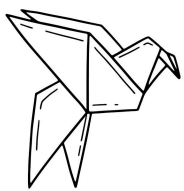
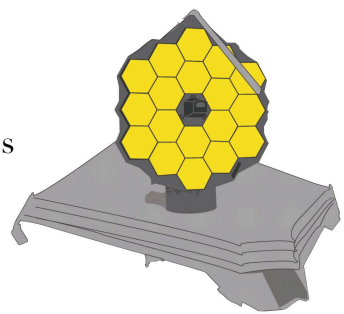
ORIGAMI: Art, Math, Engineering



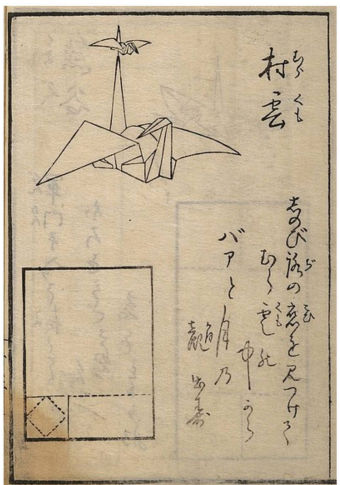
Chirantan Ganguly
MTech. Functional Materials and Devices
Roll: 23PH63R06



1/11/2023

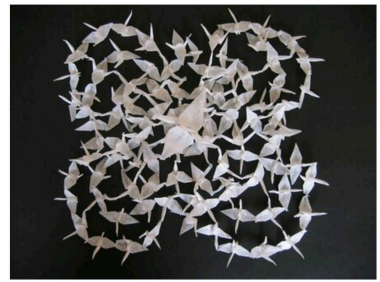
Origins of Origami (折り紙)

ori meaning "folding", and *kami* meaning "paper"



First known technical book on Origami (1797)

Art:



- One hundred cranes (百鶴, *Hyakkaku*)
- Made by folding a single sheet of paper

- Satoshi Kamiya's Ryujin 3.5
- Widely regarded as most difficult origami
- Made from a single sheet of paper

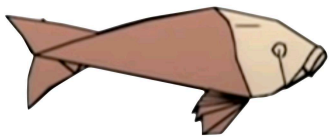
Nature:

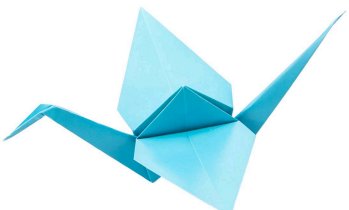
- Complex origami is also observed in nature
- For example - leaf and flower buds

Evolution of Origami (折り紙)

ori meaning "folding", and *kami* meaning "paper"

