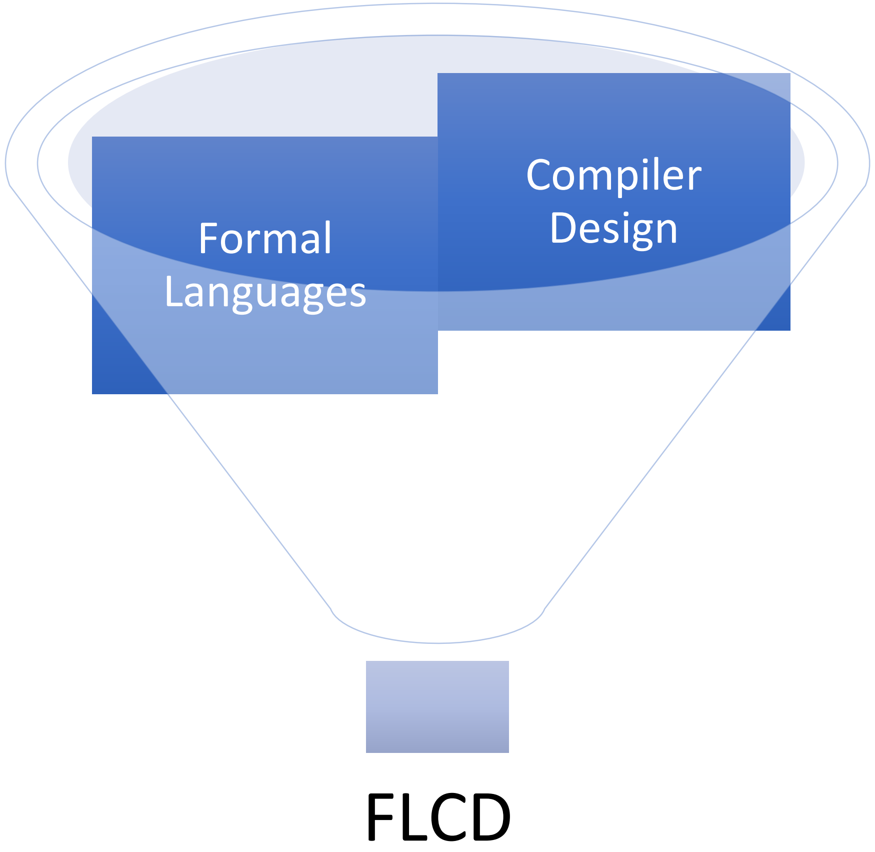
Formal Languages and Compiler Design

Simona Motogna

Why?

- Historical reasons
- Be a better programmer
- Performant algorithms



Organization Issues

- Course – 2 h/ week
- Seminar – 2h/week
- Laboratory - 2 h/week

10 presences – seminar
12 presences - lab

PRESENCE IS MANDATORY

Most interesting stuff for students

- Moodle:
  - All course resources
  - Homeworks
  - Assignments
  - Labs
  - Points / grades

- MsTeams – labs (maybe)

Minimal Conditions to Pass

- *Minimum 10 presences at seminar*
- *Minimum 12 presences at laboratory*

- *Minimum grade **6** at lab*
- *Minimum grade 5 at final exam*

Bonus

Lab work

- 10 laboratory tasks

- !!! Must be completed and loaded during lab hours

- Weighted grades:

  Lab grade

Bonus points:

- "awesome" solutions
- Extra work

I wish …

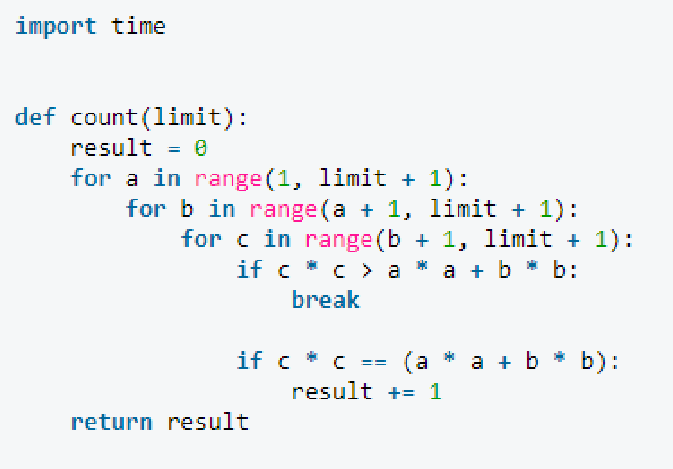
Effective communication

Interactive experience

Learning fun

References

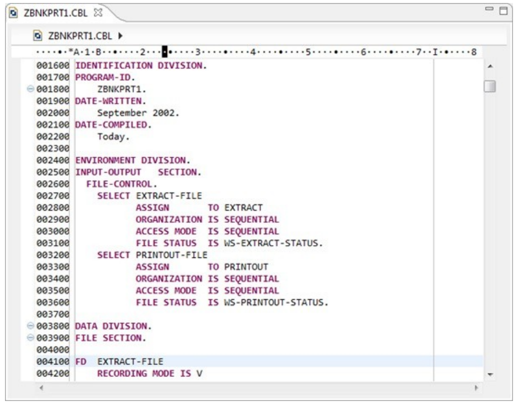
- See fișa disciplinei



```python
import time


def count(limit):
    result = 0
    for a in range(1, limit + 1):
        for b in range(a + 1, limit + 1):
            for c in range(b + 1, limit + 1):
                if c * c > a * a + b * b:
                    break

                if c * c == (a * a + b * b):
                    result += 1
    return result
```



ZBNKPRT1.CBL
ZBNKPRT1.CBL
·····•·*A·1·B··•····2···•····3····•····4····•····5····•····6····•····7··I·•····8

```cobol
001600 IDENTIFICATION DIVISION.
001700 PROGRAM-ID.
001800     ZBNKPRT1.
001900 DATE-WRITTEN.
002000     September 2002.
002100 DATE-COMPILED.
002200     Today.
002300
002400 ENVIRONMENT DIVISION.
002500 INPUT-OUTPUT   SECTION.
002600   FILE-CONTROL.
002700     SELECT EXTRACT-FILE
002800            ASSIGN       TO EXTRACT
002900            ORGANIZATION IS SEQUENTIAL
003000            ACCESS MODE  IS SEQUENTIAL
003100            FILE STATUS  IS WS-EXTRACT-STATUS.
003200     SELECT PRINTOUT-FILE
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003500            ACCESS MODE  IS SEQUENTIAL
003600            FILE STATUS  IS WS-PRINTOUT-STATUS.
003700
003800 DATA DIVISION.
003900 FILE SECTION.
004000
004100 FD  EXTRACT-FILE
004200     RECORDING MODE IS V
```
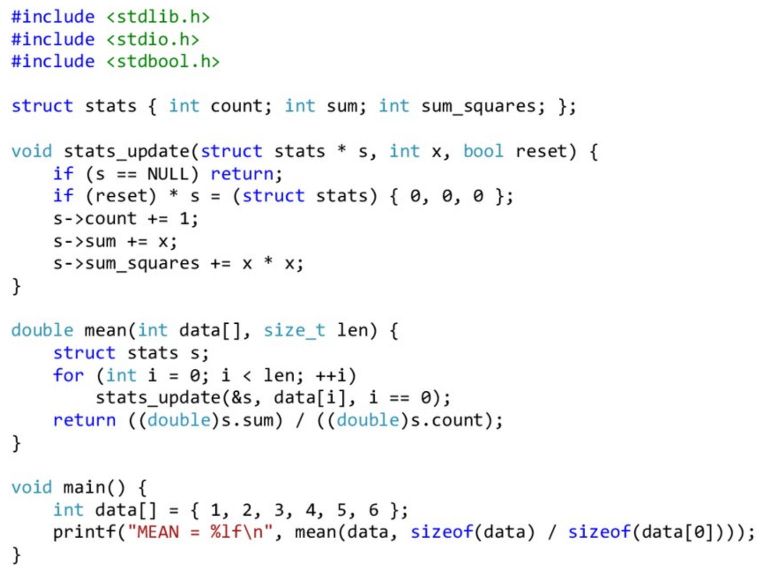
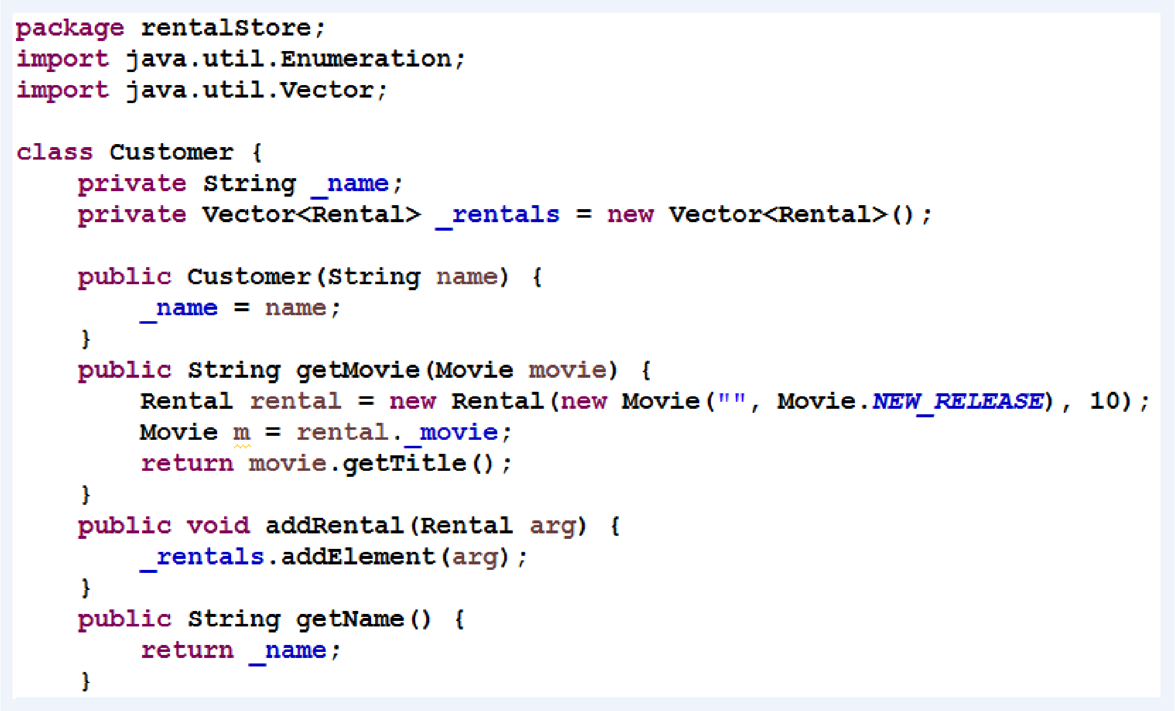
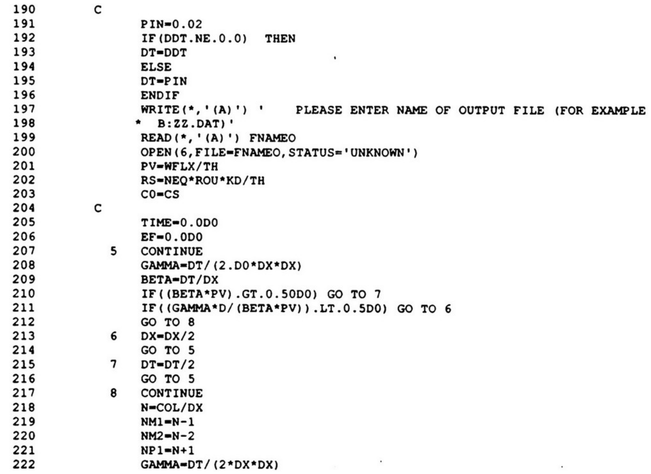



```c
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

struct stats { int count; int sum; int sum_squares; };

void stats_update(struct stats * s, int x, bool reset) {
    if (s == NULL) return;
    if (reset) * s = (struct stats) { 0, 0, 0 };
    s->count += 1;
    s->sum += x;
    s->sum_squares += x * x;
}

double mean(int data[], size_t len) {
    struct stats s;
    for (int i = 0; i < len; ++i)
        stats_update(&s, data[i], i == 0);
    return ((double)s.sum) / ((double)s.count);
}

void main() {
    int data[] = { 1, 2, 3, 4, 5, 6 };
    printf("MEAN = %lf\n", mean(data, sizeof(data) / sizeof(data[0])));
}
```

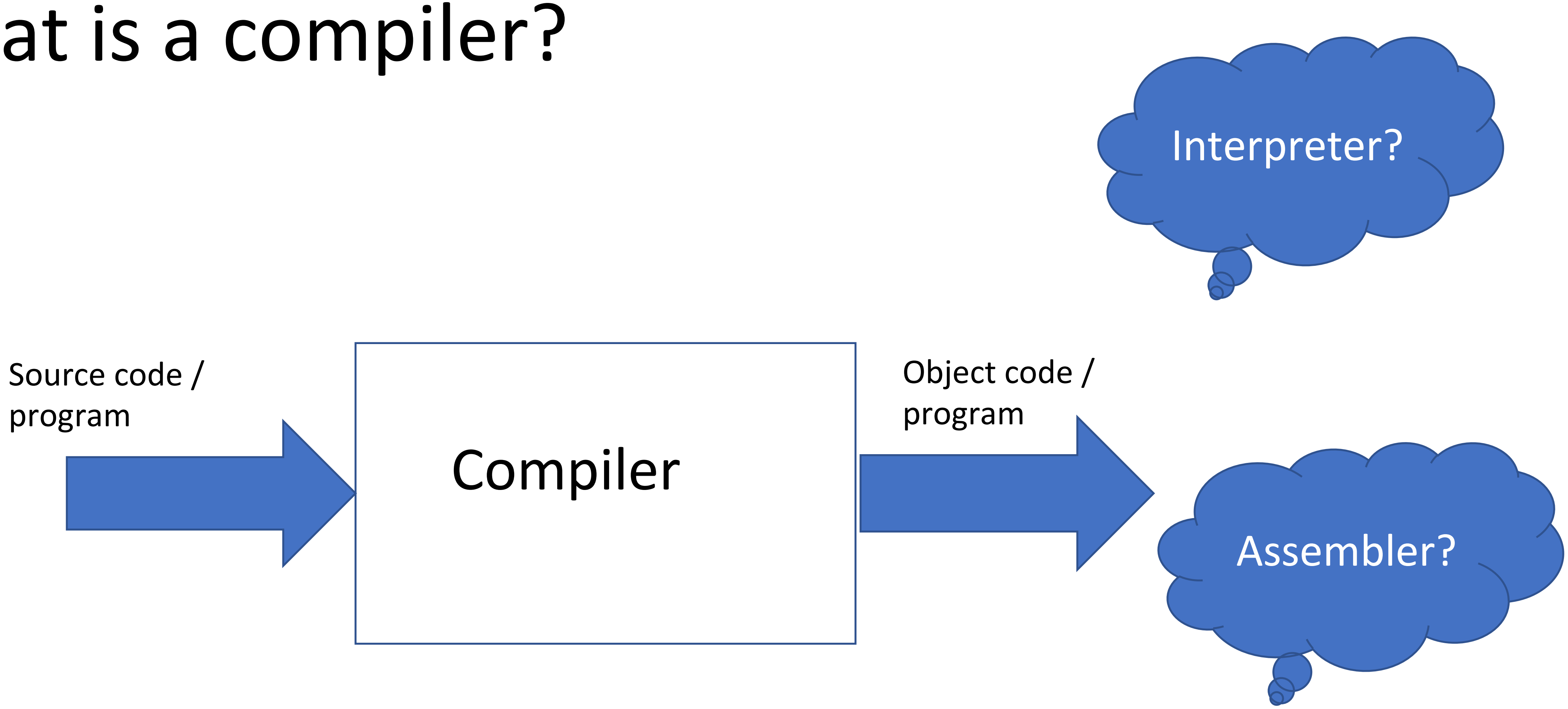


```java
package rentalStore;
import java.util.Enumeration;
import java.util.Vector;

class Customer {
    private String _name;
    private Vector<Rental> _rentals = new Vector<Rental>();

    public Customer(String name) {
        _name = name;
    }
    public String getMovie(Movie movie) {
        Rental rental = new Rental(new Movie("", Movie.NEW_RELEASE), 10);
        Movie m = rental._movie;
        return movie.getTitle();
    }
    public void addRental(Rental arg) {
        _rentals.addElement(arg);
    }
    public String getName() {
        return _name;
    }
```



```fortran
C
      PIN=0.02
      IF(DDT.NE.0.0)  THEN
      DT=DDT
      ELSE
      DT=PIN
      ENDIF
      WRITE(*,'(A)') '    PLEASE ENTER NAME OF OUTPUT FILE (FOR EXAMPLE
     *  B:ZZ.DAT)'
      READ(*,'(A)') FNAMEO
      OPEN(6,FILE=FNAMEO,STATUS='UNKNOWN')
      PV=WFLX/TH
      RS=NEQ*ROU*KD/TH
      C0=CS
C
      TIME=0.0D0
      EF=0.0D0
    5 CONTINUE
      GAMMA=DT/(2.D0*DX*DX)
      BETA=DT/DX
      IF((BETA*PV).GT.0.50D0) GO TO 7
      IF((GAMMA*D/(BETA*PV)).LT.0.5D0) GO TO 6
      GO TO 8
    6 DX=DX/2
      GO TO 5
    7 DT=DT/2
      GO TO 5
    8 CONTINUE
      N=COL/DX
      NM1=N-1
      NM2=N-2
      NP1=N+1
      GAMMA=DT/(2*DX*DX)
```

What is a compiler?



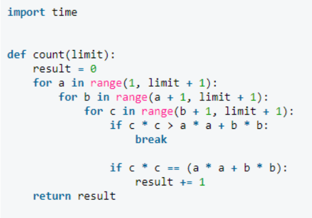
Interpreter?

Source code / program

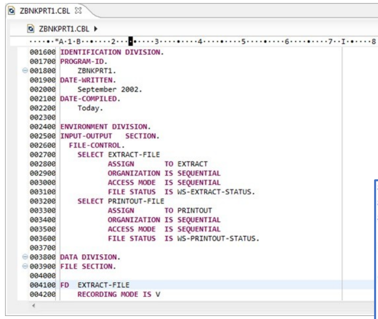
Compiler

Object code / program

Assembler?



```
import time


def count(limit):
    result = 0
    for a in range(1, limit + 1):
        for b in range(a + 1, limit + 1):
            for c in range(b + 1, limit + 1):
                if c * c > a * a + b * b:
                    break

                if c * c == (a * a + b * b):
                    result += 1
    return result
```



ZBNKPRT1.CBL
ZBNKPRT1.CBL
·····•·*A·1·B·•····2···•····3····•····4····•····5····•····6····•····7··I·•····8

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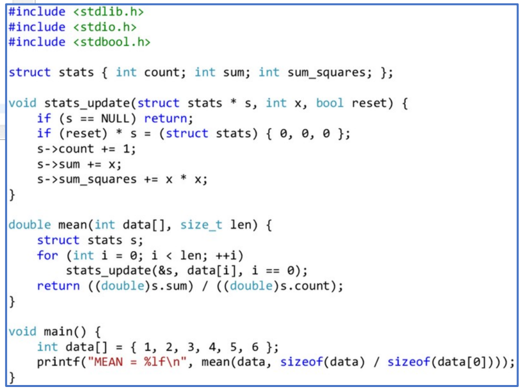

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#include <stdio.h>
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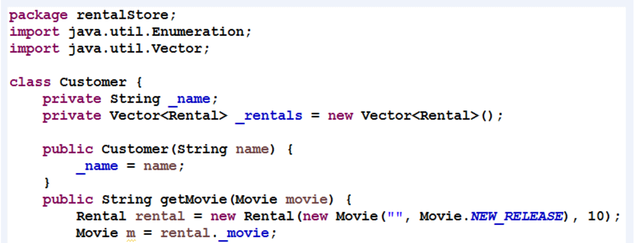
struct stats { int count; int sum; int sum_squares; };

void stats_update(struct stats * s, int x, bool reset) {
    if (s == NULL) return;
    if (reset) * s = (struct stats) { 0, 0, 0 };
    s->count += 1;
    s->sum += x;
    s->sum_squares += x * x;
}

double mean(int data[], size_t len) {
    struct stats s;
    for (int i = 0; i < len; ++i)
        stats_update(&s, data[i], i == 0);
    return ((double)s.sum) / ((double)s.count);
}

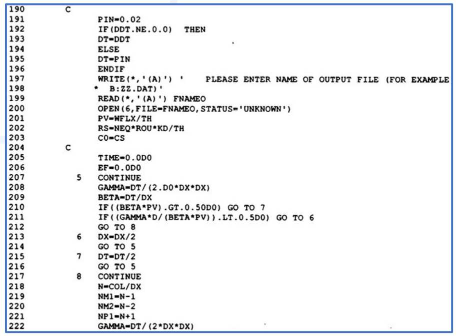
void main() {
    int data[] = { 1, 2, 3, 4, 5, 6 };
    printf("MEAN = %lf\n", mean(data, sizeof(data) / sizeof(data[0])));
}
```



```
package rentalStore;
import java.util.Enumeration;
import java.util.Vector;

class Customer {
    private String _name;
    private Vector<Rental> _rentals = new Vector<Rental>();

    public Customer(String name) {
        _name = name;
    }
    public String getMovie(Movie movie) {
        Rental rental = new Rental(new Movie("", Movie.NEW_RELEASE), 10);
        Movie m = rental._movie;
        return movie.getTitle();
    }
    public void addRental(Rental arg) {
        _rentals.addElement(arg);
    }
    public String getName() {
        return _name;
    }
```



```
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            PIN=0.02
            IF(DDT.NE.0.0)  THEN
            DT=DDT
            ELSE
            DT=PIN
            ENDIF
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            READ(*,'(A)') FNAMEO
            OPEN(6,FILE=FNAMEO,STATUS='UNKNOWN')
            PV=WFLX/TH
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            C0=CS
      C
            TIME=0.0D0
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          5 CONTINUE
            GAMMA=DT/(2.D0*DX*DX)
            BETA=DT/DX
            IF((BETA*PV).GT.0.50D0) GO TO 7
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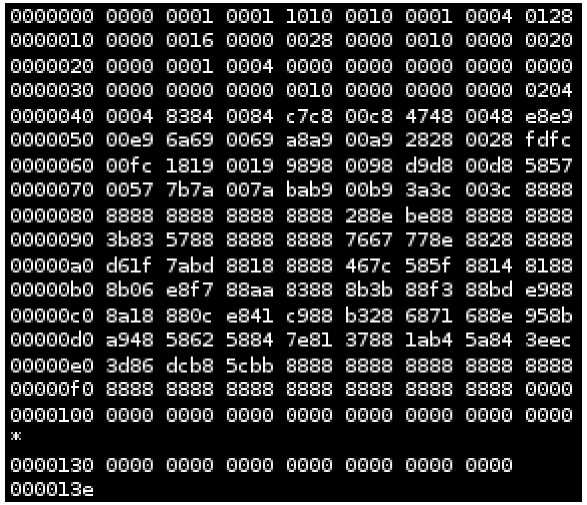
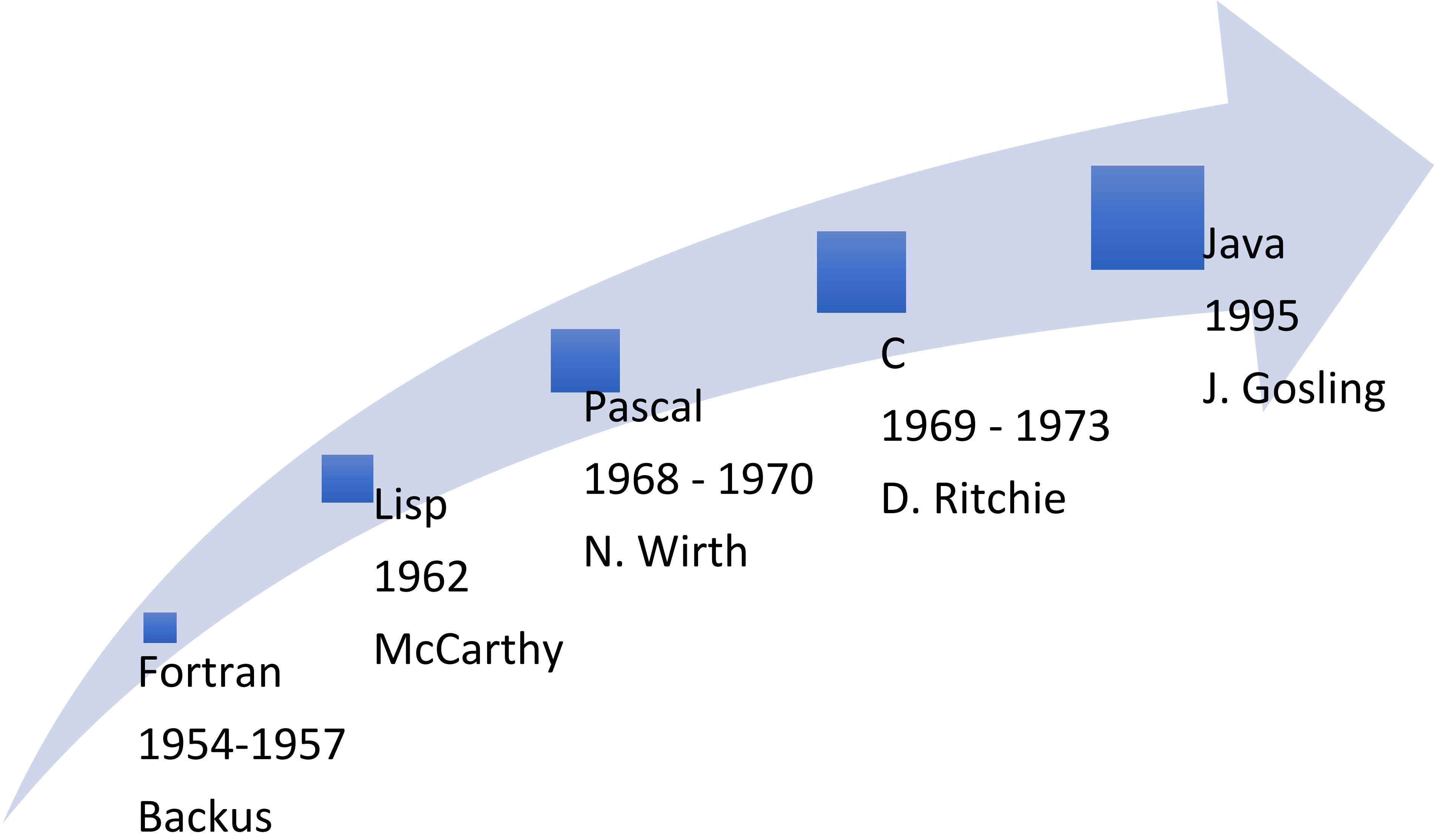


```
0000000 0000 0001 0001 1010 0010 0001 0004 0128
0000010 0000 0016 0000 0028 0000 0010 0000 0020
0000020 0000 0001 0004 0000 0000 0000 0000 0000
0000030 0000 0000 0000 0010 0000 0000 0000 0204
0000040 0004 8384 0084 c7c8 00c8 4748 0048 e8e9
0000050 00e9 6a69 0069 a8a9 00a9 2828 0028 fdfc
0000060 00fc 1819 0019 9898 0098 d9d8 00d8 5857
0000070 0057 7b7a 007a bab9 00b9 3a3c 003c 8888
0000080 8888 8888 8888 8888 288e be88 8888 8888
0000090 3b83 5788 8888 8888 7667 778e 8828 8888
00000a0 d61f 7abd 8818 8888 467c 585f 8814 8188
00000b0 8b06 e8f7 88aa 8388 8b3b 88f3 88bd e988
00000c0 8a18 880c e841 c988 b328 6871 688e 958b
00000d0 a948 5862 5884 7e81 3788 1ab4 5a84 3eec
00000e0 3d86 dcb8 5cbb 8888 8888 8888 8888 8888
00000f0 8888 8888 8888 8888 8888 8888 8888 0000
0000100 0000 0000 0000 0000 0000 0000 0000 0000
*
0000130 0000 0000 0000 0000 0000 0000 0000
000013e
```

A little bit of history …



Fortran

1954-1957

Backus

Lisp

1962

McCarthy

Pascal

1968 - 1970

N. Wirth

C

1969 - 1973

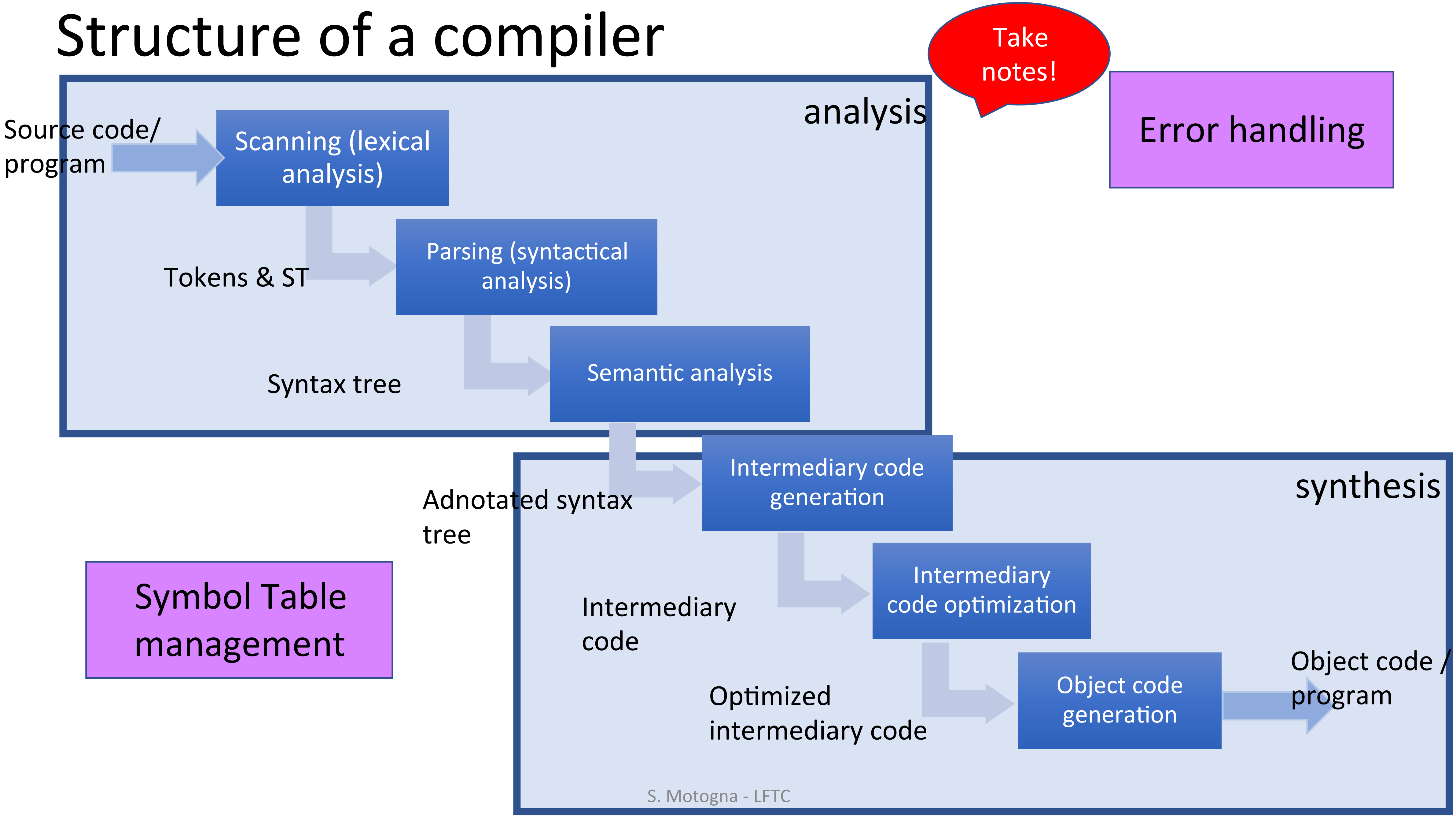
D. Ritchie

Java

1995

J. Gosling

Structure of a compiler



Take notes!

