Assignment 12

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Problem

You are given a set of n objects, where the size s_i of the i-th object satisfies $0 < s_i < 1$. Your goal is to pack all the objects into the minimum number of unit-size bins. Each bin can hold any subset of the objects whose total size does not exceed 1.
The first-fit heuristic takes each object in turn and places it into the first bin that can accommodate it, as follows. It maintains an ordered list of bins. Let b denote the number of bins in the list, where b increases over the course of the algorithm, and let $< B_1, B_2, \ldots, B_b >$ be the list of bins. Initially $b = 0$ and the list is empty. The algorithm takes each object i in turn and places it in the lowest-numbered bin that can still accommodate it. If no bin can accommodate object i, then b is incremented and a new bin B_b is opened, containing object i. Let $S = \sum_{i=1}^{n} s_i$.

1. Argue that the optimal number of bins required is at least $\lceil S \rceil$.
2. Argue that the first-fit heuristic leaves at most one bin at most half full.
3. Prove that the number of bins used by the first-fit heuristic never exceeds $\lceil 2S \rceil$.
4. Prove an approximation ratio of 2 for the first-fit heuristic.