(*This Mathematica notebook verifies the CHSH-
breaking threshold for arbitrary maximally entangled states.*)

In[109]:=

```
(* Clearing variables and declaring as real values*)
γ1 =.
Im[γ1] ^:= 0
Conjugate[γ1] ^:= θ

γ2 =.
Im[γ2] ^:= 0
Conjugate[γ2] ^:= θ

θ =.
Im[θ] ^:= 0
Conjugate[θ] ^:= θ

ϕ =.
Im[ϕ] ^:= 0
Conjugate[ϕ] ^:= ϕ

ω =.
Im[ω] ^:= 0
Conjugate[ω] ^:= ω

(* Pauli Operators *)
σx = {{0, 1}, {1, 0}};
σy = {{0, -ⅈ}, {ⅈ, 0}};
σz = {{1, 0}, {0, -1}};

(*Amplitude damping channel Kraus operators*)
K1A = {{1, 0}, {0, Sqrt[1 - γ1]}};
K1B = {{1, 0}, {0, Sqrt[1 - γ2]}};
K2A = {{0, Sqrt[γ1]}, {0, 0}};
K2B = {{0, Sqrt[γ2]}, {0, 0}};

Print["Two-Sided Amplituded Damping Kraus Operators"]
Kraus = {
KroneckerProduct[K1A, K1B], KroneckerProduct[K1A, K2B],
   KroneckerProduct[K2A, K1B], KroneckerProduct[K2A, K2B]
};
MatrixForm /@ Kraus
```

Two-Sided Amplituded Damping Kraus Operators

Out[133]=

$$\left\{\begin{pmatrix} 1 & 0 & 0 & 0 & \\ 0 & \sqrt{1-\gamma 2} & 0 & 0 & \\ 0 & 0 & \sqrt{1-\gamma 1} & 0 & \\ 0 & 0 & 0 & \sqrt{1-\gamma 1}\,\sqrt{1-\gamma 2} & \end{pmatrix},\right.$$

$$\begin{pmatrix} 0 & \sqrt{\gamma 2} & 0 & 0 & \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & \sqrt{1-\gamma 1} & \sqrt{\gamma 2} \\ 0 & 0 & 0 & 0 & \end{pmatrix}, \begin{pmatrix} 0 & 0 & \sqrt{\gamma 1} & 0 & \\ 0 & 0 & 0 & \sqrt{\gamma 1} & \sqrt{1-\gamma 2} \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & \end{pmatrix}, \left.\begin{pmatrix} 0 & 0 & 0 & \sqrt{\gamma 1}\,\sqrt{\gamma 2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

In[134]:=

```
Print["Arbitrary Qubit Unitary"]
arbQubitUnitary = {
{Exp[-ⅈ * (ϕ + ω) / 2] * Cos[θ / 2], -Exp[ⅈ * (ϕ - ω) / 2] * Sin[θ / 2]},
{Exp[-ⅈ * (ϕ - ω) / 2] * Sin[θ / 2],  Exp[ⅈ * (ϕ + ω) / 2] * Cos[θ / 2]}
};
arbQubitUnitary // MatrixForm
Print["Bell State"]
Φplus = {{1, 0, 0, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {1, 0, 0, 1}} / 2;
Φplus // MatrixForm
```

Arbitrary Qubit Unitary

Out[136]//MatrixForm=

$$\begin{pmatrix} e^{-\frac{1}{2} i (\phi+\omega)} \mathrm{Cos}\left[\frac{\theta}{2}\right] & -e^{\frac{1}{2} i (\phi-\omega)} \mathrm{Sin}\left[\frac{\theta}{2}\right] \\ e^{-\frac{1}{2} i (\phi-\omega)} \mathrm{Sin}\left[\frac{\theta}{2}\right] & e^{\frac{1}{2} i (\phi+\omega)} \mathrm{Cos}\left[\frac{\theta}{2}\right] \end{pmatrix}$$

Bell State

Out[139]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

In[140]:=

```
Print["Unitary for Arbitrary Entangled State Preparation"]
arbU = KroneckerProduct[arbQubitUnitary, {{1, 0}, {0, 1}}];
arbU // MatrixForm
```