Error handling

analysis

Source code/ program

Scanning (lexical analysis)

Tokens & ST

Parsing (syntactical analysis)

Syntax tree

Semantic analysis

Adnotated syntax tree

synthesis

Intermediary code generation

Intermediary code

Intermediary code optimization

Optimized intermediary code

Object code generation

Object code / program

Symbol Table management

Chapter 1. Scanning

Definition = treats the source program as a sequence of characters, detect lexical tokens, classify and codify them

INPUT: source program
OUTPUT: PIF + ST

```
Algorithm Scanning v1
While (not(eof)) do
    detect(token);
    classify(token);
    codify(token);
End_while
```

Detect

Take notes!

```
I am a student.I      am
Simona
```

- Separators => ***Remark 1)***

```
if (x==y) {x=y+2}
```

- Look-ahead => ***Remark 2)***

Classify

- Classes of tokens:
  - Identifiers
  - Constants
  - Reserved words (keywords)
  - Separators
  - Operators

- If a token can NOT be classified => LEXICAL ERROR

Codify

- May be codification table

OR

code for identifiers and constants

- Identifier, constant => Symbol Table (ST)

- PIF = Program Internal Form = array of pairs

- pairs (token, position in ST)

identifier, constant

```
Algorithm Scanning v2
While (not(eof)) do
    detect(token);
    if token is reserved word OR operator OR separator
        then genPIF(token, 0)
        else
        if token is identifier OR constant
            then index = pos(token, ST);
                 genPIF(token, index)
            else  message "Lexical error"
        endif
    endif
endwhile
```

a=a+b

FIP

(id,1)
(=,0)
(id,1)
(+,0)
(id,2)

ST

1	a
2	b

Remarks:

- genPIF = adds a pair (token, position) to PIF

- Pos(`token`,ST) – searches *`token`* in symbol table S*T*; if found then return position; if not found insert in SR and return position
- Order of classification (reserved word, then identifier)
- If-then-else imbricate => detect error if a token cannot be classified

Example (sem?)

- https://babeljs.io/docs/en/
- https://www.antlr.org/ and https://github.com/antlr/antlr4
- https://www.programiz.com/python-programming/online-compiler/
- https://www.w3schools.com/python/python_compiler.asp

Course 2

Algorithm Scanning v2

```
While (not(eof)) do
    detect(token);
    if token is reserved word OR operator OR separator
         then genPIF(token, 0)
         else
        if token is identifier OR constant
             then index = pos(token, ST);
                  genPIF(token_type, index)
             else  message “Lexical error”
        endif
     endif
endwhile
```

Remarks:

- Also comments are eliminated
- Most important operations: SEARCH and INSERT

Symbol Table

Definition *= contains all information collected during compiling regarding the symbolic names from the source program*

identifiers, constants, etc.

Variants:

- Unique symbol table – contains all symbolic names
- distinct symbol tables: IT (identifiers table) + CT (constants table)

ST organization

Remark: search and insert

1.	Unsorted table – in order of detection in source code	O(n)
2.	Sorted table: alphabetic (numeric)	O(lg n)
3.	Binary search tree (balanced)	O(lg n)
4.	Hash table	O(1)

Hash table

- K = set of keys (symbolic names)
- A = set of positions (|A| = m; m –prime number)

$h : K \rightarrow A$

$$h(k) = (\text{val}(k) \bmod m) + 1$$

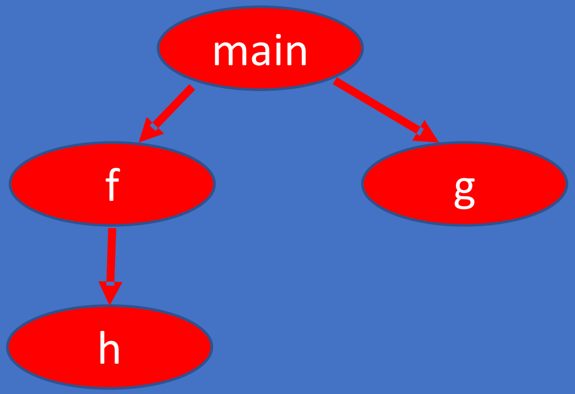
- Conflicts: $k_1 \neq k_2$, $h(k_1) = h(k_2)$

Toy hash function to use at lab:
Sum of ASCII codes of chars

Visibility domain (scope)

- Each scope – separate ST
- Structure -> inclusion tree

Hierachical structure of STs:



Example:

```
Int main(){
… int a;

void f()
  {float a;
     … int h() {…}
   }
 …
void g()
  {char a;
     …
   }
}
```

Formal Languages
- basic notions-

Examples of languages

- natural (ex. English, Romanian)
- programming (ex. C,C++, Java, Python)
- formal

A formal language is a set
Ex.:
L = {a^nb^n| n>0} L = {ab, aabb, aaabbb, …}
L' = {01^n|n>=0} L' = {0, 01, 011, …}

Example

a boy has a dog

S→PV
P→ *a* N
N→ *boy* or N→ *dog*
(N→*boy*|*dog*)
V → QC
Q → *has*
C → BN

B → *a*

- A→ α = **rule**
- S,P,V,N,Q,C,B = **nonterminal symbols**
- *a, boy,dog,has* = **terminal symbols**

Remarks

1. Sentence = word, sequence (contains only terminal symbols) ; denoted w.
2. S⇒PV⇒a NV⇒a NQC⇒a N has C - sentential form

   In general : $w=a_1a_2. . . a_n$

3. The rule guarantees syntactical correctness, but not the semantical correctness (*A dog has a boy*)

Grammar

- ***Definition***: A (formal) **grammar** is a 4-tuple: $G=(N,\Sigma,P,S)$ with the following meanings:
  - N – set of nonterminal symbols and $|N| < \infty$
  - Σ - set of terminal symbols (alphabet) and $|\Sigma|<\infty$
  - P – finite set of productions (rules), with the propriety: $P \subseteq (N \cup \Sigma)^* N (N \cup \Sigma)^* \mathbf{x}\ (N \cup \Sigma)^*$
  - $S \in N$ – start symbol /axiom

Remarks :

1. $(\alpha,\beta) \in P$ is a production denoted $\alpha \rightarrow \beta$
2. $N \cap \Sigma = \emptyset$

A* = transitive and reflexive closure = {a,aa,aaa,…} $\{a^0\}$

A = {a}

A+ = {a,aa,aaa,…}

$X^0 = \varepsilon$

Binary relations defined on $(N\cup\Sigma)^*$

- ***Direct derivation***
  $\alpha \Rightarrow \beta$, $\alpha,\beta \in (N\cup\Sigma)^*$ ***if*** $\alpha=x1xy1$, $\beta=x1yy1$ ***and*** $x\rightarrow y\in P$
  (x is transformed in y)
- ***k derivation***

$\alpha \overset{k}{\Rightarrow} \beta$,$\alpha,\beta \in (N\cup\Sigma)^*$

sequence of k direct derivations $\alpha\Rightarrow\alpha_1 \Rightarrow\alpha_2 \Rightarrow\ldots\Rightarrow\alpha_{k-1} \Rightarrow\beta$, $\alpha,\alpha_1,\alpha_2,\ldots\alpha_{k-1},\beta\in(N\cup\Sigma)^*$

- ***+ derivation***

$\alpha \overset{+}{\Rightarrow} \beta$ ***if*** $\exists$ $k>0$ ***such that*** $\alpha \overset{k}{\Rightarrow} \beta$ (there exists at least one direct derivation)

- **** derivation***

$\alpha \overset{*}{\Rightarrow} \beta$ ***if*** $\exists$ $k\geq 0$ ***such that*** $\alpha \overset{k}{\Rightarrow} \beta$ namely, $\alpha\overset{*}{\Rightarrow} \beta\Leftrightarrow\alpha\overset{+}{\Rightarrow} \beta$ ***OR*** $\alpha\overset{0}{\Rightarrow} \beta$ $(\alpha=\beta)$

Definition: ***Language generated*** by a grammar G=(N,Σ,P,S) is:

$$L(G)=\{w\in\Sigma^* \mid S \overset{*}{\Rightarrow} w\}$$

L1 = {a,b,aa}
L2 = {c,d,cd}
L1L2 = {ac,ad,acd,bc,bd,bcd,aac,aad,aacd}

Remarks:

1. $S \overset{*}{\Rightarrow} \alpha, \alpha\in(N\cup\Sigma)^*$ = sentential form
   $S \overset{*}{\Rightarrow} w, w\in\Sigma^*$ = word / sequence
2. Operations defined for languages (sets) :
   L1∪L2 , L1∩L2 , L1-L2 , $\overline{L}$ (complement) , $L^+=\cup_{!"\#} L^!$, $L^*=\cup_{!\$\#} L^!$
   Concatenation: $L=L_1L_2 = \{w_1w_2 \mid w_1\in L_1 , w_2\in L_2\}$
3. |w|=0 (empty word - denoted ε)

Definition: Two grammar G_1 and G_2 are equivalent if they generate the same language

$$L(G_1)=L(G_2)$$

Chomsky hierarchy(based on form $\alpha \rightarrow \beta \in P$)

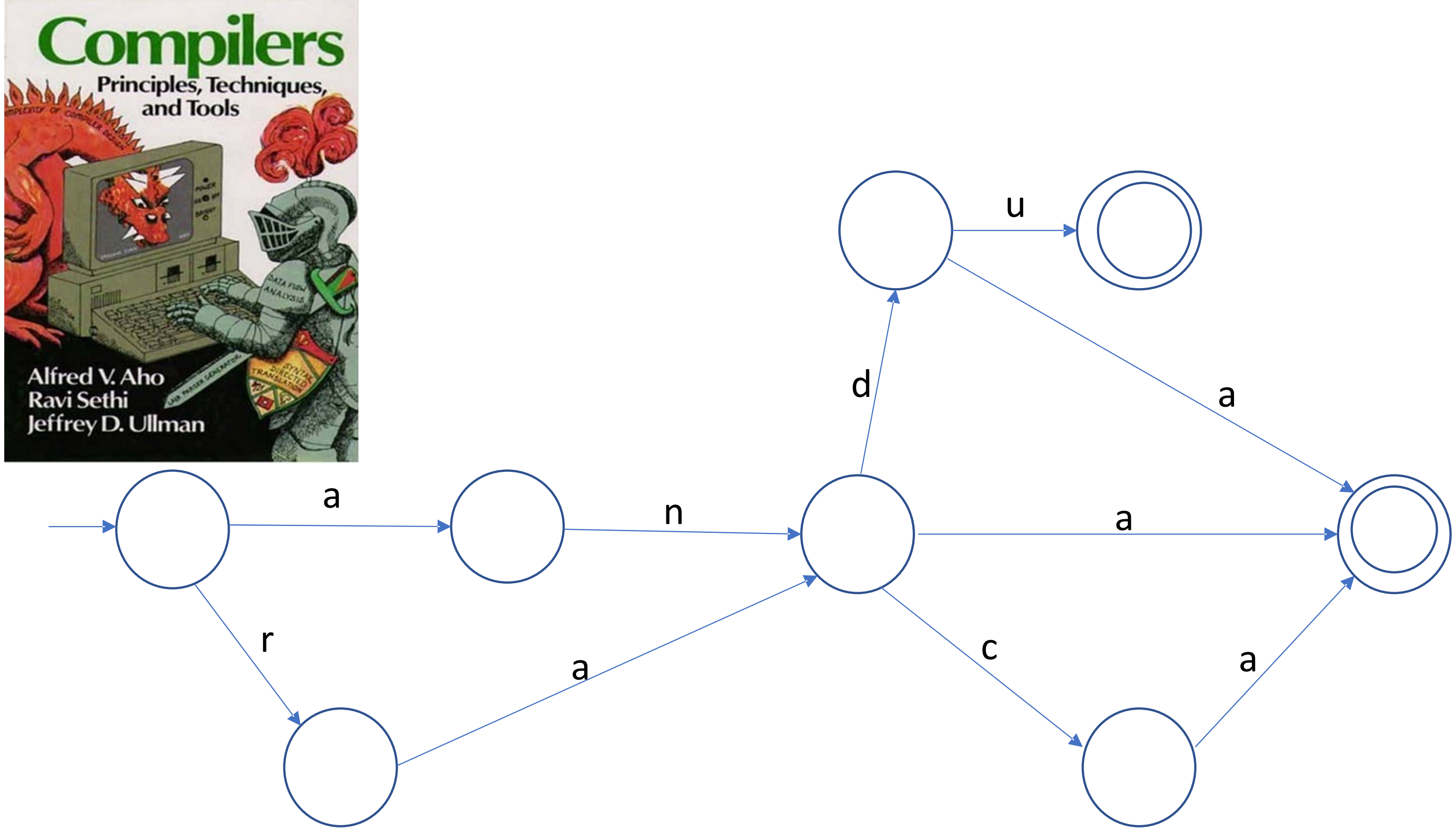
- type 0 : no restriction
- type 1 : context dependent grammar ($x_1Ay_1 \rightarrow x_1\gamma y_1$)
- type 2 : context free grammar ($A \rightarrow \alpha \in P$,where $A \in N$ and $\alpha \in (N \cup \Sigma)^*$)
- type 3 : regular grammar ($A \rightarrow aB|a \in P$)

Remark :

type 3 ⊆ type 2 ⊆ type 1 ⊆ type 0

Notations

- A,B,C,... – nonterminal symbols
- $S \in N$ – start symbol
- a,b,c,... $\in \Sigma$ – terminal symbol
- $\alpha,\beta,\gamma \in (N \cup \Sigma)^*$ - sentential forms
- ε – empty word
- $x,y,z,w \in \Sigma^*$ - words
- X,Y,U,...$\in (N \cup \Sigma)$ – grammar symbols (nonterminal or terminal)



Problem: The door to the tower is closed by the Red Dragon, using a complicated machinery. Prince Charming has managed to steal the plans and is asking for your help. Can you help him determining all the person names that can unlock the door

Course 3&4

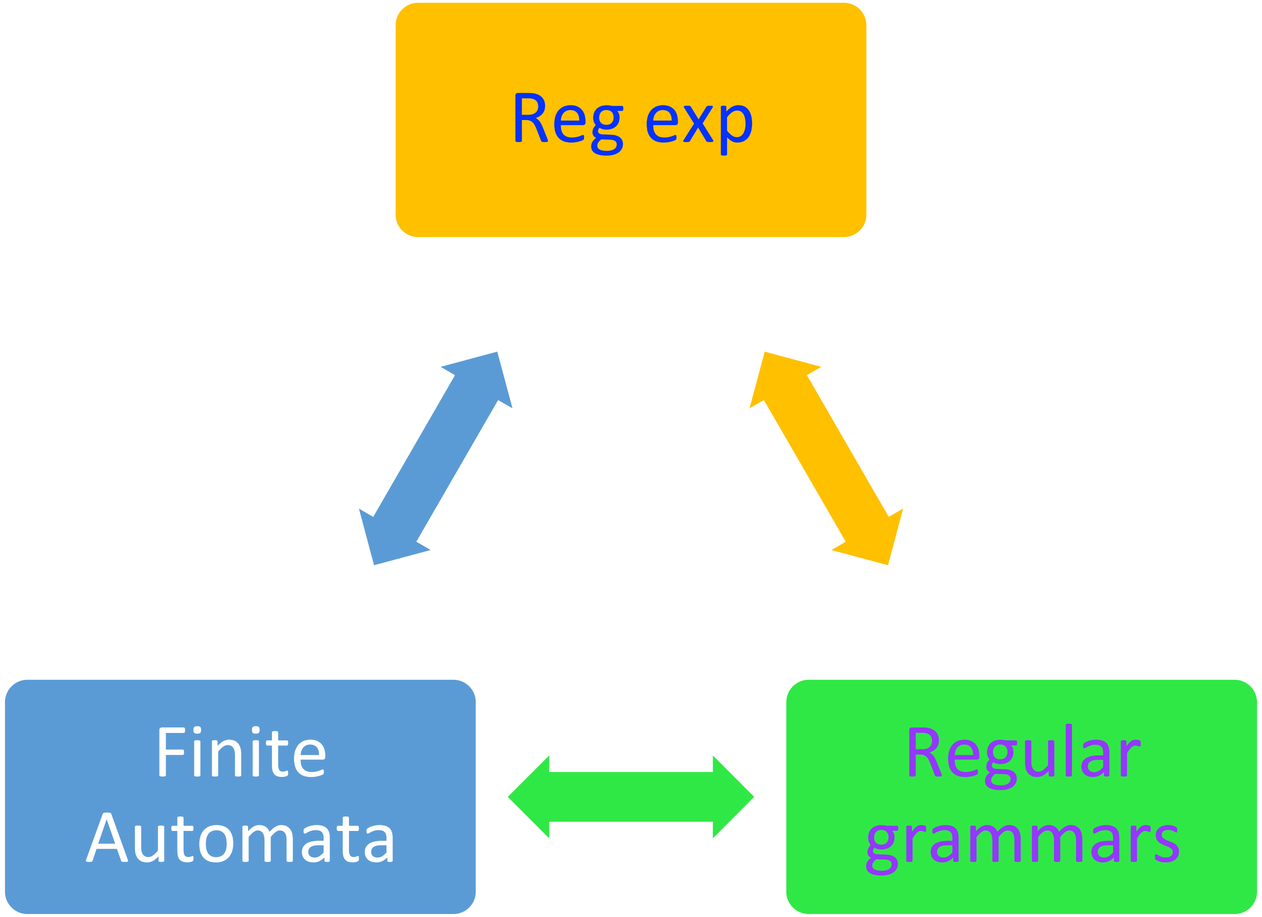
Formal Languages

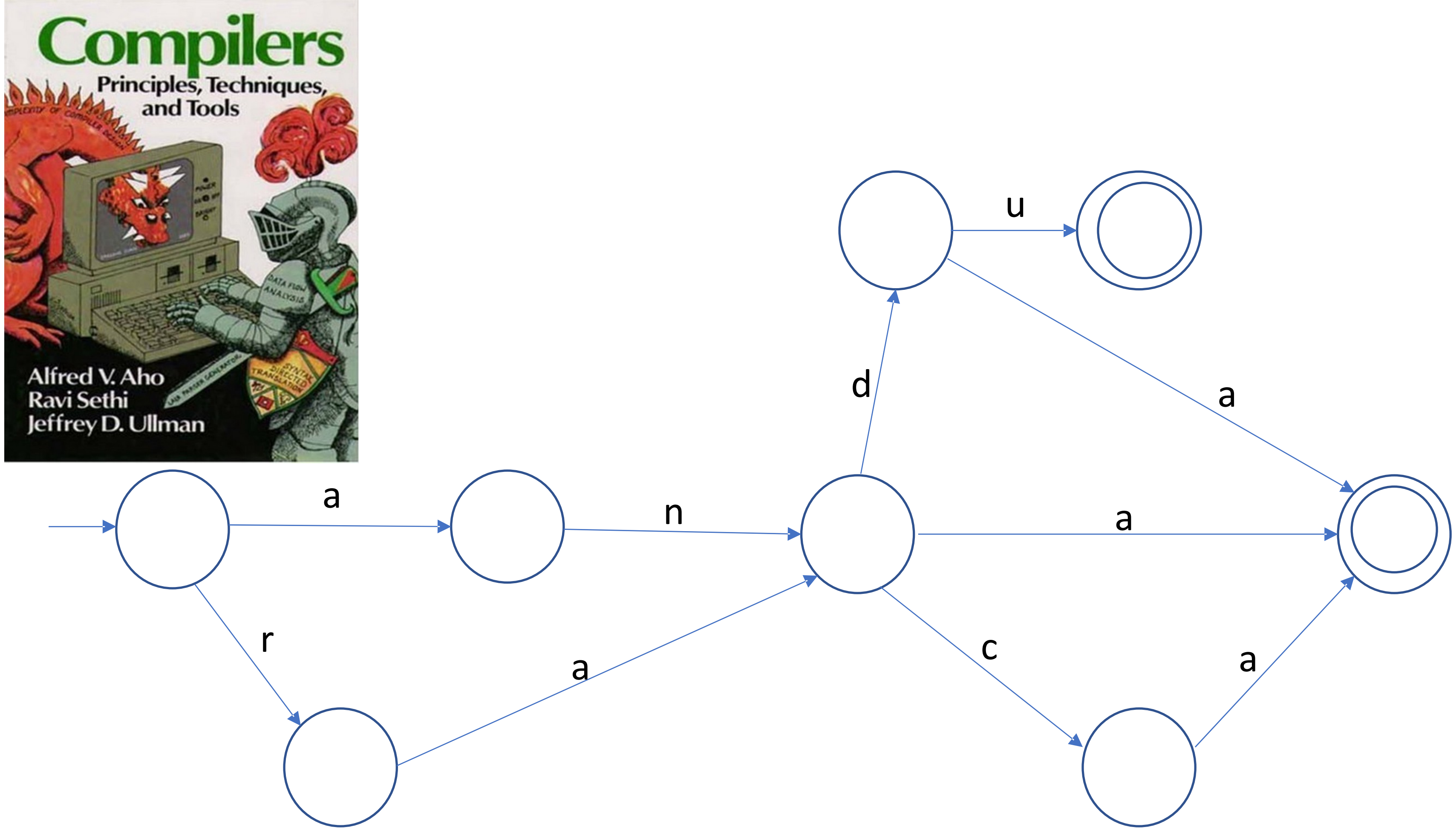
- Basic notions -

Regular languages

Why?

1. Search engine – succes of Google
2. Unix commands
3. Programming languages – new feature

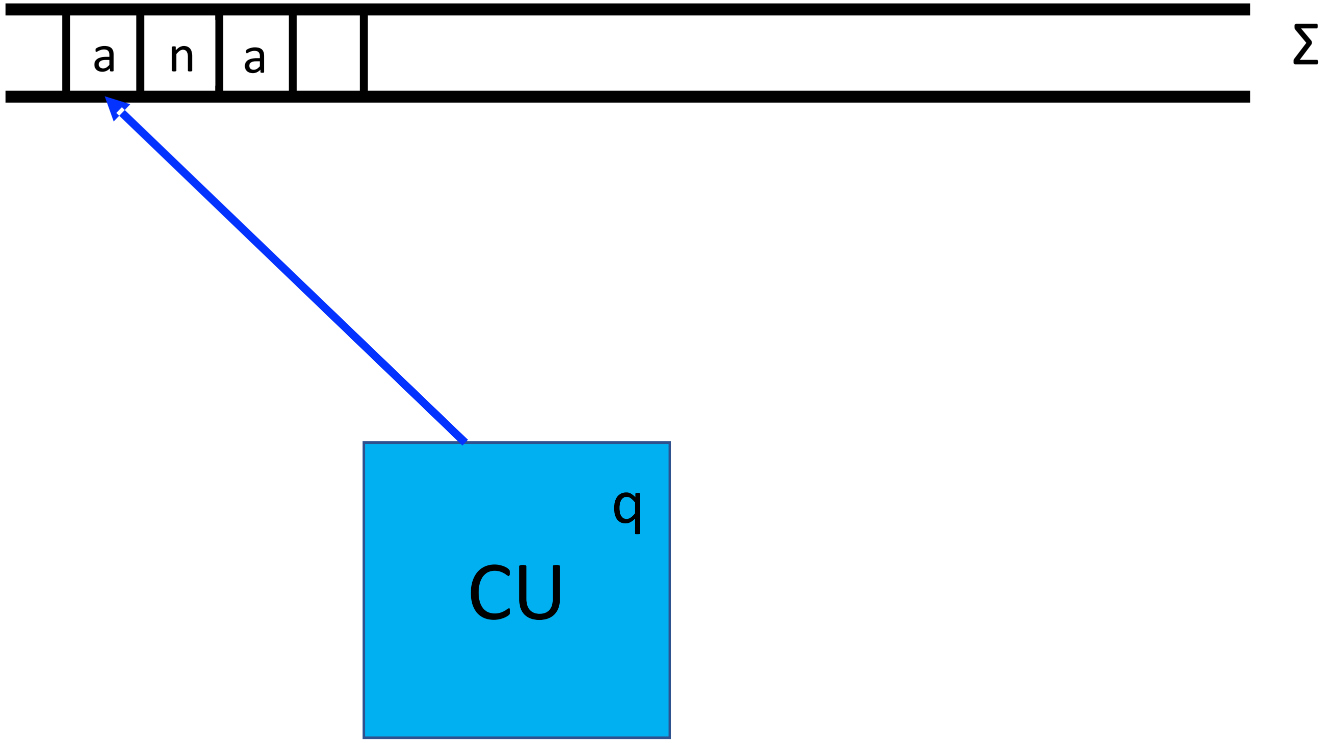

Reg exp
Finite Automata
Regular grammars



Problem: The door to the tower is closed by the Red Dragon, using a complicated machinery. Prince Charming has managed to steal the plans and is asking for your help. Can you help him determining all the person names that can unlock the door

Finite Automata

- Intuitive model



Definition: A ***finite automaton (FA)*** is a 5-tuple

$$M = (Q,\Sigma,\delta,q0,F)$$

where:

- Q - finite set of states ($|Q|<\infty$)
- Σ - finite alphabet ($|\Sigma|<\infty$)
- δ – transition function : $\delta:Q\times\Sigma\rightarrow P(Q)$
- q_0 – initial state $q_0 \in Q$
- $F\subseteq Q$ – set of final states

Remarks

1. $Q \cap \Sigma = \emptyset$
2. $\delta : Q \times \Sigma \rightarrow P(Q)$, $\varepsilon \in \Sigma^0$ - relation $\delta(q,\varepsilon)=p$ **NOT** allowed
3. If $|\delta(q,a)| \leq 1$ => deterministic finite automaton (DFA)
4. If $|\delta(q,a)| > 1$ (more than a state obtained as result) => nondeterministic finite automaton (NFA)

Property: For any NFA *M* there exists a DFA *M'* equivalent to *M*

Configuration C=(q,x)

where:

- q state
- x unread sequence from input: $x \in \sum^*$

Initial configuration : (q_0,w) , w - whole sequence

Final configuration: (q_f ,ε) , $q_f \in F$, ε –empty sequence

(corresponds to accept)

