

Hand-written Mathematical Formula Recognition Using Machine Learning

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Motivation

- Still an unsolved research topic. Even though number and character recognition systems are very mature and have human like accuracies.
- Great improvement for existing note taking apps like OneNote and Evernote. Existing handwriting recognition works great for characters but struggles with math formulae.
- Personal Motivation
 - Try to solve a real-world research problem.
 - Learn Machine Learning and apply it on a real-world problem.
 - Learn Tensorflow

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Purpose

- Solve the research problem of Mathematical formula recognition.
- Nice add-on feature to various note taking applications like OneNote and Evernote
- Extract formulae from various math related research and other documents where these formulae are in images.

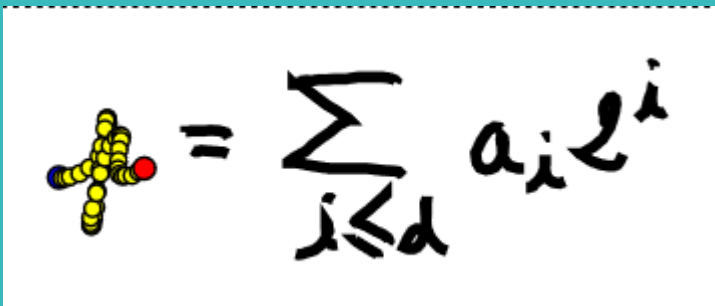
Problem Statement

- Given a formula with its corresponding stroke data, the goal of this project is to develop a system that will identify the formula and output the result in a standard text format like Latex, MathML etc.
- Alternative input for the system could be an image representation of the formula.

Background

- **INKML:** The input data format. The InkML format enables to make references between the digital ink of the expression, its segmentation into symbols and its MathML representation.
- **MATHML:** A standard (XML) format to represent a mathematical formula.
- **TensorFlow:** TensorFlow is a machine learning system that operates at large scale and in heterogeneous environments. Makes it easier to implementing Machine Learning algorithms based in existing libraries in the framework.
- **CROHME:** Competition on Recognition of Online Handwritten Mathematical Expressions

INKML vs MATHML


$$\text{stick figure} = \sum_{i \leq d} a_i 2^i$$

```
<annotationXML type="truth" encoding="Content-MathML">
  <math xmlns='http://www.w3.org/1998/Math/MathML'>
    <mrow>
      <mi xml:id="p_1">p</mi>
      <mrow>
        <mo xml:id="_1">=</mo>
        <mrow>
          <munder>
            <mo xml:id="\sum_1">\sum</mo>
            <mrow>
              <mi xml:id="i_1">i</mi>
              <mrow>
                <mo xml:id="\leq_1">\leq</mo>
                <mi xml:id="d_1">d</mi>
              </mrow>
            </mrow>
          </munder>
          <mrow>
            <msub>
              <mi xml:id="a_1">a</mi>
              <mi xml:id="i_2">i</mi>
            </msub>
            <msup>
              <mn xml:id="2_1">2</mn>
              <mi xml:id="i_3">i</mi>
            </msup>
          </mrow>
        </mrow>
      </mrow>
    </math>
  </annotationXML>
```

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Related Work

- The systems that participated in the original CROHME competitions will serve as our baseline systems to compare our results against.
- (Lu, C., et al. 2015) and (Zhang, J., et al. 2017)
- Our solution is an amalgamation of various ideas and develop a system based on modern Machine Learning frameworks and tools.
- Based on the existing literature we'll break down the problem into two sub-tasks
 - Symbol Recognition
 - Structure Recognition
- Multiple Sub-systems solve specific smaller tasks working in tandem to tackle the overall problem statement.

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Approach

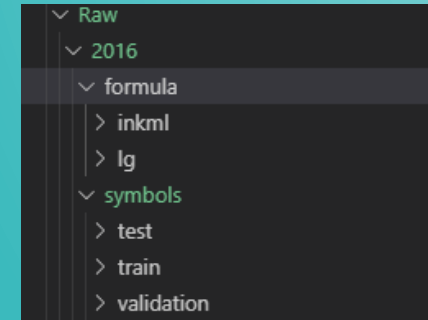
- Provide a brief sentence or two regarding how you approached the project. For example, was this a quantitative or qualitative project? Was a computer program written? Was this a research project of an existing topic?
- We'll use an iterative approach to build the systems and subsystems.
- We'll start with a quick and trivial implementation.
- Evaluate the performance, and make improvements as required
- The focus of the project will not be in trying to implement niche machine learning algorithms but instead to leverage the framework to its fullest capacity and put together a system that would compete and/or exceed the baseline performances set previously

Data Collection

- CROHME has been collecting the data by various methods and has this data available in a W3C standard format (InkML).
- We'll use this data as-is as our raw input data.
- We'll process the data to the right format as per the needs of our Machine Learning framework.

