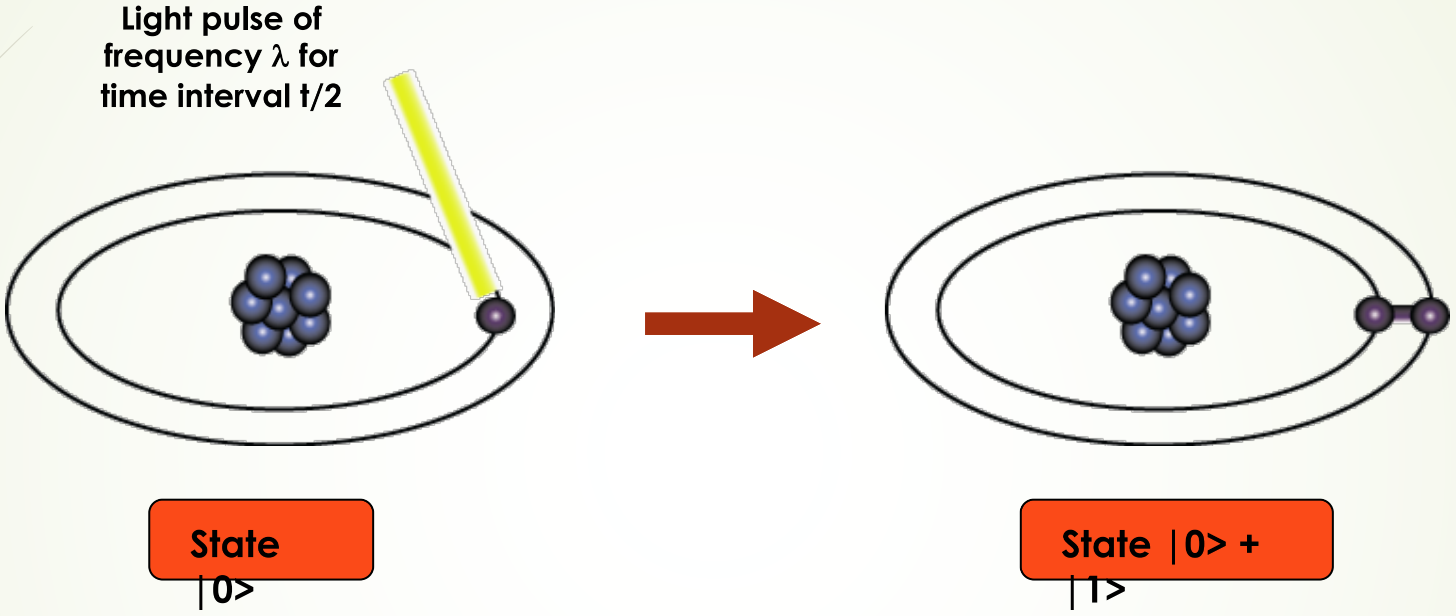
A single qubit can be forced into a ***superposition*** of the two states denoted by the addition of the state vectors:

$$|\psi\rangle = \alpha_1 |0\rangle + \alpha_2 |1\rangle$$

Where α_1 **and** α_2 are complex numbers and $|\alpha_1|^2 + |\alpha_2|^2 = 1$

A qubit in superposition is in both of the states |1> and |0 at the same time

Representation of Data - Superposition



- Consider a 3 bit qubit register. An equally weighted superposition of all possible states would be denoted by:

$$|\psi\rangle = \frac{1}{\sqrt{8}}|000\rangle + \frac{1}{\sqrt{8}}|001\rangle + \ldots + \frac{1}{\sqrt{8}}|111\rangle$$

Data Retrieval

- In general, an n qubit register can represent the numbers 0 through 2^n-1 simultaneously.
- If we attempt to retrieve the values represented within a superposition, the **superposition randomly collapses** to represent just one of the original values.

In our equation: $|\psi\rangle = \alpha_1|0\rangle + \alpha_2|1\rangle$, α_1 represents the probability of the superposition collapsing to $|0\rangle$. The α's are called probability amplitudes. In a balanced superposition, $\alpha_n = 1/\sqrt{2}$ where n is the number of qubits.

