# Classification using High Order Dissimilarities in Non-Euclidean Spaces

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## Introduction

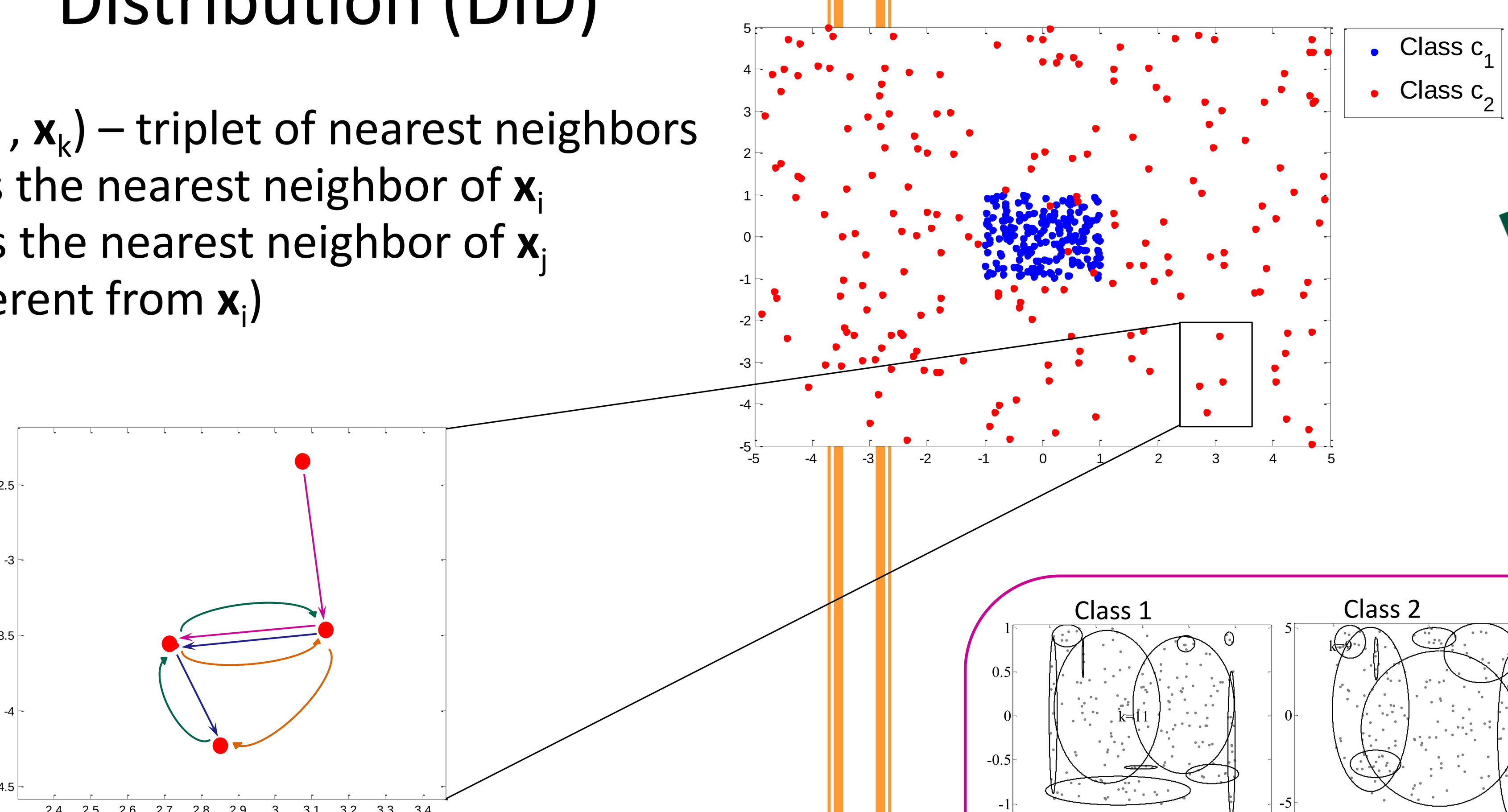
- **Dissimilarity Increments** is a high order dissimilarity exploring triplets of points.
- We **propose a novel classifier (MAP-GMDID)** that combines a maximum *a posteriori* (MAP) approach using Gaussian Mixture Models (GMM) and the **Dissimilarity Increments Distribution (DID)**.
- In this work, objects are described using **dissimilarities** between pairs of objects.
- **We build several pseudo-Euclidean feature spaces** based on the relations given by the dissimilarity matrix. We preserve only the largest eigenvalues.

