Week 1. The time value of money.

1. Interest rate and how it's formed.
2. SAR & EAR.
3. FV, PV and FoC.
4. Annuities and perpetuity.
5. TVM problems.

Glossary and abbreviations

- IR - Interest rate - процентная ставка.
- Compounding - платежи (idk)
- SAR - Stated annual rate.
- EAR - Effective annual rate.
- FoC - Frequency of Compounding
- PMT - Payment
- r - Interest rate (at the moment).

Interpretation of the Interest rate

Equilibrium rate of return - Minimum rate of return an investor must receive in order to accept the investment.

Discount rate - Rate that must be applied to a cash flow to determine its present value.

The opportunity cost - Value that investor forgos (loses) by investing.

How the Interest rate is formed

Interest rate =
Real risk-free rate (Reflects the current vs future consumption)

- **Expected inflation premium** (Money costs less over time in a real terms)
- **Default risk premium** (Compensation for possibility and probability that the borrower will default)
- **Liquidity premium** (If financial product has a low liquidity - In case if you want to sell it quickly - you will be forced to take losses by selling it under the market price)
- **Maturity premium** (Long term debts are more sensitive to the future IR falls, if you expect such).

SAR and EAR

SAR - Stated annual rate. Rate which is stated in the documents.

EAR - Effective annual rate. Rate which an investor really gets.

SAR does not account for infra-year compounding while EAR does.

FV and PV

PV - Present Value.

FV - Future Value.

FV_x - Future Value after x compounding periods (years by default)

$$FV_x = PV * (1 + SAR)^x$$

FV depends on FoC

Frequency of Compounding (FoC) - how many times during the year the transaction happens.

$$FV_x = PV * (1 + \frac{SAR}{FoC})^{x*FoC}$$

$$PV = \frac{FV_x}{(1 + \frac{SAR}{FoQ})^{x*FoQ} }$$

Annuities

Annuity is a list of *identical* cash flows

Annuities can be

1. [Finite]
2. [**Ordinary annuity** - Cash flows occur ***at the end*** of each ***compounding period***.]{.nodecor}
3. [**Annuity due** - Cash flows occur ***at the beginning*** of each ***compounding period***.]{.nodecor}
4. [Infinite]
    1. [**Perpetuity.**]{.nodecor}

PV of an annuity

$$ PV = PVofPMT_1 + PVofPMT_2 + \ldots + PVofPMT_n = \ \frac{PMT}{1 + r} + \frac{PMT}{(1 + r)^2} + \ldots + \frac{PMT}{(1 + r)^n} $$

We have a geometric sequence with

$$b = \frac{PMT}{1 + r}$$

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[and]{.nodecor} $$a = \frac{1}{1 + r}$$
[So the sum equals to]{.nodecor}
$$S = b * \frac{q^n - 1}{q - 1}$$
[substitute b and q]{.nodecor}
$$PV\textsubscript{\textnormal{annuity}} = PMT * \frac{(1 + r)^n - 1}{r * (1 + r)^n}$$

PV of a perpetuity

$$PV\textsubscript{\textnormal{perpetuiry}} = \lim_{n \to +\infty} PV\textsubscript{\textnormal{annuity}} = \lim_{n \to +\infty} PMT * \frac{(1 + r)^n - 1}{r * (1 + r)^n} = \frac{PMT}{r}$$ $$