Relationships among data - **Entanglement**

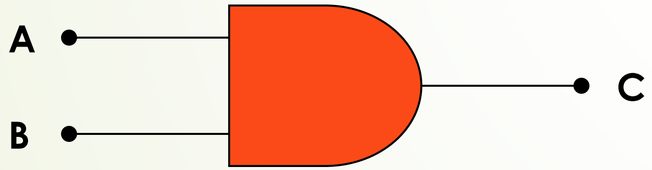
- ***Entanglement*** is the ability of quantum systems to exhibit correlations between states within a superposition.
- Imagine two qubits, each in the state |0> + |1> (a superposition of the 0 and 1.) We can entangle the two qubits such that the measurement of one qubit is always correlated to the measurement of the other qubit.

Operations on Qubits - Reversible Logic

- Due to the nature of quantum physics, the destruction of information in a gate will cause heat to be evolved which can destroy the superposition of qubits.

Ex.

The AND Gate



Input		Output
A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

In these 3 cases, information is being destroyed

- This type of gate cannot be used. We must use ***Quantum Gates***.

Quantum Gates

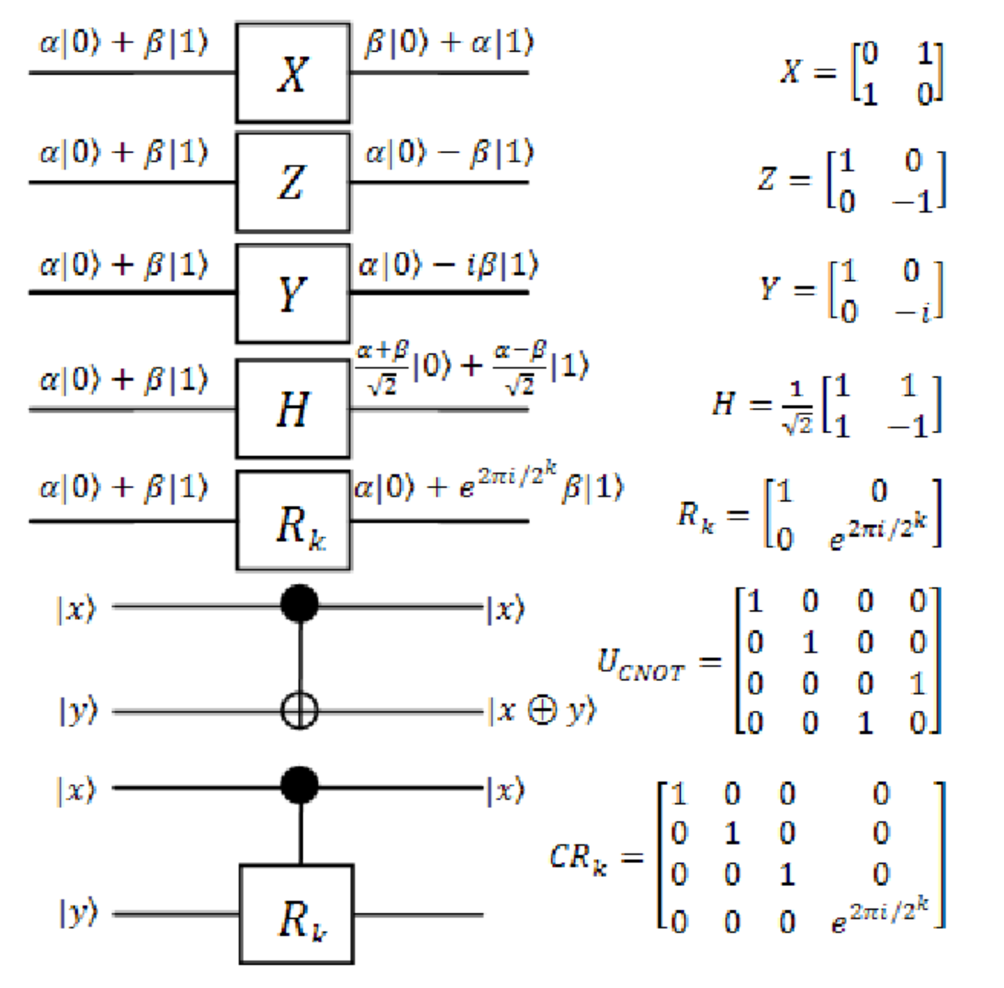
- Quantum Gates are similar to classical gates, but do not have a degenerate output. i.e. their original input state can be derived from their output state, uniquely. ***They must be reversible.***

