# 1 Lecture 1

- If you can compute anything then a turing machine must be able to solve it.
- Polynomial time: Order is $\mathcal{O}(n^\alpha)$
- Quantum computers claim to solve non-deterministic polynomial(NP) problems in polynomial time

**Theorem 1** ***Moore's Law****: The number of transistors that you can put in an IC doubles every 2 years.*

- Cant pack transistors more and more because at small transistors diameters quantum tunneling can take place
- "Lower Limit"
- Hence Quantum computers can help

## 1.1 Basics of Quantum Physics

$\psi$ denotes state of a system: I can compute anything with **only** this. In position basis, I can have

$$\langle x|\psi\rangle = \psi(x)$$

Or,

$$\langle p|\psi\rangle = \psi(p)$$

Reside in "Hilbert Space"(A linear vector space with an inner product, Infinite dimension)
Can be finite dimension hilbert space

$$L^2 \to l(l+1)h$$

$$L_z \to mh(-l \leq m \leq l)$$

Operator is an abstract object until you choose a representation. Applying a operator is same as applying a matrix multiplication. In a eigenbasis for an operator:

$$\hat{O}\,|\psi\rangle = a\,|\psi\rangle$$

$|\psi\rangle$ only increases or decreases in length
Quantum computing is all about operators on states

$$O\,|\psi\rangle = |\psi'\rangle$$

$$|\psi\rangle = \sum_{i=1}^{M} a_i\,|\phi_i\rangle \tag{1}$$

We will be dealing with TDSE:

$$ih\frac{\partial\psi(x,t)}{\partial t} = \hat{H}\psi(x,t) \tag{2}$$

**Time Evolution**

$$\psi(x,t) = e^{-\iota\hat{H}t/h}\psi(x,0) \tag{3}$$

This is the Scrodinger picture, it is for non-time dependent hamiltonian

$$\psi(x,t) = \hat{(U)}\psi(x,0)$$

Where U is unitary operator
If:

$$H \rightarrow H(t)$$

$$U(t_0,t) = \mathbf{T}e^{-\iota/h \int_{t_0}^{t} dt' H(t')}$$

It can be shown that:

$$U^\dagger U = I$$

Also:
Eigenvalues of U are of modulo 1 (TODO:Prove this)

**Theorem 2** *For any hermitian Matrix A*

$$e^{\iota A} = \hat{U}$$

*Where U is unitary operator*

For 2 non commuting operators A and B

$$e^{A+B)} \neq e^A \cdot e^B$$

For $[[A,B],A] = 0$ (TODO:See proof)

$$e^{A+B} = e^A e^B e^{-[A,B]/2}$$

$$A\left|\psi\right\rangle = a\left|\psi\right\rangle$$

$$A\left|\psi\right\rangle\left\langle\psi|\psi\right\rangle = a\left|\psi\right\rangle$$

Replace $\left|\psi\right\rangle\left\langle\psi\right|$ by

$$\hat{\rho}$$

which is the density operator

$$\left\langle\psi\right|\rho\left|\psi\right\rangle = |\psi|^2$$

$\rho$ is as important as $\psi$. Real importance is apparent in ensemble of systems.
**Ensemble** is a collection of identical systems. Example a collection of $H_2$ atoms in different states. No use if all are in the same state.
Suppose I have N states with $N_1$ of them in state $\psi_1$, $N_2$ in $\psi_2$ etc. So instead of just $\psi$, I have and ensemble of states: "Mixed states"

$$\begin{bmatrix} N_1\psi_1 \\ N_2\psi_2 \\ N_3\psi_3 \\ ... \end{bmatrix}$$

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