Classical Mechanics
PH2213

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1 Assignment 1

1.1 Solution 1

- **Virtual Displacement** is a infinitisimal small displacement consistant with the contraint relations at any given instant(i.e At frozen time.)
- With virtual displacement, we can account for the contraint relations as the work done by contraint forces is zero

Applying Newton's 2nd Law on system of particles(EQ 1):

$$\sum_i (\vec{F_i} - \dot{\vec{p}}) = 0$$

Where,

$$\vec{F_i} = \vec{F_i^{(a)}} + \vec{f_i}$$

On solving EQ 1 and putting $\sum_i \vec{f_i}.\delta\vec{r_i} = 0$,
We get **D' Alembert's** principle,

$$\sum_i (\vec{F_i^{(a)}} - \dot{\vec{p_i}}).\delta\vec{r_i} = 0$$

Which is independent of constraint forces hence its easier to work with this equation. Furthermore, we can derive **Langrange equation** using the D' Alembert's relation.

Example 1.1. For a simple pendulum with variable length l(t). The displacement of the pendulum and the tention are not perpendicular. For a virtual displavement δr consistant with l(t) at time t. If time is frozen the virtual displacement is perpendicular to the string direction hence work done is 0.

1.2 Solution 2

No
Internal Forces don't do work.

- Since internal forces cause no displacement.
- Even if we consider above statement to be false, For rigid bodies,

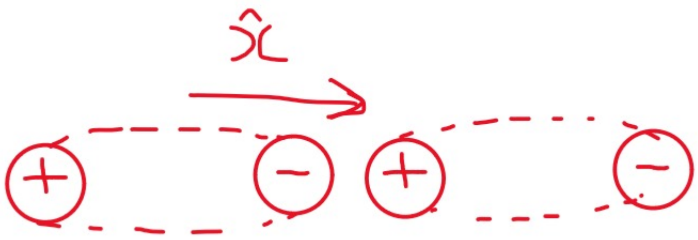
$$\text{mod } (r_a - r_b) = constant$$
$$\text{mod } (r_a - r_b)^2 = constant$$
$$(r_a - r_b).\Delta(r_a - r_b) = 0$$
$$Using\ Newton's\ 3rd\ Law$$
$$F_{ab} = -F_{ba}$$
$$F_{ab}.\Delta r_a = -F_{ba}.\Delta r_a$$
$$\Delta W = \sum_{a,b,a\neq b} F_{ab}.\Delta r_a = \sum_{a,b,b>a} (F_{ab}.\Delta r_a + F_{ba}.\Delta r_b) = \sum_{a,b,b>a} F_{ab}.\Delta(r_a - r_b)$$
$$Since\ F_{ab}\ parallel\ to\ line\ joining\ particles$$
$$F_{ab} = C_{ab}(r_a - r_b)$$
$$\Delta W = \sum_{a,b,b>a} C_{ab}(r_a - r_b).\Delta(r_a - r_b)$$

Thus Work done is 0.

1.3 Solution 3

- Newtonian Laws are only applicable for inertial frames of reference whereas Lagrange formalism is applicable to all frames.
- Newtonian Laws only work for Cartesian Coordinates whereas Lagrange equation can take any type of Coordinates.
- Its much easier to work with scalar quantities in Lagrange equation rather than vector quantities in Newtonian equations.
- In Newtonian formalism knowleadge of all forces acting on the body is required whereas in Lagrangian formalism in Kinetic and potential Energy is required.
- It can be difficult to deal with constraints in Newton formalism.
- In newton formalism the quantity $p_1 + p_2 = m_1 v_1 + m_2 v_2$ is not a constant of motion for mutually attracting particles. In Lagrange formalism the definition of canonical momentum solves this.

1.4 Solution 4



Van der Waals force is a electrostatic dipole induced dipole force such that both dipoles have their axis aligned parallel to each other.
Their axis are parallel because the electric field of the first dipole(along x) causes the induced dipole to be along x as well
According to Couloumb's Law all electrostatic forces are radial.
Using the approximation as in the figure above, all the forces are along the x direction therefore

$$\vec{F} = k.\hat{x}$$
$$\tau = \vec{r} \times \vec{F}$$
$$\tau = (r.\hat{x}) \times (k.\hat{x})$$
$$\tau = 0$$

Since torque is 0 and all forces are aligned alond the radius therefore it is a central force.