- This means that a deterministic computation can be performed on a quantum computer only if it is reversible. Luckily, it has been shown that any deterministic computation can be made reversible.(Charles Bennet, 1973)

Classical vs. Quantum logic --- gates and algorithms

QUANTUM LOGIC

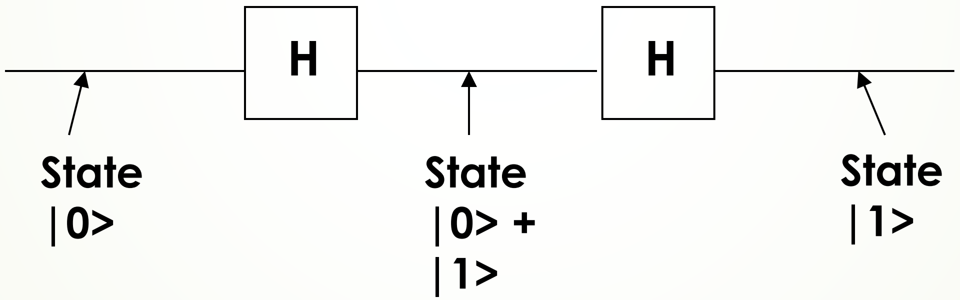
Quantum logic gates operate on single qubits and pairs of qubits



Gate name	# Qubits	Circuit Symbol	Unitary Matrix	Description
Hadamard	1	H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	Transforms a basis state into an even superposition of the two basis states.
T	1	T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	Adds a relative phase shift of $\pi/4$ between contributing basis states. Sometimes called a $\pi/8$ gate, because diagonal elements can be written as $e^{-i\pi/8}$ and $e^{i\pi/8}$.
CNOT	2	$\oplus$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	Controlled-not; reversible analogue to classical XOR gate. The input connected to the solid dot is passed through to make the operation reversible.
Toffoli (CCNOT)	3	$\oplus$	$\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{bmatrix}$	Controlled-controlled-not; a three-qubit gate that switches the third bit for states where the first two bits are 1 (that is, switches $\|110\rangle$ to $\|111\rangle$ and vice versa).
Pauli-Z	1	Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	Adds a relative phase shift of π between contributing basis states. Maps $\|0\rangle$ to itself and $\|1\rangle$ to $-\|1\rangle$. Sometimes called a "phase flip."
Z-Rotation	1	$R_z(\theta)$	$\begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$	Adds a relative phase shift of (or rotates state vector about z-axis by) θ.
NOT	1	$\oplus$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Analogous to classical NOT gate; switches $\|0\rangle$ to $\|1\rangle$ and vice versa.

Quantum Gates - Hadamard

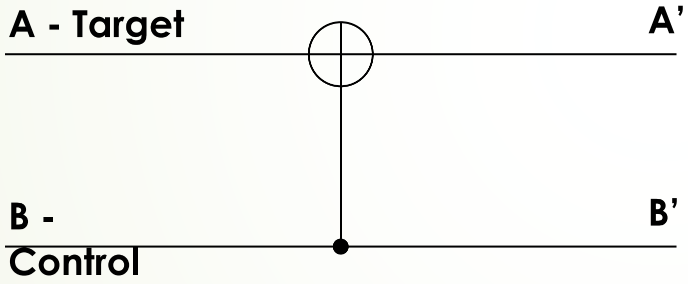
- Simplest gate involves one qubit and is called a ***Hadamard Gate*** **(**also known as a square-root of NOT gate.) Used to put qubits into superposition.



Note: Two Hadamard gates used in succession can be used as a NOT gate

Quantum Gates - Controlled NOT

- A gate which operates on two qubits is called a ***Controlled-NOT (CN) Gate.*** If the bit on the control line is 1, invert the bit on the target line.



