2 Curs 2

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(A) $\Gamma \models (\forall X)G \iff \forall \mathcal{A}\models\Gamma\ \forall h: T_\Sigma(X)\to\mathcal{A}\quad h_s(l)=h_s(r)\quad \forall l \doteq_s r \in G$

(B) $\Gamma\models(\exists X)G \iff \forall \mathcal{A}\models\Gamma\ \exists h: T_\Sigma(X)\to\mathcal{A}\quad h_s(l)\doteq h_s(r)\ \forall l\doteq_s r\in G$

$c \equiv_\Gamma^{\mathcal{A}} b \iff$
$\iff \forall h:\mathcal{A}\to\mathcal{M}\models\Gamma$
$h(c)=h(b)$

(P1) $\Gamma\models(\forall X)G \iff G\subseteq \equiv_\Gamma^{T_\Sigma(X)}$

(P2) $h:\mathcal{A}\to\mathcal{B}\quad h(\equiv_\Gamma^{\mathcal{A}})\subseteq\equiv_\Gamma^{\mathcal{B}}$

(P3) $h: T_\Sigma(X)\to T_\Sigma(Y)\quad \Gamma\models(\forall X)G \Rightarrow \Gamma\models\forall Y\, h(G)$

TM. HERBRAND echivalente pt $G\subseteq T_\Sigma(X)\times T_\Sigma(X)$

1) $\Gamma\models(\forall X)G$

2) $T_{\Sigma,\Gamma}\models(\exists X)G$

3) $\exists\psi:T_\Sigma(X)\to T_\Sigma\quad \Gamma\models(\forall\emptyset)\psi(G)\ (\iff \psi(G)\subseteq\equiv_\Gamma^{T_\Sigma})$

1.=>2) $T_{\Sigma,\Gamma}\models\Gamma$

2 => 3 $\quad h: T_\Sigma(X)\to T_{\Sigma,\Gamma}\quad \forall l\doteq_s r\in G\quad h_s(l)=h_s(r)$

$$\begin{tikzcd} T_\Sigma \arrow[r, "\eta"] & T_{\Sigma,\Gamma}=T_\Sigma/_{\equiv_\Gamma} \\ T_\Sigma(X) \arrow[u, "\psi"] \arrow[ur, "h"'] & \end{tikzcd}$$

$T_\Sigma(X)$ alg liberă $\Rightarrow$ proiectivă

$T_\Sigma(X)$ proiectiv $\exists\psi: T_\Sigma(X)\to T_\Sigma\quad \psi;\eta = h$

Fie $l \doteq_s r\in G\quad h_s(l)=h_s(r)$

$(\psi\eta)_s(l)=(\psi\eta)_s(r)\Rightarrow$

$\Rightarrow(\psi_s;\eta_s)(l)=(\psi_s;\eta_s)(r)\Rightarrow$

$\Rightarrow\eta_s(\psi_s(l))=\eta_s(\psi_s(r))\Rightarrow$

$\Rightarrow\psi_s(l)\equiv_\Gamma\psi_s(r)\qquad \boxed{\psi(G)\subseteq\equiv_\Gamma}$ (A) $\Rightarrow$

$\Rightarrow\Gamma\models(\forall\emptyset)\psi(G)$

3=>1: Se dau pt Σ-alg $\forall\mathcal{M}\models\Gamma$

~~$\mathcal{M}\models(\forall\emptyset)\psi(G)\ \forall f: T_\Sigma\to\mathcal{M}\ \forall l\doteq_s r\in G\ f(\psi(l))=f(\psi(r))$~~

$(\mathcal{M}\models(\exists X)G)\ \exists s: T_\Sigma(X)\to\mathcal{M}\quad \forall l\doteq_s r\quad s(l)=s(r)$

Fie $\mathcal{M} \models \Gamma$ $\quad T_\Sigma(X) \xrightarrow{\psi} T_\Sigma \xrightarrow{\exists! \alpha} \mathcal{M}$

$s' := \psi ; \alpha$

Fie $l \equiv_s r \in G \xrightarrow{\text{din ip.}} \psi(l) \equiv_\Gamma^{T_\Sigma} \psi(r)$