Relations between configurations

- $\vdash$ **move / transition** (simple, one step)
  $(q,ax) \vdash (p,x)$, $p \in \delta(q,a)$

- $\overset{k}{\vdash}$ **k move** = a sequence of k simple transitions) $C_0 \vdash C_1 \vdash \ldots \vdash C_k$
- $\overset{+}{\vdash}$ **+ move**
  $C \overset{+}{\vdash} C'$: $\exists$ $k>0$ such that $C \overset{k}{\vdash} C'$
- $\overset{*}{\vdash}$ *** move (star move)**
  $C \overset{*}{\vdash} C'$: $\exists$ $k \geq 0$ such that $C \overset{k}{\vdash} C'$

Definition : ***Language*** accepted by FA M = (Q,Σ,δ,q0,F) is:

$$L(M)=\{ w \in \Sigma^* \mid (q_0,w) \vdash^{*} (q_f ,\varepsilon) , q_f \in F \}$$

Remarks

1. 2 finite automata M_1 and M_2 are equivalent if and only if they accept the same language

$$L(M_1)=L(M_2)$$

1. $\varepsilon \in L(M)$ ó $q_0 \in F$ (initial state is final state)

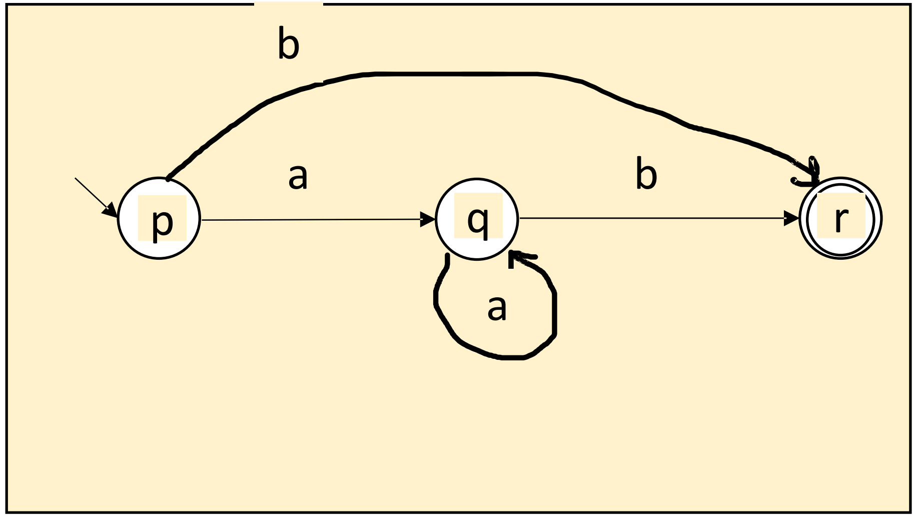
Representing FA

1. List of all elements
2. Table
3. Graphical representation

M=(Q,Σ,δ,p,F)
Q = {p,q,r}
Σ = {a,b}
δ(p,a) = q
δ(q,a)=q
δ(q,b)=r
δ(p,b)=r
F = {r}

M=(Q,Σ,δ,p,F)
F = {r}

	a	b
p	q	r
q	q	r
r	-	-



(p,aab)|-(q,ab)|-(q,b)|-(r,ε) => aab accepted
(p,aba)|-(q,ba)|-(r,a) => aba not accepted

Remember

- **Grammar**

$G=(N,\Sigma,P,S)$

$L(G)=\{w\in\Sigma^* \mid S \overset{*}{\Rightarrow} w\}$

- Finite automaton

$M = (Q,\boldsymbol{\Sigma},\delta,q_0,F)$

$L(M)=\{ w \in \Sigma^* \mid (q_0,w) \vdash (q_f ,\varepsilon) , q_f \in F \}$

Regular grammars

- G = (N, Σ, P, S) right linear grammar if

  $\forall p \in P$: A→aB or A →b, where A,B ∈N and a,b ∈ Σ

S->aA|ε; A-> a reg
S->aS|aA; A->bS|b reg
S->aA; A->aA|ε NOT reg
S->aA|ε; A->aS NOT reg

- G = (N, Σ, P, S) regular grammar if
  - G is right linear grammar

  and
  - A→ε ∉ P, with the exception that S →ε ∈ P, in which case S does not appear in the rhs (right hand side) of any other production

- L(G) = {w ∈ Σ* | S $\overset{*}{=>}$ w} - right linear language

Theorem 1: For any regular grammar $G=(N,\Sigma, P, S)$ there exists a FA $M=(Q, \Sigma, \delta, q_0,F)$ such that $L(G) = L(M)$

Proof: construct M based on G

$Q = N \cup \{K\}, K \notin N$

$q_0 = S$

$F = \{K\} \cup \{S \mid \text{if } S \rightarrow \varepsilon \in P\}$

δ: if $A \rightarrow aB \in P$ then $\delta(A,a) = B$

if $A \rightarrow a \in P$ then $\delta(A,a) = K$

Prove that L(G) = L(M) $(w \in L(G) \Leftrightarrow w \in L(M))$:

$S \overset{*}{\Rightarrow} w \Leftrightarrow (S,w) \overset{*}{\vdash} (qf\,, \varepsilon)$

$w= \varepsilon$: $S \overset{*}{\Rightarrow} \varepsilon \Leftrightarrow (S, \varepsilon) \overset{*}{\vdash} (S, \varepsilon)$ – true

$w=a_1a_2...a_n$: $S \overset{*}{\Rightarrow} w \Leftrightarrow (S,w) \overset{*}{\vdash} (K, \varepsilon)$

$S \Rightarrow a_1A_1 \Rightarrow a_1a_2A_2 \Rightarrow ... \Rightarrow a_1a_2...a_{n-1}A_{n-1} \Rightarrow a_1a_2...a_{n-1}a_n$

$S \Rightarrow a_1A_1$ exists if $S \rightarrow a_1A_1$ and then $\delta(S,a_1)=A_1$

$A_1 \rightarrow a_2A_2 : \delta(A_1,a_2)=A_2$ . . .

$A_{n-1} \rightarrow a_n : \delta(A_{n-1},a_n)=K$

$(S,a_1a_2...a_n) \vdash (A_1,a_2...a_n) \vdash (A_2,a_3...a_n) \vdash ... \vdash (A_{n-1},a_n) \vdash (K, \varepsilon)$, $K \in F$

Theorem 2: For any FA $M=(Q, \Sigma, \delta, q_0, F)$ there exists a right linear grammar $G=(N, \Sigma, P, S)$ such that $L(G) = L(M)$

Proof: construct G based on M

$N = Q$

$S = q_0$

P: if $\delta(q,a) = p$ then $q \to ap \in P$

if $p \in F$ then $q \to a \in P$

if $q_0 \in F$ then $S \to \varepsilon$

Prove that $L(M) = L(G)$ *($w \in L(M) \Leftrightarrow w \in L(G)$):*

P(i): $q \overset{i+1}{\Rightarrow} x \Leftrightarrow (q,x) \overset{i}{\vdash} (q_f , \varepsilon)$, $q_f \in F$ -prove by induction

Apply P : $q_0 \overset{i+1}{\Rightarrow} w \Leftrightarrow (q_0,w) \overset{i}{\vdash} (q_f , \varepsilon)$, $q_f \in F$

If i=0: $q \Rightarrow x$ ó $(q,x) \overset{0}{\vdash} (q_f , \varepsilon)$ $(x= \varepsilon, q=q_f)$ $q \Rightarrow \varepsilon$ ó $q_0 \to \varepsilon$, $q_0 \in F$

Assume ∀ k≤i P is true

$q \overset{i+1}{\Rightarrow} x \Leftrightarrow (q,x) \overset{i}{\vdash} (q_f , \varepsilon)$

For $q \in N$ apply ”⇒” : $q \Rightarrow ap \overset{i}{\Rightarrow} ax$

If $q \Rightarrow ap$ then $\delta(q,a)= p$; if $p \overset{i}{\Rightarrow} ax$ then $(p,x) \overset{i-1}{\vdash} (q_f , \varepsilon)$, qf∈F

THEN $(q,ax) \overset{i}{\vdash} (q_f , \varepsilon)$, qf∈F

Regular sets

Definition: Let $\mathbf{\Sigma}$ be a finite alphabet. We define regular sets over $\mathbf{\Sigma}$ recursively in the following way:

1. $\boldsymbol{\Phi}$ is a regular set over $\mathbf{\Sigma}$ (empty set)
2. $\{\boldsymbol{\varepsilon}\}$ is a regular set over $\mathbf{\Sigma}$
3. {a} is a regular set over $\mathbf{\Sigma}$, $\forall\ a \in \mathbf{\Sigma}$
4. If P, Q are regular sets over$\mathbf{\Sigma}$, then P∪Q, PQ, P* are regular sets over $\mathbf{\Sigma}$
5. Nothing else is a regular set over $\mathbf{\Sigma}$

Regular expressions

Definition: Let **Σ** be a finite alphabet. We define regular expressions over **Σ** recursively in the following way:

1. ***Φ*** is a regular expression denoting the regular set ***Φ*** (empty set)
2. ***ε*** is a regular expression denoting the regular set {***ε***}
3. **a** is a regular expression denoting the regular set {a}, ∀ a∈**Σ**
4. If **p,q** are regular expression denoting the regular sets P, Q then:
    - **p+q** is a regular expression denoting the regular set P∪Q,
    - **pq** is a regular expression denoting the regular set PQ,
    - **p*** is a regular expression denoting the regular set P*
5. Nothing else is a regular expression

Remarks:

Examples

1. $p^+ = pp^*$
2. Use paranthesis to avoid ambiguity
3. Priority of operations: *, concat, + (from high to low)
4. For each regular set we can find at least one regular exp to denote it (there is an infinity of reg exp denoting them)
5. For each regular exp, we can construct the corresponding regular set
6. 2 regular expressions are **equivalent** **iff** they denote the same regular set

Algebraic properties of regular exp

Let $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ be regular expressions.

1. $\boldsymbol{\alpha + \beta = \beta + \alpha}$
2. $\boldsymbol{\Phi^* = \varepsilon}$
3. $\boldsymbol{\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma}$
4. $\boldsymbol{\alpha(\beta\gamma) = (\alpha\beta)\gamma}$
5. $\boldsymbol{\alpha\ (\beta + \gamma) = \alpha\beta + \alpha\gamma}$
6. $\boldsymbol{(\alpha + \beta)\gamma = \alpha\gamma + \beta\gamma}$
7. $\boldsymbol{\alpha\ \varepsilon = \varepsilon\ \alpha = \alpha}$
8. $\boldsymbol{\Phi\alpha = \alpha\Phi = \Phi}$
9. $\boldsymbol{\alpha^* = \alpha + \alpha^*}$
10. $\boldsymbol{(\alpha^*)^* = \alpha^*}$
11. $\boldsymbol{\alpha + \alpha = \alpha}$
12. $\boldsymbol{\alpha + \Phi = \alpha}$

Reg exp equations

- Normal form: **X = aX + b**

  where a,b – reg exp

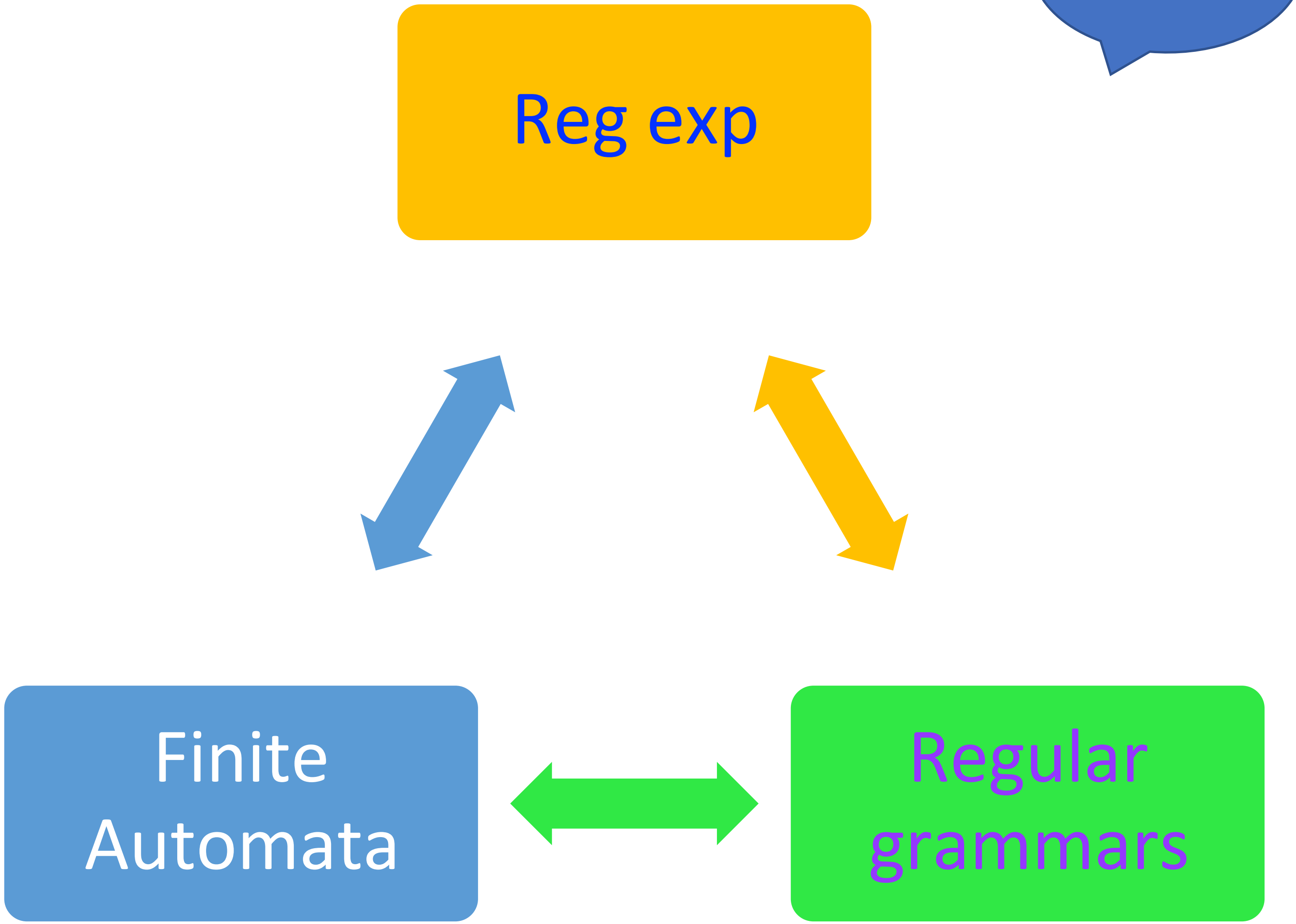
- Solution: **X = a*b**

a a*b + b = (aa* +**ε**)b = a*b

- System of reg exp equations:

$$/\begin{matrix} X = a_{!}\, X + a_{"}\, Y + a_{\#} \\ Y = b_{!}\, X + b_{"}\, Y + b_{\#} \end{matrix}$$

- Solution: Gauss method (replace X_i and solve X_n)



Reg exp

Finite Automata

Regular grammars

Why?

Maths

Prop:*Regular sets are right linear languages*

Lemma 1: $\boldsymbol{\Phi}$,{$\boldsymbol{\varepsilon}$}, {a},∀a∈$\boldsymbol{\Sigma}$ are right linear languages

Proof: constructive

i. G = ({S}, $\boldsymbol{\Sigma}$, $\boldsymbol{\Phi}$, S) – regular grammar such that L(G) = $\boldsymbol{\Phi}$

ii. G = ({S}, $\boldsymbol{\Sigma}$,{S→$\boldsymbol{\varepsilon}$}, S) – regular grammar such that L(G) ={$\boldsymbol{\varepsilon}$}

iii. G = ({S}, $\boldsymbol{\Sigma}$,{S→a}, S) – regular grammar such that L(G) ={a}

Lemma 2: If L_1 and L_2 are right linear languages then:

$L_1 \cup L_2$, L_1L_2 and $L_1{}^*$ are right linear languages.

Proof: constructive

L_1,L_2 right linear languages => $\exists G_1$, G_2 such that

$G_1 = (N_1, \boldsymbol{\Sigma}_1,P_1,S_1)$ and $L_1 = L(G_1)$

$G_2 = (N_2, \boldsymbol{\Sigma}_2,P_2,S_2)$ and $L_2 = L(G_2)$ assume $N_1 \cap N_2 = \emptyset$

i. $G_3 = (N_3, \boldsymbol{\Sigma}, P_3, S_3)$

$N_3 = N_1 U\ N_2 U\ \{S_3\}; \Sigma_3 = \Sigma_1\ U\ \Sigma_2$

$P_3 = P_1 U\ P_2 U\ \xcancel{\{S_3 \to S_1 | S_2\}}$

$\{S_3 \to \boldsymbol{\alpha}_1 |\ S_1 \to \boldsymbol{\alpha}\ _1 \in P_1\}\ U\ \{S_3 \to \boldsymbol{\alpha}_2 |\ S_2 \to \boldsymbol{\alpha}\ _2 \in P_2\}$

G_3 – right linear language

and

$L(G_3) = L(G_1)\ U\ L(G_2)$

PROOF!!! Homework

ii. $G_4 = (N_4, \boldsymbol{\Sigma}, P_4, S_4)$

$N_4 = N_1 U\ N_2;\ S_4 = S_{1;} \Sigma_4 = \Sigma_1 U \Sigma_2$

$P_4 =$ $\{A \rightarrow aB \mid \text{if } A \rightarrow aB \in P_1\}$ U

$\{A \rightarrow aS_2 \mid \text{if } A \rightarrow a \in P_1\}$ U

P_2 U

$\{S_1 \rightarrow \boldsymbol{\alpha}_2 \mid \text{if } S_1 \rightarrow \boldsymbol{\varepsilon} \in P_1 \text{ and } S_2 \rightarrow \boldsymbol{\alpha}_2 \in P_2\}$

G_4 – right linear language

and

$L(G_4) = L(G_1)\ L(G_2)$

PROOF!!! Homework

iii. $G_5 = (N_5, \boldsymbol{\Sigma}_1, P_5, S_5)$

//IDEA: concatenate L_1 with itself

$N_4 = N_1 U \{S_5\};$

$P_5 =$ P_1 U $\{S_5 \to \boldsymbol{\varepsilon}\}$ U
$\{S_5 \to \boldsymbol{\alpha}_1 \mid S_1 \to \boldsymbol{\alpha}\ _1 \in P_1\}$ U
$\{A \to aS_1 \mid \text{ if } A \to a \in P_1\}$

G_5 – right linear language
and
$\mathbf{L(G_5) = L(G_1)^*}$

PROOF!!! Homework

Theorem: *A language is a regular set if and only if is a right linear language*

Proof:

=> Apply lemma 1 and lemma 2

<= construct a system of regular exp equations where:

- Indeterminants – nonterminals
- Coefficients – terminals
- Equation for A: all the possible rewritings of A

Example: G=({S,A,B},{0,1}, P, S)

P: S → 0A | 1B | **ε**

A → 0B | 1A

B → 0S | 1

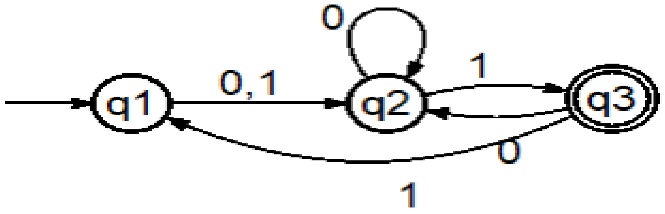
$$S = 0A + 1B + \boldsymbol{\varepsilon}$$

- $$A = 0B + 1A$$

$$B = 0S + 1$$

Regular exp = solution corresponding to S

Theorem: A language is a regular set if and only if is accepted by a FA



Proof:

=> Apply lemma 1 and lemma 2 (to follow, similar to RG)

<= construct a system of regular exp equations where:

- Indeterminants – states
- Coefficients – terminals
- Equation for A: all the possibilities that put the FA in state A
- Equation of the form: X=Xa+b => solution X=ba*

$$q_1 = q_3 0 + \boldsymbol{\varepsilon}$$
$$! \ q_2 = q_1 0 + q_1 1 + q_2 0 + q_3 0$$
$$q_3 = q_2 1$$

Regular exp = union of solutions corresponding to final states

Lemma 1’: $\boldsymbol{\Phi}$, {$\boldsymbol{\varepsilon}$}, {a}, ∀a∈$\boldsymbol{\Sigma}$ are accepted by FA

Reg exp	FA
$\boldsymbol{\Phi}$	$M = (Q, \boldsymbol{\Sigma}, \delta, q_{0,} \boldsymbol{\Phi})$
$\boldsymbol{\varepsilon}$	$M = (Q, \boldsymbol{\Sigma}, \boldsymbol{\Phi}, q_{0,} \{q_0\})$
a, ∀a∈$\boldsymbol{\Sigma}$	$M = (\{q_0,q_1\}, \boldsymbol{\Sigma}, \{\delta(q_0,a) = q_1\}, q_{0,} \{q_1\})$

Lemma 2':If L_1 and L_2 are accepted by a FA then: $L_1 \cup L_2$, L_1L_2 and L_1^* are accepted by FA

Proof:

$M_1 = (Q_1, \boldsymbol{\Sigma}_1, \delta_1, q_{01}, F_1)$ such that $L_1= L(M_1)$

$M_2 = (Q_2, \boldsymbol{\Sigma}_2, \delta_2, q_{02}, F_2)$ such that $L_2 = L(M_2)$

$M_3 = (Q_3, \boldsymbol{\Sigma}_{1U}, \delta_3, q_{03}, F_3)$

$Q_3 = Q_1 \cup Q_2 \cup \{q_{03}\}$; $\Sigma_3 = \Sigma_1 \cup \Sigma_2$

$F_3 = F_1 \cup F_2 \cup \{q_{03} \mid \text{if } q_{01} \in F_1 \text{ or } q_{02} \in F_2\}$

$\delta_3 = \delta_1 \cup \delta_2 \cup \{\delta_3(q_{03},a) = p \mid \exists\delta_1(q_{01},a) = p\} \cup$
$\{\delta_3(q_{03},a) = p \mid \exists\delta_2(q_{02},a) = p\}$

$L(M_3) = L(M_1) \cup L(M_2)$

PROOF!!! Homework

$M_4 = (Q_4, \boldsymbol{\Sigma}_4, \delta_4, q_{04}, F_4)$

$Q_4 = Q_1 \cup Q_2$; $q_{04} = q_{01}$;

$F_3 = F_2 \cup \{q \in F_1 \mid \text{if } q_{02} \in F_2\}$

$$\delta_3(q,a) = \begin{array}{l} \delta_1(q,a), \text{ if } q \in Q_1\text{-}F_1 \\ \delta_1(q,a) \cup \delta_2(q_{02},a) \text{ if } q \in F_1 \\ \delta_2(q,a), \text{ if } q \in Q_2 \end{array}$$

$L(M_3) = L(M_1)L(M_2)$

PROOF!!! Homework

$M_5 = (Q_5, \boldsymbol{\Sigma}_1, \delta_5, q_{05}, F_5)$ *//IDEA: concatenate with itself*

$Q_5 = Q_1$; $q_{05} = q_{01}$

$F_5 = F_1 \cup \{q_{01}\}$

$$\delta_5(q,a) = \begin{cases} \delta_1(q,a), \text{ if } q \in Q_1 - F_1 \\ \delta_1(q,a) \cup \delta_1(q_{01},a) \text{ if } q \in F_1 \end{cases}$$

$L(M_3) = L(M_1)^*$

PROOF!!! Homework

Course 5

Pumping Lemma

- Not all languages are regular
- How to decide if a language is regular or not?

- Idea: pump symbols

Example: L = $\{0^n1^n \mid n>= 0\}$

Theorem: (Pumping lemma, Bar-Hillel)

Let **L** be a regular language. $\exists$***p*** $\in$***N***, such that if ***w*** $\in$***L*** with ***|w|>p***, then

w = xyz, where ***0<|y|<=p***

and

$xy^iz \in L, \forall i \geq 0$

Proof

L regular => $\exists$ M = $(Q,\boldsymbol{\Sigma},\boldsymbol{\delta}, q_0, F)$ such that L= L(M)

Let |Q| = p

$$\left.\begin{array}{l} \text{If } w \in L(M)\text{: } (q_0,w) \overset{*}{\vdash} (q_f\text{ , }\boldsymbol{\varepsilon})\text{ , } q_f \in F \\ \qquad\quad\text{and} \\ |w|>p \end{array}\right\} \begin{array}{l}\text{process at least p+1 symbols}\\ \\ \text{p states}\end{array}$$

$\Rightarrow \exists$ q_1 that appear in at least 2 configurations

$(q_0,xyz) \overset{*}{\vdash} (q1,yz) \overset{+}{\vdash} (q1,z) \overset{*}{\vdash} (q_f\text{ , }\boldsymbol{\varepsilon})$, $q_f \in F$ => 0<=|y|<=p

Proof (cont)

$$
\begin{aligned}
(q_0, xy^iz) &\overset{*}{\vdash} (q_1, y^iz) \\
&\overset{*}{\vdash} (q_1, y^{i-1}z) \\
&\overset{*}{\vdash} \ldots \\
&\overset{*}{\vdash} (q_1, yz) \\
&\overset{*}{\vdash} (q_1, z) \\
&\overset{*}{\vdash} (q_f, \boldsymbol{\varepsilon}),\ q_f \in F
\end{aligned}
$$

So, if $w=xyz \in L$ then $xy^iz \in L$, for all $i>0$

If $i=0$: $(q_0,xz) \overset{*}{\vdash} (q_1, z) \overset{*}{\vdash} (q_f, \boldsymbol{\varepsilon}),\ q_f \in F$

Example: L = $\{0^n1^n \mid n>= 0\}$

Suppose L is regular => w= xyz = 0^n1^n

Consider all possible decomposition =>

Case 1. y = 0^k

$xyz = 0^{n-k}0^k1^n$; $xy^iz = 0^{n-k}0^{ik}1^n \notin L$

Case 2. y = 1^k

$xyz = 0^n1^k1^{n-k}$; $xy^iz = 0^n1^{ik}1^{n-k} \notin L$

Case 3. y = $0^k\,1^l$

$xyz = 0^{n-k}0^k1^l1^{n-l}$; $xy^iz = 0^{n-k}(0^k1^l)^i1^{n-l} \notin L$

Case 4. y = $0^k\,1^K$

$xyz = 0^{n-k}0^k1^k1^{n-k}$; $xy^iz = 0^{n-k}0^k1^k0^k1^k\ldots1^{n-l} \notin L$

=> L is not regular

Context free grammars (cfg)

Context free grammar (cfg)

- Procdutions of the form: A →α, A∈N, α∈(N∪$\boldsymbol{\Sigma}$)*

- More powerful

- Can model programming language:

  G = (N, $\boldsymbol{\Sigma}$,P,S) s.t. L(G) = programming language

Syntax tree

Definition: A syntax tree corresponding to a cfg G = (N, Σ,P,S) is a tree obtained in the following way:

1. Root is the starting symbol S
2. Nodes ∊ N∪Σ:
    1. Internal nodes ∊N
    2. Leaves ∊Σ
3. For a node A the descendants in order from left to right are X_1, X_2, ..., X_n only if $A \rightarrow X_1X_2... X_n \in P$

Remarks:

a) Parse tree = syntax tree – result of parsing (syntatic analysis)

b) Derivation tree – condition 2.2 not satisfied

c) Abstract syntax tree (AST) ≠ syntax tree (semantic analysis)

Syntax tree (cont)

Property: In a cfg $G = (N, \Sigma, P, S)$, $w \in L(G)$ <u>if and only if</u> there exists a syntax tree with frontier w.

Proof: HomeWork

Example: S-> aSbS | c; w = aacbcbc

Leftmost derivations	Rightmost derivations
S => aSbS => aaSbSbS => aacbSbS => aacbcbS => aacbcbc	S => aSbS => aSbc => aaSbSbc => aaSbcbc => aacbcbc

Definition: A cfg G = (N, Σ,P,S) is ambigous if for a w∈ L(G) there exists 2 distinct syntax tree with frontier w.

Example:

Parsing (syntax analysis) modeled with cfg:

cfg G = (N, $\boldsymbol{\Sigma}$,P,S):

- N – nonterminal: syntactical constructions: declaration, statement, expression, a.s.o.
- $\boldsymbol{\Sigma}$ – terminals; elements of the language: identifiers, constants, reserved words, operators, separators
- P – syntactical rules – expressed in BNF – simple transformation
- S – syntactical construct corresponding to program

THEN

Program syntactically correct <=> w ∈ L(G)

Equivalent transformation of cfg

- Unproductive symbols
- Inaccesible symbols

- ε - productions
- Single productions

1. Determine elements (symbols/ productions): Greedy alg
2. eliminate them: construct equivalent grammar

Unproductive symbols

Definition

A nonterminal A este ***unproductive*** in a cfg if does not generate any word: $\{w \mid A \Rightarrow^* w, w \in \Sigma^*\} = \emptyset$.