Data Collection

- Manual Data Organization
 - Organizing the raw data in the file structure in an optimal format.
 - Removed redundant data and settled on a simple file structure.
- Manual Data Validation:
 - Validated file counts from original dataset.
 - Randomly validated ~100 data points to verify the ground truth is visually similar to the stroke data.
- Manual Data Visualization:
 - Used InkML viewer to visualize provided ink data.
- Automated Data Validation:
 - Python scripts to validate data.
 - Run on each pull-request in the repository.
- Automated Data Visualization
 - Python tool to display multiple InkML files in grid format for quick and easier visual analysis.

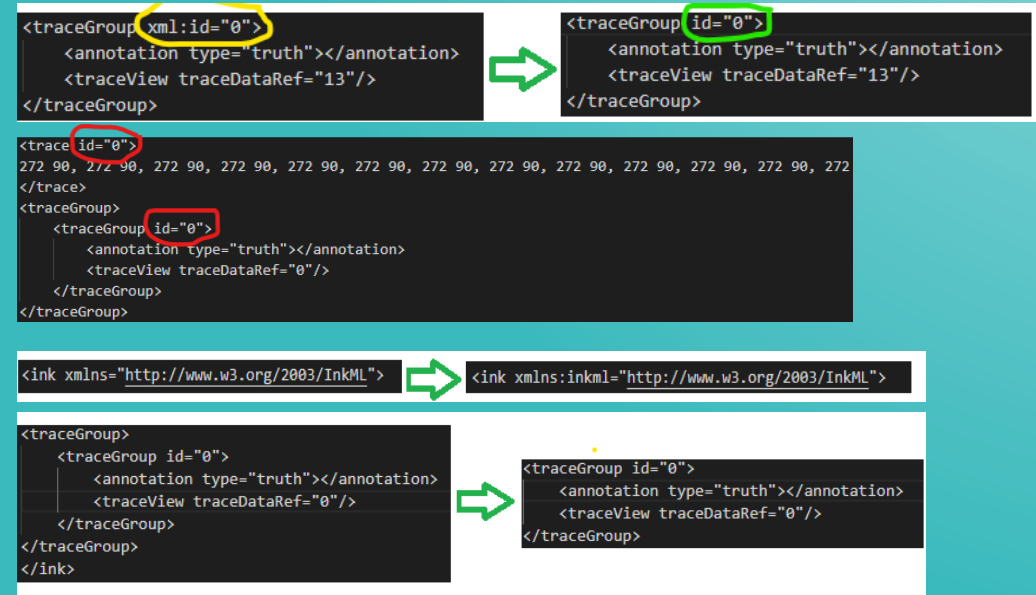


Data Analysis

- Initial visual ad-hoc analysis performed with a tool provided by CHROME. InkmlViewer
- Custom python scripts to perform bulk views and analysis.
- This was useful in to validate the correctness of the pre-processing steps performed on the data.
- Data and Models stored in MongoDB for offline analysis.

Data Cleansing

- Several issues addressed in the initial dataset.
 - “xml:id” issue
 - non-unique id attributes
 - invalid namespace
 - Redundant nesting
- Python scripts to cleanse the datasets.
- Python scripts to validate the datasets.



Data Storage

- Processed dataset stored in MongoDB for quick retrieval and advanced query capability.
- Indexes on columns for faster querying.
- Added the ability to prune datasets by soft deleting certain datasets.
- Python scripts use the pyMongo client library to retrieve data from the DB to create Machine Learning models.