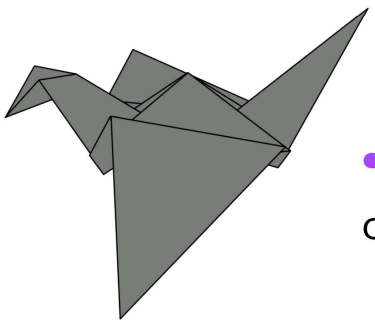
Simple



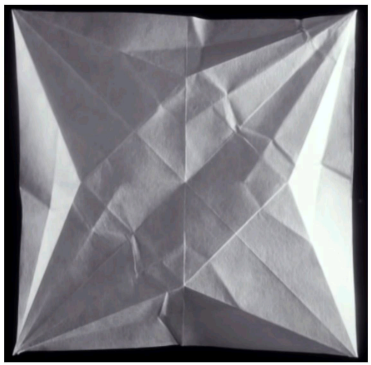


Detailed and Complex

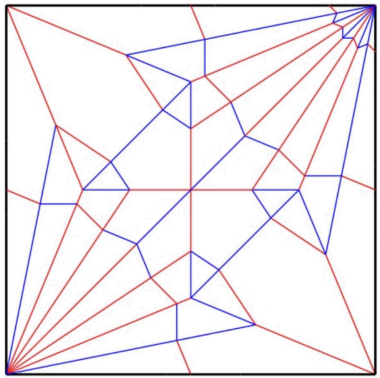
Mathematics of Origami

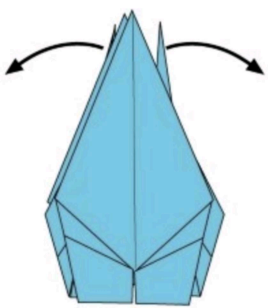


open



formalism

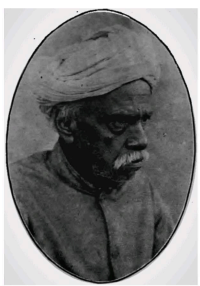




Mathematics of Origami

Modeled using a variety of mathematical techniques including:

- Geometry
- Matrix algebra (rigid folding applications)
- Combinatorics & Graph Theory
- Statistical mechanics
- Differential geometry
- Topology