Unitary for Arbitrary Entangled State Preparation

Out[142]//MatrixForm=

$$\begin{pmatrix} e^{-\frac{1}{2} i (\phi+\omega)} \mathrm{Cos}\left[\frac{\theta}{2}\right] & 0 & -e^{\frac{1}{2} i (\phi-\omega)} \mathrm{Sin}\left[\frac{\theta}{2}\right] & 0 \\ 0 & e^{-\frac{1}{2} i (\phi+\omega)} \mathrm{Cos}\left[\frac{\theta}{2}\right] & 0 & -e^{\frac{1}{2} i (\phi-\omega)} \mathrm{Sin}\left[\frac{\theta}{2}\right] \\ e^{-\frac{1}{2} i (\phi-\omega)} \mathrm{Sin}\left[\frac{\theta}{2}\right] & 0 & e^{\frac{1}{2} i (\phi+\omega)} \mathrm{Cos}\left[\frac{\theta}{2}\right] & 0 \\ 0 & e^{-\frac{1}{2} i (\phi-\omega)} \mathrm{Sin}\left[\frac{\theta}{2}\right] & 0 & e^{\frac{1}{2} i (\phi+\omega)} \mathrm{Cos}\left[\frac{\theta}{2}\right] \end{pmatrix}$$

In[144]:=

```
Print["Arbitrary Maximally Entangled State"]
ρ = arbU.Φplus.ConjugateTranspose[arbU] // FullSimplify;
ρ // MatrixForm
```

Arbitrary Maximally Entangled State

Out[146]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4} (1+\mathrm{Cos}[\theta]) & -\frac{1}{4} e^{-i \phi} \mathrm{Sin}[\theta] & \frac{1}{4} e^{-i \omega} \mathrm{Sin}[\theta] & \frac{1}{4} e^{-i (\phi+\omega)} (1+\mathrm{Cos}[\theta]) \\ -\frac{1}{4} e^{i \phi} \mathrm{Sin}[\theta] & \frac{1}{4} (1-\mathrm{Cos}[\theta]) & \frac{1}{4} e^{i (\phi-\omega)} (-1+\mathrm{Cos}[\theta]) & -\frac{1}{4} e^{-i \omega} \mathrm{Sin}[\theta] \\ \frac{1}{4} e^{i \omega} \mathrm{Sin}[\theta] & \frac{1}{4} e^{-i (\phi-\omega)} (-1+\mathrm{Cos}[\theta]) & \frac{1}{4} (1-\mathrm{Cos}[\theta]) & \frac{1}{4} e^{-i \phi} \mathrm{Sin}[\theta] \\ \frac{1}{4} e^{i (\phi+\omega)} (1+\mathrm{Cos}[\theta]) & -\frac{1}{4} e^{i \omega} \mathrm{Sin}[\theta] & \frac{1}{4} e^{i \phi} \mathrm{Sin}[\theta] & \frac{1}{4} (1+\mathrm{Cos}[\theta]) \end{pmatrix}$$

In[147]:=

```
Print["Noisy State Preparation"]
KrausApply[krausOp_] = krausOp.ρ.krausOp†;
ρNoise = Total[KrausApply /@ Kraus] // FullSimplify;
ρNoise // MatrixForm
```

