① Given the CFG below, give the leftmost and rightmost derivations for w and draw the syntax tree.

b) $G = (\{E, T, F\}, \{a, +, *, (,)\},$

$\{E \xrightarrow{1} E+T \mid \xrightarrow{2} T,$

$T \xrightarrow{3} T*F \mid \xrightarrow{4} F,$

$F \xrightarrow{5} (E) \mid \xrightarrow{6} a\}, E)$

$w = a*(a+a)$

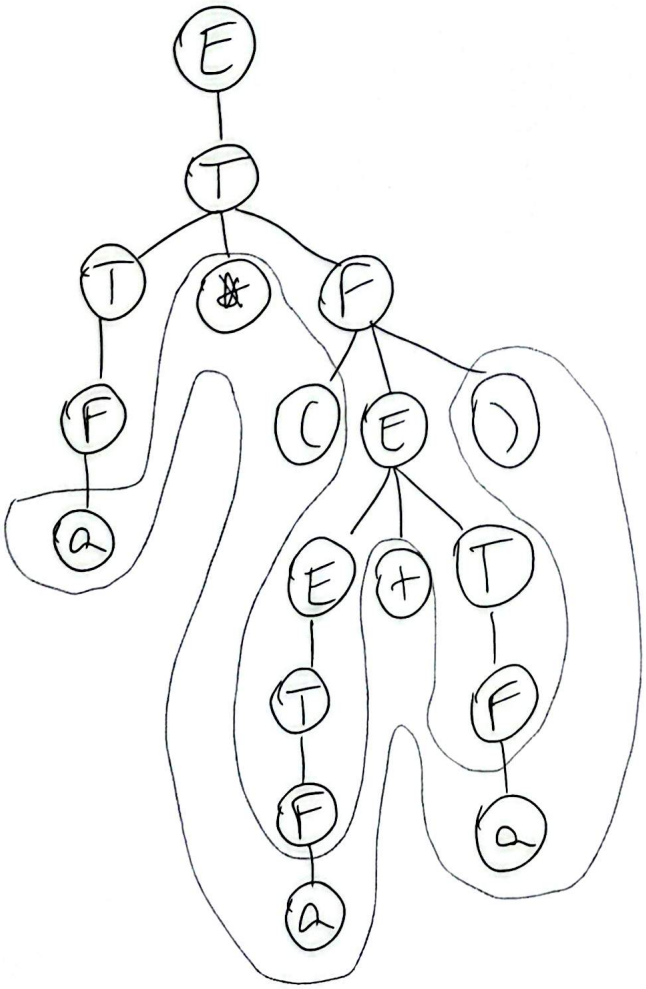
- leftmost: 2 3 4 6 5 1 2 4 6 4 6

$E \xrightarrow{2} T \xrightarrow{3} T*F \xrightarrow{4} F*F \xrightarrow{6} a*F \xrightarrow{5} a*(E) \xrightarrow{1} a*(E+T)$

$\xrightarrow{2} a*(T+T) \xrightarrow{4} a*(F+T) \xrightarrow{6} a*(a+T) \xrightarrow{4} a*(a+F)$

