

**CS 541: Artificial intelligence, Winter 2023**  
**Programming Assignment #1**

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For solving the N puzzle problem, where n=8 and 15 the following algorithms have been used - Best First search and A\* search algorithms. Manhattan Distance, Misplaced Tiles and Euclidean Distance are the three heuristics being used. Goal State is [1, 2, 3, 4, 5, 6, 7, 8, 'b']. Five different initial states are considered and the solution paths are printed for each input in reverse order.

**Steps to run the code:**

Run the attached python file with below updates as needed

**To change the input state and heuristics for 8 puzzle problem:**

- Modify the highlighted lines in the below line of code located towards the end of the python file
- Please provide one heuristic at a time by commenting the other two heuristics like shown below

```
#Input for 8 puzzle
input = ['b',1,3,4,5,8,2,6,7]
# Input for 15 puzzle
#input = [2,1,4,5,6,8,3,9,10,7,14,12,13,'b',15,11]
print("Initial state is:", input)
#Goal state for 8 puzzle
goal = [1,2,3,4,5,6,7,8,'b']
#Goal state for 15 puzzle
#goal = [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,'b']
print("Goal state is:", goal)
current_board = Puzzle(input, goal)
#heuristic = "manhattanDistance"
#heuristic = "euclideanDistance"
heuristic = "misplacedTiles"
```

**To change the input state and heuristics for 15 puzzle problem:**

- Comment out the 8 puzzle input and uncomment the 15 puzzle input
- Comment the 8 puzzle goal and uncomment the 15 puzzle goal
- Please provide one heuristic at a time by commenting the other two heuristics like shown below.

```
#Input for 8 puzzle
#input = ['b',1,3,4,5,8,2,6,7]
# Input for 15 puzzle
input = [1, 2, 3, 'b', 5, 6, 8, 4, 9, 10, 7, 11, 13, 14, 15, 12]
print("Initial state is:", input)
#Goal state for 8 puzzle
#goal = [1,2,3,4,5,6,7,8,'b']
#Goal state for 15 puzzle
goal = [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,'b']
print("Goal state is:", goal)
current_board = Puzzle(input, goal)
#heuristic = "manhattanDistance"
```

#heuristic = "euclideanDistance"

heuristic = "misplacedTiles"

### 8-puzzleProblem:

### Best-First-Search:

Number of trials = 5

Heuristic 1: Manhattan Distance	Heuristic 2: Euclidean Distance	Heuristic 3: Misplaced Tiles
Initial State 5: [4, 2, 3, 1, 5, 7, 6, 'b', 8] Solution Path:  [1, 2, 3, 4, 5, 6, 7, 8, 'b'] [1, 2, 3, 4, 5, 'b', 7, 8, 6] [1, 2, 'b', 4, 5, 3, 7, 8, 6] [1, 'b', 2, 4, 5, 3, 7, 8, 6] ['b', 1, 2, 4, 5, 3, 7, 8, 6] [4, 1, 2, 'b', 5, 3, 7, 8, 6] [4, 1, 2, 5, 'b', 3, 7, 8, 6] [4, 'b', 2, 5, 1, 3, 7, 8, 6] [4, 2, 'b', 5, 1, 3, 7, 8, 6] [4, 2, 3, 5, 1, 'b', 7, 8, 6] [4, 2, 3, 5, 1, 6, 7, 8, 'b'] [4, 2, 3, 5, 1, 6, 7, 'b', 8] [4, 2, 3, 5, 'b', 6, 7, 1, 8] [4, 2, 3, 'b', 5, 6, 7, 1, 8] [4, 2, 3, 7, 5, 6, 'b', 1, 8] [4, 2, 3, 7, 5, 6, 1, 'b', 8] [4, 2, 3, 7, 5, 6, 1, 8, 'b'] [4, 2, 3, 7, 5, 'b', 1, 8, 6] [4, 2, 3, 7, 'b', 5, 1, 8, 6] [4, 2, 3, 'b', 7, 5, 1, 8, 6] [4, 2, 3, 1, 7, 5, 'b', 8, 6] [4, 2, 3, 1, 7, 5, 8, 'b', 6] [4, 2, 3, 1, 'b', 5, 8, 7, 6] [4, 2, 3, 1, 5, 'b', 8, 7, 6] [4, 2, 3, 1, 5, 6, 8, 7, 'b'] [4, 2, 3, 1, 5, 6, 8, 'b', 7] [4, 2, 3, 1, 5, 6, 'b', 8, 7] [4, 2, 3, 'b', 5, 6, 1, 8, 7] [4, 2, 3, 5, 'b', 6, 1, 8, 7] [4, 2, 3, 5, 6, 'b', 1, 8, 7] [4, 2, 3, 5, 6, 7, 1, 8, 'b'] [4, 2, 3, 5, 6, 7, 1, 'b', 8] [4, 2, 3, 5, 'b', 7, 1, 6, 8] [4, 2, 3, 'b', 5, 7, 1, 6, 8] [4, 2, 3, 1, 5, 7, 'b', 6, 8]	Initial State 5: [4, 2, 3, 1, 5, 7, 6, 'b', 8] Solution Path:  [1, 2, 3, 4, 5, 6, 7, 8, 'b'] [1, 2, 3, 4, 5, 6, 7, 'b', 8] [1, 2, 3, 4, 'b', 6, 7, 5, 8] [1, 'b', 3, 4, 2, 6, 7, 5, 8] ['b', 1, 3, 4, 2, 6, 7, 5, 8] [4, 1, 3, 'b', 2, 6, 7, 5, 8] [4, 1, 3, 2, 'b', 6, 7, 5, 8] [4, 1, 3, 2, 5, 6, 7, 'b', 8] [4, 1, 3, 2, 5, 6, 'b', 7, 8] [4, 1, 3, 'b', 5, 6, 2, 7, 8] ['b', 1, 3, 4, 5, 6, 2, 7, 8] [1, 'b', 3, 4, 5, 6, 2, 7, 8] [1, 5, 3, 4, 'b', 6, 2, 7, 8] [1, 5, 3, 'b', 4, 6, 2, 7, 8] [1, 5, 3, 2, 4, 6, 'b', 7, 8] [1, 5, 3, 2, 4, 6, 7, 'b', 8] [1, 5, 3, 2, 'b', 6, 7, 4, 8] [1, 5, 3, 'b', 2, 6, 7, 4, 8] [1, 5, 3, 7, 2, 6, 'b', 4, 8] [1, 5, 3, 7, 2, 6, 4, 'b', 8] [1, 5, 3, 7, 2, 6, 4, 8, 'b'] [1, 5, 3, 7, 2, 'b', 4, 8, 6] [1, 5, 3, 7, 'b', 2, 4, 8, 6] [1, 5, 3, 'b', 7, 2, 4, 8, 6] [1, 5, 3, 4, 7, 2, 'b', 8, 6] [1, 5, 3, 4, 7, 2, 8, 'b', 6] [1, 5, 3, 4, 'b', 2, 8, 7, 6] [1, 5, 3, 4, 2, 'b', 8, 7, 6] [1, 5, 3, 4, 2, 6, 8, 7, 'b'] [1, 5, 3, 4, 2, 6, 8, 'b', 7] [1, 5, 3, 4, 2, 6, 'b', 8, 7] [1, 5, 3, 'b', 2, 6, 4, 8, 7] [1, 5, 3, 2, 'b', 6, 4, 8, 7] [1, 5, 3, 2, 6, 'b', 4, 8, 7] [1, 5, 3, 2, 6, 7, 4, 8, 'b'] [1, 5, 3, 2, 6, 7, 4, 'b', 8]	Initial State 5: [4, 2, 3, 1, 5, 7, 6, 'b', 8] Solution Path:  [1, 2, 3, 4, 5, 6, 7, 8, 'b'] [1, 2, 3, 4, 5, 6, 7, 'b', 8] [1, 2, 3, 4, 5, 6, 'b', 7, 8] [1, 2, 3, 'b', 5, 6, 4, 7, 8] [1, 2, 3, 5, 'b', 6, 4, 7, 8] [1, 2, 3, 5, 7, 6, 4, 'b', 8] [1, 2, 3, 5, 7, 6, 4, 8, 'b'] [1, 2, 3, 5, 7, 'b', 4, 8, 6] [1, 2, 3, 5, 'b', 7, 4, 8, 6] [1, 2, 3, 'b', 5, 7, 4, 8, 6] [1, 2, 3, 4, 5, 7, 'b', 8, 6] [1, 2, 3, 4, 5, 7, 8, 'b', 6] [1, 2, 3, 4, 'b', 7, 8, 5, 6] [1, 2, 3, 4, 7, 'b', 8, 5, 6] [1, 2, 3, 4, 7, 6, 8, 5, 'b'] [1, 2, 3, 4, 7, 6, 8, 'b', 5] [1, 2, 3, 4, 7, 6, 'b', 8, 5] [1, 2, 3, 'b', 7, 6, 4, 8, 5] ['b', 2, 3, 1, 7, 6, 4, 8, 5] [2, 'b', 3, 1, 7, 6, 4, 8, 5] [2, 7, 3, 1, 'b', 6, 4, 8, 5] [2, 7, 3, 'b', 1, 6, 4, 8, 5] [2, 7, 3, 4, 1, 6, 'b', 8, 5] [2, 7, 3, 4, 1, 6, 8, 'b', 5] [2, 7, 3, 4, 'b', 6, 8, 1, 5] [2, 'b', 3, 4, 7, 6, 8, 1, 5] ['b', 2, 3, 4, 7, 6, 8, 1, 5] [4, 2, 3, 'b', 7, 6, 8, 1, 5] [4, 2, 3, 7, 'b', 6, 8, 1, 5] [4, 2, 3, 7, 1, 6, 8, 'b', 5] [4, 2, 3, 7, 1, 6, 'b', 8, 5] [4, 2, 3, 'b', 1, 6, 7, 8, 5] [4, 2, 3, 1, 'b', 6, 7, 8, 5] [4, 2, 3, 1, 8, 6, 7, 'b', 5] [4, 2, 3, 1, 8, 6, 7, 5, 'b'] [4, 2, 3, 1, 8, 'b', 7, 5, 6]

