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Assignment - 4

Question: Random Graphs by 'Paul Erdos'.

According to the given question,
- considering a set of 'n' vertices.
- the vertices are connected randomly ~~between the~~ with the links i.e., edges. where each vertex pair is connected with the same probability 'p'.

This random graph 'G', is created by (n,p) model

Calculating the expected number of edges in a random graph G with n vertices using the (n,p) model

We assume that the number of possible edges for a vertex be represented by => 's'

Now, for a edge to be possible between the vertices, it would be 'ps'

Now, for expecting the number of edges, we take the Expectation with the probability 'p', of an edge to be possible.

So, lets define a sample space 'S', for ~~the~~ defining the happening of an event which in turn creates an edge at the vertex.

$$S\text{(Sample space event)} = \begin{cases} 1, & \text{happening of the event} \\ 0, & \text{Not happening of the event} \end{cases}$$

If we get the sample space event probability as '1' ⇒ edge is created.

If we get the sample space event probability as '0' ⇒ edge is not created.

∴

$E[ps] = p \cdot E[s]$ [property]

Now, expectation of the Sample space would be.

$$E[S] = 1 \cdot P[S(s=1)] + 0 \cdot P[S(s=0)]$$

$$\Rightarrow 1 \cdot P[S(s=1)] \quad ---- \quad (i)$$

As this is an undirected graph, the ~~probability~~ ways of choosing 2 vertices among the 'n' vertices ~~what~~ where an edge could happen will be ⇒ $^{n}C_{2}$

Now, substituting it in equ (i),

we get,

$E[S] = P[S(s=1)]$

$\Rightarrow {}^nC_2 \times$ (probability of vertex pair)

$\Rightarrow {}^nC_2 \cdot p$

Therefore, the Total possible edges in a 'n' vertices graph where, the probability of a vertex pair is 'p' is ${}^nC_2 \cdot p$.

Example: Lets consider for graph with 5 vertices. and the probability of a vertex pair be 0.2. Then the number of edges would be

And, let the sample space be

occurrence of H $\Rightarrow$ $P[H] = 1$

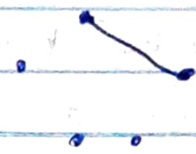
occurrence of T $\Rightarrow$ $P[T] = 0$

i.e., On occurrence of 'H', the edge is created
On occurrence of 'T', the edge is not created

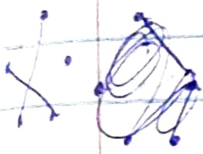
Lets analyze the possible combination of graphs.

If all 5 Tails occur.

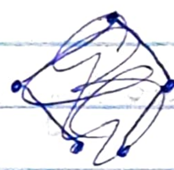
$|E| = 0$



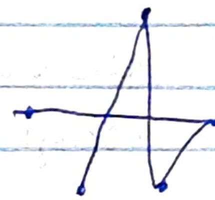
If one H & 4 Tails occur.



If 2 Heads H & 3 Tails T occur

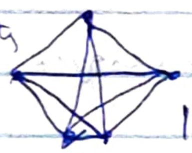


If 4 heads H & 1 Tail occur



$|E| = 4$

When probability is 1.



$|E| = 10$

no. of vertices, $n = 5$

probability, $p = 0.2$

$\therefore$ No. of possible edges $= {}^{n}C_{2} \cdot p$

$\Rightarrow {}^{5}C_{2} \times 0.2$

$\Rightarrow \frac{5 \times 4}{2} \times 0.2$

$\Rightarrow \frac{\cancel{8} \times 4^{2}}{\cancel{2}} \times \frac{2}{\cancel{10}_{2}} = \underline{2}$

$\therefore$ Therefore the total possible number of edges with 5 vertices with 0.2 probability would be '2'.

Question 2: According to the given question,

clique is a subgraph U of the graph 'G'.
If U has 'k' vertices, we call it a 'k-clique'.
Considering a random graph $G=(V,E)$
Calculating the expected number of 3-cliques in a graph 'G' created by a (n,p) model.

So, for choosing a 3-clique. of the 'n' vertices

the number of possible ways of choosing the 3-clique $\Rightarrow {}^{n}C_{3}$

The minimalistic way for 3 vertices to be connected is to have 2 edges.

And, the maximum number of edges in a 3-clique is '3'.

$$\Rightarrow {}^{3}C_{2} \Rightarrow \frac{3 \times 2}{2 \times 1} \Rightarrow 3$$

The probability of the 3-edges would be $\Rightarrow p \times p \times p$

$$\Rightarrow p^3$$

Therefore,

The expected number of 3-cliques in a graph 'G' would be.

$$\Rightarrow \begin{pmatrix}\text{number of}\\ \text{possible ways}\\ \text{of choosing}\\ \text{3-clique}\end{pmatrix} \times \begin{pmatrix}\text{probability}\\ \text{of the}\\ \text{3-edges}\end{pmatrix}$$

$$\Rightarrow \underline{\underline{{}^{n}C_{3} \times p^{3}}}$$

$\therefore$ The expected number of 3-cliques in a graph 'G' created by a (n,p) model will be $\underline{{}^{n}C_{3} \times p^{3}}$