$T_\Sigma \xrightarrow{\alpha} \mathcal{M} \models \Gamma$

$\alpha(\psi(l)) = \alpha(\psi(r))$

$(\psi;\alpha)(l) = (\psi;\alpha)(r)$

$s(l) = s(r)$

Se aplică def. congr. semantică

BULLSHIT

$x = f(y)$

$T_\Sigma(\{x, y\}) \longrightarrow T_\Sigma(\{y\})$

$\begin{cases} x \leftarrow f(y) \\ y \leftarrow y \end{cases}$

Def. Soluție pt $(\exists X) G$ în Σ-alg $\mathcal{A}$ $\quad s : T_\Sigma(X) \longrightarrow \mathcal{A}$

$s(G) \subseteq \equiv_\Gamma^{\mathcal{A}}$

Obs!

$$\begin{array}{ccc} T_\Sigma(X) & \xrightarrow{s} & \mathcal{A} \\ & \searrow^{S} & \downarrow \eta_{\mathcal{A}} \\ & & \mathcal{A}_\Gamma = \mathcal{A}/_{\equiv_\Gamma} \end{array}$$

a) $S := s ; \eta_{\mathcal{A}}$

b) Dat S, $T_\Sigma(X)$ proiectivă, η toate componentele surjective $\Big) \; \exists s : T_\Sigma(X) \to \mathcal{A}$, $\; s ; \eta_{\mathcal{A}} = S$

1) s e soluție ~~clasică~~ $\iff$ 2) S e soluție clasică : $\forall l \doteq r \in G \;\; S(l) = S(r)$

Dem:

$s(G) \subseteq \equiv_\Gamma^{\mathcal{A}} \iff \forall l \doteq r \in G$ $\quad \eta_{\mathcal{A}}(s(l)) = \eta_{\mathcal{A}}(s(l))$ $\iff \forall l \doteq r \in G \;\; (s;\eta_{\mathcal{A}})(l) = (s;\eta_{\mathcal{A}})$

$\Delta_M = \{(m, m) \mid m \in \mathcal{M}\}$

$\Delta \subseteq T_\Sigma(X) \times T_\Sigma(X) \quad : \quad 1_{T_\Sigma(X)}$ este soluție pt $(\exists X) \Delta$

$\Delta \subseteq =$

$1_{T_\Sigma(X)}(\Delta) = \Delta \subseteq\, = \,\subseteq \equiv_\Gamma^{T_\Sigma(X)}$

Prop

Compunerea unei soluții cu un mf e tot soluție

$s : T_\Sigma(X) \to \mathcal{A}$ soluție pt $(\exists X) G$ în $\mathcal{A}$

$\mathcal{A} \xrightarrow{h} \mathcal{B}$ morfism $\Big\} \implies s ; h$ este soluție pt $(\exists X) G$ în $\mathcal{B}$

Dem.

$\Delta(G) \subseteq \equiv_{\Gamma}^{\mathcal{A}}$

$h(\Delta(G)) \subseteq h(\equiv_{\Gamma}^{\mathcal{A}}) \overset{\text{Prop2}}{\subseteq} \equiv_{\Gamma}^{\mathcal{B}}$

$(s;h)(G) \subseteq \equiv_{\Gamma}^{\mathcal{B}}$

$s;h$ e sol pt $(\exists x)G$ în $\mathcal{B}$

sort nat < nlist < list
op 0 : -> nat
op s : nat -> nat
op nil : -> list
op __ : list list -> list [assoc]
op cap : nlist -> nat
op cdr : nlist -> list
var E : nat
var L : list
eq cap(E L) = E
eq cdr(E L) = L
op # : list -> nat
eq #(nil) = 0
eq #(E L) = s(#(L))

7b. Cautăm toate listele de lungime care să înceapă cu 0.

$\#(L) = s(s(0))$

$cap(L) = 0$

$T_\Sigma(\{L\})$

?

Paramodulație

$c[\underline{a}]\ s)\ G \longrightarrow_P \theta(c[r]\ s)G)$

$(\forall Y)\ l = r \in \Gamma$

Se calc. c.g.u. $T_\Sigma(X \cup Y) \longrightarrow T_\Sigma(Z$

$c([a]\ )\{l, a\}$

$\begin{cases} (\exists x)\ x + 3 = 7 \\ (\exists y)\ y + 3 = 7 \end{cases}$ la fel

(numele unor var. legate pot fi schimbate)

$(\forall x)\ l = r$ -> ex de ex. recond.