[4, 2, 3, 1, 5, 7, 6, 'b', 8]

Steps: 36

Initial State: [6,2,3,8,5,7,4,'b',1]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
[1, 2, 3, 4, 5, 6, 7, 'b', 8]  
[1, 2, 3, 4, 5, 6, 'b', 7, 8]  
[1, 2, 3, 'b', 5, 6, 4, 7, 8]  
[1, 2, 3, 5, 'b', 6, 4, 7, 8]  
[1, 2, 3, 5, 7, 6, 4, 'b', 8]  
[1, 2, 3, 5, 7, 6, 4, 8, 'b']  
[1, 2, 3, 5, 7, 'b', 4, 8, 6]  
[1, 2, 3, 5, 'b', 7, 4, 8, 6]  
[1, 2, 3, 'b', 5, 7, 4, 8, 6]  
[1, 2, 3, 4, 5, 7, 'b', 8, 6]  
[1, 2, 3, 4, 5, 7, 8, 'b', 6]  
[1, 2, 3, 4, 'b', 7, 8, 5, 6]  
[1, 2, 3, 4, 7, 'b', 8, 5, 6]  
[1, 2, 3, 4, 7, 6, 8, 5, 'b']  
[1, 2, 3, 4, 7, 6, 8, 'b', 5]  
[1, 2, 3, 4, 'b', 6, 8, 7, 5]  
[1, 2, 3, 'b', 4, 6, 8, 7, 5]  
['b', 2, 3, 1, 4, 6, 8, 7, 5]  
[2, 'b', 3, 1, 4, 6, 8, 7, 5]  
[2, 4, 3, 1, 'b', 6, 8, 7, 5]  
[2, 4, 3, 1, 6, 'b', 8, 7, 5]  
[2, 4, 3, 1, 6, 5, 8, 7, 'b']  
[2, 4, 3, 1, 6, 5, 8, 'b', 7]  
[2, 4, 3, 1, 6, 5, 'b', 8, 7]  
[2, 4, 3, 'b', 6, 5, 1, 8, 7]  
[2, 4, 3, 6, 'b', 5, 1, 8, 7]  
[2, 'b', 3, 6, 4, 5, 1, 8, 7]  
['b', 2, 3, 6, 4, 5, 1, 8, 7]  
[6, 2, 3, 'b', 4, 5, 1, 8, 7]  
[6, 2, 3, 4, 'b', 5, 1, 8, 7]  
[6, 2, 3, 4, 8, 5, 1, 'b', 7]  
[6, 2, 3, 4, 8, 5, 'b', 1, 7]  
[6, 2, 3, 'b', 8, 5, 4, 1, 7]  
[6, 2, 3, 8, 'b', 5, 4, 1, 7]  
[6, 2, 3, 8, 5, 'b', 4, 1, 7]