Algorithm 1: Elimination of unproductive symbols

input: $G = (N, \Sigma, P, S)$

output: $G' = (N', \Sigma, P', S)$, $L(G) = L(G')$

// idea: build $N_0, N_1, \ldots$ recursively (until saturation)

```
step 1: $N_0 = \emptyset$; i:=1;
step 2: $N_i = N_{i-1} \cup \{A \mid A \to \alpha \in P, \alpha \in (N_{i-1} \cup \Sigma)^*\}$
step 3: if $N_i <> N_{i-1}$   then i:=i+1; goto step 2
                              else $N' = N_i$
step 4: if $S \notin N'$      then $L(G) = \emptyset$
                              else $P' = \{A \to \alpha \mid A \to \alpha \in P \text{ and } A \in N'\}$
```

Example

G = ({S,A,B,C,D}, {a,b,c}, P,S)

P: $S \rightarrow aA \mid aC$

$A \rightarrow AB$

$B \rightarrow b$

$C \rightarrow aC \mid CD$

$D \rightarrow b$

Inaccesible symbols

Definition

A symbol $X \in N \cup \Sigma$ is ***inaccesible*** in a cfg if X does not appear in any sentential form: $\forall\ S \Rightarrow^* \alpha,\ X \notin \alpha$

Algorithm 2: Elimination of inaccessible symbols

input: $G = (N, \Sigma, P, S)$

output: $G' = (N', \Sigma, P', S)$, $L(G) = L(G')$ *and*

$\forall X \in N \cup \Sigma\ \ \exists \alpha, \beta \in (N' \cup \Sigma)^*$ *s.t.* $S \Rightarrow^*_{G'} \alpha X \beta$

```
step 1: $V_0 = \{S\}$; i:=1;
step 2: $V_i = V_{i-1} \cup \{X \mid \exists A \to \alpha X \beta \in P, A \in V_{i-1}\}$
step 3: if $V_i <> V_{i-1}$ then i:=i+1; goto step 2
                          else $N' = N \cap V_i$
                               $\Sigma' = \Sigma \cap V_i$
                               $P' = \{A \to \alpha \mid A \to \alpha \in P, A \in N', \alpha \in (N \cup \Sigma)^* \}$
```

Example

G = ({S,A,B,C,D}, {a,b,c,d}, P,S)

P: $S \rightarrow aA \mid aC$

$A \rightarrow AB$

$B \rightarrow b$

$C \rightarrow aC \mid bCb$

$D \rightarrow bB \mid d$

ε-productions

Definition

A cfg $G=(N,\Sigma,P,S)$ is without ε***productions*** if

1. $P \not\ni A \to \varepsilon$ (ε-productions)

OR

2. $\exists\, S\to\varepsilon$ si $S \notin rhs(p), \forall p \in P$

Algorithm 3: Elimination of ε***productions***

input: cfg $G = (N, \Sigma, P, S)$

output: cfg $G' = (N', \Sigma, P', S')$

```
step 1: construct $\overline{N} = \{A \mid A \in N, A \Rightarrow^{+} \varepsilon\}$
        1.a.    $N_0 := \{A \mid A\to\varepsilon \in P\}$;
                $i := 1$;
        1.b. $N_i := N_{i-1} \cup \{A \mid A\to\alpha \in P, \alpha \in N^{*}_{i-1}\}$
        1.c. if $N_i <> N_{i-1}$ then $i:=i+1$; goto step 1.b
                    else $\overline{N} = N_i$
```

A->BC

B->ε

C->ε

```
step 2: Let P' = set of productions built:
    2.a. if $A\to\alpha_0 B_1\alpha_1 B_2\alpha_2 \dots B_k\alpha_k \in P$, $k>=0$
            and for $i := 1,k$ $B_i \in \overline{N}$
            and $\alpha_j \notin \overline{N}$, $j:=0,k$
        then add to P' all prod of the form
            $A\to\alpha_0 X_1\alpha_1 X_2\alpha_2 \dots X_k\alpha_k$
            where $X_i$ is $B_i$ or $\varepsilon$ (not $A\to\varepsilon$ )
    2.b if $S \in N'$ then add S' to N' and $S'\to S \mid \varepsilon$ to P
            else $N' := N$; $S' := S$.
```

Example

G = ({S,A,B}, {a,b},P,S)

P: $S \rightarrow aA \mid aAbB$

$A \rightarrow aA \mid B$

$B \rightarrow bB \mid \boldsymbol{\varepsilon}$

Single productions

Definition

O production of the form $A \rightarrow B$ is called single production or renaming rule.

Algorithm 4 : Elimination of single productions

Input: cfg G, without ε-productions

Output: G' s.t. $L(G) = L(G')$

```
For each $A \in N$ build the set $N_A = \{B \mid A \Rightarrow^* B\}$ :
1.a. $N_0 := \{A\}$, $i := 1$
1.b. $N_i := N_{i-1} \cup \{C \mid B \rightarrow C \in P$ si $B \in N_{i-1}\}$
1.c. if $N_i \neq N_{i-1}$ then $i := i+1$ goto 1.b.
                    else $N_A := N_i$
P': for all $A \in N$ do
        for all $B \in N_A$ do
         if $B \rightarrow \alpha \in P$ and not "single" then $A \rightarrow \alpha \in P'$
```

$G' = (N, \Sigma, P', S)$

Example

$G = (\{E,T,F\},\{a,(,),+,*\},P,E)$

P: $E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid a$

Parsing

- Cfg G = (N, **Σ**, P,S) check if w ∊ L(G)
- Construct parse tree

- How:
  1. Top-down vs. Bottom-up
  2. Recursive vs. linear

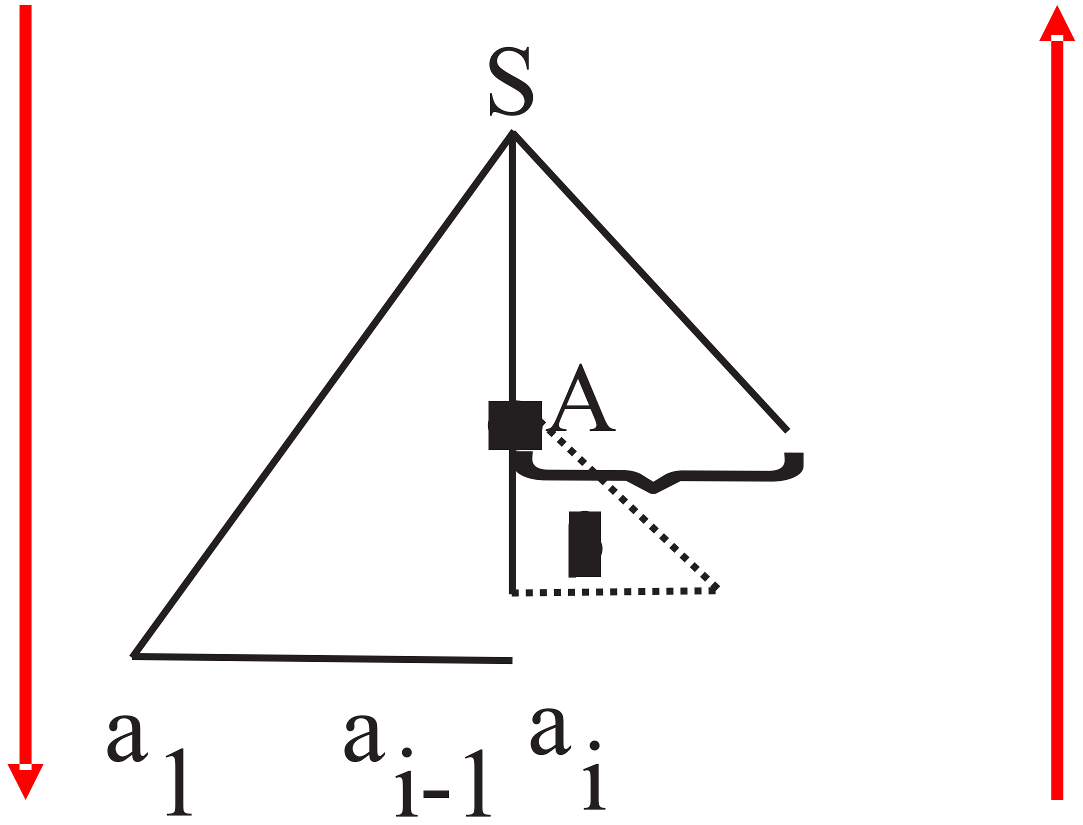


Figura 3.2:Construçţa arborelui prin analiza sintactic˘a LL(1)

3.2.1. Gramatici de tip LL(k)

Course 6

Problem: Parsing (construct the parsee tree)

```
if the source program is sintactically correct
        then construct syntax tree
        else ”syntax error”
```

source program is sintactically correct = w ∈ L(G) ó $S \overset{*}{=>} w$

Parsing

- How:
  1. Top-down vs. Bottom-up
  2. Recursive vs. linear

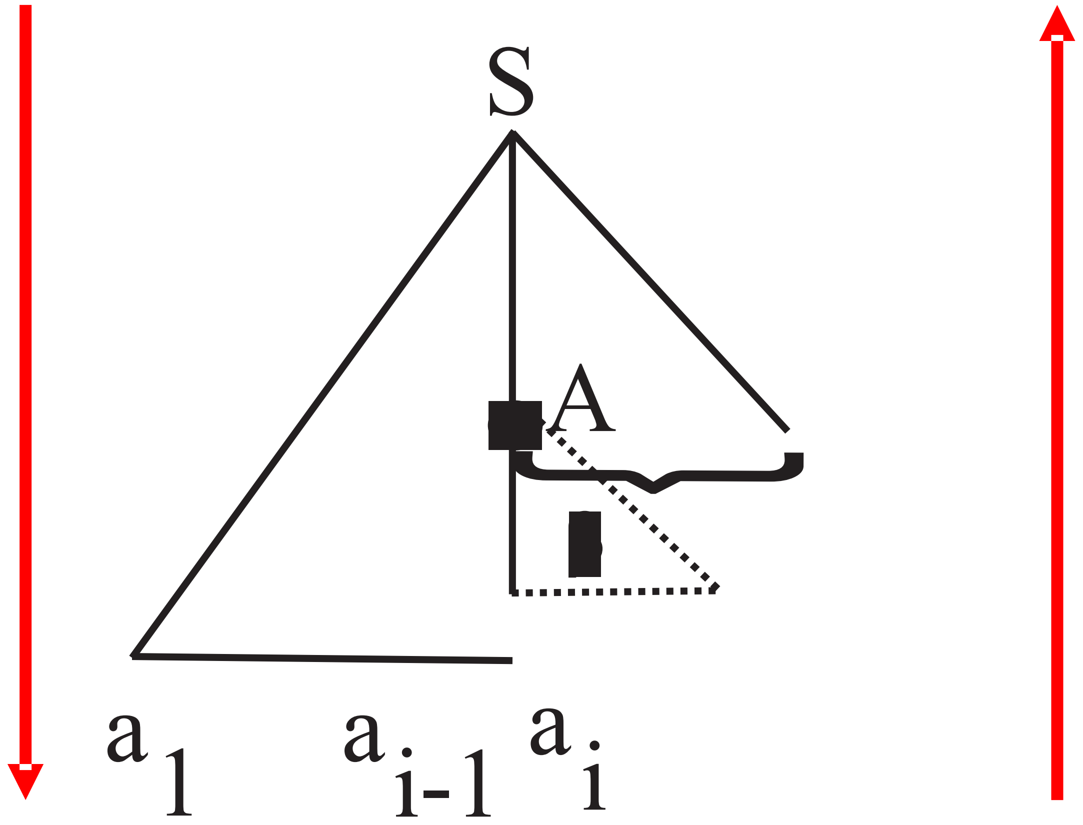


Figura 3.2:Construcţia arborelui prin analiza sintactic˘a LL(1)

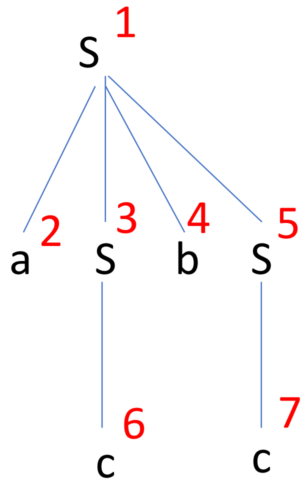
3.2.1. Gramatici de tip LL(k)

	Descendent	Ascendent
Recursive	Descendent recursive parser	Ascendent recursive parser
Linear	LL(k): LL(1)	LR(k): LR(0), SLR, LR(1), LALR

Result – parse tree -representation

- Arbitrary tree – child sybling representation
- Sequence of derivations S => $\boldsymbol{\alpha}_1$ => $\boldsymbol{\alpha}_2$ =>… => $\boldsymbol{\alpha}_n$ = w
- String of production – index associated to prod – which prod is used at each derivation step: 1,4,3,…

index	Info	Parent	Right sibling
1	S	0	0
2	a	1	0
3	S	1	2
4	b	1	3
5	S	1	4
6	c	3	0
7	c	5	0



Descendent recursive parser

- Example

S -> aSbS | aS | c

Formal model

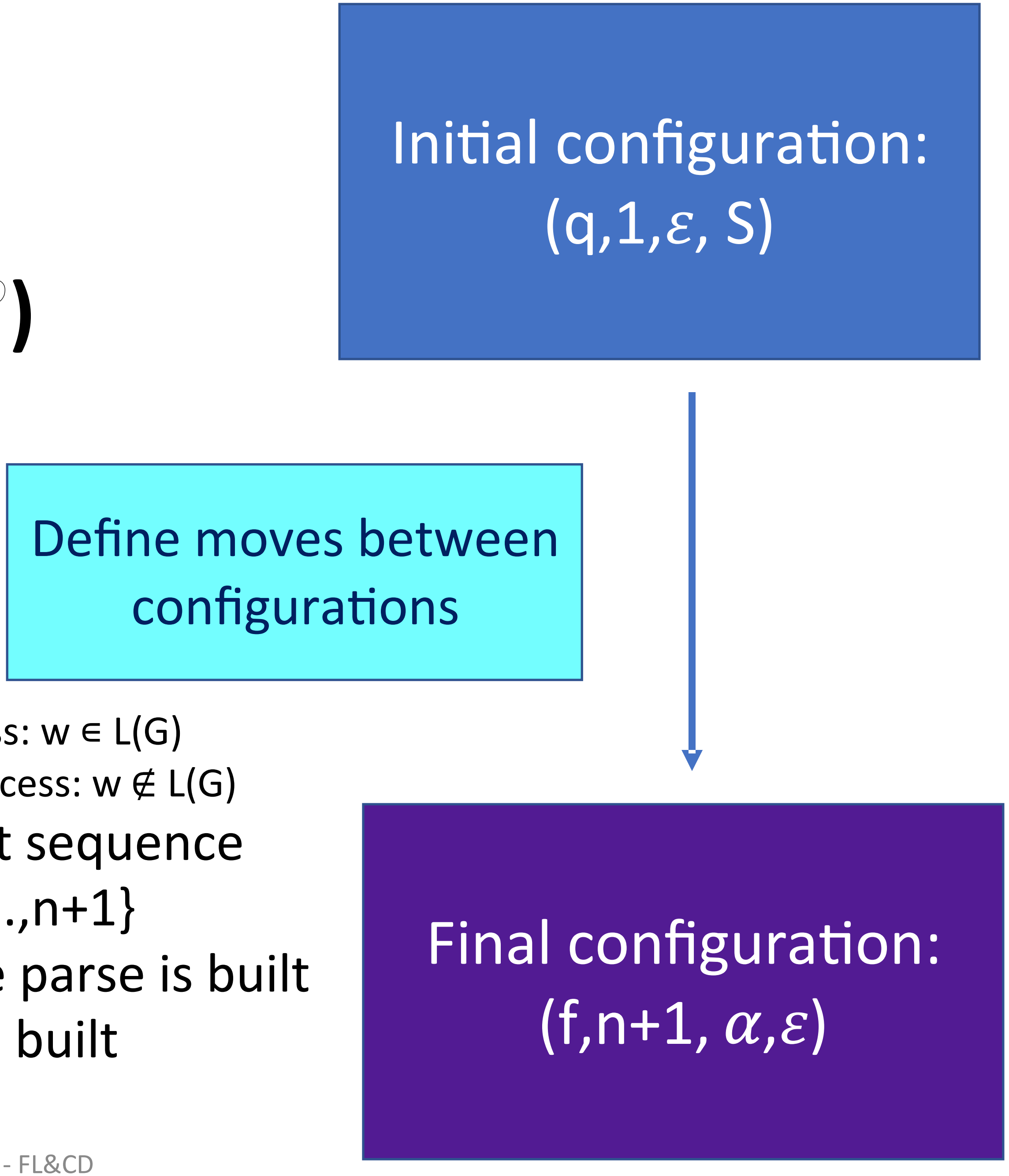
- Configuration

$$(\mathbf{s}, \mathbf{i}, \alpha, \beta)$$

where:

- s = state of the parsing, can be:
  - q = normal state
  - b = back state
  - f = final state - corresponding to success: w ∊ L(G)
  - e = error state – corresponding to insuccess: w ∉ L(G)
- i – position of current symbol in input sequence

  $w = a_1a_2\ldots a_n$, $i \in \{1,\ldots,n+1\}$
- α = working stack, stores the way the parse is built
- β = input stack, part of the tree to be built



Initial configuration:
$(q,1,\varepsilon, S)$

Define moves between configurations

Final configuration:
$(f,n+1, \alpha,\varepsilon)$

Expand

WHEN: head of input stack is a nonterminal

$(q,i, \boldsymbol{\alpha}, A\boldsymbol{\beta}) \vdash (q,i, \boldsymbol{\alpha}A_1, \boldsymbol{\gamma}_1\boldsymbol{\beta})$

where:

$A \rightarrow \boldsymbol{\gamma}_1 \mid \boldsymbol{\gamma}_2 \mid \ldots$ represents the productions corresponding to A

1 = first prod of A

Advance

WHEN: head of input stack is a terminal = current symbol from input

$(q,i, \boldsymbol{\alpha}, a_i\boldsymbol{\beta}) \vdash (q,i+1, \boldsymbol{\alpha} a_i, \boldsymbol{\beta})$

Momentary insuccess

WHEN: head of input stack is a terminal ≠ current symbol from input

$(q,i,\boldsymbol{\alpha},a_i\boldsymbol{\beta}) \vdash (b,i,\boldsymbol{\alpha},a_i\boldsymbol{\beta})$

Back

WHEN: head of working stack is a terminal

$(b, i, \boldsymbol{\alpha} a, \boldsymbol{\beta}) \vdash (b, i-1, \boldsymbol{\alpha}, a\boldsymbol{\beta})$

Another try

WHEN: head of working stack is a nonterminal

$(b,i, \boldsymbol{\alpha}\ A_j, \boldsymbol{\gamma}_j\boldsymbol{\beta}) \vdash$ $(q,i, \boldsymbol{\alpha}A_{j+1}, \boldsymbol{\gamma}_{j+1}\boldsymbol{\beta})$, if $\exists\ A \rightarrow \boldsymbol{\gamma}_{j+1}$

$(b,i, \boldsymbol{\alpha}, A\ \boldsymbol{\beta})$, otherwise with the exception

$(e,i, \boldsymbol{\alpha}, \boldsymbol{\beta})$, if i=1, A =S, **ERROR**

Success

$(q, n+1, \boldsymbol{\alpha}, \varepsilon) \vdash (f, n+1, \boldsymbol{\alpha}, \varepsilon)$

Algorithm

Algorithm Descendent Recursive

INPUT: G, w $=a_1a_2\ldots a_n$

OUTPUT: string of productions and message

```
config = (q,1, $\varepsilon$,S);                                   //initial configuration ($s,i,\alpha,\beta$)
while (s $\neq$ f) and (s $\neq$ e) do
   if s = q
     then if (i=n+1) and IsEmpty($\beta$)
               then Success(config)
               else
                    if Head($\beta$) = A
                       then Expand(config)
                       else
                          if  Head($\beta$) = $a_i$
                              then Advance(config)
                              else MomentaryInsuccess(config)
     else
          if s = b
             then
                  if Head($\alpha$) = a
                    then Back(config)
                    else AnotherTry(config)
endWhile
if s = e  then message"Error"
          else message "Sequence accepted";
               BuildStringOfProd($\alpha$)
```

w ∈ L(G) - HOW

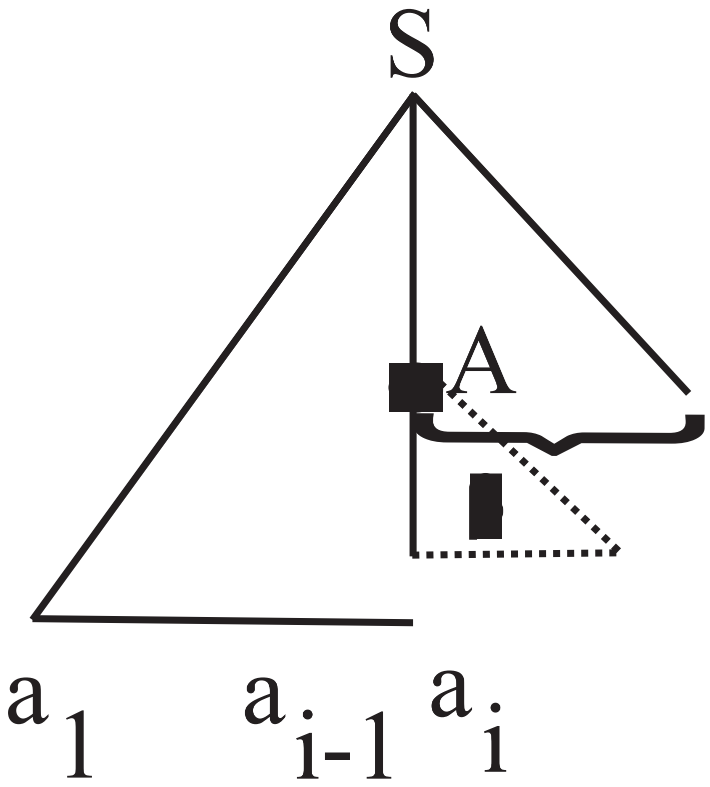
- Process $\boldsymbol{\alpha}$:
  - From left to right (reverse if stored as stack)
  - Skip terminal symbols
  - Nonterminals – index of prod

- Example: $\boldsymbol{\alpha}$ = S_1 a S_2 a S_3 c b S_3 c

When the algorithm never stops?

- S->S**α** – expand infinitely (left recursive)

LL(1) Parser



Linear algorithm

Figura 3.2:Construcţia arborelui prin analiza sintactic˘a LL(1)

3.2.1. Gramatici de tip LL(k)

Definiţia 3.1. *[AU73]O gramatic˘a G = (N, Σ, P, S) este de tip LL(k) dac˘a pentru oricare dou˘a deriv˘ari de st^anga:*

- predic¸tia de lungime k: $a_{i+1} . . . a_{i+k}$,

FIRST$_k$

dup˘a cum se observ˘a ¸si din alegerea produc¸tiei $A ! Ø$ în figura 3.2.

Predic¸tia de lungime k reprezint˘a urm˘atoarele k simbolur

bui generate din configura¸tia curent˘a. Pentru aceasta se introduce

$F\,IRST_k$ [ASU86], care calculeaz˘a primele k simboluri ce ¸se încep

deriv˘ari succesive dintr-o anumit˘a form˘a propozi¸tional˘a:

- ≈ first k terminal symbols that can be generated from *α*
- **Definition**:

$F\,IRST_k : (N \,[\, ß)^{§} \,!\, P(ß^{k})$

$F\,IRST_k(Æ) = \{u|u\ 2\ ß^{k}, Æ\ )^{§}\ ux, |u| = k$ sau $Æ\ )^{§}\ u, |u| ∑ k\}$

(primele k simboluri ale lui $Æ$)

LL(1) Parser

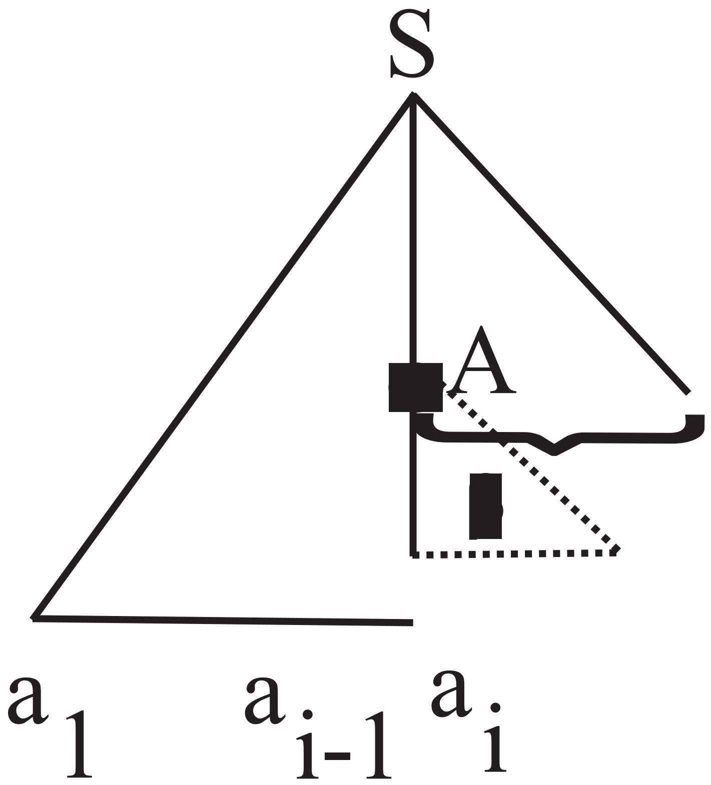


Figura 3.2: Construcţia arborelui prin analiza sintactic˘a LL(1)

Linear algorithm

3.2.1. Gramatici de tip LL(k)

Definiţia 3.1. *[AU73]O gramatic˘a G = (N, Σ, P, S) este de tip LL(k) dac˘a pentru oricare dou˘a deriv˘ari de stˆanga:*

Operation: $\oplus$ = concatenation of length 1

L1 = {aa,ab,ba}

L2 = {00,01}

L1$\oplus$L2 = {a,0}

L1={a, $\boldsymbol{\varepsilon}$}

L2={0,1}

L1$\oplus$L2 ={a,0,1}

- predic¸tia de lungime k: $a_{i+1} . . . a_{i+k}$,

FIRST$_k$

dup˘a cum se observ˘a ¸si din alegerea produc¸tiei A ! Ø ˆın figura 3.2.

Predic¸tia de lungime *k* reprezint˘a urm˘atoarele k simbolur

bui generate din configura¸tia curent˘a. Pentru aceasta se introduce

- ~ first k terminal symbols that can be generated from *α*

F IRST$_k$ [ASU86], care calculeaz˘a primele k simboluri ce ¸se p

- **Definition**:

deriv˘ari succesive dintr-o anumit˘a form˘a propozi¸tional˘a:

F IRST$_k$: (*N* [ß)§ ! *P*(ß^k)

F IRST$_k$(*Æ*) = {*u*|*u* 2 ß^k, *Æ*)§ *ux*, |*u*| = *k* sau *Æ*)§ *u*, |*u*| ∑ *k*}

(primele k simboluri ale lui *Æ*)

FIRST$_k$

- Which are the first k terminal symbols that can be generated from A?

- https://forms.office.com/r/kNHNGW7XtC

3.2.3. Construirea tabelului de analiz˘a LL(1)

Construct FIRST

Calculul elementelor din tabel depinde de valorile funcţiei $F\ IRST$.