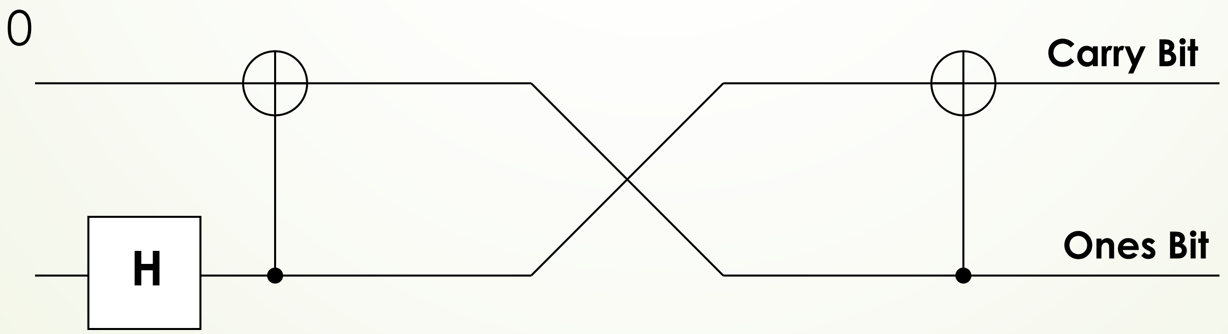
Input		Output	
A	B	A'	B'
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1

Note: The CN gate has a similar behavior to the XOR gate with some extra information to make it reversible.

Example Operation - Multiplication By 2

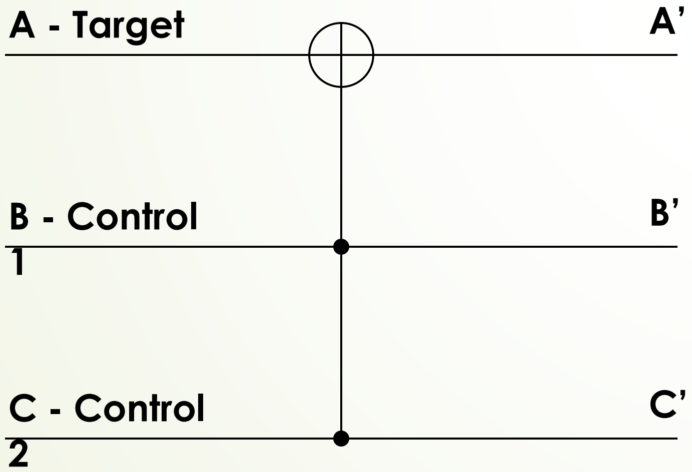
- We can build a reversible logic circuit to calculate multiplication by 2 using CN gates arranged in the following manner:

Input		Output	
Carry Bit	Ones Bit	Carry Bit	Ones Bit
0	0	0	0
0	1	1	0



Quantum Gates - Controlled Controlled NOT (CCN)

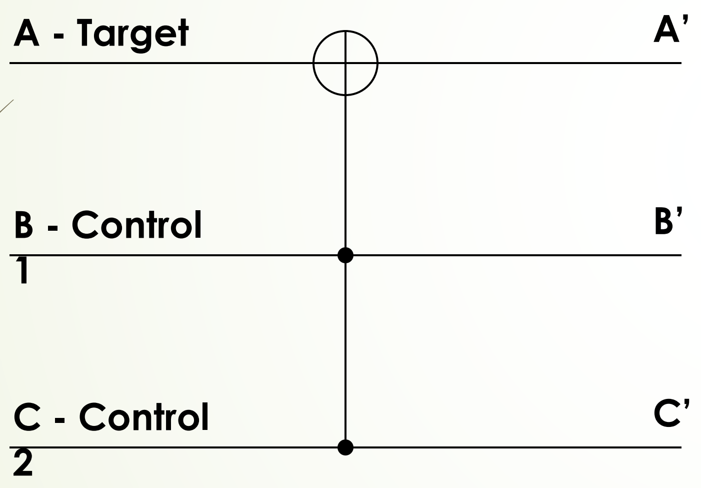
- A gate which operates on three qubits is called a ***Controlled Controlled NOT (CCN) Gate.*** Iff the bits on both of the control lines is 1,then the target bit is inverted.



Input			Output		
A	B	C	A’	B’	C’
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	0
1	1	1	0	1	1

A Universal Quantum Computer

- The CCN gate has been shown to be a ***universal*** reversible logic gate as it can be used as a NAND gate.



Input			Output		
A	B	C	A'	B'	C'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	0
1	1	1	0	1	1

When our target input is 1, our target output is a result of a NAND of B and C.