[1, 5, 3, 2, 'b', 7, 4, 6, 8]  
[1, 'b', 3, 2, 5, 7, 4, 6, 8]  
['b', 1, 3, 2, 5, 7, 4, 6, 8]  
[2, 1, 3, 'b', 5, 7, 4, 6, 8]  
[2, 1, 3, 4, 5, 7, 'b', 6, 8]  
[2, 1, 3, 4, 5, 7, 6, 'b', 8]  
[2, 1, 3, 4, 5, 7, 6, 8, 'b']  
[2, 1, 3, 4, 5, 'b', 6, 8, 7]  
[2, 1, 3, 4, 'b', 5, 6, 8, 7]  
[2, 'b', 3, 4, 1, 5, 6, 8, 7]  
['b', 2, 3, 4, 1, 5, 6, 8, 7]  
[4, 2, 3, 'b', 1, 5, 6, 8, 7]  
[4, 2, 3, 1, 'b', 5, 6, 8, 7]  
[4, 2, 3, 1, 5, 'b', 6, 8, 7]  
[4, 2, 3, 1, 5, 7, 6, 8, 'b']  
[4, 2, 3, 1, 5, 7, 6, 'b', 8]

steps: 52

Initial State: [6,2,3,8,5,7,4,'b',1]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
[1, 2, 3, 4, 5, 'b', 7, 8, 6]  
[1, 2, 3, 4, 'b', 5, 7, 8, 6]  
[1, 2, 3, 'b', 4, 5, 7, 8, 6]  
['b', 2, 3, 1, 4, 5, 7, 8, 6]  
[2, 'b', 3, 1, 4, 5, 7, 8, 6]  
[2, 4, 3, 1, 'b', 5, 7, 8, 6]  
[2, 4, 3, 1, 5, 'b', 7, 8, 6]  
[2, 4, 'b', 1, 5, 3, 7, 8, 6]  
[2, 'b', 4, 1, 5, 3, 7, 8, 6]  
['b', 2, 4, 1, 5, 3, 7, 8, 6]  
[1, 2, 4, 'b', 5, 3, 7, 8, 6]  
[1, 2, 4, 5, 'b', 3, 7, 8, 6]  
[1, 'b', 4, 5, 2, 3, 7, 8, 6]  
[1, 4, 'b', 5, 2, 3, 7, 8, 6]  
[1, 4, 3, 5, 2, 'b', 7, 8, 6]  
[1, 4, 3, 5, 'b', 2, 7, 8, 6]  
[1, 4, 3, 'b', 5, 2, 7, 8, 6]  
[1, 4, 3, 7, 5, 2, 'b', 8, 6]  
[1, 4, 3, 7, 5, 2, 8, 'b', 6]  
[1, 4, 3, 7, 'b', 2, 8, 5, 6]  
[1, 4, 3, 7, 2, 'b', 8, 5, 6]  
[1, 4, 3, 7, 2, 6, 8, 5, 'b']  
[1, 4, 3, 7, 2, 6, 8, 'b', 5]  
[1, 4, 3, 7, 2, 6, 'b', 8, 5]  
[1, 4, 3, 'b', 2, 6, 7, 8, 5]  
['b', 4, 3, 1, 2, 6, 7, 8, 5]  
[4, 'b', 3, 1, 2, 6, 7, 8, 5]  
[4, 2, 3, 1, 'b', 6, 7, 8, 5]  
[4, 2, 3, 1, 6, 'b', 7, 8, 5]  
[4, 2, 'b', 1, 6, 3, 7, 8, 5]  
[4, 'b', 2, 1, 6, 3, 7, 8, 5]  
[4, 6, 2, 1, 'b', 3, 7, 8, 5]  
[4, 6, 2, 'b', 1, 3, 7, 8, 5]  
['b', 6, 2, 4, 1, 3, 7, 8, 5]  
[6, 'b', 2, 4, 1, 3, 7, 8, 5]

[4, 2, 3, 1, 'b', 8, 7, 5, 6]  
[4, 2, 3, 1, 5, 8, 7, 'b', 6]  
[4, 2, 3, 1, 5, 8, 'b', 7, 6]  
[4, 2, 3, 'b', 5, 8, 1, 7, 6]  
[4, 2, 3, 5, 'b', 8, 1, 7, 6]  
[4, 2, 3, 5, 7, 8, 1, 'b', 6]  
[4, 2, 3, 5, 7, 8, 1, 6, 'b']  
[4, 2, 3, 5, 7, 'b', 1, 6, 8]  
[4, 2, 3, 5, 'b', 7, 1, 6, 8]  
[4, 2, 3, 'b', 5, 7, 1, 6, 8]  
[4, 2, 3, 1, 5, 7, 'b', 6, 8]  
[4, 2, 3, 1, 5, 7, 6, 'b', 8]

steps: 48

Initial State: [6,2,3,8,5,7,4,'b',1]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
[1, 2, 3, 4, 5, 6, 7, 'b', 8]  
[1, 2, 3, 4, 5, 6, 'b', 7, 8]  
[1, 2, 3, 'b', 5, 6, 4, 7, 8]  
[1, 2, 3, 5, 'b', 6, 4, 7, 8]  
[1, 2, 3, 5, 7, 6, 4, 'b', 8]  
[1, 2, 3, 5, 7, 6, 4, 8, 'b']  
[1, 2, 3, 5, 7, 'b', 4, 8, 6]  
[1, 2, 3, 5, 'b', 7, 4, 8, 6]  
[1, 2, 3, 'b', 5, 7, 4, 8, 6]  
[1, 2, 3, 4, 5, 7, 'b', 8, 6]  
[1, 2, 3, 4, 5, 7, 8, 'b', 6]  
[1, 2, 3, 4, 'b', 7, 8, 5, 6]  
[1, 2, 3, 4, 7, 'b', 8, 5, 6]  
[1, 2, 3, 4, 7, 6, 8, 5, 'b']  
[1, 2, 3, 4, 7, 6, 8, 'b', 5]  
[1, 2, 3, 4, 7, 6, 'b', 8, 5]  
[1, 2, 3, 'b', 7, 6, 4, 8, 5]  
['b', 2, 3, 1, 7, 6, 4, 8, 5]  
[2, 'b', 3, 1, 7, 6, 4, 8, 5]  
[2, 7, 3, 1, 'b', 6, 4, 8, 5]  
[2, 7, 3, 'b', 1, 6, 4, 8, 5]  
[2, 7, 3, 4, 1, 6, 'b', 8, 5]  
[2, 7, 3, 4, 1, 6, 8, 'b', 5]  
[2, 7, 3, 4, 'b', 6, 8, 1, 5]  
[2, 'b', 3, 4, 7, 6, 8, 1, 5]  
['b', 2, 3, 4, 7, 6, 8, 1, 5]  
[4, 2, 3, 'b', 7, 6, 8, 1, 5]  
[4, 2, 3, 7, 'b', 6, 8, 1, 5]  
[4, 2, 3, 7, 6, 'b', 8, 1, 5]  
[4, 2, 3, 7, 6, 5, 8, 1, 'b']  
[4, 2, 3, 7, 6, 5, 8, 'b', 1]  
[4, 2, 3, 7, 6, 5, 'b', 8, 1]  
[4, 2, 3, 'b', 6, 5, 7, 8, 1]  
[4, 2, 3, 6, 'b', 5, 7, 8, 1]  
[4, 2, 3, 6, 5, 'b', 7, 8, 1]