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handwritingdata.Symbols

Documents Aggregations Explain Plan Indexes

FILTER {truth : { \$ne: "junk" } && { \$ne: "!" } }

INSERT DOCUMENT VIEW LIST TABLE

```

_id: "IVC_2014_FS14_E329_10219"
ui: "IVC_2014_FS14_E329_10219"
type: "test"
truth: ""
traces: Binary('gA2jbnvtchkuY29yZS5tdix8awFycmF5C19yZmVibnN0cnVjdApwAGNudwIweGpuZGFycmF5CnEBSWFCQzJX2hVZGVjcWp1bmVw...')
stroke_count: 1
soft_delete: false

_id: "IVC_2014_FS14_E331_10248"
ui: "IVC_2014_FS14_E331_10248"
type: "test"
truth: ""
traces: Binary('gA2jbnvtchkuY29yZS5tdix8awFycmF5C19yZmVibnN0cnVjdApwAGNudwIweGpuZGFycmF5CnEBSWFCQzJX2hVZGVjcWp1bmVw...')
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soft_delete: false

_id: "IVC_2014_FS14_E331_10252"
ui: "IVC_2014_FS14_E331_10252"
type: "test"
truth: ""
traces: Binary('gA2jbnvtchkuY29yZS5tdix8awFycmF5C19yZmVibnN0cnVjdApwAGNudwIweGpuZGFycmF5CnEBSWFCQzJX2hVZGVjcWp1bmVw...')
stroke_count: 1
soft_delete: false

_id: "IVC_2014_FS14_E334_10300"
ui: "IVC_2014_FS14_E334_10300"
type: "test"
truth: ""
traces: Binary('gA2jbnvtchkuY29yZS5tdix8awFycmF5C19yZmVibnN0cnVjdApwAGNudwIweGpuZGFycmF5CnEBSWFCQzJX2hVZGVjcWp1bmVw...')
stroke_count: 1
soft_delete: false

```

handwritingdata.Symbols

Documents Aggregations Explain Plan **Indexes**

CREATE INDEX

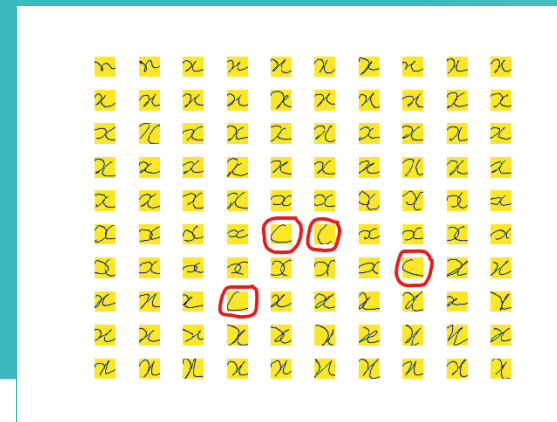
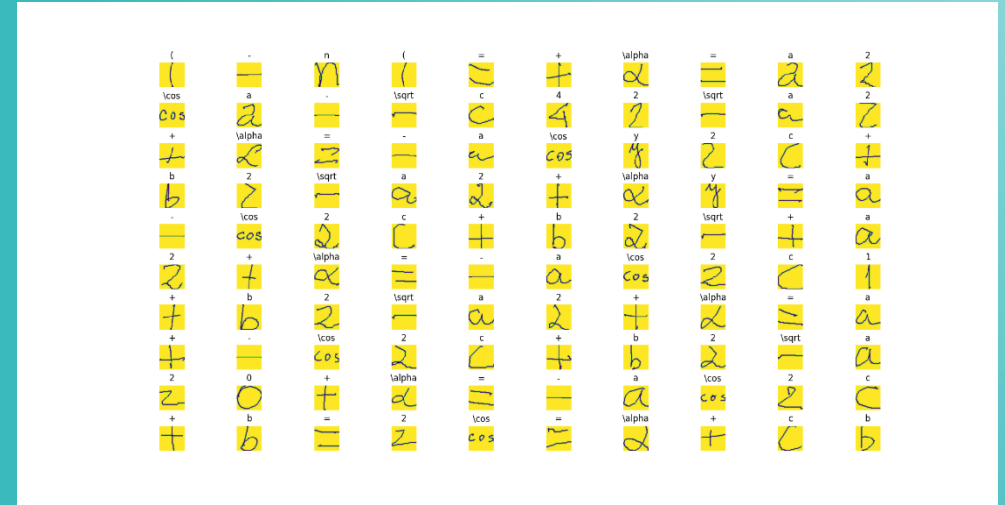
DOCUMENTS 191.0k TOTAL SIZE 941.9MB AVG. SIZE 5.0KB INDEXES 5 TOTAL SIZE 6.5MB AVG. SIZE 1.3MB