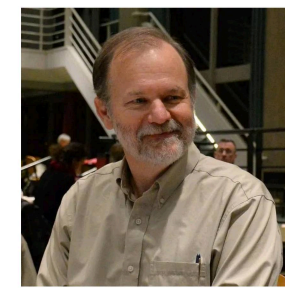
T. Sundara Row

Harry Houdini

Margherita Beloch

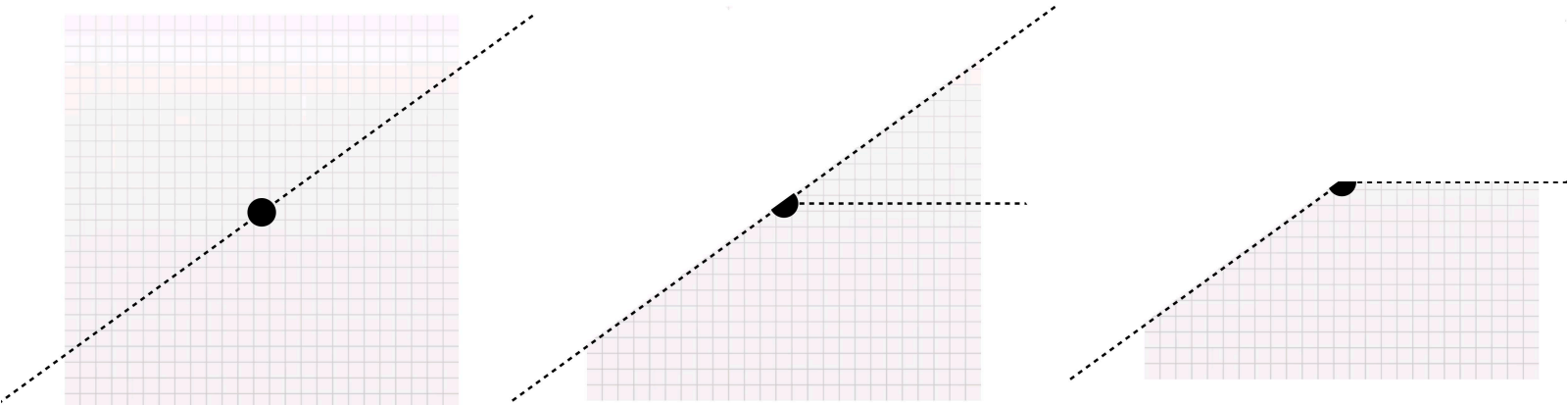
Akira Yoshizawa

Humiaki Huzita



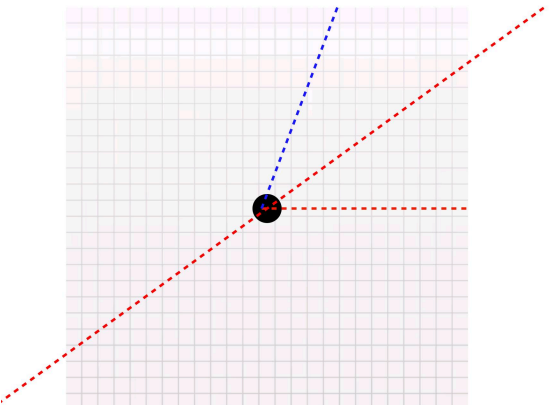
Robert J Lang

Geometry in Origami



Fold it any way you want through a fixed vertex

Geometry in Origami



Valley Fold

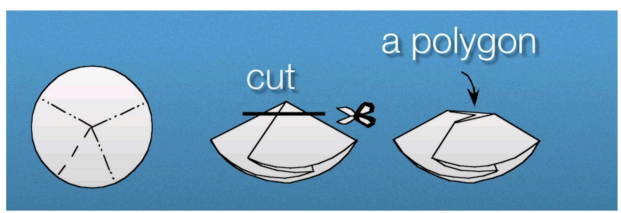
Valley Creases	Mountain Creases
3	1
5	3
3	5
4	2

Mountain Fold

$$M - V = \pm 2$$

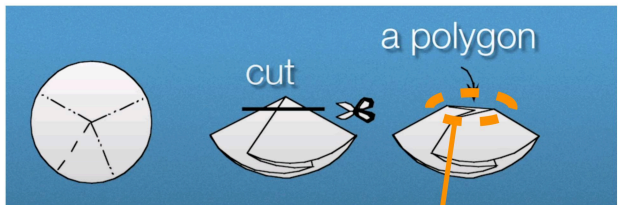
Maekawa's Theorem: Let v be a vertex in a flat origami crease pattern. Let **M** and **V** be the number of mountain and valley creases at v, respectively. Then $M - V = \pm 2$.

A simple proof:



Geometry in Origami

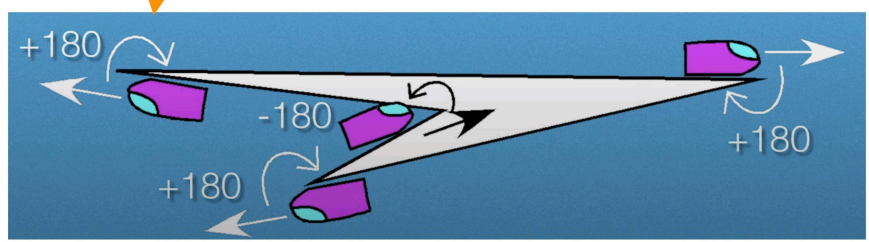
A simple proof:



Imagine a monorail train traveling clockwise around the cross-section.