[6, 2, 3, 8, 5, 7, 4, 1, 'b']  
 [6, 2, 3, 8, 5, 7, 4, 'b', 1]  
 steps: 38

Initial State:

[2,1,3,8,5,'b',4,6,7]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 'b', 7, 8, 6]  
 [1, 2, 'b', 4, 5, 3, 7, 8, 6]  
 [1, 'b', 2, 4, 5, 3, 7, 8, 6]  
 [1, 5, 2, 4, 'b', 3, 7, 8, 6]  
 [1, 5, 2, 'b', 4, 3, 7, 8, 6]  
 ['b', 5, 2, 1, 4, 3, 7, 8, 6]  
 [5, 'b', 2, 1, 4, 3, 7, 8, 6]  
 [5, 2, 'b', 1, 4, 3, 7, 8, 6]  
 [5, 2, 3, 1, 4, 'b', 7, 8, 6]  
 [5, 2, 3, 1, 4, 6, 7, 8, 'b']  
 [5, 2, 3, 1, 4, 6, 7, 'b', 8]  
 [5, 2, 3, 1, 'b', 6, 7, 4, 8]  
 [5, 'b', 3, 1, 2, 6, 7, 4, 8]  
 ['b', 5, 3, 1, 2, 6, 7, 4, 8]  
 [1, 5, 3, 'b', 2, 6, 7, 4, 8]  
 [1, 5, 3, 2, 'b', 6, 7, 4, 8]  
 [1, 'b', 3, 2, 5, 6, 7, 4, 8]  
 ['b', 1, 3, 2, 5, 6, 7, 4, 8]  
 [2, 1, 3, 'b', 5, 6, 7, 4, 8]  
 [2, 1, 3, 7, 5, 6, 'b', 4, 8]  
 [2, 1, 3, 7, 5, 6, 4, 'b', 8]  
 [2, 1, 3, 7, 5, 6, 4, 8, 'b']  
 [2, 1, 3, 7, 5, 'b', 4, 8, 6]

[6, 2, 'b', 4, 1, 3, 7, 8, 5]  
 [6, 2, 3, 4, 1, 'b', 7, 8, 5]  
 [6, 2, 3, 4, 'b', 1, 7, 8, 5]  
 [6, 2, 3, 4, 8, 1, 7, 'b', 5]  
 [6, 2, 3, 4, 8, 1, 'b', 7, 5]  
 [6, 2, 3, 'b', 8, 1, 4, 7, 5]  
 [6, 2, 3, 8, 'b', 1, 4, 7, 5]  
 [6, 2, 3, 8, 1, 'b', 4, 7, 5]  
 [6, 2, 3, 8, 1, 5, 4, 7, 'b']  
 [6, 2, 3, 8, 1, 5, 4, 'b', 7]  
 [6, 2, 3, 8, 'b', 5, 4, 1, 7]  
 [6, 2, 3, 8, 5, 'b', 4, 1, 7]  
 [6, 2, 3, 8, 5, 7, 4, 1, 'b']  
 [6, 2, 3, 8, 5, 7, 4, 'b', 1]  
 steps: 50

Initial State:

[2,1,3,8,5,'b',4,6,7]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 6, 7, 'b', 8]  
 [1, 2, 3, 4, 'b', 6, 7, 5, 8]  
 [1, 'b', 3, 4, 2, 6, 7, 5, 8]  
 ['b', 1, 3, 4, 2, 6, 7, 5, 8]  
 [4, 1, 3, 'b', 2, 6, 7, 5, 8]  
 [4, 1, 3, 2, 'b', 6, 7, 5, 8]  
 [4, 1, 3, 2, 5, 6, 7, 'b', 8]  
 [4, 1, 3, 2, 5, 6, 'b', 7, 8]  
 [4, 1, 3, 'b', 5, 6, 2, 7, 8]  
 ['b', 1, 3, 4, 5, 6, 2, 7, 8]  
 [1, 'b', 3, 4, 5, 6, 2, 7, 8]  
 [1, 5, 3, 4, 'b', 6, 2, 7, 8]  
 [1, 5, 3, 'b', 4, 6, 2, 7, 8]  
 [1, 5, 3, 2, 4, 6, 'b', 7, 8]  
 [1, 5, 3, 2, 4, 6, 7, 'b', 8]  
 [1, 5, 3, 2, 'b', 6, 7, 4, 8]  
 [1, 5, 3, 'b', 2, 6, 7, 4, 8]  
 [1, 5, 3, 7, 2, 6, 'b', 4, 8]  
 [1, 5, 3, 7, 2, 6, 4, 'b', 8]  
 [1, 5, 3, 7, 2, 6, 4, 8, 'b']  
 [1, 5, 3, 7, 2, 'b', 4, 8, 6]  
 [1, 5, 3, 7, 'b', 2, 4, 8, 6]  
 [1, 5, 3, 7, 8, 2, 4, 'b', 6]

[4, 2, 'b', 6, 5, 3, 7, 8, 1]  
 [4, 'b', 2, 6, 5, 3, 7, 8, 1]  
 ['b', 4, 2, 6, 5, 3, 7, 8, 1]  
 [6, 4, 2, 'b', 5, 3, 7, 8, 1]  
 [6, 4, 2, 5, 'b', 3, 7, 8, 1]  
 [6, 'b', 2, 5, 4, 3, 7, 8, 1]  
 [6, 2, 'b', 5, 4, 3, 7, 8, 1]  
 [6, 2, 3, 5, 4, 'b', 7, 8, 1]  
 [6, 2, 3, 5, 4, 1, 7, 8, 'b']  
 [6, 2, 3, 5, 4, 1, 7, 'b', 8]  
 [6, 2, 3, 5, 'b', 1, 7, 4, 8]  
 [6, 2, 3, 'b', 5, 1, 7, 4, 8]  
 [6, 2, 3, 7, 5, 1, 'b', 4, 8]  
 [6, 2, 3, 7, 5, 1, 4, 'b', 8]  
 [6, 2, 3, 7, 'b', 1, 4, 5, 8]  
 [6, 2, 3, 'b', 7, 1, 4, 5, 8]  
 [6, 2, 3, 4, 7, 1, 'b', 5, 8]  
 [6, 2, 3, 4, 7, 1, 5, 'b', 8]  
 [6, 2, 3, 4, 7, 1, 5, 8, 'b']  
 [6, 2, 3, 4, 7, 'b', 5, 8, 1]  
 [6, 2, 3, 4, 'b', 7, 5, 8, 1]  
 [6, 2, 3, 4, 8, 7, 5, 'b', 1]  
 [6, 2, 3, 4, 8, 7, 'b', 5, 1]  
 [6, 2, 3, 'b', 8, 7, 4, 5, 1]  
 [6, 2, 3, 8, 'b', 7, 4, 5, 1]  
 [6, 2, 3, 8, 5, 7, 4, 'b', 1]  
 steps: 62