Pentru a putea descrie o metod˘a de calcul a lui $F\ IRS$… e de urm˘atoarea proprietate:

Concatenation of length 1

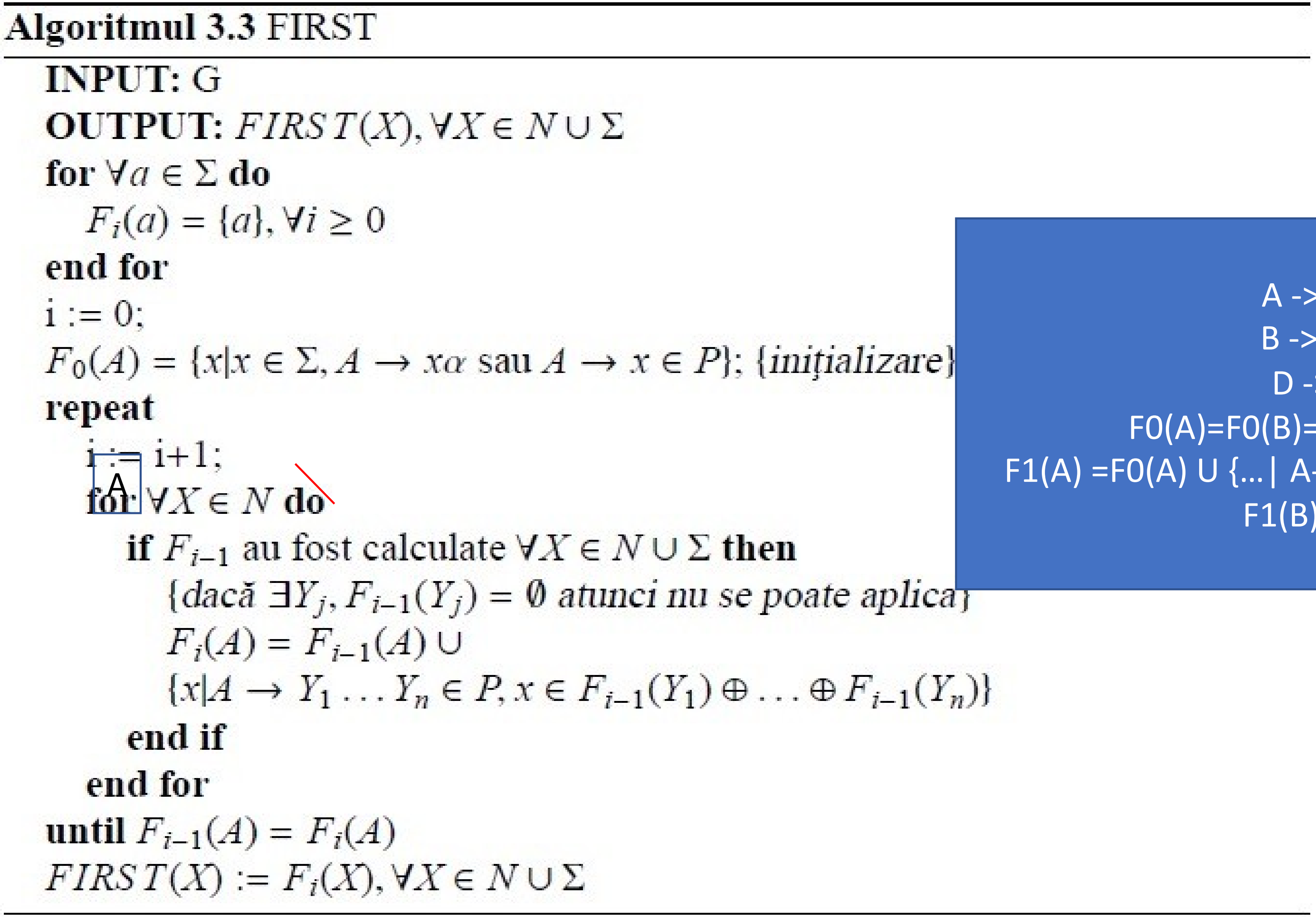
ØFIRST$_1$ denoted FIRST

Observaţii [GJ90]:

ØRemarks:

- If a L_1, L_2 are 2 languages over alphabet Σ, then atunci: $L_1 © L_2 =$ $\{w|x\ 2\ L_1, y\ 2\ L_2, xy = w, |w| \sum 1$ sau $xy = wz, |w|$ and ¸si
- $F\ IRST\ (ÆØ) = F\ IRST\ (Æ)\ ©\ F\ IRST\ (Ø)$

$F\ IRST\ (X_1 \ldots X_n) = F\ IRST\ (X_1)\ ©\ \ldots\ ©\ F\ IRST\ (X_n)$



Algoritmul 3.3 FIRST

```
INPUT: G
OUTPUT: $FIRST(X), \forall X \in N \cup \Sigma$
for $\forall a \in \Sigma$ do
    $F_i(a) = \{a\}, \forall i \geq 0$
end for
i := 0;
$F_0(A) = \{x | x \in \Sigma, A \to x\alpha$ sau $A \to x \in P\}$; {inițializare}
repeat
    i := i+1;
    for $\forall X \in N$ do
        if $F_{i-1}$ au fost calculate $\forall X \in N \cup \Sigma$ then
            {dacă $\exists Y_j, F_{i-1}(Y_j) = \emptyset$ atunci nu se poate aplica}
            $F_i(A) = F_{i-1}(A) \cup$
            $\{x | A \to Y_1 \ldots Y_n \in P, x \in F_{i-1}(Y_1) \oplus \ldots \oplus F_{i-1}(Y_n)\}$
        end if
    end for
until $F_{i-1}(A) = F_i(A)$
$FIRST(X) := F_i(X), \forall X \in N \cup \Sigma$
```

A

A -> BC
B -> DA
D -> a
F0(A)=F0(B)=Ø; F0(D)={a}
F1(A) =F0(A) U {…| A->BC F0(B)⨁F(D)}= Ø
F1(B) ={a}

predic¸tia q urm atorul simbol de pe banda de in

determin˘a ˆın mod unic alegerea unei¸ i produc A ! Æt.

FOLLOW

Teorema 3.2. *[S¸ er87] O gramatic˘a este de tip LL(1) d pentru fiecare neterminal A cu produc¸tiile A ! Æ$_1$|Æ$_2$| . . . |Æ$_n$ F \F IRST $_k$(Æ$_j$) = ; ¸si dac˘a Æ$_i$)§ ≤, F IRST (Æ$_j$)\F OLLOW (A) 1, n, i 6= j.*

A → ε

Ø FOLLOW$_k$(A)≈ next k symbols generated after/ following A

Dup˘a cum sugereaz˘a teorema, ˆın situa special˘a a ap primulsimboldin ... F IRST (A) ... secven¸a liz ˆand arborele ... va c˘a ˆ lu˘am ˆın consi ... ea ce ”u *A.* Pentru aceasta se introduce o nou˘a func¸tie [AU73]:

F OLLOW : (*N* [ß) § ! *P*(ß)

F OLLOW (Ø) = {*w* 2 ß|*S*)§ ÆØ∞, *w* 2 *F IRST* (∞)}.

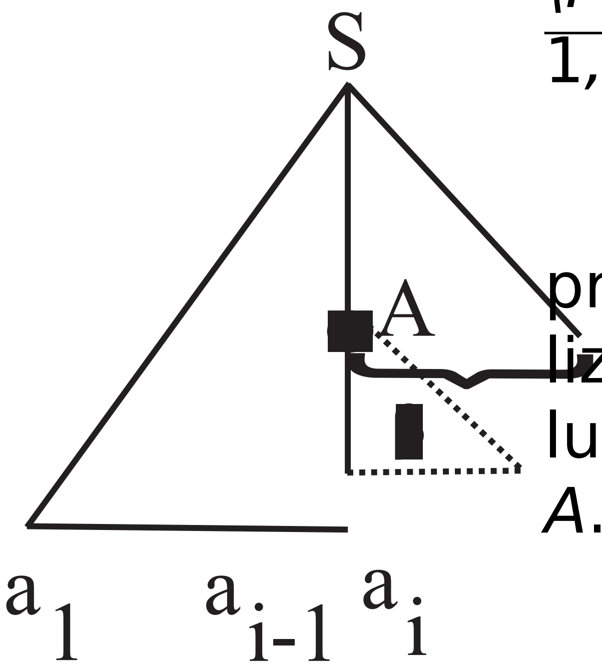


Figura 3.2: Construc¸tia arborelui prin analiza sintactic˘a LL(1)

Follow(A)
S=>* xBy => xaAy
What if B->uA

Pentru a construi un analizor sintactic LL(1) avem n care pot ap˘area pe parcursul analizei ¸si care se men numit ***tabela de analiz˘a LL(1)***.

3.2.1. Gramatici de tip LL(k)

Defini¸tia 3.1. *[AU73]O gramatic˘a G = (N, ß, P, S) este de tip LL(k)*

Algorithm FOLLOW

INPUT: G, FIRST(X), $\forall X \in N \cup \Sigma$

OUTPUT: FOLLOW(A), $\forall A \in N$

```
for $A \in N - \{S\}$ do                                {init}
        $L_0(A) = \Phi$;
endFor;
$L_0(S) = \{\varepsilon\}$;                                {init}
i = 0;
repeat
   i = i+1;
   for $B \in N$ do
        for $A \to \alpha B y \in P$ do
           for $\forall a \in FIRST(y)$ do
                if $a = \varepsilon$ then $F_i(B) = F_i(B) \cup F_{i-1}(A)$
                        else $F_i(B) = F_{i-1}(B) \cup First(y)$
                endif
           endFor
        endFor
   endfor
until $Fi(X) = Fi-1(X), \forall X \in N$
$FOLLOW(X) = Fi(X), \forall X \in N$
```

$S \Rightarrow^0 S$ // ε after S

S => aAc=> abBc
A -> bB

FIRST

- ≈ first terminal symbols that can be generated from α

FOLLOW

- ≈ next symbol generated after/ following A

LL(k)

- L = left (sequence is read from left to right)
- L = left (use leftmost derivation)
- Prediction of length **k**

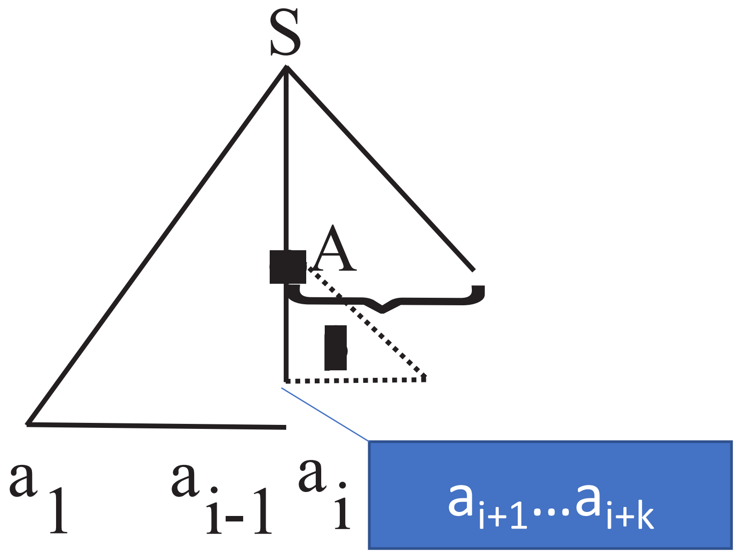


Figura 3.2:Constructia arborelui prin analiza sintactic˘a LL(1)

LL(k) Principle

- In any moment of parsing, action is uniquely determinde by:
- Closed part ($a_1…a_i$)
- Current symbol A
- Prediction $a_{i+1}…a_{i+k}$ (length k)

Definition

3.2.1. Gramatici de tip LL(k)

Definiţia 3.1. *[AU73]O gramatic˘a G =* (*N*, *ß*, *P*, *S*) *este de tip LL(k*

dac˘a pentru oricare dou˘a deriv˘ari de stˆanga:

- *A cfg is LL(k) if for any 2 leftmost derivation we have:*

1. $S \,)^{§}_{st}\, wAÆ \,)_{st}\, wØÆ \,)^{§}_{st}\, wx;$

2. $S \,)^{§}_{st}\, wAÆ \,)_{st}\, w∞Æ \,)^{§}_{st}\, wy;$

such that ˆ*at* $F\,IRST_k(x) = F\,IRST_k(y)$ *then* ´*Ø* = ∞.

Definiţia poate fi reformulat˘a astfel: pentru orice form˘a propoziţional˘a *wAÆ*, primele *k* simboluri derivabile din *AÆ* definesc ˆın mod unic ţie care se poate aplica lui *A* pentru a obţine o derivare a unui cuvˆan secvenţ˘a de simboluri terminale) care ˆıncepe cu *w* ¸si se continu simboluri. Aceast˘a condiţie este uneori dificil de verificat ¸si ˆın majo

Theorem

The necessary and sufficient condition for a grammar to be LL (
that for any pair of distinct productions of a nonterminal (A→ β,
A→ γ, $\beta \neq \gamma$*) the condition holds:*

$$\text{FIRST}_k(\beta\alpha) \cap \text{FIRST}_k(\gamma\alpha) = \Phi, \forall\alpha \quad \text{such that} \quad S \overset{*}{=>} uA\alpha$$

Theorem: A grammar is LL(1) if and only if for any nonterminal A with productions A →α_1| α_2|…| α_n , FIRST(α_i) ∩ FIRST(α_j) = Ø and if $\alpha_i \Rightarrow \varepsilon$, FIRST($\alpha_i$) ∩ FOLLOW(A)= Ø, ∀i,j = 1,n,i≠j

LL(1) Parser

- Prediction of length 1

- Steps:
  1) construct FIRST, FOLLOW
  2) Construct LL(1) parse table
  3) Analyse sequence based on moves between configurations

Executed 1 time

Step 2: Construct LL(1) parse table

- Possible action depend on:
  - Current symbol ∈ **N**∪**Σ**
  - Possible prediction ∈ **Σ**
- Add a special character "$" (∉ **N**∪**Σ**) – marking for "empty stack"

= > table:

- One line for each symbol ∈ **N**∪**Σ** ∪{$}
- One column for each symbol ∈ **Σ** ∪{$}

Rules LL(1) table

pentru fiecare pre¸dic posibil În plus, se adaug˘a un caracter special, de
obicei notat '$' 2(N [ß), al c˘arui scop este s˘a marcheze sfˆar¸situl secvent
¸si c˘aruia i se aloc˘a o linie ¸si o coloan˘a în tabel. Efectul acestui simbol ˆın
faza de analiz˘a propriu-zis˘a este de a elimina verific˘arile de stiv˘a goal
Regulile de completare a tabelului sunt:

1. *M* (*A, a*) = (*Æ, i*), *8a 2 F IRST* (*Æ*), *a 6=* ≤, *A* production in P u with number i

   M (*A, b*) = (*Æ, i* if a ≤ *2 F IRST* (*Æ*), *8b 2 F OLLOW* (*A*), *A ! Æ* production in P with number i ul i;

57

2. *M* (*a, a*) = *pop, 8a 2* ,
3. *M* ($, $) = *acc*;
4. M(x,a)=err (error) otherwise cazuri.

Pentru gramatica din exemplul precedent, construc¸tia tabelului de ana-
liz˘a LL(1) necesit˘a ¸si calculul mul¸timilor *F OLLOW* pentru neterminalele
A ¸si *C*, deoarece ≤ *2 F IRST* (*A*) ¸si ≤ *2 F IRST* (*C*). Aplicarea algoritmului

Remark

A grammar is LL(1) if the LL(1) parse table does NOT contain conflicts – there exists at most one value in each cell of the table M(A,a)

Step 3: Definire configurations and moves

- INPUT:
  - Language grammar G = (N, **Σ**, P,S)
  - LL(1) parse table
  - Sequence to be parsed w =a_1…a_n
- OUTPUT:

  If (w ∈L(G)) *then* **string of productions**

  else **error** & **location of error**

LL(1) configurations

(α, β, π)

where:

- α = input stack
- β = working stack
- π = output (result)

Initial configuration:
$(w\$,S\$,\varepsilon)$

Final configuration:
$(\$, \$, \pi)$

producţiile ... se face pe baza criteriului stivei vide.

Tranziţiile se definesc în felul următor:

1. **push** - operaţia de punere în stivă:
   $(ux, A\AE\$, \underline{o}) \vdash (ux, \O\AE\$, \underline{o}i)$, dacă $M(A, u) = (\O, i)$;
   de fapt, în stiva de lucru se efectuează următoarele operaţii: se scoate A din stivă şi se pune $\O$ în stivă;
2. **pop** - operaţia de scoatere din stivă, se elimină vârfurile ambelor stive (dacă ele coincid):
   $(ux, a\AE\$, \underline{o}) \vdash (x, \AE\$, \underline{o})$, dacă M(a,u)=pop ;
3. tranziţia de acceptare, dacă s-a obţinut configuraţia finală, notată **acc**:
   $(\$, \$, \underline{o}) \vdash acc$;
4. în celelalte cazuri eroare, notată **err**:
   $(n\AE, x\O\$, \underline{o}) \vdash err$.

Corespunzător tranziţiilor de mai sus, analiza sintactică LL(1) se face conform algoritmului 3.5.

Moves

1. Push - put in stack
2. Pop - pop from stacks (from both stacks)
3. Accept
4. Error - otherwise

Algorithm LL(1) parsing

- INPUT:
  - LL(1) table with NO conflicts;
  - G –grammar (productions)
  - Input sequence w = $a_1a_2 \ldots a_n$
- OUTPUT:
  - sequence accepted or not?
  - If yes then string of productions

Algorithm LL(1) parsing (cont)

```
alpha := w$;beta := S$;pi := ε; config =(alpha,beta, pi)
go := true;
while go do
    if M(head(beta),head(alfa))=(b,i) then
            ActionPush(config)
    else
        if M(head(beta),head(alfa))=pop then
            ActionPop(config)
        else
            if M(head(beta),head(alfa))=acc then
                go:=false; s:="acc";
            else go:=false; s:="err";
            end if
        end if
    end if
end while
```

```
if s="'acc"' then
        write("Sequence accepted");
        write(pi)
    else
        write(" Sequence not accepted")
```

Remarks

1) LL(1) parser provides location of the error

2) Grammars can be transformed to be LL(1)

   example:

   I -> if C then S | if C then S else S // is not LL(1)

   I -> if C then S T

   T -> ε | else S // is LL(1)

Play time!!!

- Menti.com cod: 42 60 49

Curs 8

LR(k) parsing

Terms

Reminder:
rhp = right handside of production
lhp = left handside of production

- Prediction – see LL(1)
- Handle = symbols from the head of the working stack that form (in order) a rhp

- ***Shift – reduce*** parser:
  - **shift** symbols to form a handle
  - When a rhp is formed – **reduce** to the corresponding lhp

LR(k)

- L = left – sequence is read from left to right
- R = right – use rightmost derivations
- k = length of prediction

- <u>Enhanced grammar</u>

- G = (N, Σ,P,S)
- G' =(N ∪ {S'},Σ,P ∪ {S' → S},S'), S'∉ N

S' does NOT appear in any rhp

LR(k)

- Ascendent

- Linear – COST? – what we compute to obtain linear algorithm?

- **Definition 1**: If in a cfg $G = (N, \Sigma, P, S)$ we have

  $S \overset{*}{\Rightarrow}_r \alpha A w \Rightarrow_r \alpha\beta w$, where $\alpha \in (N \cup \Sigma)^*, A \in N, w \in \Sigma^*$, then

  any prefix of sequence $\alpha\beta$ is called ***live prefix*** in G.

- **Definition 2**: ***LR(k) item*** is defined as $[A \to \alpha.\beta,u]$, where $A \to \alpha\beta$ is a production, $u \in \Sigma^k$ and describe the moment in which, considering the production $A \to \alpha\beta$, α was detected (α is in head of stack) and it is expected to detect β.

- **Definition 3**: LR(k) item $[A \to \alpha.\beta,u]$ is ***valid for the live prefix*** $\gamma\alpha$ if:

  $S \overset{*}{\Rightarrow}_r \gamma A w \Rightarrow_r \gamma\alpha\beta w$

  $u = FIRST_k(w)$

Definition 4: A cfg G = (N, Σ, P, S) is LR(k), for k>=0, if

$$\left.\begin{array}{ll} 1. & S' \overset{*}{\Rightarrow}_r \alpha A w \Rightarrow_r \alpha\beta w \\[2ex] 2. & S' \overset{*}{\Rightarrow}_r \gamma B x \Rightarrow_r \alpha\beta y \\[2ex] 3. & FIRST_k(w) = FIRST_k(y) \end{array}\right\} \quad => \ \alpha = \gamma \ \textit{AND} \ A = B \ \textit{AND} \ x = y$$

- $[A \rightarrow \alpha\beta.,u]$ – special case: prefix is all rhp - apply reduce

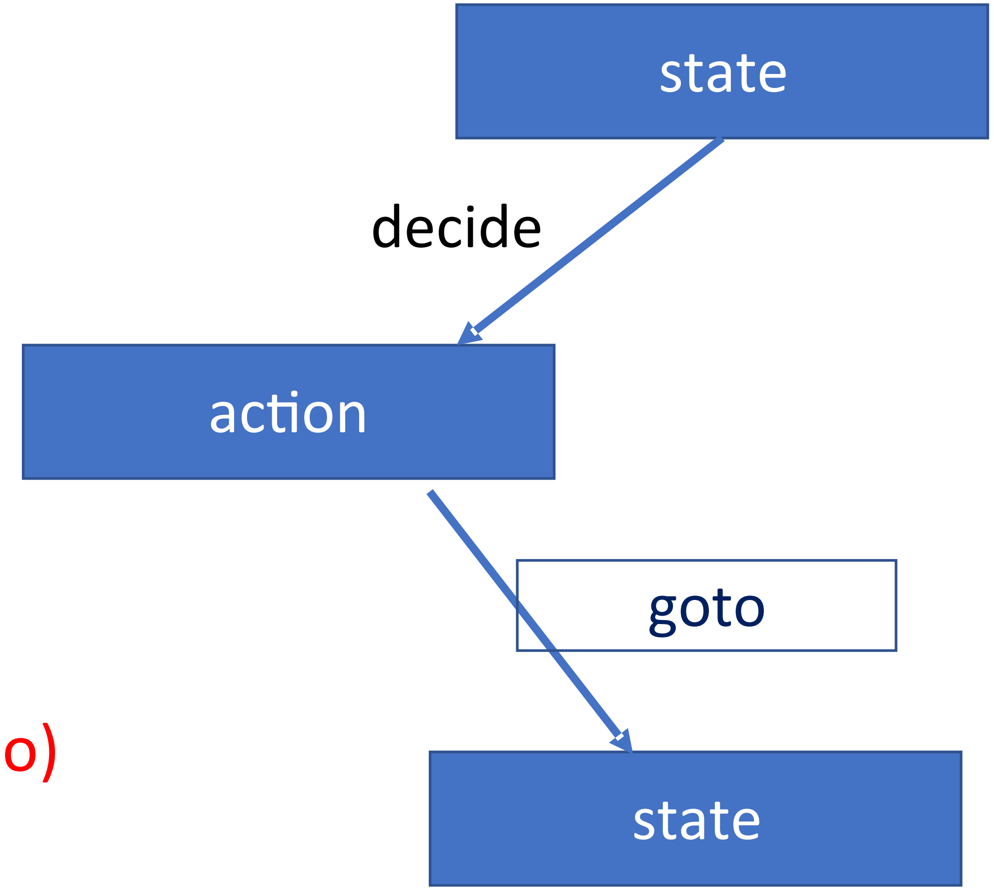
- Otherwise $[A \rightarrow \alpha.\beta,u]$ – apply shift

Consequence 1: state is important –
should be stored by parsing method

⇒ Working stack:
$\$s_{init}X_1s_1 \ldots X_m s_m$

where: \$ - mark empty stack
$X_i \in N \cup \Sigma$
s_i - states

Consequence 2: the action takes the
parsing process to another state (goto)



LR(k) principle

- Current state
- Current symbol
- prediction

<u>uniquely</u> determines:

- Action to be applied
- Move to a new state

=> LR(k) table – 2 parts: **action** part + **goto** part

States

What a state contains?

- LR items – all items corresponding to same live prefix
- *closure*

How to go from one state to another state? How many states?

- *goto*
- *Canonical collection*

What LR item will be in the same state?

- [A → α.Bβ,u] valid for live prefix γα
- B → δ(P) => S =>* γα Bβw =>dr γαδβw

=> [B → .δ,u] valid for live prefix γα

- tranziţia în alt˘a stare.

De aceea tabelele de analiz˘a LR(k) au dou˘a compone

- acţiunea care se va efectua ¸si

de deplasare, numit˘a ”goto“.

Care sunt ¸si cum se determin˘a aceste st˘ari? Pentru a r˘asp

consider˘am elementul de analiz˘a [*A* ! *Æ*.*BØ*, *u*] care conform d

implic˘a:

S)§dr ∞*Aw*)dr ∞*ÆBØw* ¸si

u = *F IRST*k(*w*) valabil pentru prefixul viabil ∞*Æ*.

Dac˘a ˆın gramatic˘a exist˘a o producţie *B* ! ± atunci elemen

[*B* ! .±, *u*] are, de asemenea, *u* valid pentru prefixul vi

LR(k) parsing:
LR(0), SLR, LR(1), LALR

- Define item
- Construct set of states
- Construct table

Executed 1 time

- Parse sequence based on moves between configurations

LR(0) Parser

- Prediction of length 0 (ignored)

1. LR(0) item: $[A \to \alpha.\beta]$

2. Construct set of states

- What a state contains – Algorithm *closure_LR(0)*
- How to move from a state to another – Function *goto_LR(0)*
- Construct set of states – Algorithm *ColCan_LR(0)*

Canonical collection

2. closure(I) = I [{[B ! .∞]|[A ! Æ.BØ] 2 I}, conform observaţiei din paragraful anterior.

Algorithm *Closure_LR(0)*

Algoritmul 3.8 ClosureLR0

```
INPUT: I-element de analizˇa; G'- gramatica ˆımbogˇat
OUTPUT: C = closure(I);
C := {I};
repeat
  for 8[A ! Æ.BØ] 2 C do
    for 8B ! ∞ 2 P do
      if [B ! .∞] 62 C then
        C = C [ [B ! .∞]
      end if
    end for
  end for
until C nu se mai modificˇa
```

Pentru a determina stˇarile ¸si cum se deplaseazˇa automatul din

Function *goto_LR(0)*

goto : $P(\mathcal{E}_0) \times (N \cup \Sigma) \rightarrow P(\mathcal{E}_0)$

where $\mathcal{E}_0$ = set of LR(0) items

goto(s, X) = closure($\{[A \rightarrow \alpha X.\beta] \mid [A \rightarrow \alpha.X\beta] \in s\}$)

Algorithm *ColCan_LR(0)*

Algoritmul 3.9 Col stariLR0

```
INPUT: G'- gramatica ˆımbog˛ăţat
OUTPUT: C - colec̦tia canonicˇa de stˇari
$C := ;;$
$s_0 := closure(\{[S^0! .S]\})$
$C := C [ \{s_0\};$
repeat
   for $8s 2 C$ do
      for $8X 2 N [ ß$ do
         if $goto(s, X)=6 ;$ and $goto(s, X)2/C$ then
            $C = C [ goto(s, X)$
         end if
      end for
   end for
until C nu se mai modificˇa
```

S-> aS|bSc|dA
A -> dc

Goto(s0,S)
Goto(s0,A)
Goto(s0,a)
Goto(s0,b)
Goto(s0,c) =∅
Goto(so,d)

A ! c

3. Construct LR(0) table

- one line for each state

- 2 parts:
  - Action: one column (for a state, action is unique because prediction is ignored)
  - Goto: one column for each symbol X ∈ N ∪ Σ

Rules LR(0) table

1. *if* $[A \to \alpha.\beta] \in s_i$ *then* **action(s_i)=shift**
2. *if* $[A \to \beta.] \in s_i$ and $A \neq S'$ *then* **action(s_i)=reduce l**, where l = number of production $A \to \beta$
3. *if* $[S' \to S.] \in s_i$ *then* **action(s_i)=acc**
4. *if* goto(s_i, X) = s_j *then* **goto(s_i, X) = s_j**
5. otherwise = **error**

Remarks

1) Initial state of parser = state containing [S’ → .S]
2) No shift from accept state:

   if s is accept state *then* goto(s, X) = Ø, ∀X ∈ N ∪ Σ.

3) *If* in state ***s*** action is reduce *then* goto(s, X) = Ø, ∀X ∈ N ∪ Σ.
4) Argument G’: Let G = ({S},{a,b,c},{S → aSbS,S → c},S)

   states [S → aSbS.] and [S → c.] – accept / reduce ?

Remarks (cont)

5) A grammar is NOT LR(0) if the LR(0) table contains conflicts:

- shift – reduce conflict: a state contains items of the form [A → α.β] and [B → γ.], yielding to 2 distinct actions for that state

- reduce – reduce conflict: when a state contains items of the form [A → αβ.] and [B → γ.], in which the action is reduce, but with distinct productions

4. Define configurations and moves

- INPUT:
  - Grammar G' = (NU{S'}, **Σ**, P U {S'->S},S')
  - LR(0) table
  - Input sequence w =$a_1 \ldots a_n$
- OUTPUT:

if (w ∈L(G)) *then* **string of productions**
else **error & location of error**

LR(0) configurations

(α, β, π)

where:

- α = working stack
- β = input stack
- π = output (result) stack

Initial configuration:
($\$s_0$,w\$,ε)

Final configuration:
($\$s_{acc}$, \$, π)

Moves

1. **Shift**

if action(s_m)= shift AND head(β)=a_i AND goto(s_m,a_i)=S_j **then**

$(\$s_0x_1 \ldots x_m s_m, a_i \ldots a_n\$, \pi) \vdash (\$s_0x_1 \ldots x_m s_m a_i s_j, a_{i+1} \ldots a_n\$, \pi)$

2. **Reduce**

if action(s_m) = reduce l AND (l) A → $x_{m-p+1} \ldots x_m$ AND goto(s_{m-p},A) = s_j **then**

$(\$s_0 \ldots x_m s_m, a_i \ldots a_n\$, \pi) \vdash (\$s_0 \ldots x_{m-p} s_{m-p} A s_j, a_i \ldots a_n\$, l\pi)$

3. **Accept**

if action(s_m) = accept **then** ($\$s_m$,\$, π)=acc

4. **Error** - otherwise

LR(0) Parsing Algorithm

INPUT:

- LR(0) table – conflict free
- grammar G’: production numbered
- - sequence = Input sequence w =$a_1 … a_n$

- OUTPUT:

if (w ∈L(G)) *then* **string of productions**

else **error & location of error**

LR(0) Parsing Algorithm

```
state :=0;
alpha := ‘$s0’; beta :=‘w$’; phi := “; end:= false
Config := (alpha,beta,phi);
Repeat
      if action(state)=‘shift’ then
              ActionShift(config)
      else
           if action(state) =’reduce l” then
              ActionReduce(config)
           else
              if action(state)=‘accept’ then
                 write(” success”,); write(phi);
                 end := true;
              if action(state) = ‘error’ then
                 write(” error”)
                end := true
Until end
```

Course 9

LR(k) Parsing (cont.)

