Reguli de deducție. Corectitudinea lor

$G \longrightarrow_R G'$

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y) \xrightarrow{S} A$

Paramodulație

Pararescriere

Paramod. extinsă

R. narrowing

R. reflexie extinsă

R. reflexie

eliminare egalități adevărate

$R_{n} \perp EEA \Rightarrow n$

$G \cup \{l = r\} \rightarrow_{n}$ ⊛

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y)$

$\theta(l) =_\Delta \theta(r)$

⊛ $\rightarrow_{n} \theta(G) \cup \{\theta(l) \doteq \theta(r')\} \rightarrow \theta(G)$

$R_n \quad G \longrightarrow \theta(G)$

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y)$

EEA $\quad G \cup \{l \doteq l\} \rightarrow G$

Reg. refl. extinsă

$G \cup \{l \doteq r\} \rightarrow_{RE} \theta(G)$

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y) \quad \theta(l) =_\Delta \theta(r)$

Reg. reflexivă $\quad G \cup \{l \doteq_\Delta r\} \rightarrow_r \theta(G)$

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y)$

$\theta = c\,G \cup \{l, r\}$

R_r

$\Downarrow$

EEA

$c\,G \cup \{l, l\} = 1$

Regula e corectă dc

$\forall\, S : T_\Sigma(Y) \rightarrow A$ sol pt $(\exists Y)\, G'$

$\theta ; S$ este soluție pt $(\exists X)\, G$

Dem că regulile sunt corecte

Def $\quad S : T_\Sigma(X) \rightarrow A$ este sol pt $(\exists X)\, G$ dacă $S(G) \subseteq \equiv_\Gamma^A$

$G \xrightarrow[R]{} G' \xrightarrow[R']{} G''$ (aplic 2 reg consec)

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y) \xrightarrow{\theta'} T_\Sigma(Z)$

$G \xrightarrow[R+R']{} G''$. $\quad R$ și R' corecte at și aplicarea lor succesivă e corectă

$T_\Sigma(X) \xrightarrow[\theta;\theta']{} T_\Sigma(Z)$

Dem

$\forall s: T_\Sigma(Z) \to \mathcal{A}$ sol pt $(\exists Z)\, G''$ (ip.)

$(\theta ; \theta') ; s$ sol pt $(\exists X)\, G$ (de dem)

Fie $S: T_\Sigma(Z) \to \mathcal{A}$ sol pt $(\exists Z)\, G''$

R' corect : $\theta' ; S$ sol pt $(\exists Y)\, G'$

R corect : $\theta ; (\theta' ; S)$ sol pt $(\exists X)\, G$

$\|$ asoc.

$(\theta ; \theta') ; S$

Rafinarea e corectă

$G \xrightarrow[\theta]{} \theta(G)$ (orice sol pt $\theta(G')$ e sol pt ?)

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y) \xrightarrow{S} \mathcal{A}$

Fie S soluție pt $(\exists Y)\, \theta(G)$

$S(\theta(G)) \subseteq \equiv_\rho^{\mathcal{A}}$

$(\theta ; S)$ e sol pt $(\exists X)\, G$

Eliminarea eg adev. e corectă Dem.

$G \cup \{ l \doteq l \} \to G$

$T_\Sigma(X) \xrightarrow{1} T_\Sigma(X) \xrightarrow{S} \mathcal{A}$

S sol pt $(\exists X)\, G \Rightarrow$

$S(G) \subseteq \equiv_\rho^{\mathcal{A}}$

$\equiv_\rho^{\mathcal{A}} \Rightarrow$ $S(G \cup \{ l \doteq l \}) =$

$= S(G) \cup \{ S(l) = S(l) \} \subseteq \equiv_\rho^{\mathcal{A}}$

Deci S sol pt $(\exists X)\, G \cup \{ l \doteq l \}$

Corolar

1) Reg refl. extinse e corectă

2) Reg. refl. e corectă (caz partic de refl. extinsă)

Pararescrierea (egalități, perechi de termeni → para)

$G \cup \{c[h_s(l)]\}$ (context extras) $\Rightarrow_{pr} G \cup \{c[h_s(r)]\} \cup h(H)$

$(\forall Y)\ l \doteq_s r$ if $H \in \Gamma$; $h: T_\Sigma(X) \to T_\Sigma(Y)$

Morfismul calculat e $1_{T_\Sigma(X)}$

$H \Rightarrow l \doteq r$

$\forall\ u \doteq v \in H$

$u \equiv v$ (congr. semantic)

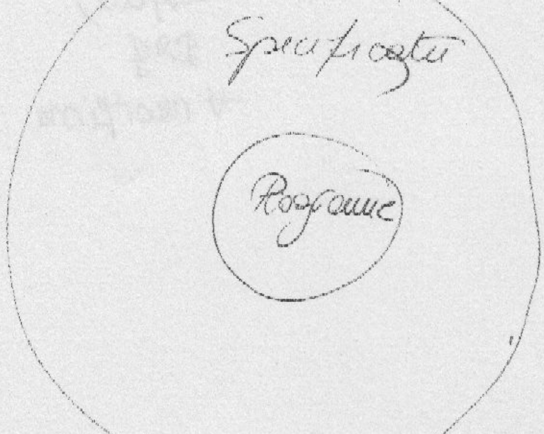
$\Updownarrow$

(~~rescriere~~ $u == v$

$fn(u) = fn(v)$

$u \Rightarrow fn(u)$

$v \Rightarrow fn(v)$



Thm. Pararescrierea e o regulă corectă.