$\xrightarrow{6} a*(a+a)$

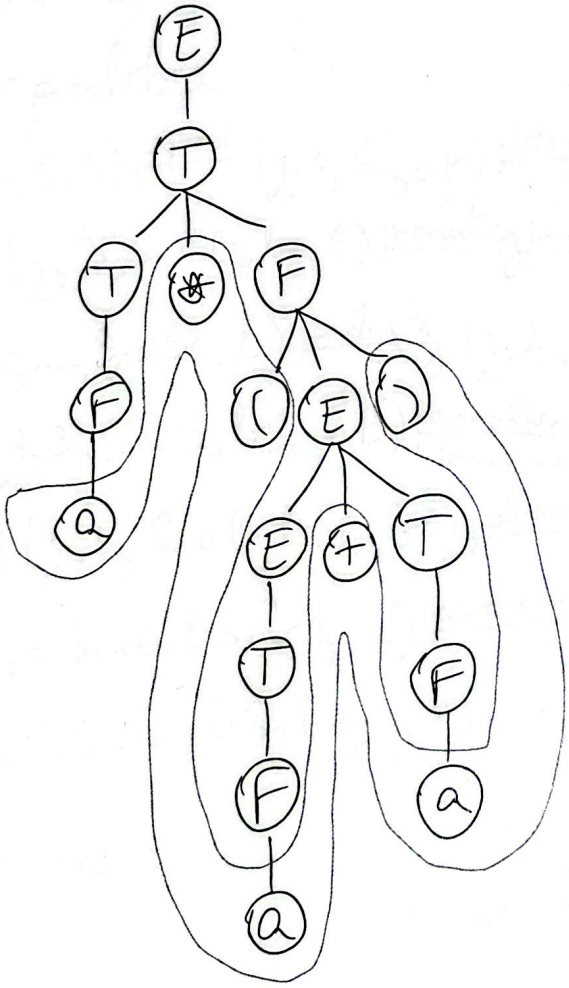
parse / syntax tree



- Rightmost: 2351462646

$E \xrightarrow{2} \underline{T} \xrightarrow{3} T*\underline{F} \xrightarrow{5} T*(\underline{E}) \xrightarrow{1} T*(E+\underline{T}) \xrightarrow{4} T*(E+\underline{F})$
$\xrightarrow{6} T*(\underline{E}+a) \xrightarrow{2} T*(\underline{T}+a) \xrightarrow{4} T*(\underline{F}+a) \xrightarrow{6} \underline{T}*(a+a)$
$\xrightarrow{4} F*(a+a) \xrightarrow{6} a*(a+a)$

parse/syntax tree



② Build the parse tree using the recursive descent parser (last exercise from Seminar 7) for:

$G = (N, \Sigma, P, S)$

$N = \{S, A, B\}$, $\Sigma = \{a, b\}$

P: $S \to \overset{1}{aB} \mid \overset{2}{bA}$

$A \to \overset{3}{a} \mid \overset{4}{aS} \mid \overset{5}{bAA}$

$B \to \overset{6}{b} \mid \overset{7}{bS} \mid \overset{8}{aBB}$

a) $w = abba$

b) $w = abaabb$

c) $w = bb$

a) $w = abba$

$(q, 1, \varepsilon, S) \vdash_{\text{expand}} (q, 1, S_1, aB) \vdash_{\text{advance}} (q, 2, S_1 a, B) \vdash_{\text{expand}} (q, 2, S_1 aB_6, b)$

$\vdash_{\text{advance}} (q, 3, S_1 aB_6 b, \varepsilon) \vdash_{\text{momentary insuccess}} (b, 3, S_1 aB_6 b, \varepsilon)$

$\vdash_{\text{back}} (b, 2, S_1 aB_6, b) \vdash_{\text{another try}} (q, 2, S_1 aB_7, bS) \vdash_{\text{advance}} (q, 3, S_1 aB_7 b, S)$

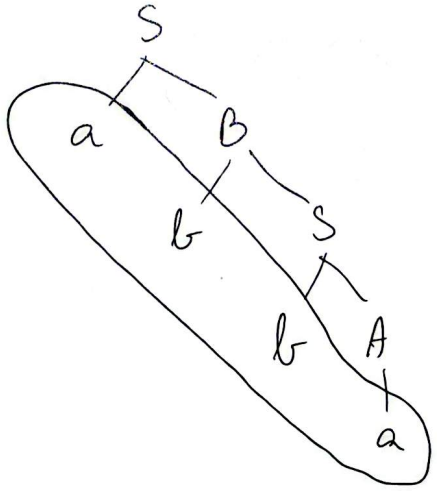
$\vdash_{\text{expand}} (q, 3, S_1 aB_7 bS_1, aB) \vdash_{\text{momentary insuccess}} (b, 3, S_1 aB_7 bS_1, aB)$

$\vdash_{\text{another try}} (q, 3, S_1 aB_7 bS_2, bA) \vdash_{\text{advance}} (q, 4, S_1 aB_7 bS_2 b, A)$

$\vdash_{\text{expand}} (q, 4, S_1 aB_7 bS_2 bA_3, a) \vdash_{\text{advance}} (q, 5, S_1 aB_7 bS_2 bA_3 a, \varepsilon)$

$\vdash_{\text{success}} (f, 5, S_1 aB_7 bS_2 bA_3 a, \varepsilon) \Rightarrow w$ is syntactically correct

parse tree: $S_1 B_7 S_2 A_3$



4/6

b) $w = abaabb$

$G = (N, \Sigma, P, S)$

$N = \{S, A, B\}$, $\Sigma = \{a, b\}$

P: $S \to \overset{1}{a}B \mid \overset{2}{b}A$

$A \to \overset{3}{a} \mid \overset{4}{a}S \mid \overset{5}{b}AA$

$B \to \overset{6}{b} \mid \overset{7}{b}S \mid \overset{8}{a}BB$

$(q, 1, \varepsilon, S) \vdash_{\text{expand}} (q, 1, S_1, aB) \vdash_{\text{advance}} (q, 2, S_1 a, B) \vdash_{\text{expand}} (q, 2, S_1 aB_6, b)$

$\vdash_{\text{advance}} (q, 3, S_1 aB_6 b, \varepsilon) \vdash_{\text{momentary insuccess}} (b, 3, S_1 aB_6 b, \varepsilon) \vdash_{\text{back}} (b, 2, S_1 aB_6, b)$