Every time it comes to a **M** -> rotates by 180°

Every time it comes to a **V** -> rotates by -180°



So, for one full circle, we have

$$180^\circ M - 180^\circ V = \pm 360^\circ$$

$$M - V = \pm 2$$

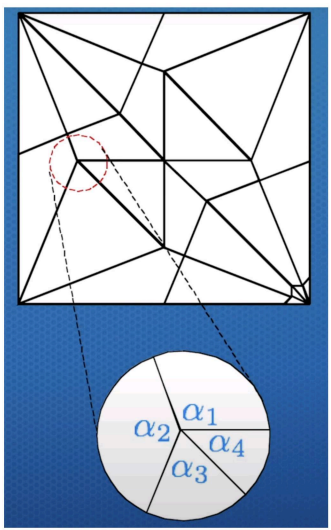
Geometry in Origami

Kawasaki's Theorem: Angles around a flat folded vertex have

$$\sum_i \alpha_{2i-1} = 180^\circ$$

$$\sum_i \alpha_{2i} = 180^\circ$$

Example



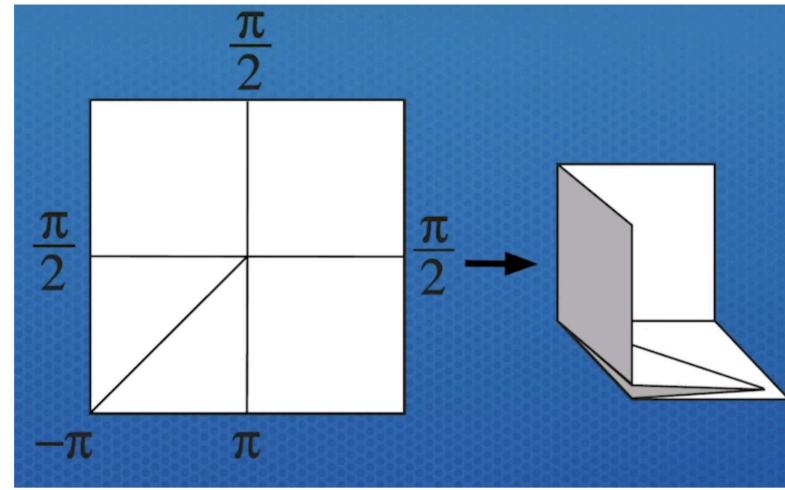
Here

$$\alpha_1 + \alpha_3 = 180^\circ$$

$$\alpha_2 + \alpha_4 = 180^\circ$$

Matrix Algebra in Origami

Origami folds = piecewise isometries, which are just matrix transformations in space.



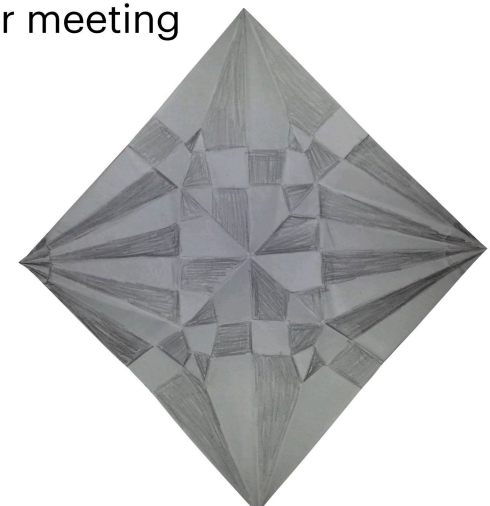
The transformation by folds can therefore be modeled by rotation matrices along the folds.

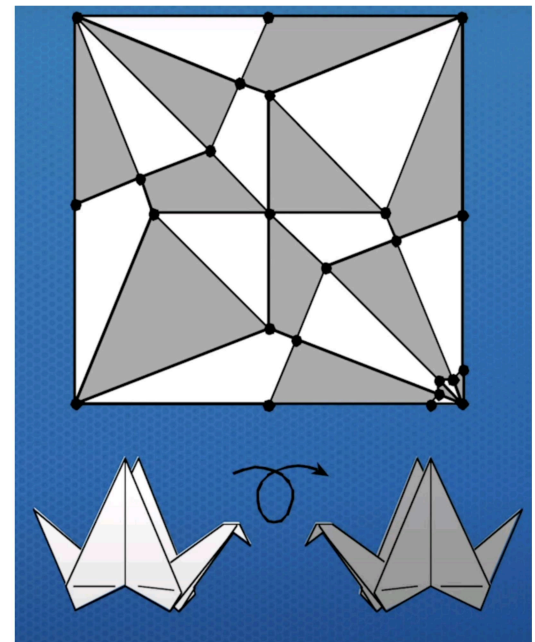
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

Graph Theory in Origami

Origami crease patterns = planar graphs (networks)

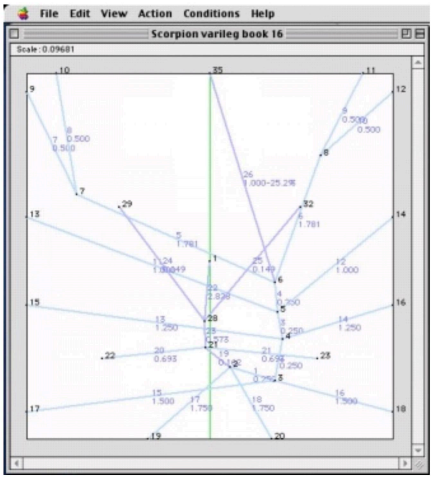
Two colourability: Any crease pattern can be colored with only two colors without the same color meeting