LR(k) parsing:
LR(0), SLR, LR(1), LALR

- Define item
- Construct set of states
- Construct table

Executed 1 time

- Parse sequence based on moves between configurations

Algorithm *ColCan_LR(0)*

Algoritmul 3.9 Col stariLR0

```
INPUT: G'- gramatica ˆımbog˘at¸it˘a
OUTPUT: C - colecţia canonic˘a de st˘ari
C := ;;
$s_0$ := closure({[S0! .S]})          // state corresponding to prod. of S' = initial state
C := C [ {$s_0$};                      //initialize collection with $s_0$
repeat
  for 8s 2 C do
    for 8X 2 N [ ß do
      if goto(s, X)6=; and goto(s, X)2/C then
        C = C [ goto(s, X)             //add new state
      end if
    end for
  end for
until C nu se mai modific˘a
```

A ! c

2. closure(I) = I [{[B ! .±]|[A ! Æ.BØ] 2 I}, conform observa¸tiei
din paragraful anterior.

Algorithm *Closure*

I = LR(0) item of the form [A->$\alpha.\beta$]

Algoritmul 3.8 ClosureLR0

```
INPUT: I-element de analiz˘a; G'- gramatica ˆımbog˘a¸tit
OUTPUT: C = closure(I);
C := {I};            //initialize Closure with the LR(0) item
repeat
   for 8[A ! Æ.BØ] 2 C do    //search productions with dot in front of nonterminal
      for 8B ! ∞ 2 P do      //search productions of that nonterminal
         if [B ! .∞] 62 C then
            C = C [ [B ! .∞]    //adds item formed from production with dot in
         end if                 //front of right hand side of the production
      end for
   end for
until C nu se mai modific˘a
```

Pentru a determina st˘arile ¸si cum se deplaseaz˘a automatul din

Function *goto*

goto : $P(\mathcal{E}_0) \times (N \cup \Sigma) \rightarrow P(\mathcal{E}_0)$ //creates new states

where $\mathcal{E}_0$ = set of LR(0) items

$$goto(s, X) = closure(\{[A \rightarrow \alpha X.\beta] \mid [A \rightarrow \alpha.X\beta] \in s\})$$

goto(s,X): in state **s**, search LR(0) item that has dot in front of symbol **X**. Move the dot after symbol **X** and call closure for this new item.

SLR Parser

Prediction = next symbols on input sequence

- SLR = Simple LR

- Remark:

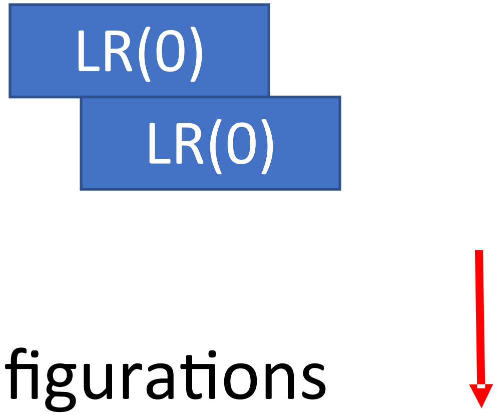
  LR(0) – lots of conflicts – solved if considering prediction

=>

1. LR(0) canonical collection of states– prediction of length 0
2. Table and parsing sequence – prediction of length 1

SLR Parsing:

- define item
- Construct set of states
- Construct table
- Parse sequence based on moves between configurations



LR(0)

LR(0)

Construct SLR table

Remarks:

1. Prediction = next symbol from input sequence => FOLLOW

   - see LL(1)

2. Structure – LR(k):
   - Lines - states
   - action + goto

action – a column for each prediction ∈ **Σ**

goto – a column for each symbol X ∈N∪**Σ**

Optimize table structure: merge *action* and *goto* columns for Σ

Remark (LR(0) table):

- *if* s is accept state *then* goto(s, X) = Ø, ∀X ∈ N ∪ Σ.
- *If* in state ***s*** action is reduce *then* goto(s, X) = Ø, ∀X ∈ N ∪ Σ.

SLR table

And goto

	Action		GOTO	
	a_1 …	a_n	B_1 …	B_m
s_0				
s_1				
…				
s_k				

$a_1,\ldots,a_n \in \Sigma$

$B_1,\ldots,B_m \in N$

$s_0,\ldots,s_k$ - states

Rules for SLR table

1. If $[A \to \alpha.\beta] \in s_i$ and $goto(s_i,a) = s_j$ then **action(s_i,a)**=**shift s_j**

   // dot is not at the end

2. if $[A \to \beta.] \in s_i$ and $A \neq S'$ then **action(s_i,u)**=**reduce l**, where l – number of production $A \to \beta$, $\forall u \in FOLLOW(A)$

   //dot is at the end, but not for S'

3. if $[S' \to S.] \in s_i$ then **action(s_i,\$)**=**acc**

   // dot is at the end, prod. of S'

4. if $goto(s_i, X) = s_j$ then **goto(s_i, X) = s_j** , $\forall X \in N$

5. otherwise **error**

Remarks

1. Similarity with LR(0)

2. A grammar is SLR if the SLR table does not contain conflicts (more than one value in a cell)

Parsing sequences

- INPUT:
  - Grammar G' = (NU{S'}, **Σ**, P U {S'->S},S')
  - SLR table
  - Input sequence w =$a_1 \ldots a_n$
- OUTPUT:

  if (w ∈L(G)) *then* **string of productions**

  else **error & location of error**

SLR = LR(0) configurations

(α, β, π)

where:

- α = working stack
- β = input stack
- π = output (result)

Initial configuration:
($\$s_0$,w\$,ε)

Final configuration:
($\$s_{acc}$, \$, π)

Moves

head(β) = prediction

1. **Shift**

if action(s_m,a_i)= shift s_j **then**

$(\$s_0x_1 \ldots x_m s_m, a_i \ldots a_n\$, \pi) \vdash (\$s_0x_1 \ldots x_m s_m a_i s_j, a_{i+1} \ldots a_n\$, \pi)$

2. **Reduce**

if action(s_m,a_i) = reduce t AND (t) $A \rightarrow x_{m-p+1} \ldots x_m$ AND goto(s_{m-p},A) = s_j

then

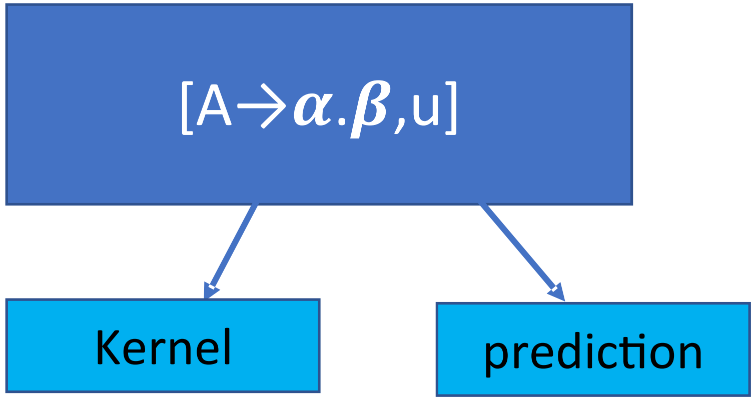
$(\$s_0 \ldots x_m s_m, a_i \ldots a_n\$, \pi) \vdash (\$s_0 \ldots x_{m-p} s_{m-p} A s_j, a_i \ldots a_n\$, t\ \pi)$

3. **Accept**

if action(s_m,\$) = accept **then** **(**$\$s_m,\$, \pi$)=acc

4. **Error** - otherwise

LR(1) Parser



1. Define item
2. Construct set of states
3. Construct table
4. Parse sequence based on moves between configurations

Construct LR(1) set of states

- Alg *ColCan_LR1*
- Function *goto_LR1*
- Alg *Closure_LR1*

Algorithm *ColCan_LR1*

```
INPUT: G' – enhanced grammar
OUTPUT: $\mathbb{C}_1$ – cannonical collection of states
$\mathbb{C}_1 = \emptyset$
S0 = Closure_LR1({[S'→.S,$]})
$\mathbb{C}_1 := \mathbb{C}_1 \cup \{s_0\}$
Repeat
    for $\forall s \in \mathbb{C}_1$ do
        for $\forall X \in N \cup \Sigma$ do
              T = goto_LR1(s,X)
              if $T \neq \emptyset$ and $T \notin \mathbb{C}_1$ then
                        $\mathbb{C}_1 = \mathbb{C}_1 \cup T$
              endif
        endfor
    endfor
Until $\mathbb{C}_1$ unchanged
```

Function *goto_LR1*

Goto_LR1 : $P(\mathcal{E}_1) \times (N \cup \Sigma) \to P(\mathcal{E}_1)$

where $\mathcal{E}_1$ = set ofLR(1) items

Goto_LR1(s, X) = *Closure_LR1*$(\{[A \to \alpha X.\beta,u] \mid [A \to \alpha.X\beta,u] \in s\})$

• tranziţia în alt˘a stare.

De aceea tabelele de analiz˘a LR(k) au dou˘a compone

De aceea tabelele de analiz˘a LR(k) au dou˘a compone

Algorithm *Closure_LR1*

de deplasare, numit˘a "goto".

de deplasare, numit˘a "goto".

Care sunt ¸si cum se determin˘a aceste st˘ari? Pentru a r˘asp

Care sunt ¸si cum se determin˘a aceste st˘ari? Pentru a r˘asp

consider˘am elementul de analiz˘a [*A ! Æ.BØ, u*] care, conform d

consider˘am elementul de analiz˘a [*A ! Æ.BØ, u*] care, conform d

• [A → α.Bβ,u] valid for live prefix γα =>

implic˘a:

implic˘a:

S)§ dr ∞Aw) dr ∞ÆBØw ¸si

S)§ dr ∞Aw) dr ∞ÆBØw ¸si

u = FIRST_k(w) valabil pentru prefixul viabil ∞Æ.

u = FIRST_k(w) valabil pentru prefixul viabil ∞Æ.

Dac˘a în gramatic˘a exist˘a o produc¸tie *B ! ±* atunci elementul

Dac˘a în gramatic˘a exist˘a o produc¸tie *B ! ±* atunci elementul

[*B ! .±, u*] are, de asemenea, *u* valid pentru prefixul viabil

[*B ! .±, u*] are, de asemenea, *u* valid pentru prefixul viabil

• [B → .δ, *smth*] ∈ P => $S \Rightarrow^* \gamma A w \Rightarrow_{dr} \gamma\alpha B\beta w \Rightarrow_{dr} \gamma\alpha\delta\beta w$.

Aceast˘a observa¸tie sugereaz˘a faptul c˘a el

punz˘atoare unui acela¸si prefix viabil ar

=> [B → .δ,b] valid for live prefix γα,

aceast˘a mul¸time caracterizeaz˘a un pas al analiz

∀b ∈ FIRST(βu) // First(βw) = First(βu)

Mul¸timea care va con¸tine toate elementele c

prefix viabil va forma o ***stare*** a automatu

Starea ini¸tial˘a este cea care corespunde

8[*A ! Æ.BØ, a*] *2 closure*(*C*), *8B ! ± 2 P*, [*B ! .±, b*] *2 closure*(*C*)
pentru *8b 2 F IRST* (*Øa*)

Algorithm *Closure_LR1*

Algoritmul 3.11 ClosureLR1

```
INPUT: I-element de analiz˘a; G'- gramatica ˆımbog˘at¸it˘a;
          F IRST (X), 8X 2 N [ ß;
OUTPUT: C 1 = closure(I);
C1 := {I};
repeat
  for 8[A ! Æ.BØ, a] 2 C1 do
    for 8B ! ∞ 2 P do
      for 8b 2 F IRST (Øa) do
        if [B ! .∞, b]2 C1 then
          C1 = C1 [ [B ! .∞, b]
        end if
      end for
    end for
  end for
until C1 nu se mai modific˘a
```

Defini¸tia func¸tiei *goto* se actualizeaz˘a ˆın:

goto(*s, X*) = *closure*({[*A ! ÆX.Ø, a*]|[*A ! Æ.XØ, a*] *2 s*})

Construct LR(1) table

- Structure – SLR
- Rules:

1. if $[A \rightarrow \alpha.\beta,u] \in s_i$ and $\text{goto}(s_i,a) = s_j$ then **action(s_i,a)**=**shift s_j**
2. if $[A \rightarrow \beta.,u] \in s_i$ and $A \neq S'$ then **action(s_i,u)**=**reduce l**, where l – number of production $A \rightarrow \beta$
3. if $[S' \rightarrow S.,\$] \in s_i$ then **action(s_i,\$)**=**acc**
4. if $\text{goto}(s_i, X) = s_j$ then **goto(s_i, X) = s_j** , $\forall X \in N$
5. otherwise = **error**

Remarks

1. A grammar is LR(1) if the LR(1) table does not contain conflicts

2. Number of states – significantly increase

4. Define configurations and moves

- INPUT:
  - Grammar G' = (NU{S'}, **Σ**, P U {S'->S},S')
  - LR(1) table
  - Input sequence w =$a_1…a_n$
- OUTPUT:

  if (w ∈L(G))   *then* **string of productions**

  else **error & location of error**

LR(1) configurations

(α, β, π)

where:

- α = working stack
- β = input stack
- π = output (result)

Initial configuration:
$(\$s_0, w\$, \varepsilon)$

Final configuration:
$(\$s_{acc}, \$, \pi)$

Moves

head(β) = prediction

1. **Shift**

if action(s_m,a_i)= shift s_j **then**

$(\$s_0x_1 \ldots x_m s_m, a_i \ldots a_n\$, \pi) \vdash (\$s_0x_1 \ldots x_m s_m a_i s_j, a_{i+1} \ldots a_n\$, \pi)$

2. **Reduce**

if action(s_m,a_i) = reduce t AND (t) $A \rightarrow x_{m-p+1} \ldots x_m$ AND goto(s_{m-p},A) = s_j

then

$(\$s_0 \ldots x_m s_m, a_i \ldots a_n\$, \pi) \vdash (\$s_0 \ldots x_{m-p} s_{m-p} A s_j, a_i \ldots a_n\$, t\,\pi)$

3. **Accept**

if action(s_m,\$) = accept **then** ($\$s_m,\$, \pi$)=acc

4. **Error** - otherwise

LALR Parser

- LALR = Look Ahead LR(1)

- why?

LALR principle

$[A \rightarrow \alpha\beta.,u] \in s_i$ apply reduce (k) then goto(s_i,A) =s_m
$[A \rightarrow \alpha\beta.,v] \in s_j$ apply reduce (k) then goto(s_j,A) =s_n

$[A \rightarrow \alpha.\beta,u] \in s_i$

=> $[A \rightarrow \alpha.\beta,u|v] \in s_{i,j}$

$[A \rightarrow \alpha.\beta,v] \in s_j$

- Merge states with the same kernel, conserving all predictions, if **no conflict** is created

LALR Parsing

- Same as LR(1)
- Number of LALR states = number of SLR / LR(0) states

- How? - LR(1) states

LR(k) Parsers

- LR(0):
  - Items ignore prediction
  - Reduce can be applied only in singular states (contain one item)
  - Lot of conflicts
- SLR:
  - Use same items as LR(0)
  - When reduce consider prediction
  - Eliminate several LR(0) conflicts (not all)
- LR(1):
  - Performant algorithm for set of states
  - Generate few conflicts
  - Generate lot of states
- LALR:
  - Merge LR(1) states ccorresponding to same kernel
  - Most used algorithm (most performant)

Quiz time

Parsing - recap

	Descendent	Ascendent
Recursive	Descendent recursive parser	Ascendent recursive parser
Linear	LL(1)	LR(0), SLR, LR(1), LALR

Parsing – recap

Eliminarea conflictelor nu este ˆıntotdeauna u¸sor de realizat ¸si de aceea se dore¸ste evitarea lor. Cea mai pu¸tin restrictiv˘a clas˘a este cea a gramaticilor LR(1), dar analizorul sintactic are alte dezavantaje, asupra c˘arora vom reveni. Figura 3.4 ilustreaz˘a incluziunea dintre tipurile de gramatici luate ˆın considerare ˆın analiza sintactic˘a. Se observ˘a c˘a nu exist˘a o corela¸tie evident˘a ˆıntre gramaticile LL(1) ¸si gramaticile LR(k), o gramatic˘a LL(1) poate s˘a fie LR(1), LALR, SLR sau chiar LR(0), dar orice gramatic˘a LL(1) este LR(1).

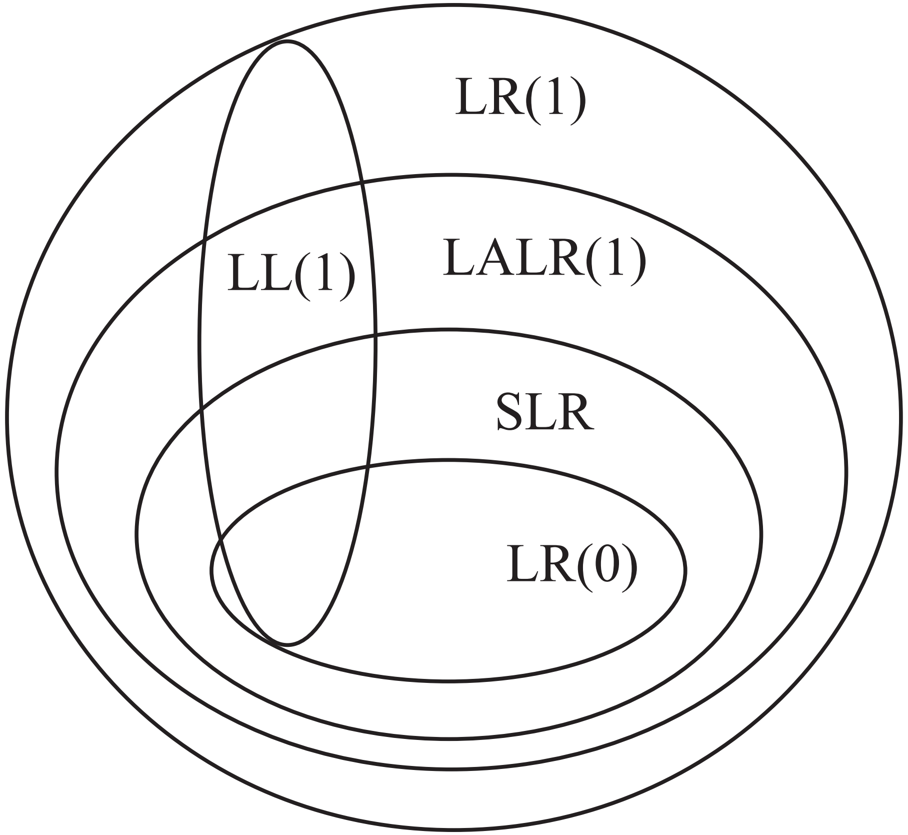
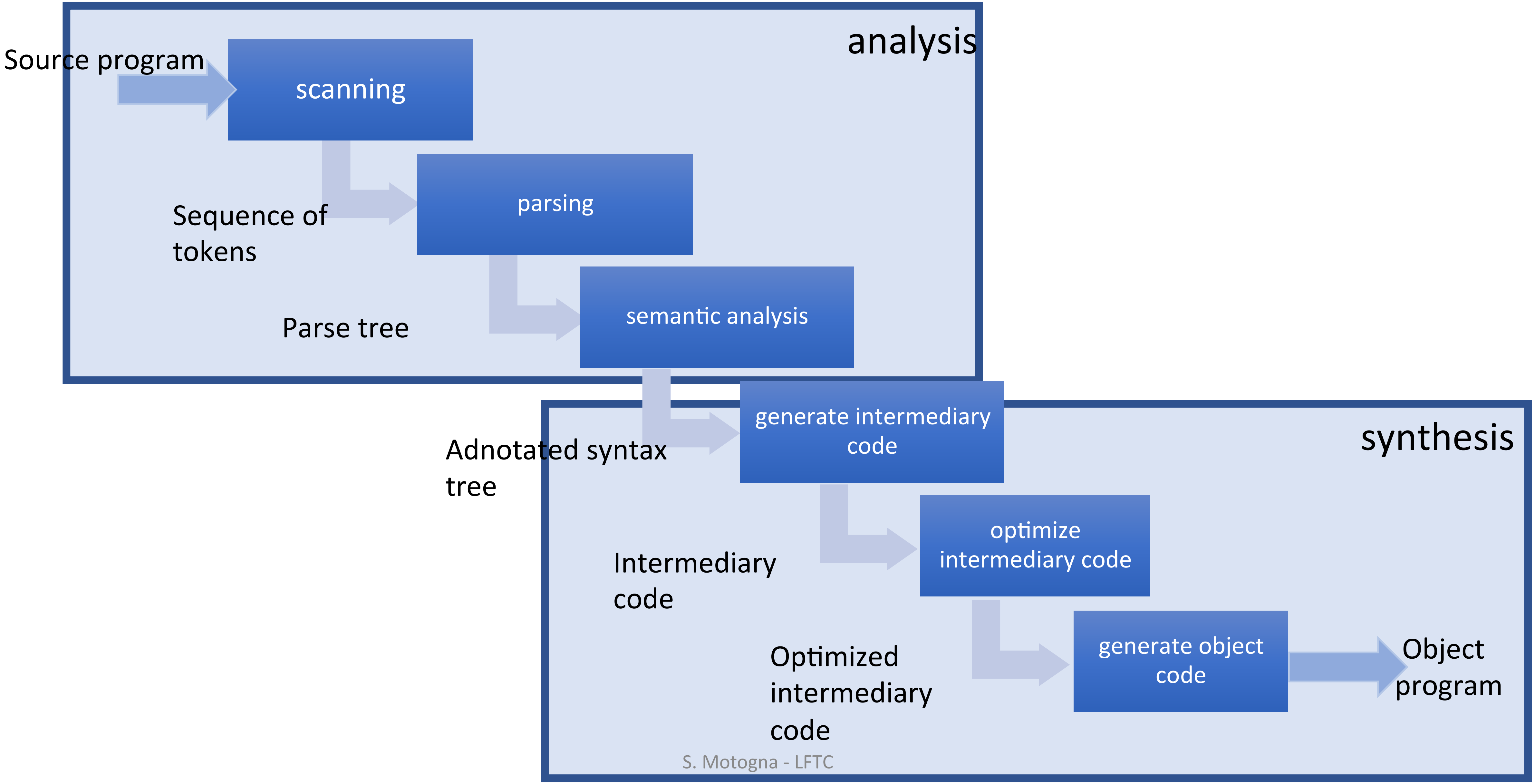


Figura 3.5: Rela¸tia dintre diferite clase de gramatici ˆın func¸tie de metoda de analiz˘a sintactic˘a

Structure of compiler



analysis

Source program

scanning

Sequence of tokens

parsing

Parse tree

semantic analysis

Adnotated syntax tree

generate intermediary code

synthesis

Intermediary code

optimize intermediary code

Optimized intermediary code

generate object code

Object program

Course 10

Important notice

Ø9.12.2021

7.30 - Course Formal Languages and Compiler Design

9.20 - Course Formal Languages and Compiler Design

Ø16.12.2021

7.30 – Course Parallel and Distributed Programming

9.20 – Course Parallel and Distributed Programming

LEX & YACC

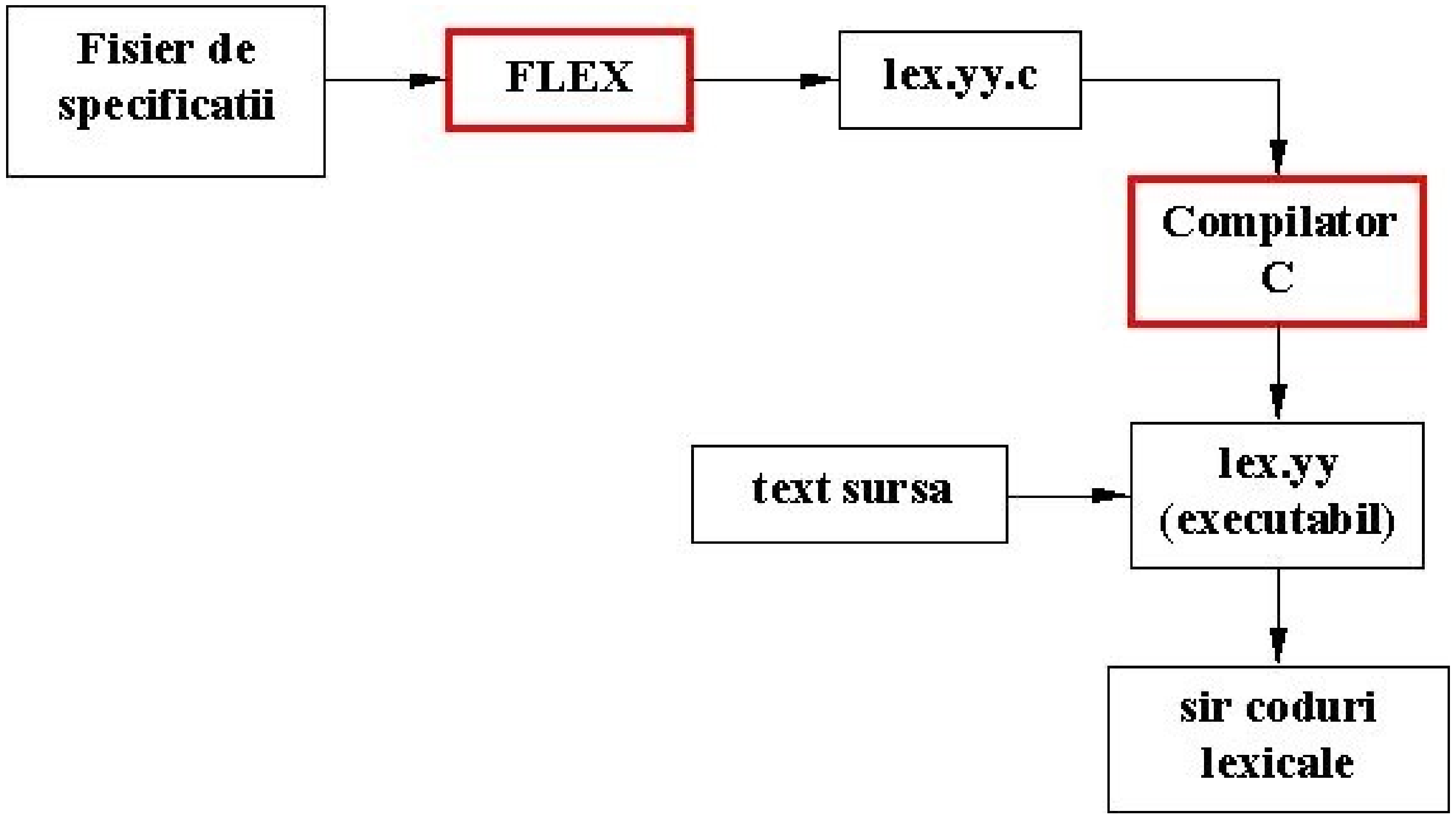
1. Have you heard about these tools?

2. Have you used any of them?

Scanning & Parsing Tools

- Scanning => lex
- Parsing => yacc

Lex – Unix utilitary (flex – Windows version)



INPUT FILE FORMAT

- The file containing the specification is a text file, that can have any name. Due to historic reasons we recommend the extension **.lxi**.
- Consists of 3 sections separated by a line containing %%:

```
definitions
%%
rules
%%
user code
```

Example 1:

```
%%
 username printf( "%s", getlogin() );
```

specifies a scanner that, when finding the string “`username`”, will replace it with the user login name

Definition Section:

- C declarations

+

- declarations of simple *name definitions* (used to simplify the scanner specification), of the form

```
        name definition
```

- where:
  - **name** is a word formed by one or more letters, digits, '_' or '-', with the remark that the first character MUST be letter or '_' and must be written on the FIRST POSITION OF THE LINE.
  - **definition** is a regular expression and is starting with the first nonblank character after name until the end of line.
  - declarations of *start conditions*.

Rules Section

- to associate semantic actions with regular expressions. It may also contain user defined C code, in the following way:

pattern action

where:

- **pattern** is a regular expression, whose first character MUST BE ON THE FIRST POSITION OF THE LINE;

- **action** is a sequence of one or more C statements that MUST START ON THE SAME LINE WITH THE PATTERN. If there are more than one statements they will be nested between {}. In particular, the action can be a void statement.