## Pseudo-Euclidean Spaces

- **Pseudo-Euclidean Spaces:**
  - **PES:** *p+q* eigenvectors (*p* largest positive and *q* largest negative)
  - **PPES:** *p* positive eigenvectors
  - **NPES:** *q* negative eigenvectors
  - **CES:** add 2|*a*| (|*a*| largest negative eigenvalue) to all eigenvalues
- **Dissimilarity Spaces:** we compute pairwise Euclidean distances between data points of the previous spaces.

## Dissimilarity Increments Distribution (DID)

- $(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k)$ – triplet of nearest neighbors
  - $\mathbf{x}_j$ is the nearest neighbor of $\mathbf{x}_i$
  - $\mathbf{x}_k$ is the nearest neighbor of $\mathbf{x}_j$ (different from $\mathbf{x}_i$)




- The **dissimilarity increments** between neighboring patterns is defined as

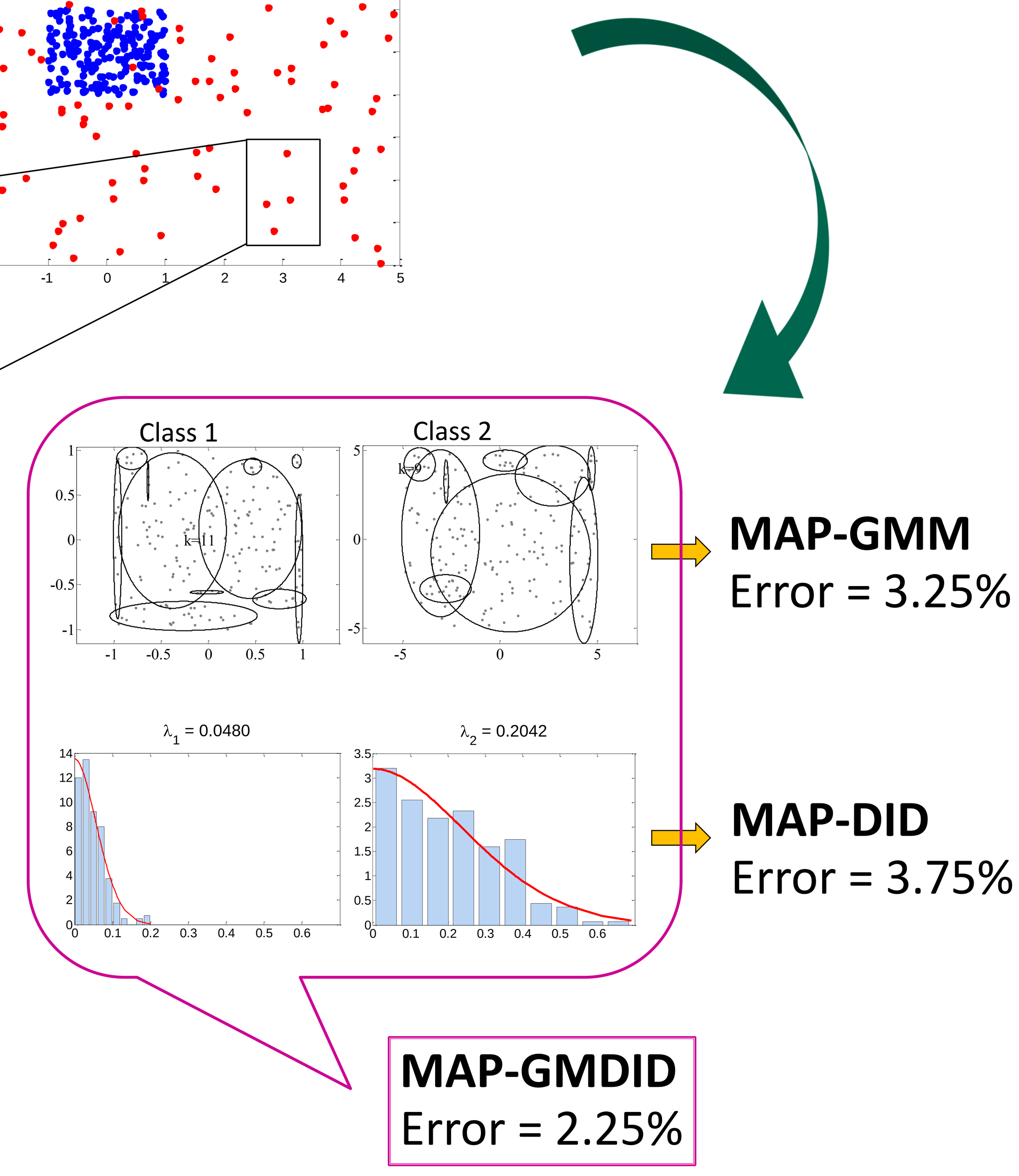
$$d_{inc}(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k) = |d(\mathbf{x}_i, \mathbf{x}_j) - d(\mathbf{x}_j, \mathbf{x}_k)|$$