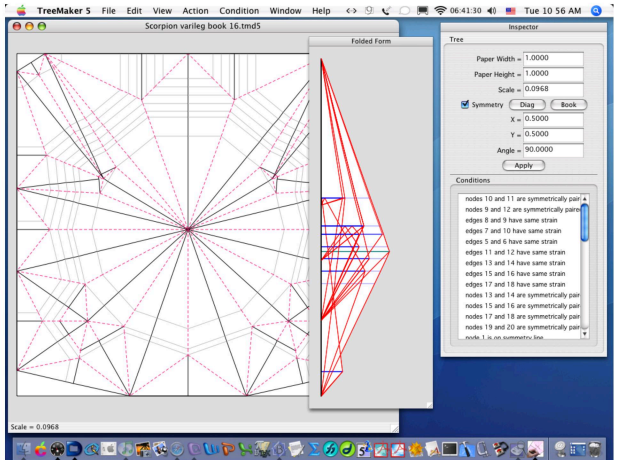


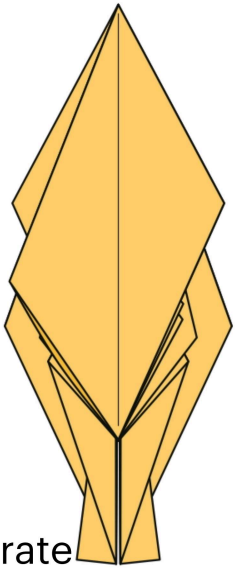


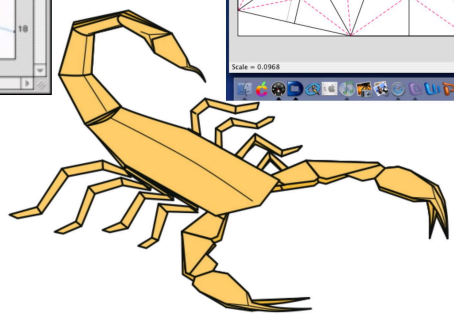
Computer Aided Origami

Uses mathematical models to generate crease patterns from stick figure inputs







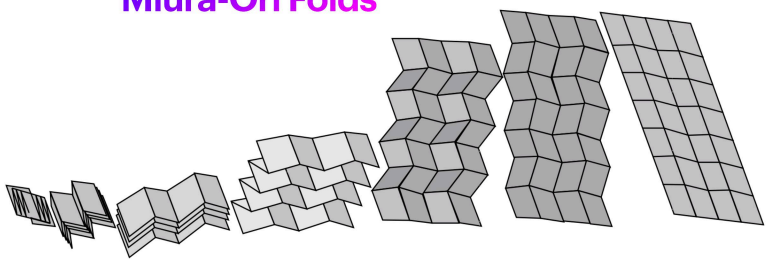


There is NO 100% accurate computer software for origami yet

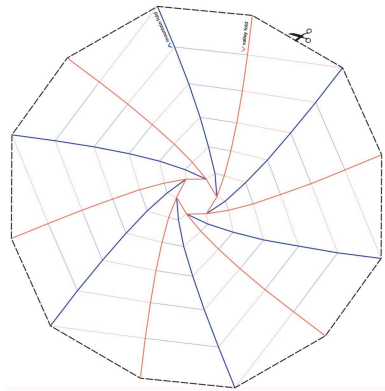
Ref:
[1] Tason. (n.d.). TreeMaker. https://langorigami.com/article/treemaker/