User Defined Code Section:

- Is optional (if is missing, then the separator %% following the rules section can also miss). If it exists, then its containing user defined C code is copied without any change at the end of the file lex.yy.c.
- Normally, in the user defined code section, one may have:

  - function *main()* containing call(s) to *yylex()*, if we want the scanner to work autonomously (for ex., to test it);

  - other called functions from *yylex()* (for ex. *yywrap()* or functions called during actions); in this case, the user code from definitions section must contain: either prototypes, either ***#include*** directives of the headers containing the prototypes

Launching the execution:

lex [*option*] [*name_specification _file*]

where *name_specification _file is an input file (implicitly,* stdin)

```
$ lex spec.lxi
$ gcc lex.yy.c -o your_lex
$ your_lex<input.txt
```

options: http://dinosaur.compilertools.net/flex/manpage.html

Example

yacc

Parsing (syntax analysis) modeled with cfg:

cfg G = (N, $\boldsymbol{\Sigma}$,P,S):

- N – nonterminal: syntactical constructions: declaration, statement, expression, a.s.o.
- $\boldsymbol{\Sigma}$ – terminals; elements of the language: identifiers, constants, reserved words, operators, separators
- P – syntactical rules – expressed in BNF – simple transformation
- S – syntactical construct corresponding to program

THEN

Program syntactical correct <=> w ∈ L(G)

yacc – Unix tool (Bison – Window version)

- **Yet Another Compiler Compiler**

- LALR
- C code

A yacc grammar file has four main sections

```
%{
C declarations
%}

yacc declarations

%%
Grammar rules
%%

Additional C code
```

contains declarations that define terminal and nonterminal symbols, specify precedence, and so on.

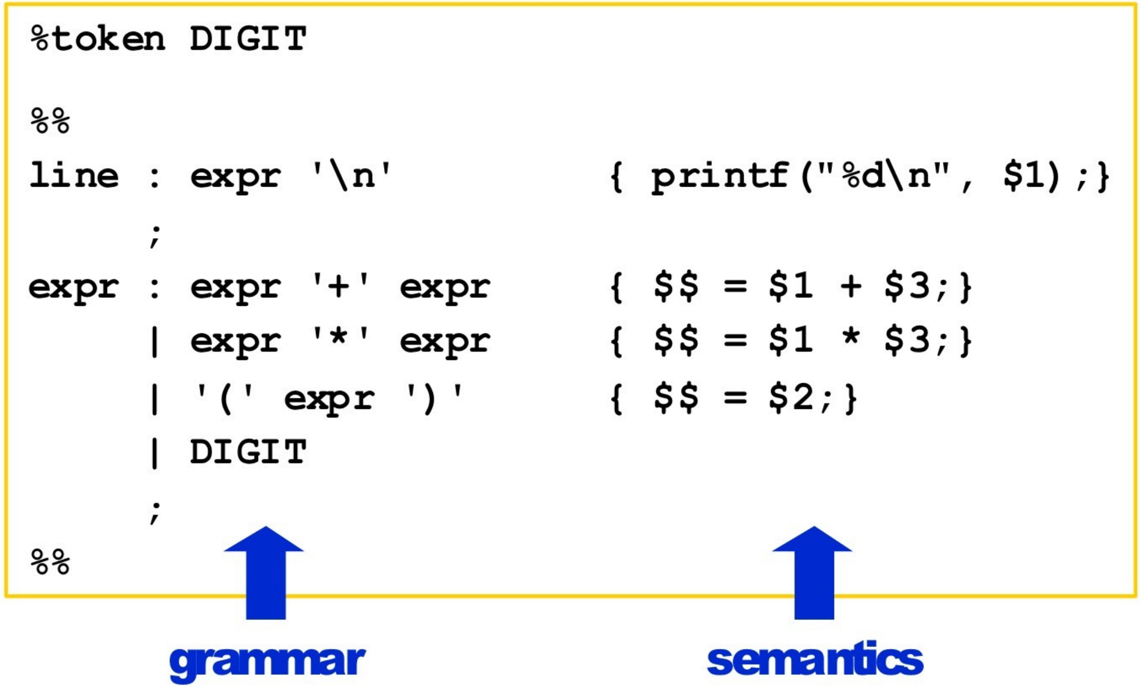
The grammar rules section

- contains one or more yacc grammar rules of the following general form:

```
result  :  components. . .        {C statements  }

           ;

exp:       exp '+' exp
  ;

result  :      rule1-components  . . .
        | rule2-components  . . .
        . . .
        ;
result  :                    /*empty */
        | rule2-components  . . .
        ;
```

Example: expression interpreter

- input



```
%token DIGIT

%%
line : expr '\n'          { printf("%d\n", $1);}
     ;
expr : expr '+' expr      { $$ = $1 + $3;}
     | expr '*' expr      { $$ = $1 * $3;}
     | '(' expr ')'       { $$ = $2;}
     | DIGIT
     ;
%%
```

grammar
semantics

- Yacc has a stack of values - referenced ‘$i’ in semantic actions

- Input file (desk0)

```
%%
line : expr '\n'          { printf("%d\n", $1);}
     ;
expr : expr '+' expr      { $$ = $1 + $3;}
     | expr '*' expr      { $$ = $1 * $3;}
     | '(' expr ')'       { $$ = $2;}
     | DIGIT
     ;
```

```
> make desk0
bison -v desk0.y
desk0.y contains 4 shift/reduce conflicts.
gcc -o desk0 desk0.tab.c
>
```

Conflict resolution in yacc

- Conflict **shift-reduce** – prefer **shift**

- Conflict **reduce-reduce** – chose first production

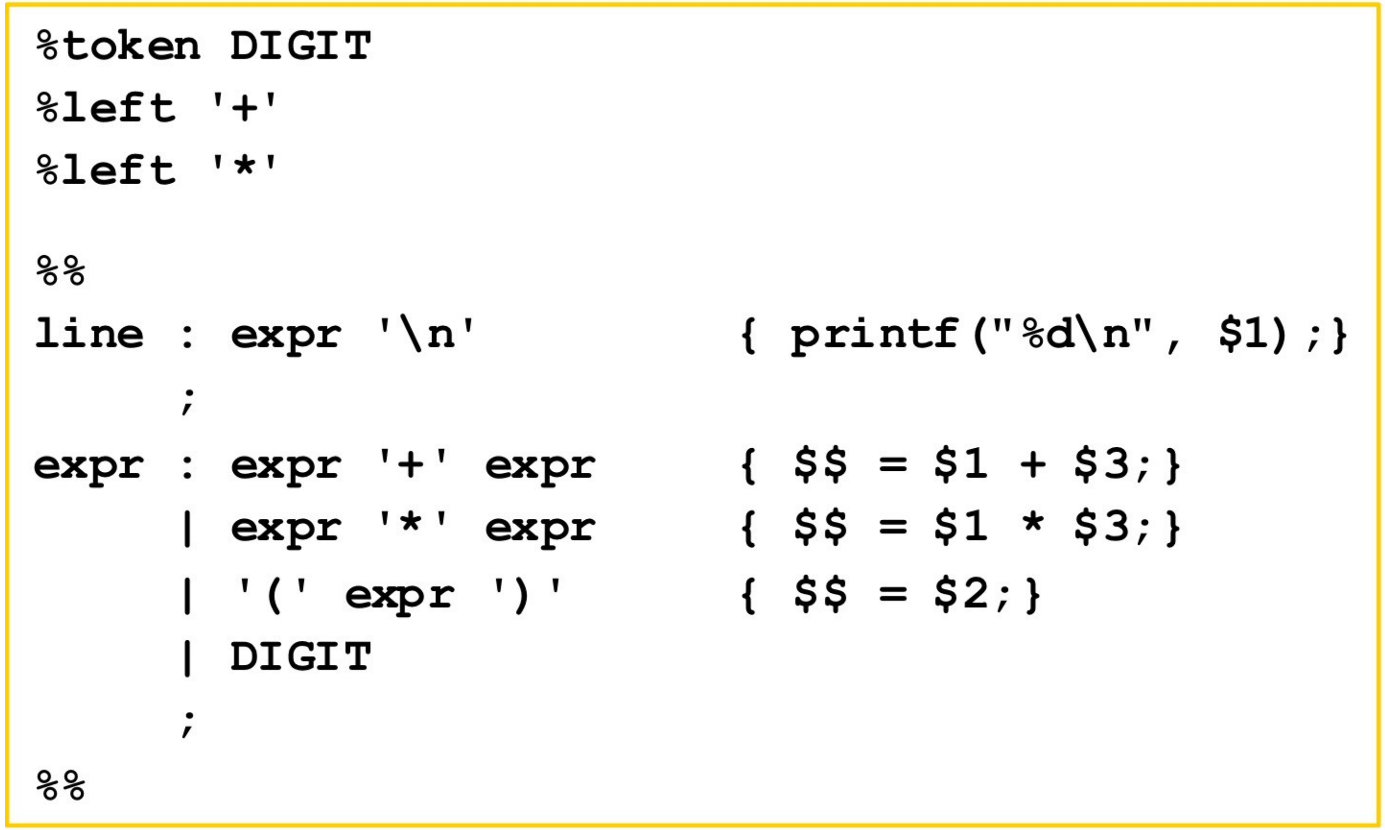
```
%%
line : expr '\n'              { printf("%d\n", $1);}
     ;
expr : expr '+' expr          { $$ = $1 + $3;}
     | expr '*' expr          { $$ = $1 * $3;}
     | '(' expr ')'           { $$ = $2;}
     | DIGIT
     ;
%%
```

- Run yacc
- Run desk0

```
> desk0
2*3+4
14
```

Operator priority in yacc

- From low to great



```
%token DIGIT
%left '+'
%left '*'

%%
line : expr '\n'               { printf("%d\n", $1);}
     ;
expr : expr '+' expr           { $$ = $1 + $3;}
     | expr '*' expr           { $$ = $1 * $3;}
     | '(' expr ')'            { $$ = $2;}
     | DIGIT
     ;
%%
```

- Use

```
>lex spec.lxi
>yacc –d spec.y
>gcc lex.yy.c y.tab.c -o result –lfl
>result<InputProgram
```

- More on

  http://catalog.compilertools.net/lexparse.html

Example

Course 11

Push-Down Automata (PDA)

Intuitive Model

Definition

- A push-down automaton (APD) is a 7-tuple M = $(Q,\boldsymbol{\Sigma},\boldsymbol{\Gamma},\boldsymbol{\delta},q_0,Z_0,F)$ where:
  - Q – finite set of states
  - $\boldsymbol{\Sigma}$ - alphabet (finite set of input symbols)
  - $\boldsymbol{\Gamma}$ – stack alphabet (finite set of stack symbols)
  - $\boldsymbol{\delta} : Q \times (\boldsymbol{\Sigma} \cup \{\boldsymbol{\varepsilon}\}) \times \boldsymbol{\Gamma} \rightarrow \mathcal{P}(Q\times \boldsymbol{\Gamma}^*)$ –transition function
  - $q_0 \in Q$ – initial state
  - $Z_0 \in \boldsymbol{\Gamma}$ – initial stack symbol
  - $F \subseteq Q$ – set of final states

Push-down automaton

Transition is determined by:

- Current state
- Current input symbol
- Head of stack

Reading head -> input band:

- Read symbol
- No action

Stack:

- Zero symbols => pop
- One symbol => push
- Several symbols => repeated push

Configurations and transition / moves

- Configuration:

$$(q, x, \alpha) \in Q \times \Sigma^* \times \Gamma^*$$

where:

  - PDA is in state **q**
  - Input band contains **x**
  - Head of stack is ***α***

- Initial configuration (q_0, w, Z_0)

Configurations and moves(cont.)

- Moves between configurations:

$p,q \in Q$, $a \in \boldsymbol{\Sigma}$, $Z \in \boldsymbol{\Gamma}$, $w \in \boldsymbol{\Sigma^*}$, $\boldsymbol{\alpha}, \boldsymbol{\gamma} \in \boldsymbol{\Gamma^*}$

$(q,aw,Z\boldsymbol{\alpha}) \vdash (p,w,\boldsymbol{\gamma}Z\boldsymbol{\alpha})$ iff $\boldsymbol{\delta}(q,a,Z) \ni (p,\boldsymbol{\gamma}Z)$

$(q,aw,Z\boldsymbol{\alpha}) \vdash (p,w,\boldsymbol{\alpha})$ iff $\boldsymbol{\delta}(q,a,Z) \ni (p,\boldsymbol{\varepsilon})$

$(q,aw,Z\boldsymbol{\alpha}) \vdash (p,aw,\boldsymbol{\gamma}Z\boldsymbol{\alpha})$ iff $\boldsymbol{\delta}(q,\boldsymbol{\varepsilon},Z) \ni (p,\boldsymbol{\gamma}Z)$

($\boldsymbol{\varepsilon}$-move)

- $\overset{k}{\vdash}$ **,** $\overset{+}{\vdash}$ **,** $\overset{*}{\vdash}$

Language accepted by PDA

- Empty stack principle:

$L_{\boldsymbol{\varepsilon}}(M) = \{w|w\in\boldsymbol{\Sigma}^*, (q_0,w,Z_0) \vdash^* (q,\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}), q\in Q\}$

- Final state principle:

$L_f(M) = \{w|w\in\boldsymbol{\Sigma}^*, (q_0,w,Z_0) \vdash^* (q_f,\boldsymbol{\varepsilon},\boldsymbol{\gamma}), q_f\in F\}$

Representations

- Enumerate
- Table
- Graphic

Construct PDA

- L = $\{0^n1^n \mid n \geq 1\}$
- States, stack, moves?

1. States:
   - Initial state:q_0 – beginning and process symbols ‘0’
   - When first symbol ‘1’ is found – move to new state => q_1
   - Final: final state q_2
2. Stack:
   - Z_0 – initial symbol
   - X – to count symbols:
     - When reading a symbol ’0’ – push X in stack
     - When reading a symbol ‘1’ – pop X from stack

Exemple 1 (enumerate)

$M = (\{q_0,q_1,q_2\}, \{0,1\}, \{Z_0,X\},\boldsymbol{\delta},q_0,Z_0,\{q_2\})$

$\boldsymbol{\delta}(q_0,0,Z_0) = (q_0,XZ_0)$

$\boldsymbol{\delta}(q_0,0,X) = (q_0,XX)$

$\boldsymbol{\delta}(q_0,1,X) = (q_1,\boldsymbol{\varepsilon})$

$\boldsymbol{\delta}(q_1,1,X) = (q_1,\boldsymbol{\varepsilon})$

~~$\boldsymbol{\delta}(q_1,\boldsymbol{\varepsilon},Z_0) = (q_2,Z_0)$~~

$\boldsymbol{\delta}(q_1,\boldsymbol{\varepsilon},Z_0) = (q_1, \boldsymbol{\varepsilon})$

Empty stack

$\vdash (q_1, \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon})$

$(q_0,0011,Z_0) \vdash (q_0,011,XZ_0) \vdash (q_0,11,XXZ_0) \vdash (q_1,1,XZ_0) \vdash (q_1, \boldsymbol{\varepsilon}, Z_0) \vdash (q_2, \boldsymbol{\varepsilon}, Z_0)$

Final state

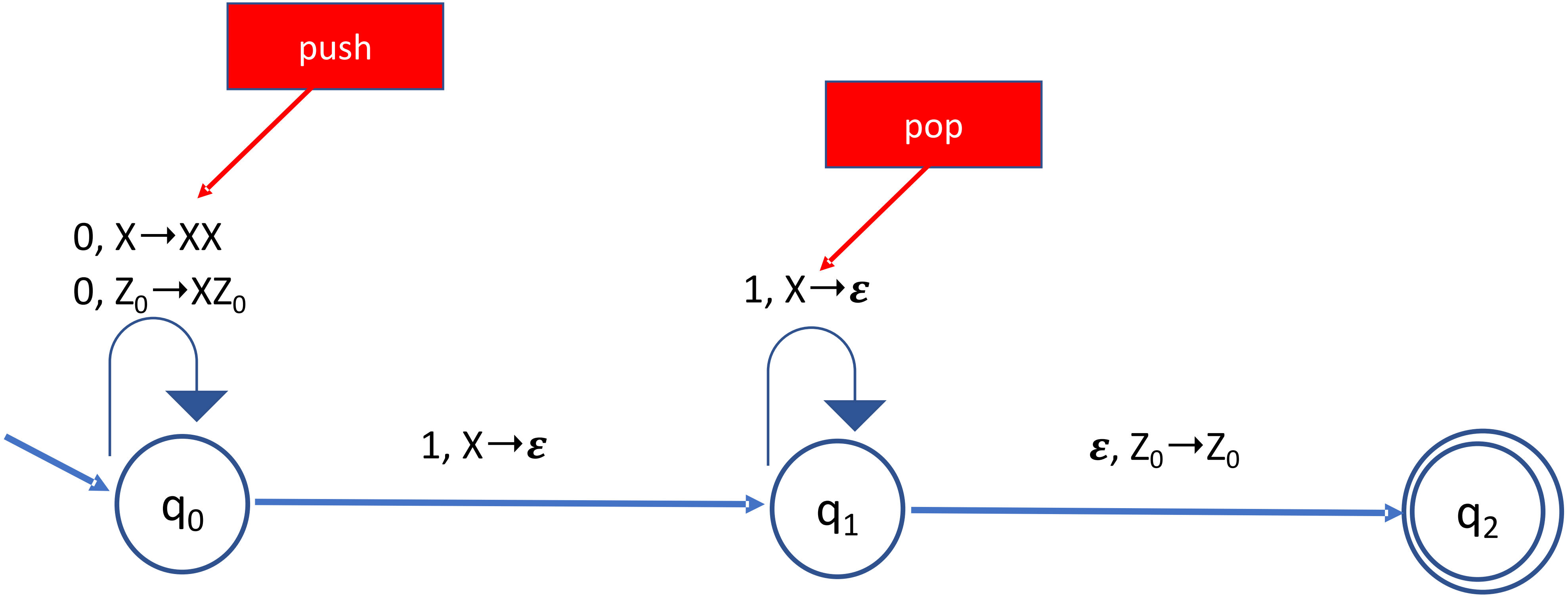
Exemple 1 (table)

		0	1	ε
q_0	Z_0	q_0,XZ_0		
	X	q_0,XX	q_1,ε	
q_1	Z_0			q_2,Z_0 (q_1, ε)
	X		q_1,ε	
q_2	Z_0			
	X			

(q0,0011,Z0) |- (q0,011,XZ0) |- (q0,11,XXZ0) |- (q1,1,XZ0)
|- (q1, ε,Z0) |- (q2, ε,Z0) q2 final seq. is acc based on final state

(q0,0011,Z0) |- (q0,011,XZ0) |- (q0,11,XXZ0) |- (q1,1,XZ0)
|- (q1, ε,Z0) |-(q1, ε, ε) seq is acc based on empty stack

Exemple 1 (graphic)



Properties

Theorem 1: For any PDA M, there exists a PDA M' such that

$$L_{\varepsilon}(M) = L_f(M')$$

Theorem 2: For any PDA M, there exists a context free grammar such that

$$L_{\varepsilon}(M) = L(G)$$

Theorem 3: For any context free grammar there exists a PDA M such that

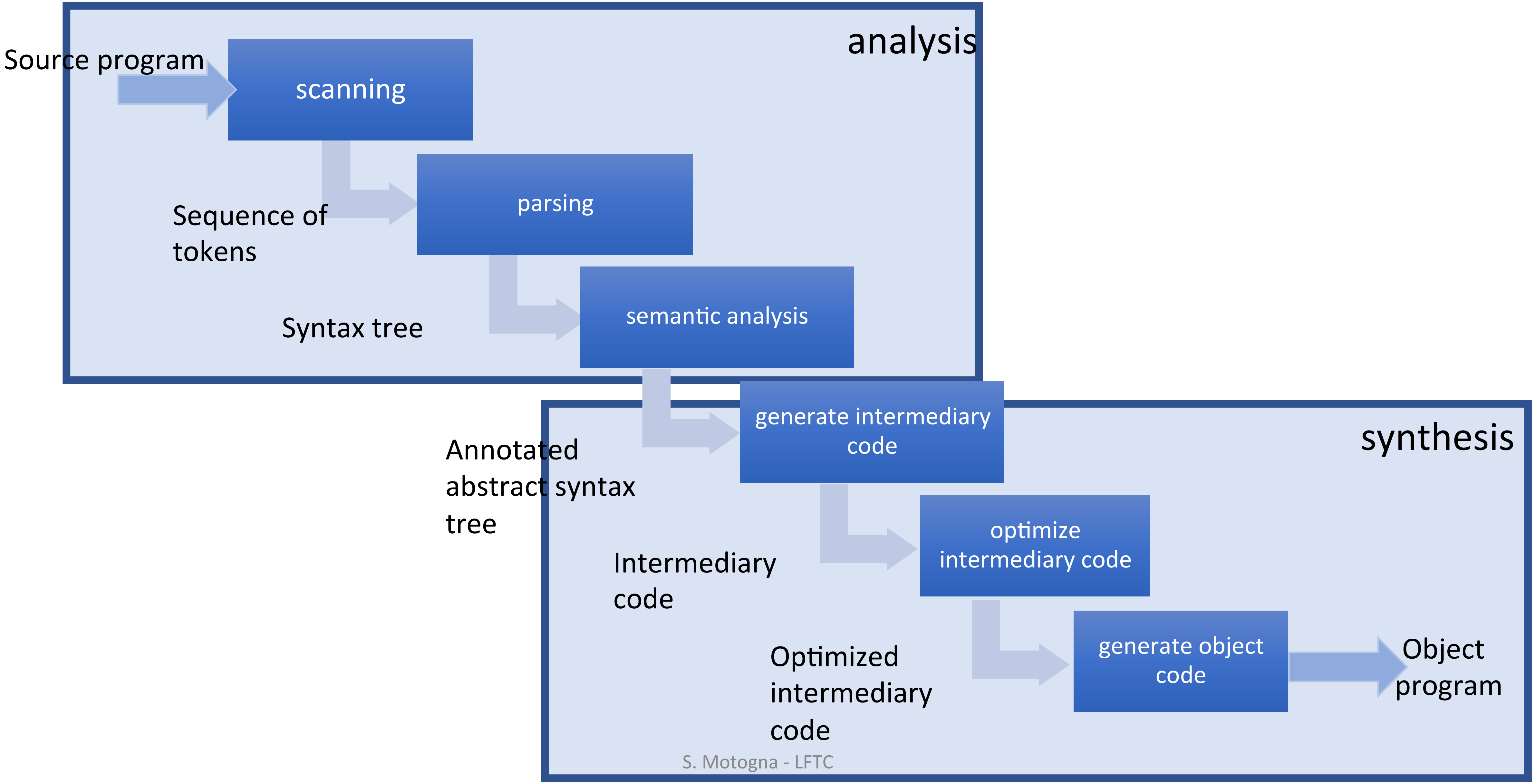
$$L(G) = L_{\varepsilon}(M)$$

HW

- Parser:
  - Descendent recursive
  - LL(1)
  - LR(0), SLR, LR(1)

Corresponding PDA

Structure of compiler



Semantic analysis

- Parsing – result: syntax tree (ST)

- Simplification: abstract syntax tree (AST)

- Annotated abstract syntax tree (AAST)
    - Attach semantic info in tree nodes

Example

Semantic analysis

- Attach meanings to syntactical constructions of a program
- What:
  - Identifiers -> values / how to be evaluated
  - Statements -> how to be executed
  - Declaration -> determine space to be allocated and location to be stored
- Examples:
  - Type checkings
  - Verify properties
- How:
  - **Attribute grammars**
  - Manual methods

Attribute grammar

- Syntactical constructions (nonterminals) – attributes

$$\forall\, X \in N \cup \Sigma: A(X)$$

- Productions – rules to compute/ evaluate attributes

$$\forall\, p \in P: R(p)$$

Definition

AG = (G,A,R) is called ***attribute grammar*** where:

- G = (N,$\boldsymbol{\Sigma}$,P,S) is a context free grammar
- A = {A(X) | X ∈N U $\boldsymbol{\Sigma}$} – is a finite set of attributes
- R = {R(p) | p ∈P} – is a finite set of rules to compute/evaluate attributes

Example 1

- G = ({N,B},{0,1}, P, N}

P:

$N_1 \to N_2B$

$\underline{N \to B}$

B -> 0

B -> 1

$N_1.v = \underbrace{2 * N_2.v} + \underbrace{B.v}$

$\underline{N.v = B.v}$

B.v = 0

$\underline{B.v} = 1$

Attribute – value of number = **v**

- **Synthetized attribute: A(lhp) depends on rhp**
- **Inherited attribute: A(rhp) depends on lhp**

Evaluate attributes

- Traverse the tree: can be an infinite cycle

- Special classes of AG:
  - L-attribute grammars: for any node the depending attributes are on the "*left*";
    - can be evaluated in one left-to-right traversal of syntax tree
    - Incorporated in top-down parser (LL(1))
  - S-attribute grammars: synthetized attributes
    - Incorporated in bottom-up parser (LR)

Steps

- What? - decide what you want to compute (type, value, etc.)
- Decide attributes:
  - How many
  - Which attribute is defined for which symbol
- Attach evaluation rules:
  - For each production – which rule/rules

Example 2 (L-attribute grammar)

Decl -> DeclTip ListId

ListId -> Id

ListId -> ListId, Id

ListId.type = DeclTip.type

Id.type = ListId.type

$\text{ListId}_2\text{.type} = \text{ListId}_1\text{.type}$

$\text{Id.type} = \text{ListId}_1\text{.type}$

Attribute – type

int i,j

Example 3 (S-attribute grammar)

ListDecl -> ListDecl; Decl

ListDecl -> Decl

Decl -> Type ListId

Type -> int

Type -> long

ListId -> Id

ListId -> ListId, Id

ListDecl_1.dim = ListDecl_2.dim + Decl.dim

ListDecl.dim = Decl.dim

Decl.dim = Type.dim * ListId.no

Type.dim = 4

Type.dim =8

ListId.no = 1

ListId_1.no = ListId_2.no + 1

Attributes – dim + no – **for which symbols**

int i,j; long k

Proposed problems (HW):

1) Define an attribute grammar for arithmetic expressions
2) Define an attribute grammar for logical expressions
3) Define an attribute grammar for if statement

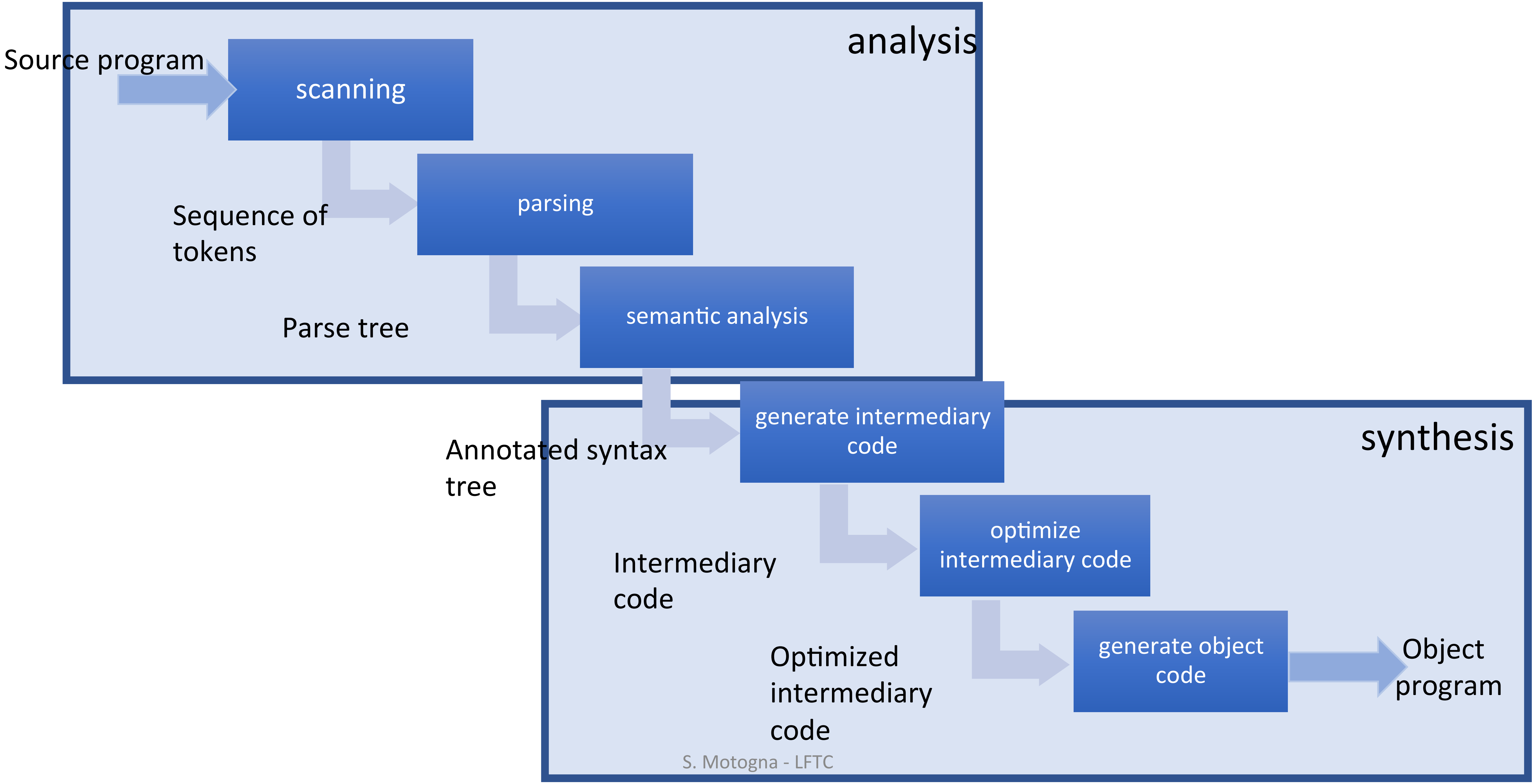
Manual methods

- Symbolic execution
  - Using control flow graph, simulate on stack how the program will behave
  - [Grune – Modern Compiler Design]

- Data flow equations
  - Data flow – associate equations based on data consumed in each node (statement) of the control flow graph: In, Out, Generated, Killed
  - [Grune – Modern Compiler Design], [Kildall], [course]

Course 12

Structure of compiler



Generate intermediary code

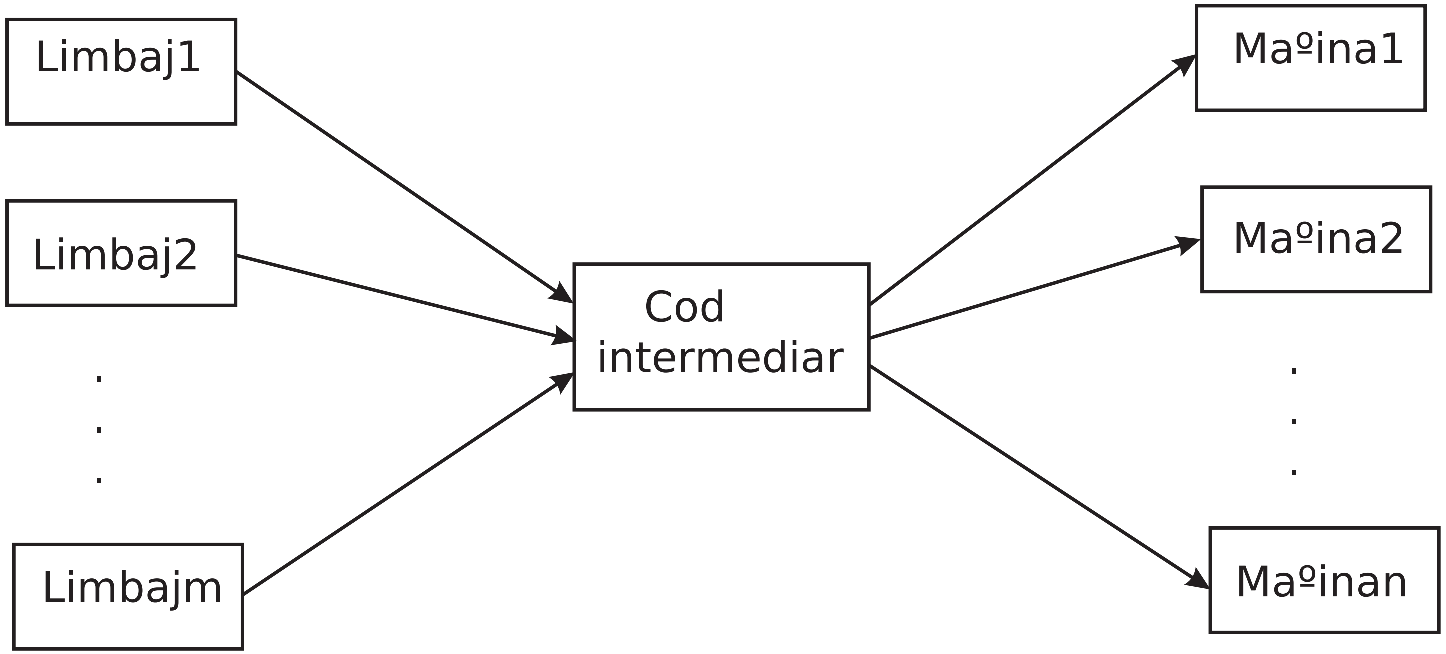


Figura 5.1: Crearea de compilatoare pentru *m* limbaje ¸si *n* ma¸sini folosind cod intermediar

Forms of intermediary code

- Java bytecode – source language: Java

  – machine language (dif. platforms) JVM
- MSIL (Microsoft Intermediate Language)

  – source language: C#, VB, etc.