Noisy State Preparation

Out[150]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4} ((1+\gamma 1)(1+\gamma 2)+(-1+\gamma 1)(-1+\gamma 2)\mathrm{Cos}[\theta]) & \frac{1}{4} e^{-i \phi} (-1+\gamma 1) \sqrt{1-\gamma 2}\ \mathrm{Sin}[\theta] & -\frac{1}{4} e^{-i \omega} \sqrt{1-\gamma 1} \\ \frac{1}{4} e^{i \phi} (-1+\gamma 1) \sqrt{1-\gamma 2}\ \mathrm{Sin}[\theta] & -\frac{1}{4} (-1+\gamma 2)(1+\gamma 1+(-1+\gamma 1)\mathrm{Cos}[\theta]) & \frac{1}{4} e^{i (\phi-\omega)} \sqrt{1-\gamma 1} \\ -\frac{1}{4} e^{i \omega} \sqrt{1-\gamma 1}\ (-1+\gamma 2)\mathrm{Sin}[\theta] & \frac{1}{4} e^{-i (\phi-\omega)} \sqrt{1-\gamma 1} \sqrt{1-\gamma 2}\ (-1+\mathrm{Cos}[\theta]) & -\frac{1}{4} (-1+\gamma 1)(1+\gamma \\ \frac{1}{4} e^{i (\phi+\omega)} \sqrt{1-\gamma 1} \sqrt{1-\gamma 2}\ (1+\mathrm{Cos}[\theta]) & \frac{1}{4} e^{i \omega} \sqrt{1-\gamma 1}\ (-1+\gamma 2)\mathrm{Sin}[\theta] & -\frac{1}{4} e^{i \phi} (-1+\gamma 1) \end{pmatrix}$$

In[151]:=

```
Print["Correlation Matrix for Noisy Two-Qubit State"]
Tρ = {
{Tr[ρNoise.KroneckerProduct[σx, σx]],
      Tr[ρNoise.KroneckerProduct[σx, σy]], Tr[ρNoise.KroneckerProduct[σx, σz]]},
{Tr[ρNoise.KroneckerProduct[σy, σx]],
      Tr[ρNoise.KroneckerProduct[σy, σy]], Tr[ρNoise.KroneckerProduct[σy, σz]]},
{Tr[ρNoise.KroneckerProduct[σz, σx]],
      Tr[ρNoise.KroneckerProduct[σz, σy]], Tr[ρNoise.KroneckerProduct[σz, σz]]}
} // TrigToExp // FullSimplify;
Tρ // MatrixForm
```

```
Correlation Matrix for Noisy Two-Qubit State
```

Out[153]//MatrixForm=

$$\left(\begin{array}{ll} \sqrt{1-\gamma1}\ \sqrt{1-\gamma2}\ (\mathrm{Cos}[\theta]\ \mathrm{Cos}[\phi]\ \mathrm{Cos}[\omega]-\mathrm{Sin}[\phi]\ \mathrm{Sin}[\omega]) & \sqrt{1-\gamma1}\ \sqrt{1-\gamma2}\ (\mathrm{Cos}[\theta]\ \mathrm{Cos}[\omega]\ \mathrm{Sin}[\phi]+\mathrm{Cos}[\phi]\ \mathrm{S} \\ \sqrt{1-\gamma1}\ \sqrt{1-\gamma2}\ (\mathrm{Cos}[\omega]\ \mathrm{Sin}[\phi]+\mathrm{Cos}[\theta]\ \mathrm{Cos}[\phi]\ \mathrm{Sin}[\omega]) & \sqrt{1-\gamma1}\ \sqrt{1-\gamma2}\ (-\mathrm{Cos}[\phi]\ \mathrm{Cos}[\omega]+\mathrm{Cos}[\theta]\ \mathrm{Sin}[\phi] \\ (-1+\gamma1)\ \sqrt{1-\gamma2}\ \mathrm{Cos}[\phi]\ \mathrm{Sin}[\theta] & (-1+\gamma1)\ \sqrt{1-\gamma2}\ \mathrm{Sin}[\theta]\ \mathrm{Sin}[\phi] \end{array}\right.$$

```
Uρ = Tρᵀ.Tρ // FullSimplify;
Uρ // MatrixForm
```