Fie $S: T_\Sigma(Y) \to \mathcal{A}$ soluție pt $(\exists Y) G \cup c[h_s(r)] \cup h(H)$

$S(G \cup c[h_s(r)] \cup h(H)) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

1) $S(G) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

2) $S(c[h_s(r)]) \in \equiv^{\mathcal{A}}_{\Gamma}$

3) $S(h(H)) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

Nu $\forall$ specif. e program; specif. e program $\Leftrightarrow$ sistemul de rescriere e confluent și are propr. de terminare

(ip)

Tb dem că $1_{T_\Sigma(X)}$; $S: T_\Sigma(X) \to \mathcal{A}$ este sol pt $(\exists X)\ G \cup \{c[h_s(l)]\}$

$$S(G \cup \{c[h_s(l)]\}) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$$

a) $S(G) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

b) $S(\{c[h_s(l)]\}) \in \equiv^{\mathcal{A}}_{\Gamma}$

Dem b)

$a = b$ în $\mathcal{A} \iff \forall\, h: \mathcal{A} \to \mathcal{M} \models \Gamma$ $h(a) = h(b)$
Γ-algebră

Fie $\theta : \mathcal{A} \to \mathcal{M} \models$

$\theta(S(C[h_s(r)]))$ este o egalitate adevărată

$\theta(S(h(H)))$ (o mulț. de egalități adevărate) $\to h;S;\theta : T_\Sigma(Y) \to \mathcal{M} \models \Gamma$.

$\mathcal{M} \models (\forall Y)\, l \doteq_s r$ if H și $(h;S;\theta)(H)$ o mulț. de egalități adevărate
(satisface)
Def
$\forall$ morfism $m: T_\Sigma(Y) \to \mathcal{M}$ dacă $m(H)$ sunt egalități $\Rightarrow$
$\Rightarrow m(l) = m(r)$

$(h;S;\theta)_s(l) = (h;S;\theta)_s(r)$

$$\theta(S(C[h_s(l)])) = (S;\theta)(C[h_s(l)]) =$$

(regulă $m(C[a]) = m^*(c)[m(a)]$)

$$= (S;\theta)^*(C)[(S;\theta)_s(h_s(l))] = (S;\theta)^*(c)[(h;S;\theta)_s(l)] =$$

$$= (S;\theta)^*(c)[(h;S;\theta)_s(r)] = (S;\theta)^*(c)[(S;\theta)(h_s(r))] =$$

$$= (S;\theta)(C[h_s(r)]) = \theta(S(C[h_s(r)]))$$

$\Downarrow$

$\theta(S(C[h_s(l)]))$ este o egalitate $\quad S(C[h_s(l)]) \in \,=_\Gamma^{\mathcal{A}}$

Paramodulație

$G \cup \{C[a]\} \xrightarrow[P]{}$

$(\forall Y)\, l =_{\Delta} r$ if $H \in P$

$Y \cap X = \emptyset$

egale

$\theta = CGU\{a, l\} : T_\Sigma(X \cup Y) \to T_\Sigma(Z)$

Paramodulație extinsă

$\theta(a) = \theta(b)$ (doar unificator, nu cel mai general)

Dem. R.unif + Param rescriere = Paramodulație extinsă $T_\Sigma(X) \xrightarrow{\theta/_{T_\Sigma(X)}} T_\Sigma(Z)$

$(\forall Y)\, l =_{\Delta} r$ if $H \in P$ $Y \cap X = \emptyset$

$\theta : T_\Sigma(X \cup Y) \to T_\Sigma(Z)$

$\theta(a) = \theta(l)$

$G \cup C[a] \xrightarrow[u]{\theta/_{T_\Sigma(X)}} \theta(G) \cup \{\theta(C[a])\} = \theta(G) \cup \{\theta^{\circ}(c)\,[\theta(a)]\} =$

$= \theta(G) \cup \{\theta^{\circ}(c)\,[\theta_\Delta(l)]\} \xrightarrow[R_1]{} \theta(G) \cup \{\theta^{\circ}(c)\,[\theta_\Delta(r)]\} \cup \theta(H)$

$= \theta(G) \cup \{\theta(C[r])\} \cup \theta(H)$

$= \theta(G \cup \{C[r]\} \cup H)$

Corolar

1) Reg. param. extinse e corectă. (reg unif + reg param rescriere corecte)

2) Paramodulația e corectă (caz particular)

$G \xrightarrow[R_1]{} G_1 \xrightarrow[R_2]{} G_2 \to \dots \xrightarrow[R_n]{} G_n$ ← mulț. de ec. adev. (identitatea e soluție)

$T_\Sigma(X) \xrightarrow{\theta_1} \quad \xrightarrow{\theta_2} \dots \xrightarrow{\theta_n}$

$\theta_n; id$ sol pt G_{n-1} etc.

$\theta_1; \theta_2; \dots \theta_n$ sol pt $(\exists X)G$

(compunere unif. calculate)