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Assignment - 5

Question:- The original SELECT algorithm is defined as below :-

- → Dividing the 'n' elements of the input array into $\lceil n/5 \rceil$ groups of 5 elements each and at most one group made up of the remaining n mod 5 elements.
- → Finding the median of each of the $\lceil n/5 \rceil$ groups and then picking the median from the sorted list of group elements.
- → Now, using the 'SELECT' recursively to find the median 'x' of the $\lceil n/5 \rceil$ medians found in step above.
- → Now, partition the input array around the median-of-medians 'x' using the modified version of PARTITION. Let 'K' be one more than the number of elements on the low side of the partition.
- → If $i = K$,
  then return x.

else:

use SELECT recursively to find the i^{th} smallest element on the low side if $i < K$, or the $(i-K)^{th}$ smallest element on the high side if $i > K$.

Now, from the SELECT() algorithm.

- From the given input array elements size 'n'.
- We get $\lceil n/5 \rceil$ - 5 element groups and one group of n mod 5 elements.
- Now, extensively applying the SELECT algorithm. we get set of medians ⇒ whose median is 'x'.
- [As we know, there are 5 elements so the median would be technically 3^{rd} element]. So, the number of elements less or equal to median are $\underline{\underline{3}}$.

So, now the least number of elements greater than the median (x) are ⇒ $3\left(\frac{1}{2}\left(\frac{n}{5}\right)-2\right)$

$$\Rightarrow \frac{3n}{10} - 6$$

As,

If median-of-median = m^*

The, lower bound on the number of elements that are greater than m^* ~~and a lower bound on the number of elements smaller than m^*~~ for 'n' elements dividing by 'k' groups

$$\Rightarrow \left\lceil \frac{n}{k} \right\rceil \left(\frac{1}{2}\left(\left\lceil \frac{n}{k} \right\rceil\right) - 2\right)$$

Now, when elements are divided into groups of '7'.

We know that,

Atleast 50% of the $\lceil n/7 \rceil$ groups gives 4 elements greater than the median-of-median 'x'. $\Rightarrow 4\left(\frac{1}{2}\left(\frac{n}{7}\right)-2\right)$

$$\Rightarrow \left(\frac{2n}{7}\right)-8$$

So, now recursively calling $\Rightarrow$ Select on a problem of size at most

$$n - \frac{2n}{7} - 8 \Rightarrow \frac{5n}{7} - 8$$

Therefore, now the recurrence run-time becomes

$$T(n) \leq T\left(\left\lceil \frac{n}{7} \right\rceil\right) + T\left(\frac{5n}{7}+8\right) + O(n)$$

If it has to be linear with the run-time,

Then, we know

$$T(n) = O(n)$$

such That $T(n) \leq cn$ where

$O(n) = 6n$; $6, c \rightarrow$ constant

$$T(n) \le T\left(\left\lceil \frac{n}{7} \right\rceil\right) + T\left(\frac{5n}{7} + 8\right) + c_1 n$$

$$\le c\left\lceil \frac{n}{7} \right\rceil + c\left(\frac{5n}{7} + 8\right) + c_1 n$$

$$\le \frac{cn}{7} + \frac{5cn}{7} + \cancel{40}\,8c + c_1 n$$

$$\le \frac{6cn}{7} + \cancel{40}\,8c + c_1 n$$

$$= n + \left[-\frac{6cn}{7} + \cancel{40}\,8c + c_1 n\right] \quad \text{--- (i)}$$

$$= \frac{cn}{7} - \cancel{40}\,8c - c_1 n$$

This would be linear if the term $\left[\frac{6cn}{7} + \cancel{40}\,8c + c_1 n\right]$ in equation (i) is ≤ 0.

$$\therefore \frac{6cn}{7} + \cancel{40}\,8c + c_1 n \le 0$$

$$c_1 n \le -\frac{6cn}{7} - \cancel{40}\,8c$$

$$\frac{7(c_1 n + \cancel{40}\,8c)}{6n} \le c$$

$$\cancel{c_1 \le -\frac{6c}{7} - \frac{40}{n}}$$

$\therefore$ Therefore 'c' should be $\frac{7(c_1 n + \cancel{40}\,8c)}{6n}$

$$\Rightarrow \frac{7c_1 n + 56c}{6n}$$

$$\Rightarrow \cancel{-\frac{6cn}{7}} + \cancel{\frac{8c}{n}} + \cancel{c_1 n} \leq 0$$

$$\Rightarrow \cancel{c\left[-\frac{6n}{7} + 8\right]} + \cancel{c_1 n} \leq 0$$

$$\Rightarrow c_1 n \leq$$

$$\Rightarrow \frac{6cn}{7} + 8c + c_1 n \leq cn$$

$$\Rightarrow 8c + c_1 n \leq \frac{cn}{7}$$

$$\Rightarrow c_1 n \leq \frac{cn}{7} - 8c$$

$$\Rightarrow c_1 \leq \frac{c}{7} - \frac{8c}{n} \Rightarrow c_1 \leq c\left[\frac{n}{7} - 8\right]$$

$$\Rightarrow \frac{7c_1}{cn - 56c} \geq c$$

$$\boxed{\frac{7c_1}{n-56} \geq c_1}$$

Therefore $(n-56)$ the denominator cannot be '0'

$$\therefore n - 56 > 0$$

$$\Rightarrow \boxed{n > 56}$$

So, let 'n' be multiple of '56'.

Then $n = 56 \cdot k$

$\therefore 1 - \frac{56}{n} \Rightarrow 1 - \frac{1}{k} \Rightarrow \frac{k-1}{k}$

And. $c > \frac{7a}{1 - \frac{56}{n}} \Rightarrow \boxed{c > \frac{7ak}{k-1}}$

$\therefore$ Thence, it still runs in linear time.

Now, when the elements are divided into group of 3 elements.

Similarly, applying the above formulae,

The least number of elements greater than the median-of-median 'x' $\Bigg\} \Rightarrow 2\left(\frac{1}{2}\left(\left\lceil \frac{n}{3} \right\rceil\right) - 2\right)$

$\Rightarrow \frac{n}{3} - 4$

Now, recursively calling the SELECT() algorithm we are left with $\Bigg\} \Rightarrow n - \left(\frac{n}{3} - 4\right)$

$\Rightarrow \frac{2n+4}{3}$ elements

Now, with this recursive calling, the run-time complexity would be

$T(n) \leq T(\lceil n/3 \rceil) + T\left(\frac{2n}{3} + 4\right) + O(n)$

Now, for $T(n)$ to be linearly running

$T(n) = O(n)$ iff $T(n) \leq cn$ $\quad O(n) = bn$

b, c → const

$\Rightarrow T(n) \leq c(n/3) + c\left(\frac{2n}{3} + 4\right) + bn$

$\Rightarrow T(n) \leq \frac{cn}{3} + \frac{2nc}{3} + 8 + bn$

$\leq cn + 8 + bn$

$\therefore$ $8 + bn$ should be ≤ 0. for $T(n) \leq cn$

but b, c are constants and

$8 + bn$ is definitely greater than '0'.

$\therefore$ Hence, $T(n)$ when divided into elements of 3 does not run linearly.