Initial State:

[2,1,3,8,5,'b',4,6,7]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 6, 7, 'b', 8]  
 [1, 2, 3, 4, 5, 6, 'b', 7, 8]  
 [1, 2, 3, 'b', 5, 6, 4, 7, 8]  
 [1, 2, 3, 5, 'b', 6, 4, 7, 8]  
 [1, 2, 3, 5, 7, 6, 4, 'b', 8]  
 [1, 2, 3, 5, 7, 6, 4, 8, 'b']  
 [1, 2, 3, 5, 7, 'b', 4, 8, 6]  
 [1, 2, 3, 5, 'b', 7, 4, 8, 6]  
 [1, 2, 3, 'b', 5, 7, 4, 8, 6]  
 [1, 2, 3, 4, 5, 7, 'b', 8, 6]  
 [1, 2, 3, 4, 5, 7, 8, 'b', 6]  
 [1, 2, 3, 4, 'b', 7, 8, 5, 6]  
 [1, 2, 3, 4, 7, 'b', 8, 5, 6]  
 [1, 2, 3, 4, 7, 6, 8, 5, 'b']  
 [1, 2, 3, 4, 7, 6, 8, 'b', 5]  
 [1, 2, 3, 4, 7, 6, 'b', 8, 5]  
 [1, 2, 3, 'b', 7, 6, 4, 8, 5]  
 ['b', 2, 3, 1, 7, 6, 4, 8, 5]  
 [2, 'b', 3, 1, 7, 6, 4, 8, 5]  
 [2, 7, 3, 1, 'b', 6, 4, 8, 5]  
 [2, 7, 3, 'b', 1, 6, 4, 8, 5]  
 [2, 7, 3, 4, 1, 6, 'b', 8, 5]  
 [2, 7, 3, 4, 1, 6, 8, 'b', 5]

[2, 1, 3, 7, 'b', 5, 4, 8, 6]  
 [2, 1, 3, 'b', 7, 5, 4, 8, 6]  
 [2, 1, 3, 4, 7, 5, 'b', 8, 6]  
 [2, 1, 3, 4, 7, 5, 8, 'b', 6]  
 [2, 1, 3, 4, 'b', 5, 8, 7, 6]  
 [2, 1, 3, 4, 5, 'b', 8, 7, 6]  
 [2, 1, 3, 4, 5, 6, 8, 7, 'b']  
 [2, 1, 3, 4, 5, 6, 8, 'b', 7]  
 [2, 1, 3, 4, 5, 6, 'b', 8, 7]  
 [2, 1, 3, 'b', 5, 6, 4, 8, 7]  
 [2, 1, 3, 5, 'b', 6, 4, 8, 7]  
 [2, 1, 3, 5, 8, 6, 4, 'b', 7]  
 [2, 1, 3, 5, 8, 6, 4, 7, 'b']  
 [2, 1, 3, 5, 8, 'b', 4, 7, 6]  
 [2, 1, 3, 5, 'b', 8, 4, 7, 6]  
 [2, 1, 3, 'b', 5, 8, 4, 7, 6]  
 [2, 1, 3, 4, 5, 8, 'b', 7, 6]  
 [2, 1, 3, 4, 5, 8, 7, 'b', 6]  
 [2, 1, 3, 4, 'b', 8, 7, 5, 6]  
 [2, 1, 3, 4, 8, 'b', 7, 5, 6]  
 [2, 1, 3, 4, 8, 6, 7, 5, 'b']  
 [2, 1, 3, 4, 8, 6, 7, 'b', 5]  
 [2, 1, 3, 4, 8, 6, 'b', 7, 5]  
 [2, 1, 3, 'b', 8, 6, 4, 7, 5]  
 [2, 1, 3, 8, 'b', 6, 4, 7, 5]  
 [2, 1, 3, 8, 6, 'b', 4, 7, 5]  
 [2, 1, 3, 8, 6, 5, 4, 7, 'b']  
 [2, 1, 3, 8, 6, 5, 4, 'b', 7]  
 [2, 1, 3, 8, 'b', 5, 4, 6, 7]  
 [2, 1, 3, 8, 5, 'b', 4, 6, 7]

steps: 54

Initial state:

['b',1,3,8,4,5,2,6,7]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 'b', 7, 8, 6]  
 [1, 2, 'b', 4, 5, 3, 7, 8, 6]  
 [1, 'b', 2, 4, 5, 3, 7, 8, 6]  
 ['b', 1, 2, 4, 5, 3, 7, 8, 6]  
 [4, 1, 2, 'b', 5, 3, 7, 8, 6]  
 [4, 1, 2, 5, 'b', 3, 7, 8, 6]  
 [4, 'b', 2, 5, 1, 3, 7, 8, 6]  
 [4, 2, 'b', 5, 1, 3, 7, 8, 6]  
 [4, 2, 3, 5, 1, 'b', 7, 8, 6]  
 [4, 2, 3, 5, 1, 6, 7, 8, 'b']  
 [4, 2, 3, 5, 1, 6, 7, 'b', 8]  
 [4, 2, 3, 5, 'b', 6, 7, 1, 8]

[1, 5, 3, 7, 8, 2, 'b', 4, 6]  
 [1, 5, 3, 'b', 8, 2, 7, 4, 6]  
 [1, 5, 3, 8, 'b', 2, 7, 4, 6]  
 [1, 5, 3, 8, 2, 'b', 7, 4, 6]  
 [1, 5, 3, 8, 2, 6, 7, 4, 'b']  
 [1, 5, 3, 8, 2, 6, 7, 'b', 4]  
 [1, 5, 3, 8, 2, 6, 'b', 7, 4]  
 [1, 5, 3, 'b', 2, 6, 8, 7, 4]  
 [1, 5, 3, 2, 'b', 6, 8, 7, 4]  
 [1, 5, 3, 2, 7, 6, 8, 'b', 4]  
 [1, 5, 3, 2, 7, 6, 8, 4, 'b']  
 [1, 5, 3, 2, 7, 'b', 8, 4, 6]  
 [1, 5, 3, 2, 'b', 7, 8, 4, 6]  
 [1, 'b', 3, 2, 5, 7, 8, 4, 6]  
 ['b', 1, 3, 2, 5, 7, 8, 4, 6]  
 [2, 1, 3, 'b', 5, 7, 8, 4, 6]  
 [2, 1, 3, 8, 5, 7, 'b', 4, 6]  
 [2, 1, 3, 8, 5, 7, 4, 'b', 6]  
 [2, 1, 3, 8, 5, 7, 4, 6, 'b']  
 [2, 1, 3, 8, 5, 'b', 4, 6, 7]

steps: 44

Initial state:

['b',1,3,8,4,5,2,6,7]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 'b', 7, 8, 6]  
 [1, 2, 3, 4, 'b', 5, 7, 8, 6]  
 [1, 2, 3, 'b', 4, 5, 7, 8, 6]  
 ['b', 2, 3, 1, 4, 5, 7, 8, 6]  
 [2, 'b', 3, 1, 4, 5, 7, 8, 6]  
 [2, 4, 3, 1, 'b', 5, 7, 8, 6]  
 [2, 4, 3, 1, 5, 'b', 7, 8, 6]  
 [2, 4, 'b', 1, 5, 3, 7, 8, 6]  
 [2, 'b', 4, 1, 5, 3, 7, 8, 6]  
 ['b', 2, 4, 1, 5, 3, 7, 8, 6]  
 [1, 2, 4, 'b', 5, 3, 7, 8, 6]  
 [1, 2, 4, 5, 'b', 3, 7, 8, 6]

[2, 7, 3, 4, 'b', 6, 8, 1, 5]  
 [2, 'b', 3, 4, 7, 6, 8, 1, 5]  
 ['b', 2, 3, 4, 7, 6, 8, 1, 5]  
 [4, 2, 3, 'b', 7, 6, 8, 1, 5]  
 [4, 2, 3, 7, 'b', 6, 8, 1, 5]  
 [4, 2, 3, 7, 1, 6, 8, 'b', 5]  
 [4, 2, 3, 7, 1, 6, 'b', 8, 5]  
 [4, 2, 3, 'b', 1, 6, 7, 8, 5]  
 [4, 2, 3, 1, 'b', 6, 7, 8, 5]  
 [4, 2, 3, 1, 8, 6, 7, 'b', 5]  
 [4, 2, 3, 1, 8, 6, 7, 5, 'b']  
 [4, 2, 3, 1, 8, 'b', 7, 5, 6]  
 [4, 2, 3, 1, 'b', 8, 7, 5, 6]  
 [4, 2, 3, 1, 5, 8, 7, 'b', 6]  
 [4, 2, 3, 1, 5, 8, 'b', 7, 6]  
 [4, 2, 3, 'b', 5, 8, 1, 7, 6]  
 [4, 2, 3, 5, 'b', 8, 1, 7, 6]  
 [4, 2, 3, 5, 7, 8, 1, 'b', 6]  
 [4, 2, 3, 5, 7, 8, 1, 6, 'b']  
 [4, 2, 3, 5, 7, 'b', 1, 6, 8]  
 [4, 2, 3, 5, 'b', 7, 1, 6, 8]  
 [4, 2, 3, 'b', 5, 7, 1, 6, 8]  
 [4, 2, 3, 1, 5, 7, 'b', 6, 8]  
 [4, 2, 3, 1, 5, 7, 6, 'b', 8]  
 [4, 2, 3, 1, 5, 7, 6, 8, 'b']  
 [4, 2, 3, 1, 5, 'b', 6, 8, 7]  
 [4, 2, 3, 1, 'b', 5, 6, 8, 7]  
 [4, 2, 3, 'b', 1, 5, 6, 8, 7]  
 ['b', 2, 3, 4, 1, 5, 6, 8, 7]  
 [2, 'b', 3, 4, 1, 5, 6, 8, 7]  
 [2, 1, 3, 4, 'b', 5, 6, 8, 7]  
 [2, 1, 3, 4, 8, 5, 6, 'b', 7]  
 [2, 1, 3, 4, 8, 5, 'b', 6, 7]  
 [2, 1, 3, 'b', 8, 5, 4, 6, 7]  
 [2, 1, 3, 8, 'b', 5, 4, 6, 7]  
 [2, 1, 3, 8, 5, 'b', 4, 6, 7]

steps: 60

Initial state:

['b',1,3,8,4,5,2,6,7]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 6, 7, 'b', 8]  
 [1, 2, 3, 4, 5, 6, 'b', 7, 8]  
 [1, 2, 3, 'b', 5, 6, 4, 7, 8]  
 [1, 2, 3, 5, 'b', 6, 4, 7, 8]  
 [1, 2, 3, 5, 7, 6, 4, 'b', 8]  
 [1, 2, 3, 5, 7, 6, 4, 8, 'b']  
 [1, 2, 3, 5, 7, 'b', 4, 8, 6]  
 [1, 2, 3, 5, 'b', 7, 4, 8, 6]  
 [1, 2, 3, 'b', 5, 7, 4, 8, 6]  
 [1, 2, 3, 4, 5, 7, 'b', 8, 6]  
 [1, 2, 3, 4, 5, 7, 8, 'b', 6]  
 [1, 2, 3, 4, 'b', 7, 8, 5, 6]