Name and Definition ^	Type	Size	Usage	Properties	Drop
GroundTruthIndex truth	REGULAR	970.8 KB	38 since Mon Sep 02 2019		
_id _id	REGULAR	3.1 MB	28 since Sun Sep 01 2019	UNIQUE	
soft_delete_1 soft_delete type	REGULAR	876.5 KB	80 since Tue Sep 03 2019	COMPOUND	
soft_delete_truth soft_delete truth	REGULAR	938.0 KB	21 since Tue Sep 03 2019	COMPOUND	
type_1 type	REGULAR	909.3 KB	1 since Mon Sep 02 2019		

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Findings - Visualizing the data

- Python's matplotlib and custom python scripts to view the data.
- Training data may not be clean. For example: datapoints circled in red.
- The system could be robust to handle these outliers due to the large dataset size. Example (5042 training samples for x)
- Worst performing symbols could be ones with low number of training samples. These will need to be looked up closer and prune the training data if necessary.



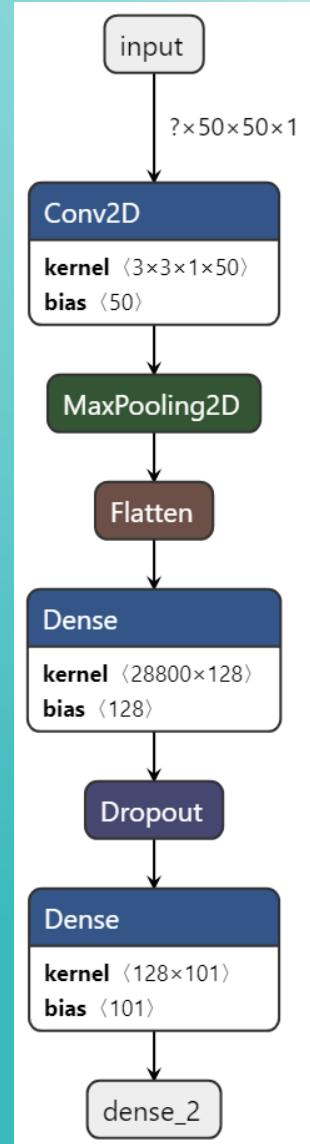