$\#(E\ L1) = s(\#(L1))$

$CGU\{\#L, \#(E\ L1)\}$ $L \leftarrow E\ L1$

$T_\Sigma(\{E, L1\})$

$s(\#(L1)) = s(s(0))$, $cap(E\ L1) = 0$

$\#(E1\ L2) = s(\#(L2))$ $CGU\{\#(L1), \#(E1\ L2)\}$

$C: s(Z) = s(s(0))$

$L1 \leftarrow E1\ L2$

$s(s(\#(L2))) = s(s(0))$

$cap(E\ E1\ L2) = 0$

$T_\Sigma(\{E, E1, L2\})$

$L3 \leftarrow E1L2$

$rap(E2\ L3) = E2$

$T_\Sigma(\{E, L1, L2\})$

$\Big\downarrow L2 \leftarrow nil$

$T_\Sigma(\{E, L1\})$ $E = 0$.

$L = E\ L1 =$
$= E\ E1\ L2$
$= E\ E1\ nil$
$= 0\ E1\ nil$

$s(s(\#(L)) = s(s(0))$ $E = 0$

$\#(nil) = 0$

$s(s(0)) = s(s(0))$ $E = 0$

eliminarea egalităților adevărate $G \cup \{l \doteq l\} \to G$. nu calculat e identitatea

Reguli de deducție

- regula morfismului $h: T_\Sigma(X) \to T_\Sigma(Y)$

$G \xrightarrow{m} h(G)$ nu calculat e h

- Regula reflexiei extinse

$h: T_\Sigma(X) \to T_\Sigma(X)$ $h(l) = h(r)$

$G \cup \{l \doteq r\} \to_{re} h(G)$ nu calc. e h.

- R. reflexiei $h: T_\Sigma(X) \to T_\Sigma(Y)$ $h = CGU\{l, r\}$

$G \cup \{l \doteq r\} \to_r h(G)$ nu calc. e h.

~~Regula nuf, reflexie extinsă~~

Regula nuf. $\longrightarrow$ Reflexiei extinse

$\downarrow$

Eliminarea egalităților adevărate $\longleftarrow$ Reflexie

$G \cup \{l \doteq r\} \xrightarrow{m} h(G) \cup \{h(l) \doteq h(r)\} \xrightarrow{\text{elimin. eg.}} h(G)$

$G \cup \{l = l\}$

$CGU\{l, l\} = 1$

$\mathcal{A} = (A_s, A_\sigma)$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad =$

$c \in T_\Sigma(A \cup \{\bullet\})$ $\quad c$ **context** $\Leftrightarrow nr_\bullet(c) = 1$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \doteq$

(bulina apare o singură dată) $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \stackrel{\circ}{=}$

$nr_\bullet(\bullet) = 1 \qquad nr_\bullet(a) = 0$

$nr_\bullet(\sigma(t_1, \ldots, t_n)) = \sum_{i=1}^{n} nr_\bullet(t_i)$

$\bullet \leftarrow d$ (un el. din A de ac. sort cu $\bullet$)

$\hat{A} : T_\Sigma(A \cup \{\bullet\}) \to \mathcal{A}$

$$\begin{cases} (\bullet \leftarrow d)(\bullet) = d \\ (\bullet \leftarrow d)(a) = \underset{\hat{A}}{a} \end{cases}$$

Not.: $(\bullet \leftarrow d)(t) = t[d]$

$$\begin{tikzcd}
\mathcal{A} \arrow[r, "h"] & \mathcal{B} \\
T_\Sigma(\mathcal{A} \cup \{\bullet\}) \arrow[u, "\bullet \to d"] \arrow[r, "h^\circ"] & T_\Sigma(\mathcal{B} \cup \{\bullet\}) \arrow[u, "\bullet \leftarrow h(d)"']
\end{tikzcd}$$

$h^\circ(\bullet) = \bullet$

$h^\circ(a) = \underset{\hat{\mathcal{A}}}{h(a)}$

$h(t[d]) = h^\circ(t)[h(d)]$

$h : \mathcal{A} \to \mathcal{B}$

$h(a \doteq a) = h(a) \doteq h(a)$

$\mathcal{P}(\mathcal{A} \times \mathcal{A}) \xrightarrow{h} \mathcal{P}(\mathcal{B} \times \mathcal{B})$

$(c \doteq a)[d] = c[d] \doteq a$

$c \stackrel{\circ}{=} a \quad h(c \doteq a) = h(c) \doteq h(a)$

$h(c[d]) = h^\circ(c)[h(a)]$