Functional Origami Designs

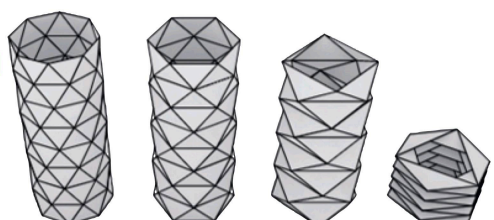
Miura-Ori Folds

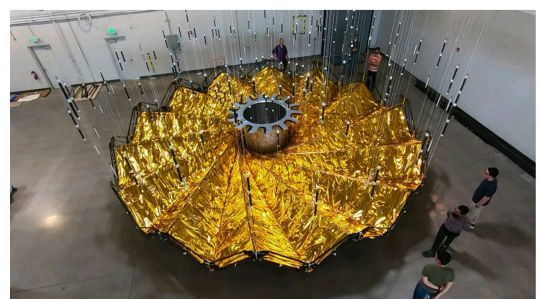


Starshed Origami

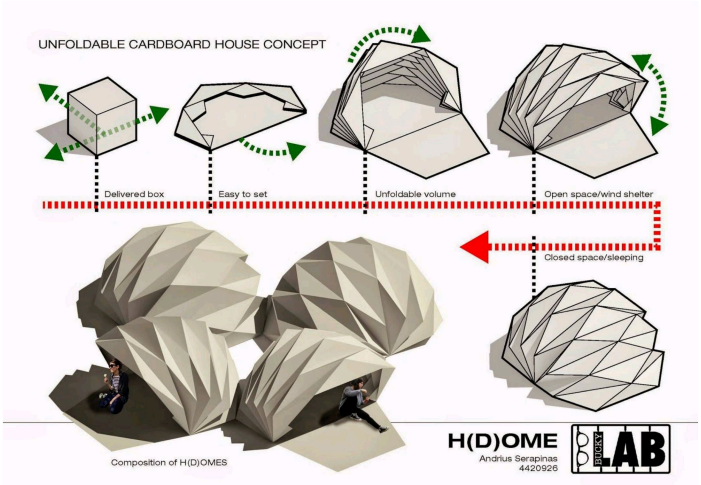


Yoshimura Buckling



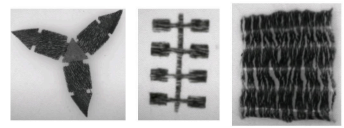


Applications of Origami

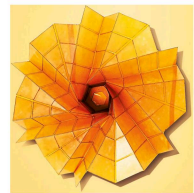


Foldable Emergency tents

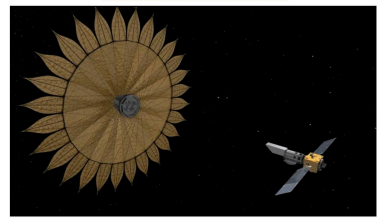
Origami stent to open constricted blood vessels



Magnetic origami nanobots for drug delivery



Deployment of bulky equipments in spacecrafts



Ref:

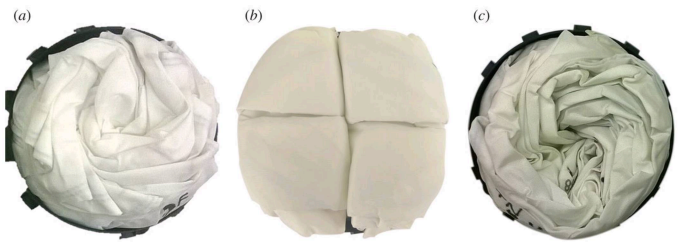
[1] Fox, A. (2019). These origami robots could one day deliver drugs inside your body. Science. https://doi.org/10.1126/science.aax8069
[2] Tillman, N. T. (2015, September 4). FLOWER POWER: Giant "Starshades" prepped for exoplanet hunting. Space.com. https://www.space.com/30465-alien-planet-search-starshade-technology.html
[3] Lufkin, B. (2014, May 29). Origami solar panels to power the space stations of tomorrow. WIRED UK. https://www.wired.co.uk/article/origami-in-space
[4] From origami to a prototype stent. (2017, August 7). mddionline.com. https://www.mddionline.com/implants/origami-prototype-stent

Applications of Origami

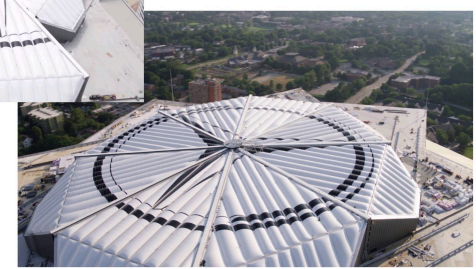
Shock absorbers

Bulletproof Shields

Airbag packing



Origami Architecture



Engineering is where art meets Math and Science

Thank You