Out[46]//MatrixForm=

$$\left(\begin{array}{ll} \frac{1}{4}\ (-1+\gamma1)\ (-1+\gamma2)\ \left(4-\gamma1+\gamma1\ \mathrm{Cos}[2\ \theta]-2\ \gamma1\ \mathrm{Cos}[2\ \phi]\ \mathrm{Sin}[\theta]^2\right) & -((-1+\gamma1)\ \gamma1\ (-1+\gamma2)\ \mathrm{Cos}[\phi]\ \mathrm{Si} \\ -((-1+\gamma1)\ \gamma1\ (-1+\gamma2)\ \mathrm{Cos}[\phi]\ \mathrm{Sin}[\theta]^2\ \mathrm{Sin}[\phi]) & \frac{1}{4}\ (-1+\gamma1)\ (-1+\gamma2)\ \left(4-\gamma1+\gamma1\ \mathrm{Cos}[2\ \theta]+\right. \\ (-1+\gamma1)\ \gamma1\ \sqrt{1-\gamma2}\ (\gamma2+(-1+\gamma2)\ \mathrm{Cos}[\theta])\ \mathrm{Cos}[\phi]\ \mathrm{Sin}[\theta] & (-1+\gamma1)\ \gamma1\ \sqrt{1-\gamma2}\ (\gamma2+(-1+\gamma2)\ \mathrm{Co} \end{array}\right.$$

In[154]:=

```
Print["Eigenvalues of Uρ"]
evalsUρ = Eigenvalues[Uρ] // FullSimplify;
evalsUρ[[1]]
evalsUρ[[2]]
evalsUρ[[3]]
```

Eigenvalues of Uρ

Out[156]=

$$(-1+\gamma 1)(-1+\gamma 2)$$

Out[157]=

$$\frac{1}{4}\Big(2(-2+\gamma 2)(-1+\gamma 2)-\gamma 1(-1+\gamma 2)(-6+3\gamma 2+4\gamma 2\,\mathrm{Cos}[\theta]+\gamma 2\,\mathrm{Cos}[2\theta])+\gamma 1^2(2+3(-1+\gamma 2)\gamma 2+(-1+\gamma 2)\gamma 2(4\,\mathrm{Cos}[\theta]+\mathrm{Cos}[2\theta]))-\frac{1}{2}\sqrt{\Big(-64(-1+\gamma 1)(-1+\gamma 2)((-1+\gamma 1)(-1+\gamma 2)+\gamma 1\,\gamma 2\,\mathrm{Cos}[\theta])^2+4\big(4+2(-3+\gamma 1)\gamma 1-6\gamma 2-3(-3+\gamma 1)\gamma 1\,\gamma 2+(2+3(-1+\gamma 1)\gamma 1)\gamma 2^2+(-1+\gamma 1)\gamma 1(-1+\gamma 2)\gamma 2(4\,\mathrm{Cos}[\theta]+\mathrm{Cos}[2\theta])\big)^2\Big)}\Big)$$

Out[158]=

$$\frac{1}{4}\Big(2(-2+\gamma 2)(-1+\gamma 2)-\gamma 1(-1+\gamma 2)(-6+3\gamma 2+4\gamma 2\,\mathrm{Cos}[\theta]+\gamma 2\,\mathrm{Cos}[2\theta])+\gamma 1^2(2+3(-1+\gamma 2)\gamma 2+(-1+\gamma 2)\gamma 2(4\,\mathrm{Cos}[\theta]+\mathrm{Cos}[2\theta]))+\frac{1}{2}\sqrt{\Big(-64(-1+\gamma 1)(-1+\gamma 2)((-1+\gamma 1)(-1+\gamma 2)+\gamma 1\,\gamma 2\,\mathrm{Cos}[\theta])^2+4\big(4+2(-3+\gamma 1)\gamma 1-6\gamma 2-3(-3+\gamma 1)\gamma 1\,\gamma 2+(2+3(-1+\gamma 1)\gamma 1)\gamma 2^2+(-1+\gamma 1)\gamma 1(-1+\gamma 2)\gamma 2(4\,\mathrm{Cos}[\theta]+\mathrm{Cos}[2\theta])\big)^2\Big)}\Big)$$

In[51]:=

```
(*No CHSH violations are found beyond the critical noise threshold*)
Maximize[{evalsUρ[[3]] + evalsUρ[[1]],
  0 ≤ (1 - γ1) * (1 - γ2) ≤ 0.5, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[2]] + evalsUρ[[1]],
  0 ≤ (1 - γ1) * (1 - γ2) ≤ 0.5, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[3]] + evalsUρ[[2]],
  0 ≤ (1 - γ1) * (1 - γ2) ≤ 0.5, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
```