- The **DID** in a class is given by

$$p_{d_{inc}}(w;\lambda) = \frac{\pi\beta^2}{4\lambda^2} w \exp\left(-\frac{\pi\beta^2}{4\lambda^2}w^2\right) + \frac{\pi^2\beta^3}{8\sqrt{2}\lambda^3} \times \left(\frac{4\lambda^2}{\pi\beta^2} - w^2\right) \exp\left(-\frac{\pi\beta^2}{8\lambda^2}w^2\right) \operatorname{erfc}\left(\frac{\sqrt{\pi}\beta}{2\sqrt{2}\lambda}w\right)$$

  - erfc(.) is the complementary error function
  - $\beta = 2 - \sqrt{2}$
  - $\lambda = \mathbb{E}[w]$

## MAP-GMDID Classifier




- $\{\mathbf{x}_i, c_i, inc_i\}_{i=1}^{N}$ is labeled dataset
  - $\mathbf{x}_i$ is a feature vector in $\mathbb{R}^d$
  - $c_i$ is the class label
  - $inc_i$ is the set of increments yielded by all the triplets of points containing $\mathbf{x}_i$
- MAP rule:

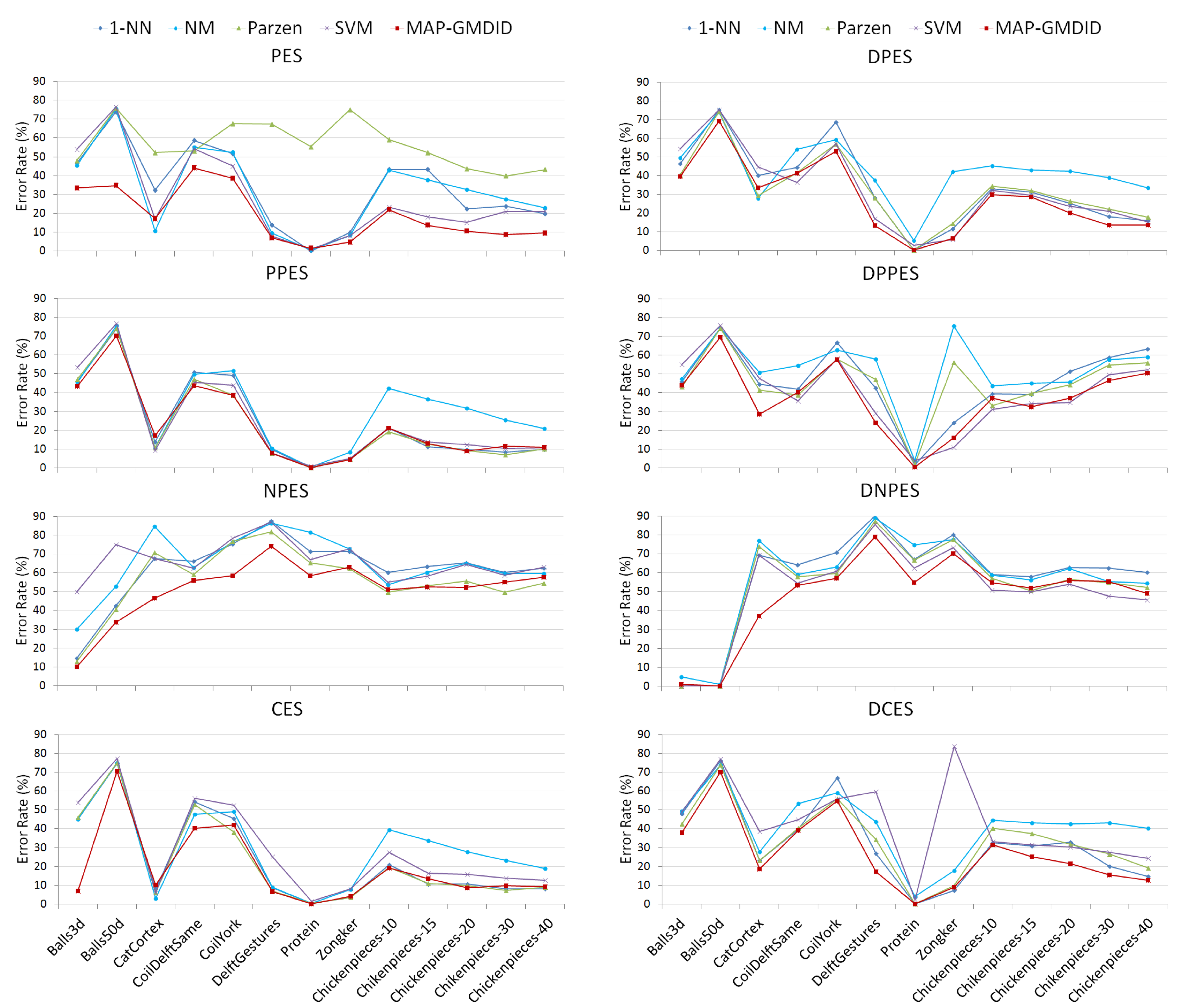
$$\max_{c_j} p(c_j|\mathbf{x}_i, inc_i) = \max_{c_j} p(\mathbf{x}_i, inc_i|c_j)p(c_j)$$

- MAP-GMDID combines the GMM and DID assuming that $\mathbf{x}_i$ and $inc_i$ are conditionally independent:

$$p(\mathbf{x}_i, inc_i|c_j) = \left(\sum_{l=1}^{K} p(\mathbf{x}_i|g_l)\right) p(inc_i|c_j)$$

$$= \left(\sum_{l=1}^{K} \alpha_l p(\mathbf{x}_i|\Sigma_l, \mu_l)\right)\left(\frac{\sum_{n=1}^{M} p(inc_i^n|\lambda_l)}{M}\right)$$

- $p(c_j) = |c_j|/N$




## Conclusions

- MAP-GMDID can be interpreted as a GMM with an operator that forces a class to have a common increment structure.
- MAP-GMDID outperforms other classifiers, especially in NPES and PES.
- CES and PPES have the lowest error rates for most of the classifiers.
- MAP-GMDID as similar performance in PES, PPES and CES.

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