  – machine language (dif. platforms) Windows
- GNU RTL (Register Transfer Language)

  – source language: C, C++, Pascal, Fortran etc.

  – machine language (dif. platforms)

Representations of intermediary code

- Annotated tree: intermediary code is generated in semantic analysis
- Polish postfix form:
  - No parenthesis
  - Operators appear in the order of execution
  - Ex.: MSIL

Exp = a + b * c	ppf = abc*+
Exp = a * b + c	ppf = ab*c+
Exp = a * (b + c)	ppf = abc+*

- 3 address code

3 address code

= sequence of simple format statements, close to object code, with the following general form:

< result >=< arg1 >< op >< arg2 >

Represented as:

- Quadruples
- Triples
- Indirected Triples

- Quadruples:

  < op > < arg1 > < arg2 > < result >

- Triples:

  < op > < arg1 > < arg2 >

(considered that the triple is storing the result)

Special cases:

1. Expressions with unary operator**: < result >=< op >< arg2 >**
2. Assignment of the form **a := b** => the 3 addresss code is **a = b** (no operatorand no 2nd argument)
3. Unconditional jump: statement is **goto L**, where L is the label of a 3 address code
4. Conditional jump: **if c goto L**: if **c** is evaluated to **true** then unconditional jump to statement labeled with L, else (if c is evaluated to false), execute the next statement
5. Function call p(x1, x2, ..., xn) – sequence of statements: **param x1, param x2 , param xn, call p, n**
6. Indexed variables: < arg1 >,< arg2 >,< result > can be array elements of the form **a[i]**
7. Pointer, references: **&x, *x**

Example: b∗b–4∗a∗c

op	arg1	arg2	rez
*	b	b	t1
*	4	a	t2
*	t2	c	t3
-	t1	t3	t4

nr	op	arg1	arg2
(1)	*	b	b
(2)	*	4	a
(3)	*	(2)	c
(4)	-	(1)	(3)

Example 2

If (a<2) then a=b else a=b*b

Optimize intermediary code

- Local optimizations:
  - Perform computation at compile time – constant values
  - Eliminate redundant computations
  - Eliminate inaccessible code – if…then...else...

- Loop optimizations:
  - Factorization of loop invariants
  - Reduce the power of operations

comune $C § B$ ¸si $D + C § B$.

D:=D+C*B

A:=D+C*B

C:=D+C*B

Secvenţa corespunz˘atoare de cod cu trei adrese, reprezen

este:

Eliminate redundant computations

Example:

```
D:=D+C*B
A:=D+C*B
C:=D+C*B
```

(1)	*	C	B
(2)	+	D	(1)
(3)	:=	(2)	D
~~(4)~~	~~*~~	~~C~~	~~B~~
(5)	+	D	(4)
(6)	:=	(5)	A
~~(7)~~	~~*~~	~~C~~	~~B~~
~~(8)~~	~~+~~	~~D~~	~~(7)~~
(9)	:=	(8)	C

Aceste subexpresii comune se reg˘asesc ˆın triplete identice

(4), (7), dar ¸si unele mai greu de observat ca (2), (5), (8). Ideea es

un astfel de calcul se poate efectua o singur˘a dat˘a

Determine redundant operations

- Operation (j) is redudant to operation (i) with i<j if the 2 operations are identical and if the operands in (j) did not change in any operation between (i+1) and (j-1)
- Algorithm [Aho]

constă în a scoate această instrucţiune înaintea ciclului, ea ex

astfel o singură dată.

Factorization of loop invariants

Exemplul 6.3. O secvenţă de prog invariant înainte şi după optimizare:

What is a loop invariant?

```
for(i=0; i<=n;i++)                    x=y+z;
   { x=y+z;                           for(i=0; i<=n;i++)
   a[i]=i*x}                             { a[i]=i*x}
```

Reducerea puterii operaţiilor

Această optimizare are ca scop înlocuirea operaţiilor

exemplu înmulţirea) cu operaţii mai ieftine (adunarea) în

Challenge

```
V1:
P = a[0]
For i=1 to n
   P = P + a[i]*v^i
```

```
V2:
P = a[0]
Q=v
For i=1 to n
   P = P + a[i]*Q
   Q = Q*v
```

Consider n, and a[i] i=0,n the coefficients of a polynomial P.

Given v, write an algorithm that computes the value of P(v)

```
V3
P=a[n]
For i=1 to n
  P = P*v + a[n-i]
```

3 solutions

P(x) = a[n]*x^n+ … + a[1]*x + a[0] = (a[n]*x^(n-1)+ … + a[1])*x + a[0]

Reduce the power of operations

valoarea calculat ̆a la iterat ̧ia 1.

Exemplul 6.4 Consider ̂and ciclul urm ̆ator, ̂ın care v este un

de ciclu, el poate fi optimizat astfel:

```
for(i=k;i<=n;i++)              t1=k*v;
      { t=i*v;                 for(i=k;i<=n;i++)
      . . .}                         { t=t1;
                                     t1=t1+v;...}
```

valoarea calculat ̆a la iterat ̧ia 1.

Exemplul 6.4 Consider ̂and ciclul urm ̆ator, ̂ın care v este

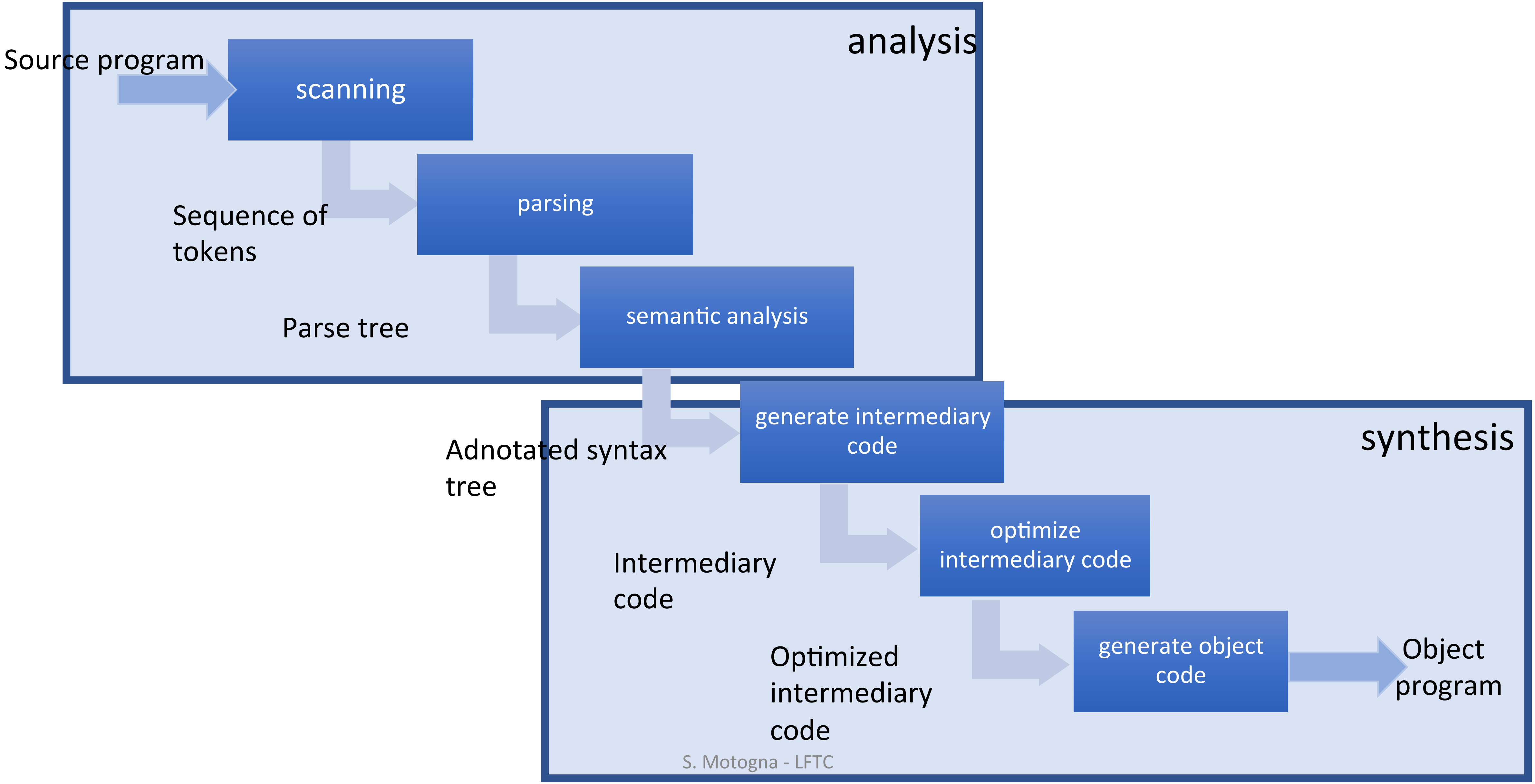
de ciclu, el poate fi optimizat astfel:

for(i=k;i<=n;i++) { t=i*v; . . .}

t1=k*v; for(i=k;i<=n;i++) { t=t1; t1=t1+v;...}

Course 13

Structure of compiler



Generate object code

= translate intermediary code statements into statements of object code (machine language)

- Depend on “machine”: architecture and OS

Computer with accumulator

- A **stack machine** consists of:
- a stack for storing and manipulating values (store subexpressions and results)
- Accumulator – to execute operation
- 2 types of statements:
  - move and copy values in and from head of stack to accumulator
  - Operations on stack head, functioning as follows: operands are popped from stack, execute operation and then put the result in stack

Example: 4 * (5+1)

Code	acc	stack
acc ← 4	4	<>
push acc	4	<4>
acc ← 5	5	<4>
push acc	5	<5,4>
acc ← 1	1	<5,4>
acc ← acc + head	6	<5,4>
pop	6	<4>
acc ← acc * head	24	<4>
pop	24	<>

Computer with registers

- Registers +
- Memory

- Instructions:
    - LOAD v,R – load value **v** in register **R**
    - STORE R,v – put value **v** from register **R** in memory
    - ADD R1,R2 – add to the value from register **R1**, value from register **R2** and store the result in **R1** (initial value is lost!)

2 aspects:

- Register allocation – way in which variable are stored and manipulated;

- Instruction selection – way and order in which the intermediary code statements are mapped to machine instructions

Remarks:

1. A register can be available or occupied =>

   VAR(R) = set of variables whose values are stored in register R

2. For every variable, the place (register, stack or memory) in which the current value of the value exists=>

   MEM(x)= set of locations in which the value of variable x exists (will be stored in Symbol Table)

Example: F := A ∗ B – (C + B) ∗ (A * B)

Intermediary code	Object code	VAR	MEM

		VAR(R0) = {}
		VAR(R1) = {}	

(1) T1 = A * B			
(2) T2 = C + B			
(3) T3 = T2 * T1			
(4) F:= T1 – T3			

Example: F := A ∗ B – (C + B) ∗ (A * B)

Intermediary code	Object code	VAR	MEM

		VAR(R0) = {}
		VAR(R1) = {}	

(1) T1 = A * B	LOAD A, R0
	MUL R0, B
		VAR(R0) = {A}
		VAR(R0) = {T1}
			MEM(T1) = {R0}

(2) T2 = C + B			
(3) T3 = T2 * T1			
(4) F:= T1 – T3			

Example: F := A ∗ B – (C + B) ∗ (A * B)

Intermediary code	Object code	VAR	MEM

		VAR(R0) = {}
		VAR(R1) = {}	

(1) T1 = A * B	LOAD A, R0 MUL R0, B	VAR(R0) = {T1}	MEM(T1) = {R0}
(2) T2 = C + B	LOAD C, R1 ADD R1, B	VAR(R1) = {T2}	MEM(T2) = {R1}
(3) T3 = T2 * T1			
(4) F:= T1 – T3			

Example: F := A ∗ B – (C + B) ∗ (A * B)

Intermediary code	Object code	VAR	MEM

		VAR(R0) = {}
		VAR(R1) = {}	

(1) T1 = A * B	LOAD A, R0 MUL R0, B	VAR(R0) = {T1}	MEM(T1) = {R0}
(2) T2 = C + B	LOAD C, R1 ADD R1, B	VAR(R1) = {T2}	MEM(T2) = {R1}

(3) T3 = T2 * T1	MUL R1,R0	VAR(R1) = {T3}	MEM(T2) = {}
			MEM(T3) = {R1}

(4) F:= T1 – T3			

Example: F := A ∗ B – (C + B) ∗ (A * B)

Intermediary code	Object code	VAR	MEM

		VAR(R0) = {}
		VAR(R1) = {}	

(1) T1 = A * B	LOAD A, R0 MUL R0, B	VAR(R0) = {T1}	MEM(T1) = {R0}
(2) T2 = C + B	LOAD C, R1 ADD R1, B	VAR(R1) = {T2}	MEM(T2) = {R1}

(3) T3 = T2 * T1	MUL R1,R0	VAR(R1) = {T3}	MEM(T2) = {}
			MEM(T3) = {R1}

(4) F:= T1 – T3	SUB R0,R1
	STORE RO, F
		VAR(R0) = {F}
		VAR(R1) = {}
			MEM(T1) = {}
			MEM(F) = {R0, F}

More about Register Allocation

- Registers – **limited resource**
- Registers – perform operations / computations
- Variables **much more** than registers

IDEA: *assigning a large number of variables to a reduced number of registers*

Live variables

- Determine the number of variables that are live (used)

Example:

a = b + c

d = a + e

e = a + c

	op	op1	op2	rez
1	+	b	c	a
2	+	a	e	d
3	+	a	c	e

	1	2	3
a	x	x	x
b	x		
c	x	x	x
d		x	
e		x	x

Graph coloring allocation (Chaitin a.o. 1982)

- Graph:
    - nodes = live variables that should be allocated to registers
    - edges = live ranges simultaneously live

Register allocation = graph coloring: colors (registers) are assigned to the nodes such that two nodes connected by an edge do not receive the same color

Disadvantage:

- NP complete problem

Linear scan allocation (Poletto a.o., 1999)

- determine all live range, represented as an interval
- intervals are traversed chronologically
- greedy algorithm

Advantage: speed – code is generated faster (speed in code generation)

Disadvantage: generated code is slower (NO speed in code execution)

Instruction selection

Example: F := A ∗ B – (C + B) ∗ (A * B)

Intermediary code	Object code	VAR	MEM

		VAR(R0) = {}
		VAR(R1) = {}	

(1) T1 = A * B	LOAD A, R0 MUL R0, B	VAR(R0) = {T1}	MEM(T1) = {R0}
(2) T2 = C + B	LOAD C, R1 ADD R1, B STORE R0,T1	VAR(R1) = {T2}	MEM(T2) = {R1}

(3) T3 = T2 * T1	MUL R1,R0
	MUL R0,R1
		VAR(R1) = {T3}	MEM(T2) = {}
			MEM(T3) = {R1}

(4) F:= T1 – T3	LOAD T1,R1		

Decide which register to use for an instruction

Turing Machines

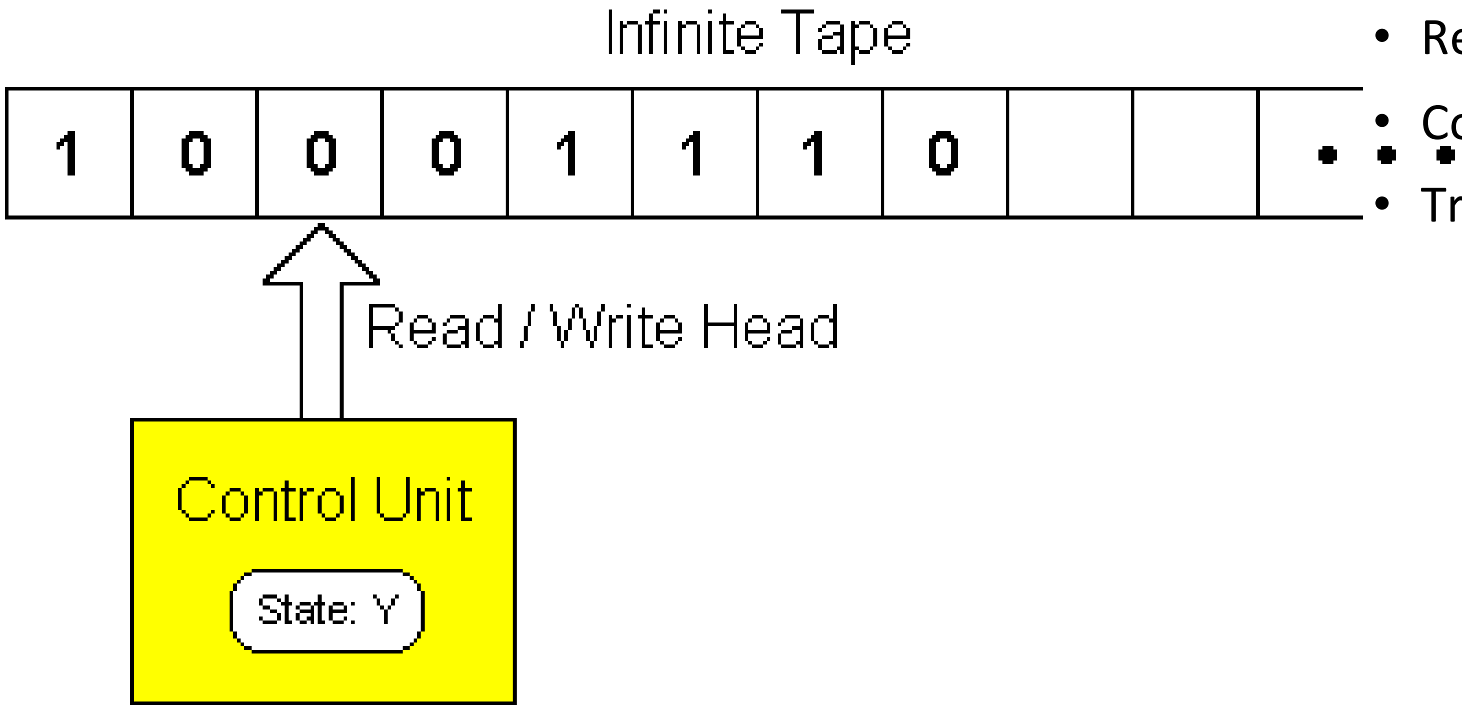
Alan Turing

- Enigma (criptography)
- Turing test
- Turing machine (1937)

Turing Machine

- Mathematical model for computation
- Abstract machine
- Can simulate any algorithm

Turing Machine



- Input band (infinite)
- Reading head
- Control Unit: states
- Transitions / moves

Turing machine – definition

7-tuple M = (Q, $\boldsymbol{\Gamma}$,b,$\boldsymbol{\Sigma}$,$\boldsymbol{\delta}$,q_0, F) where:

- Q – finite set of states
- $\boldsymbol{\Gamma}$ - alphabet (finite set of band symbols)
- b $\in \boldsymbol{\Gamma}$ - blank (symbol)
- $\boldsymbol{\Sigma} \subseteq \boldsymbol{\Gamma} \setminus \{b\}$ – input alphabet
- $\boldsymbol{\delta}$: $(Q\setminus F) \times \boldsymbol{\Gamma} \rightarrow Q \times \boldsymbol{\Gamma} \times \{L,R\}$ –transition function
- $q_0 \in Q$ – initial state
- $F \subseteq Q$ – set of final states

L = left
R = right

Example – palindrome over {0,1}

- 001100, 00100, 101101 a.s.o. accepted
- 00110, 1011 a.s.o. not accepted

$$\underline{0}0110\underline{0}$$

Example – palindrome over {0,1}

	0	1	b
q_0	(p_1,b,R)	(p_2,b,R)	(q_f,b,R)
p_1	$(p_1,0,R)$	$(p_1,1,R)$	(q_1,b,L)
p_2	$(p_2,0,R)$	$(p_2,1,R)$	(q_2,b,L)
q_1	(q_r,b,L)		(q_f,b,R)
q_2		(q_r,b,L)	(q_f,b,R)
q_r	$(q_r,0,L)$	$(q_r,1,L)$	(q_0,b,R)
q_f			

Delete 0 in left side;
search 0 in right side

Delete 1 in left side;
search 1 in right side

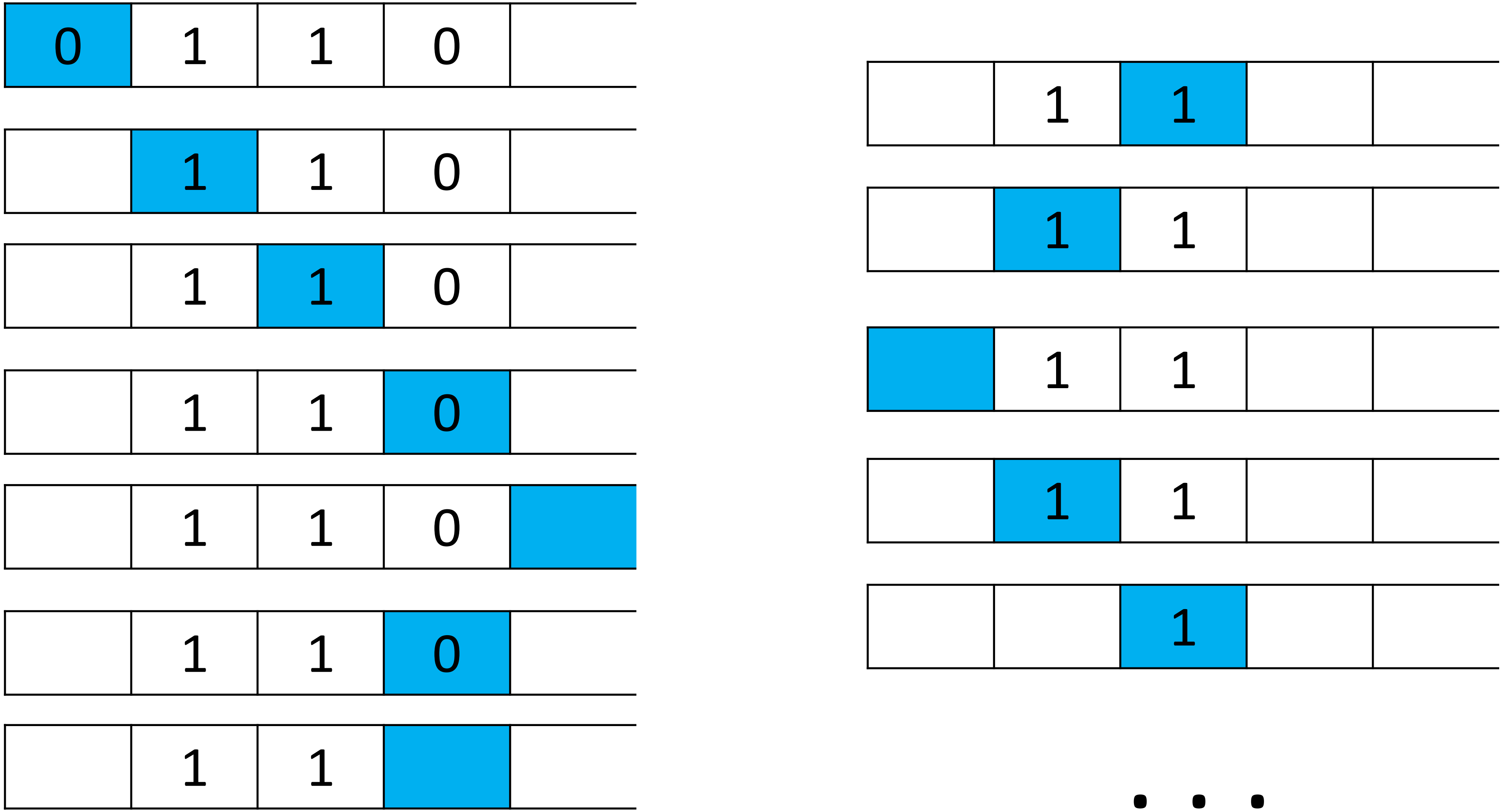
On right is 0 or 1?

Shift right

q_1 and q_2 – process 0 and 1 on the right

qf –final state

0110



$(q_0,\underline{0}110)$ |- $(p_1, \underline{1}10)$ |- $(p_1, 1\underline{1}0)$

|- $(p_1, 11\underline{0})$ |- $(p_1, 110\underline{b})$ |- $(q_1, 11\underline{0})$

|- $(q_r, 1\underline{1})$ |- $(q_r, \underline{1}1)$ |- $(q_r, \underline{b}11)$

|- $(q_0, \underline{1}1)$ |- . . .

	0	1	b
q_0	(p_1,b,R)	(p_2,b,R)	(q_f,b,R)
p_1	$(p_1,0,R)$	$(p_1,1,R)$	(q_1,b,L)
p_2	$(p_2,0,R)$	$(p_2,1,R)$	(q_2,b,L)
q_1	(q_r,b,L)		(q_f,b,R)
q_2		(q_r,b,L)	(q_f,b,R)
q_r	$(q_r,0,L)$	$(q_r,1,L)$	(q_0,b,R)
q_f			

https://turingmachinesimulator.com

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