Out[51]= {1., {γ1 → 1., γ2 → 1., θ → 0.}}

Out[52]= {0.87868, {γ1 → 0.292893, γ2 → 0.292893, θ → 5.13947}}

Out[53]= {1., {γ1 → 1., γ2 → 1., θ → 0.}}

In[54]:=
```
(*Besides the γ1=γ2=1 point,
M(ρNoise)=1 is maximum found along the critical noise threshold*)
Maximize[{evalsUρ[[3]] + evalsUρ[[1]],
  0.1 ≤ (1 - γ1) * (1 - γ2) ≤ 0.5, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[2]] + evalsUρ[[1]],
  0.1 ≤ (1 - γ1) * (1 - γ2) ≤ 0.5, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[3]] + evalsUρ[[2]],
  0.1 ≤ (1 - γ1) * (1 - γ2) ≤ 0.5, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
```

Out[54]= $\{1., \{\gamma1 \to 0.0605868, \gamma2 \to 0.467753, \theta \to 1.517 \times 10^{-7}\}\}$

Out[55]= $\{0.87868, \{\gamma1 \to 0.292893, \gamma2 \to 0.292893, \theta \to 1.14372\}\}$

Out[56]= $\{0.843146, \{\gamma1 \to 0.292893, \gamma2 \to 0.292893, \theta \to 7.65125 \times 10^{-8}\}\}$

In[57]:=
```
(*Extending slightly beyond the critical boundary allows for nonlocality*)
Maximize[{evalsUρ[[3]] + evalsUρ[[1]],
  0.1 ≤ (1 - γ1) * (1 - γ2) ≤ 0.51, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[2]] + evalsUρ[[1]],
  0.1 ≤ (1 - γ1) * (1 - γ2) ≤ 0.51, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[3]] + evalsUρ[[2]],
  0.1 ≤ (1 - γ1) * (1 - γ2) ≤ 0.51, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
```

Out[57]= $\{1.02, \{\gamma1 \to 0.195036, \gamma2 \to 0.366432, \theta \to 1.64578 \times 10^{-7}\}\}$

Out[58]= $\{0.897571, \{\gamma1 \to 0.285857, \gamma2 \to 0.285857, \theta \to 1.15897\}\}$

Out[59]= $\{0.860126, \{\gamma1 \to 0.285857, \gamma2 \to 0.285857, \theta \to 1.73622 \times 10^{-7}\}\}$

In[78]:=
```
(*In the range γ1,γ2 in [0,0.5] no arbitrary maximally
 entangled state performs better than the Bell state.
In this range, the Bell state's sum of maximal eigenvalues is 2*(1-γ1)*(1-γ2)*)
Maximize[{evalsUρ[[3]] + evalsUρ[[1]] - 2 * (1 - γ1) * (1 - γ2),
  0.5 ≤ (1 - γ1) * (1 - γ2) ≤ 1, 0 ≤ γ1 ≤ 0.5, 0 ≤ γ2 ≤ 0.5, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[2]] + evalsUρ[[1]] - 2 * (1 - γ1) * (1 - γ2),
  0.5 ≤ (1 - γ1) * (1 - γ2) ≤ 1, 0 ≤ γ1 ≤ 0.5, 0 ≤ γ2 ≤ 0.5, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[3]] + evalsUρ[[2]] - 2 * (1 - γ1) * (1 - γ2),
  0.5 ≤ (1 - γ1) * (1 - γ2) ≤ 1, 0 ≤ γ1 ≤ 0.5, 0 ≤ γ2 ≤ 0.5, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
```

Out[78]= $\{1.88738 \times 10^{-15}, \{\gamma1 \to 0.229651, \gamma2 \to 0.0919844, \theta \to 8.34177 \times 10^{-8}\}\}$

Out[79]= $\{0., \{\gamma1 \to 0., \gamma2 \to 0., \theta \to 4.70961\}\}$

Out[80]= $\{0., \{\gamma1 \to 0., \gamma2 \to 0., \theta \to 3.65059 \times 10^{-6}\}\}$