Findings – Generating Models

- Keras to model the Neural Networks.
 - High-level neural network API.
 - Simplifies Neural Network creation.
- Tensorflow as backend for Keras.
- GPU vs CPU
 - Training performed on Intel Core i7 4710MQ (CPU) vs NVIDIA GeForce GTX 880M (GPU)
 - GPU Performance: 975us/step or ~1024steps/sec
 - CPU Performance: 8ms/step or ~125steps/sec

```
model = Sequential()

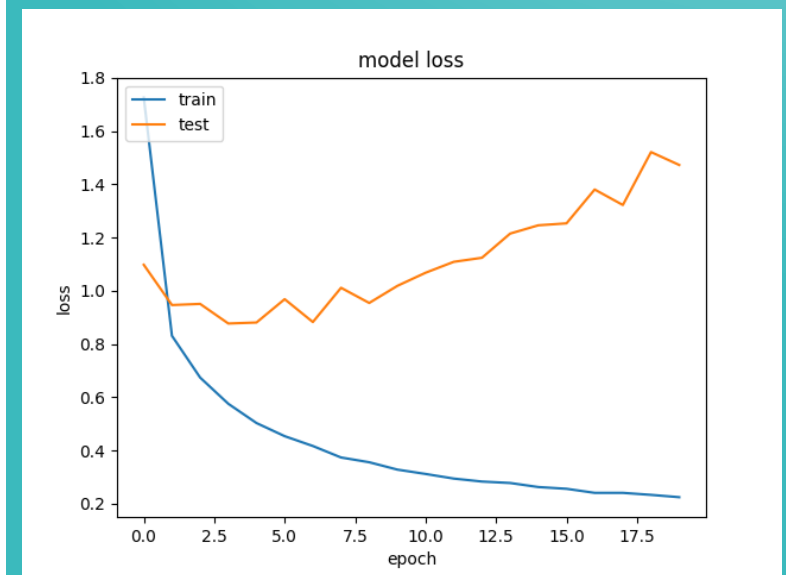
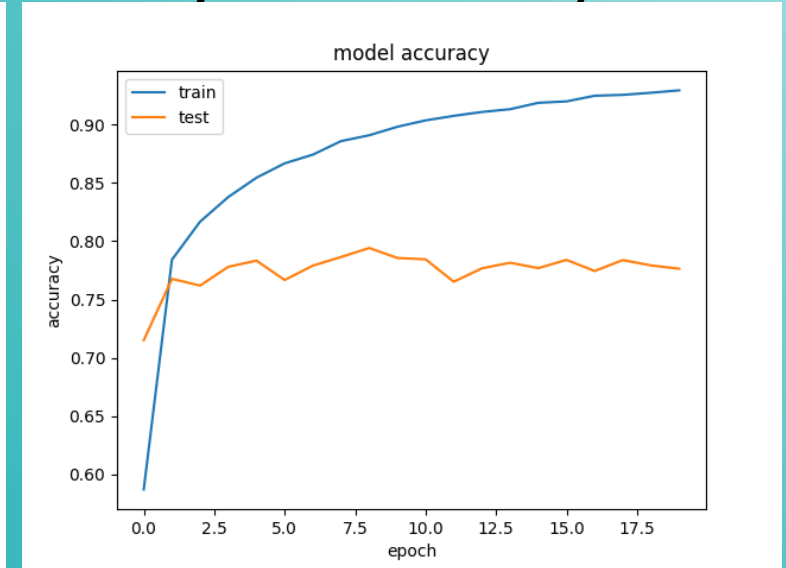
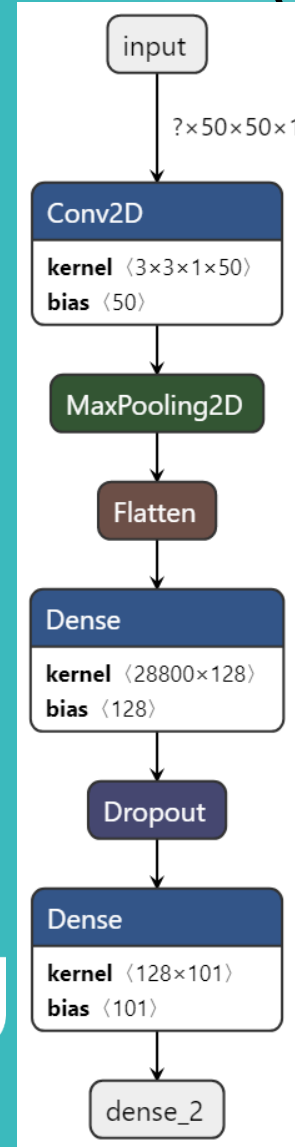
model.add(Conv2D(50, kernel_size=(3,3), input_shape=symbol_input_shape))
model.add(MaxPooling2D(pool_size=(2, 2)))
model.add(Flatten()) # Flattening the 2D arrays for fully connected layers
model.add(Dense(128, activation=tf.nn.relu))
model.add(Dropout(0.2))
model.add(Dense(101, activation=tf.nn.softmax))

model.compile(optimizer='adam',
              loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
```