$\vdash_{\text{another try}} (q, 2, S_1 aB_7, bS) \vdash_{\text{advance}} (q, 3, S_1 aB_7 b, S)$

$\vdash_{\text{expand}} (q, 3, S_1 aB_7 bS_1, aB) \vdash_{\text{advance}} (q, 4, S_1 aB_7 bS_1 a, B)$

$\vdash_{\text{expand}} (q, 4, S_1 aB_7 bS_1 aB_6, b) \vdash_{\text{momentary insuccess}} (b, 4, S_1 aB_7 bS_1 aB_6, b)$

$\vdash_{\text{another try}} (q, 4, S_1 aB_7 bS_1 aB_7, bS) \vdash_{\text{momentary insuccess}} (b, 4, S_1 aB_7 bS_1 aB_7, bS)$

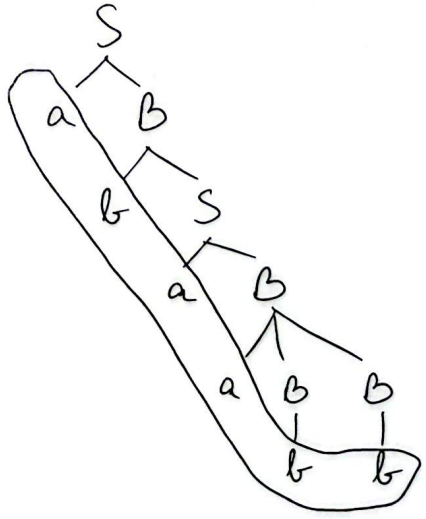
$\vdash_{\text{another try}} (q, 4, S_1 aB_7 bS_1 aB_8, aBB) \vdash_{\text{advance}} (q, 5, S_1 aB_7 bS_1 aB_8 a, BB)$

$\vdash_{\text{expand}} (q, 5, S_1 aB_7 bS_1 aB_8 aB_6, bB) \vdash_{\text{advance}} (q, 6, S_1 aB_7 bS_1 aB_8 aB_6 b, B)$

$\vdash_{\text{expand}} (q, 6, S_1 aB_7 bS_1 aB_8 aB_6 bB_6, b) \vdash_{\text{advance}} (q, 7, S_1 aB_7 bS_1 aB_8 aB_6 bB_6 b, \varepsilon)$

$\vdash_{\text{success}} (f, 7, S_1 aB_7 bS_1 aB_8 aB_6 bB_6 b, \varepsilon) \Rightarrow w$ is syntactically correct

parse tree: $S_1 B_7 S_1 B_8 B_6 B_6$



c) $w = bb$

$G = (N, \Sigma, P, S)$

$N = \{S, A, B\}$, $\Sigma = \{a, b\}$

P: $S \to \overset{1}{a}B \mid \overset{2}{b}A$

$A \to \overset{3}{a} \mid \overset{4}{a}S \mid \overset{5}{b}AA$

$B \to \overset{6}{b} \mid \overset{7}{b}S \mid \overset{8}{a}BB$

$(q, 1, \varepsilon, S) \vdash_{\text{expand}} (q, 1, S_1, aB) \vdash_{\substack{\text{momentary} \\ \text{insuccess}}} (b, 1, S_1, aB)$

$\vdash_{\text{another try}} (q, 1, S_2, bA) \vdash_{\text{advance}} (q, 2, S_2 b, A) \vdash_{\text{expand}} (q, 2, S_2 b A_3, a)$

$\vdash_{\substack{\text{momentary} \\ \text{insuccess}}} (b, 2, S_2 b A_3, a) \vdash_{\text{another try}} (q, 2, S_2 b A_4, aS)$

$\vdash_{\substack{\text{momentary} \\ \text{insuccess}}} (b, 2, S_2 b A_4, aS) \vdash_{\text{another try}} (q, 2, S_2 b A_5, bAA)$

$\vdash_{\text{advance}} (q, 3, S_2 b A_5 b, AA)$

We parsed the entire word and still have elements in β

$\Rightarrow$ we have to go back

$\vdash_{\substack{\text{momentary} \\ \text{insuccess}}} (b, 3, S_2 b A_5 b, AA) \vdash_{\text{back}} (b, 2, S_2 b A_5, bAA)$

$\vdash_{\text{another try}} (b, 2, S_2 b, A) \vdash_{\text{back}} (b, 1, S_2, bA)$

$\vdash_{\text{another try}} (b, 1, \varepsilon, S) \vdash_{\text{error}} (e, 1, \varepsilon, S)$

$\Rightarrow w$ is syntactically incorrect

$\Rightarrow$ we cannot parse it