Undergraduate Algebra
MT2213

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Contents

1 Definitions

1.1 Group Action

G is group
A is set
$G \times A \mapsto A$

- $e.a = a \ \forall a$
- $g_1.(g_2.a) = (g_1 g_2).a \ \forall g_1, g_2, a$

1.2 Orbit

$O_a = G.a = \{g.a | g \in G\} \subset A$

1.3 Stabilizer

$G_a = \{g \in G \| g.a = a\} \subset G$

2 Theorems

Theorem 2.1. Any finite group of prime order is cyclic. Hence, there is only one possible group Z/pZ upto isomorphism.

Theorem 2.2. G is group or order p^2.Then, G is Abelian. G isomorphic to either Z/p^2Z or $Z/pZ \times Z/pZ$

Theorem 2.3. Orbit Stabilizer Theorem
G is a finite group acting on a finite set A
Then,

- $|A| = \sum_{a \in A|G} |O_a|$