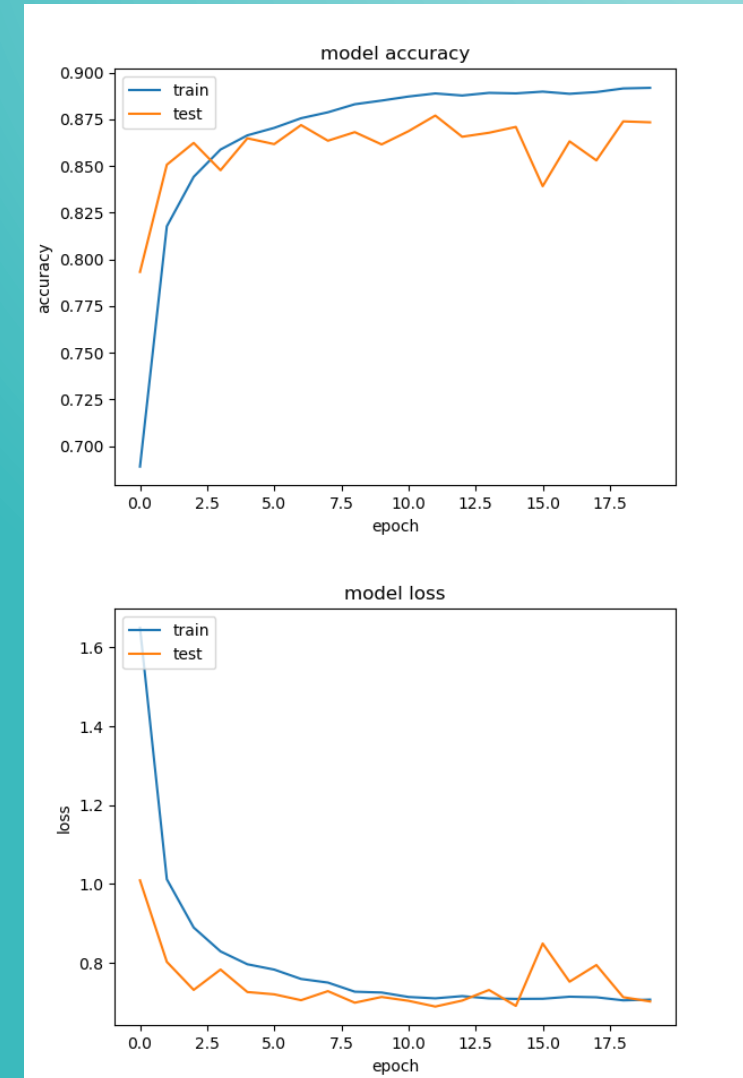
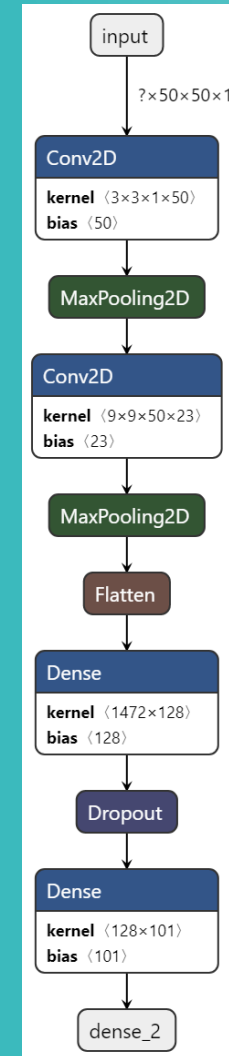
Findings – Model Performance (1-Layer CNN)

- Performance on a Convolution Neural Network (CNN).
- Performance on Validation and Test datasets going flat at ~80%
- Model is overfitting the training data and unable to generalize.
- Clearly visible from increase in the loss function on the validation data while the training loss is going down.
- Next steps:
 - Tune other parameters like kernel sizes in Conv2D and first dense layer.
 - Look into the top error labels. Model could be having issues differentiating similar mathematical symbols like '/' vs '|' or '0' vs 'o'.
 - Tune other parameters like optimizer and loss functions or the dropout factor.
 - Compare the performance against the baseline systems' performances.



Findings – Model Performance (2-Layer CNN)

- ~82% validation and test accuracy on 1-layer even with tuning hyper parameters
- Validation loss always increasing after ~6 epochs.
- Extend the CNN with multiple convolutional layers.
- ~88% validation and test accuracy. Comparable to baseline models.
- Model loss on validation set clearly contained.
- Model is generalizing better



Findings – Formula segmentation

- Open CV contour API
- Can handle spatial relations with ease
- Can iteratively implement complex relations like subscripts, superscripts, fractions and square roots.
- Simple linear model is currently assumed.

Conclusion

- Designed a Machine Learning system end-to-end on commercial hardware
- Dedicated hardware is useful to improve performance
- Simple CNNs match performance of existing baseline systems
- Further scope for improvement with more complex CNN architecture
- Valuable data collected for further analysis on understanding the hyper parameters.

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Future Work

- Improve Symbol Recognition:
 - Further scope to improve the performance of symbol recognition
 - Complex CNNs could improve performance to ~95%
 - Multiple predictions per symbol for visually similar symbols like x and \times
- Formula Segmentation:
 - Current implementation is trivial and assumes linear structure
 - Wide variety of formula un-supported. Ex: polynomial equations
 - Add support for complex spatial relations in symbols. Example: Superscripts, subscripts, fractions and square roots.
- Demo tool:
 - Real test for the model.
 - Establish a feedback loop to drive future improvements.