In[81]:=

```
(*In the range γ1,γ2 in [0,1] there exists arbitrary maximally
 entangled states that perform better than the Bell state,
but these stated do not violate the CHSH inequality.*)
Maximize[
 {evalsUρ[[3]] + evalsUρ[[1]] - (1 - γ1) * (1 - γ2) - Max[(1 - γ1) * (1 - γ2), (1 - γ1 - γ2 + 2 * γ1 * γ2)^2],
  0 ≤ (1 - γ1) * (1 - γ2) ≤ 1.0, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[
 {evalsUρ[[2]] + evalsUρ[[1]] - (1 - γ1) * (1 - γ2) - Max[(1 - γ1) * (1 - γ2), (1 - γ1 - γ2 + 2 * γ1 * γ2)^2],
  0 ≤ (1 - γ1) * (1 - γ2) ≤ 1.0, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[
 {evalsUρ[[3]] + evalsUρ[[2]] - (1 - γ1) * (1 - γ2) - Max[(1 - γ1) * (1 - γ2), (1 - γ1 - γ2 + 2 * γ1 * γ2)^2],
  0 ≤ (1 - γ1) * (1 - γ2) ≤ 1.0, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
```

Out[81]= $\{0.0274935, \{\gamma 1 \to 0.267255, \gamma 2 \to 0.841128, \theta \to 4.77277\}\}$

NMaximize: The function value $1.97397 \times 10^{-10} + 6.53379 \times 10^{-10}\, i$ is not a real number at $\{\gamma 1, \gamma 2, \theta\}$ = $\{0.994678, 0.0658896, 0.\}$.

Out[82]= $\{-7.0157 \times 10^{-9}, \{\gamma 1 \to 0.994686, \gamma 2 \to 0.0658438, \theta \to 1.21651 \times 10^{-6}\}\}$

Out[83]= $\{3.19189 \times 10^{-16}, \{\gamma 1 \to 0.990223, \gamma 2 \to 0.208649, \theta \to 7.7204 \times 10^{-10}\}\}$

In[75]:=

```
(*Along the CHSH-Breaking boundary the maximimum value is M(ρNoise)=1.
That is, arbitrary entangled states do not improve the violation
 of the CHSH inequality along the critical noise threshold.*)
Maximize[{evalsUρ[[3]] + evalsUρ[[1]],
  (1 - γ1) * (1 - γ2) == 1 / 2, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[2]] + evalsUρ[[1]],
  (1 - γ1) * (1 - γ2) == 1 / 2, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
Maximize[{evalsUρ[[3]] + evalsUρ[[2]],
  (1 - γ1) * (1 - γ2) == 1 / 2, 0 ≤ γ1 ≤ 1, 0 ≤ γ2 ≤ 1, 0 ≤ θ ≤ 2 * π}, {γ1, γ2, θ}]
```

Out[75]= $\left\{1, \left\{\gamma 1 \to \frac{11}{27}, \gamma 2 \to \frac{5}{32}, \theta \to 0\right\}\right\}$

Out[76]= $\left\{-\frac{3}{2}\left(-2+\sqrt{2}\right), \left\{\gamma 1 \to \frac{-\frac{1}{2}+\frac{1}{2}\left(2-\sqrt{2}\right)}{-1+\frac{1}{2}\left(2-\sqrt{2}\right)}, \gamma 2 \to \frac{1}{2}\left(2-\sqrt{2}\right), \theta \to 2\pi + 2\,\mathrm{ArcTan}[-0.644\ldots]\right\}\right\}$

Out[77]= $\left\{\frac{1}{2}\left(13-8\sqrt{2}\right), \left\{\gamma 1 \to \frac{-\frac{1}{2}+\frac{1}{2}\left(2-\sqrt{2}\right)}{-1+\frac{1}{2}\left(2-\sqrt{2}\right)}, \gamma 2 \to \frac{1}{2}\left(2-\sqrt{2}\right), \theta \to 0\right\}\right\}$