[4, 2, 3, 'b', 5, 6, 7, 1, 8]  
 [4, 2, 3, 7, 5, 6, 'b', 1, 8]  
 [4, 2, 3, 7, 5, 6, 1, 'b', 8]  
 [4, 2, 3, 7, 5, 6, 1, 8, 'b']  
 [4, 2, 3, 7, 5, 'b', 1, 8, 6]  
 [4, 2, 3, 7, 'b', 5, 1, 8, 6]  
 [4, 2, 3, 'b', 7, 5, 1, 8, 6]  
 [4, 2, 3, 1, 7, 5, 'b', 8, 6]  
 [4, 2, 3, 1, 7, 5, 8, 'b', 6]  
 [4, 2, 3, 1, 'b', 5, 8, 7, 6]  
 [4, 2, 3, 1, 5, 'b', 8, 7, 6]  
 [4, 2, 3, 1, 5, 6, 8, 7, 'b']  
 [4, 2, 3, 1, 5, 6, 8, 'b', 7]  
 [4, 2, 3, 1, 5, 6, 'b', 8, 7]  
 [4, 2, 3, 'b', 5, 6, 1, 8, 7]  
 [4, 2, 3, 5, 'b', 6, 1, 8, 7]  
 [4, 2, 3, 5, 8, 6, 1, 'b', 7]  
 [4, 2, 3, 5, 8, 6, 1, 7, 'b']  
 [4, 2, 3, 5, 8, 'b', 1, 7, 6]  
 [4, 2, 3, 5, 'b', 8, 1, 7, 6]  
 [4, 2, 3, 'b', 5, 8, 1, 7, 6]  
 [4, 2, 3, 1, 5, 8, 'b', 7, 6]  
 [4, 2, 3, 1, 5, 8, 7, 'b', 6]  
 [4, 2, 3, 1, 'b', 8, 7, 5, 6]  
 [4, 2, 3, 1, 8, 'b', 7, 5, 6]  
 [4, 2, 3, 1, 8, 6, 7, 5, 'b']  
 [4, 2, 3, 1, 8, 6, 7, 'b', 5]  
 [4, 2, 3, 1, 8, 6, 'b', 7, 5]  
 [4, 2, 3, 'b', 8, 6, 1, 7, 5]  
 [4, 2, 3, 8, 'b', 6, 1, 7, 5]  
 [4, 2, 3, 8, 6, 'b', 1, 7, 5]  
 [4, 2, 3, 8, 6, 5, 1, 7, 'b']  
 [4, 2, 3, 8, 6, 5, 1, 'b', 7]  
 [4, 2, 3, 8, 'b', 5, 1, 6, 7]  
 [4, 2, 3, 'b', 8, 5, 1, 6, 7]  
 [4, 2, 3, 1, 8, 5, 'b', 6, 7]  
 [4, 2, 3, 1, 8, 5, 6, 'b', 7]  
 [4, 2, 3, 1, 'b', 5, 6, 8, 7]  
 [4, 'b', 3, 1, 2, 5, 6, 8, 7]  
 ['b', 4, 3, 1, 2, 5, 6, 8, 7]  
 [1, 4, 3, 'b', 2, 5, 6, 8, 7]  
 [1, 4, 3, 6, 2, 5, 'b', 8, 7]  
 [1, 4, 3, 6, 2, 5, 8, 'b', 7]  
 [1, 4, 3, 6, 'b', 5, 8, 2, 7]  
 [1, 4, 3, 'b', 6, 5, 8, 2, 7]  
 [1, 4, 3, 8, 6, 5, 'b', 2, 7]  
 [1, 4, 3, 8, 6, 5, 2, 'b', 7]  
 [1, 4, 3, 8, 'b', 5, 2, 6, 7]  
 [1, 'b', 3, 8, 4, 5, 2, 6, 7]  
 ['b', 1, 3, 8, 4, 5, 2, 6, 7]

steps: 63

Initial State:

['b',1,3,4,5,8,2,6,7]

[1, 'b', 4, 5, 2, 3, 7, 8, 6]  
 [1, 4, 'b', 5, 2, 3, 7, 8, 6]  
 [1, 4, 3, 5, 2, 'b', 7, 8, 6]  
 [1, 4, 3, 5, 2, 6, 7, 8, 'b']  
 [1, 4, 3, 5, 2, 6, 7, 'b', 8]  
 [1, 4, 3, 5, 2, 6, 'b', 7, 8]  
 [1, 4, 3, 'b', 2, 6, 5, 7, 8]  
 [1, 4, 3, 2, 'b', 6, 5, 7, 8]  
 [1, 4, 3, 2, 7, 6, 5, 'b', 8]  
 [1, 4, 3, 2, 7, 6, 5, 8, 'b']  
 [1, 4, 3, 2, 7, 'b', 5, 8, 6]  
 [1, 4, 3, 2, 'b', 7, 5, 8, 6]  
 [1, 4, 3, 'b', 2, 7, 5, 8, 6]  
 [1, 4, 3, 5, 2, 7, 'b', 8, 6]  
 [1, 4, 3, 5, 2, 7, 8, 'b', 6]  
 [1, 4, 3, 5, 'b', 7, 8, 2, 6]  
 [1, 4, 3, 'b', 5, 7, 8, 2, 6]  
 [1, 4, 3, 8, 5, 7, 'b', 2, 6]  
 [1, 4, 3, 8, 5, 7, 2, 'b', 6]  
 [1, 4, 3, 8, 5, 7, 2, 6, 'b']  
 [1, 4, 3, 8, 5, 'b', 2, 6, 7]  
 [1, 4, 3, 8, 'b', 5, 2, 6, 7]  
 [1, 'b', 3, 8, 4, 5, 2, 6, 7]  
 ['b', 1, 3, 8, 4, 5, 2, 6, 7]

steps: 37

Initial State:

['b',1,3,4,5,8,2,6,7]

[1, 2, 3, 4, 7, 'b', 8, 5, 6]  
 [1, 2, 3, 4, 7, 6, 8, 5, 'b']  
 [1, 2, 3, 4, 7, 6, 8, 'b', 5]  
 [1, 2, 3, 4, 7, 6, 'b', 8, 5]  
 [1, 2, 3, 'b', 7, 6, 4, 8, 5]  
 [1, 2, 3, 7, 'b', 6, 4, 8, 5]  
 [1, 2, 3, 7, 8, 6, 4, 'b', 5]  
 [1, 2, 3, 7, 8, 6, 'b', 4, 5]  
 [1, 2, 3, 'b', 8, 6, 7, 4, 5]  
 [1, 2, 3, 8, 'b', 6, 7, 4, 5]  
 [1, 'b', 3, 8, 2, 6, 7, 4, 5]  
 ['b', 1, 3, 8, 2, 6, 7, 4, 5]  
 [8, 1, 3, 'b', 2, 6, 7, 4, 5]  
 [8, 1, 3, 2, 'b', 6, 7, 4, 5]  
 [8, 1, 3, 2, 4, 6, 7, 'b', 5]  
 [8, 1, 3, 2, 4, 6, 'b', 7, 5]  
 [8, 1, 3, 'b', 4, 6, 2, 7, 5]  
 [8, 1, 3, 4, 'b', 6, 2, 7, 5]  
 [8, 1, 3, 4, 6, 'b', 2, 7, 5]  
 [8, 1, 3, 4, 6, 5, 2, 7, 'b']  
 [8, 1, 3, 4, 6, 5, 2, 'b', 7]  
 [8, 1, 3, 4, 'b', 5, 2, 6, 7]  
 [8, 1, 3, 'b', 4, 5, 2, 6, 7]  
 ['b', 1, 3, 8, 4, 5, 2, 6, 7]

steps: 37

Initial State:

['b',1,3,4,5,8,2,6,7]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 6, 7, 'b', 8]  
 [1, 2, 3, 4, 'b', 6, 7, 5, 8]  
 [1, 'b', 3, 4, 2, 6, 7, 5, 8]  
 ['b', 1, 3, 4, 2, 6, 7, 5, 8]  
 [4, 1, 3, 'b', 2, 6, 7, 5, 8]  
 [4, 1, 3, 2, 'b', 6, 7, 5, 8]  
 [4, 1, 3, 2, 5, 6, 7, 'b', 8]  
 [4, 1, 3, 2, 5, 6, 'b', 7, 8]  
 [4, 1, 3, 'b', 5, 6, 2, 7, 8]  
 ['b', 1, 3, 4, 5, 6, 2, 7, 8]  
 [1, 'b', 3, 4, 5, 6, 2, 7, 8]  
 [1, 5, 3, 4, 'b', 6, 2, 7, 8]  
 [1, 5, 3, 4, 6, 'b', 2, 7, 8]  
 [1, 5, 3, 4, 6, 8, 2, 7, 'b']  
 [1, 5, 3, 4, 6, 8, 2, 'b', 7]  
 [1, 5, 3, 4, 'b', 8, 2, 6, 7]  
 [1, 'b', 3, 4, 5, 8, 2, 6, 7]  
 ['b', 1, 3, 4, 5, 8, 2, 6, 7]

steps: 19

Manhattan Distance - Average  
 steps=(36+38+54+63+19)/5=**42**

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 6, 7, 'b', 8]  
 [1, 2, 3, 4, 'b', 6, 7, 5, 8]  
 [1, 'b', 3, 4, 2, 6, 7, 5, 8]  
 ['b', 1, 3, 4, 2, 6, 7, 5, 8]  
 [4, 1, 3, 'b', 2, 6, 7, 5, 8]  
 [4, 1, 3, 2, 'b', 6, 7, 5, 8]  
 [4, 1, 3, 2, 5, 6, 7, 'b', 8]  
 [4, 1, 3, 2, 5, 6, 'b', 7, 8]  
 [4, 1, 3, 'b', 5, 6, 2, 7, 8]  
 ['b', 1, 3, 4, 5, 6, 2, 7, 8]  
 [1, 'b', 3, 4, 5, 6, 2, 7, 8]  
 [1, 5, 3, 4, 'b', 6, 2, 7, 8]  
 [1, 5, 3, 4, 6, 'b', 2, 7, 8]  
 [1, 5, 3, 4, 6, 8, 2, 7, 'b']  
 [1, 5, 3, 4, 6, 8, 2, 'b', 7]  
 [1, 5, 3, 4, 'b', 8, 2, 6, 7]  
 [1, 'b', 3, 4, 5, 8, 2, 6, 7]  
 ['b', 1, 3, 4, 5, 8, 2, 6, 7]

steps: 19

Euclidean distance- Average  
 steps=(52+50+44+37+19)/5=**40.4**

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 6, 7, 'b', 8]  
 [1, 2, 3, 4, 'b', 6, 7, 5, 8]  
 [1, 'b', 3, 4, 2, 6, 7, 5, 8]  
 ['b', 1, 3, 4, 2, 6, 7, 5, 8]  
 [4, 1, 3, 'b', 2, 6, 7, 5, 8]  
 [4, 1, 3, 2, 'b', 6, 7, 5, 8]  
 [4, 1, 3, 2, 5, 6, 7, 'b', 8]  
 [4, 1, 3, 2, 5, 6, 'b', 7, 8]  
 [4, 1, 3, 'b', 5, 6, 2, 7, 8]  
 ['b', 1, 3, 4, 5, 6, 2, 7, 8]  
 [1, 'b', 3, 4, 5, 6, 2, 7, 8]  
 [1, 5, 3, 4, 'b', 6, 2, 7, 8]  
 [1, 5, 3, 4, 7, 6, 2, 'b', 8]  
 [1, 5, 3, 4, 7, 6, 2, 8, 'b']  
 [1, 5, 3, 4, 7, 'b', 2, 8, 6]  
 [1, 5, 3, 4, 'b', 7, 2, 8, 6]  
 [1, 5, 3, 4, 8, 7, 2, 'b', 6]  
 [1, 5, 3, 4, 8, 7, 2, 6, 'b']  
 [1, 5, 3, 4, 8, 'b', 2, 6, 7]  
 [1, 5, 3, 4, 'b', 8, 2, 6, 7]  
 [1, 'b', 3, 4, 5, 8, 2, 6, 7]  
 ['b', 1, 3, 4, 5, 8, 2, 6, 7]

steps: 23

Misplaced tiles - Average  
 steps=(48+62+60+37+23)/5=**46**

**A\* search :**

<u>Heuristic 1: Manhattan Distance</u>	<u>Heuristic 2: Euclidean Distance</u>	<u>Heuristic 3: Misplaced Tiles</u>
<p>Initial State 1: [4,2,3,1,5,7,6,'b',8]</p> <p>Final solution path:  [1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 6, 7, 'b', 8]  [1, 2, 3, 4, 'b', 6, 7, 5, 8]  [1, 2, 3, 4, 6, 'b', 7, 5, 8]  [1, 2, 'b', 4, 6, 3, 7, 5, 8]  [1, 'b', 2, 4, 6, 3, 7, 5, 8]  ['b', 1, 2, 4, 6, 3, 7, 5, 8]  [4, 1, 2, 'b', 6, 3, 7, 5, 8]  [4, 1, 2, 6, 'b', 3, 7, 5, 8]  [4, 'b', 2, 6, 1, 3, 7, 5, 8]  [4, 2, 'b', 6, 1, 3, 7, 5, 8]  [4, 2, 3, 6, 1, 'b', 7, 5, 8]  [4, 2, 3, 6, 1, 8, 7, 5, 'b']  [4, 2, 3, 6, 1, 8, 7, 'b', 5]  [4, 2, 3, 6, 1, 8, 'b', 7, 5]  [4, 2, 3, 'b', 1, 8, 6, 7, 5]  [4, 2, 3, 1, 'b', 8, 6, 7, 5]  [4, 2, 3, 1, 7, 8, 6, 'b', 5]  [4, 2, 3, 1, 7, 8, 6, 5, 'b']  [4, 2, 3, 1, 7, 'b', 6, 5, 8]  [4, 2, 3, 1, 'b', 7, 6, 5, 8]  [4, 2, 3, 1, 5, 7, 6, 'b', 8]  Steps = 22  Initial state: [6,2,3,8,5,7,4,'b',1]</p>	<p>Initial State1 : [4,2,3,1,5,7,6,'b',8]</p> <p>Final solution path:  [1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 'b', 7, 8, 6]  [1, 2, 3, 4, 'b', 5, 7, 8, 6]  [1, 2, 3, 4, 8, 5, 7, 'b', 6]  [1, 2, 3, 4, 8, 5, 7, 6, 'b']  [1, 2, 3, 4, 8, 'b', 7, 6, 5]  [1, 2, 3, 4, 'b', 8, 7, 6, 5]  [1, 2, 3, 'b', 4, 8, 7, 6, 5]  [1, 2, 3, 7, 4, 8, 'b', 6, 5]  [1, 2, 3, 7, 4, 8, 6, 'b', 5]  [1, 2, 3, 7, 4, 8, 6, 5, 'b']  [1, 2, 3, 7, 4, 'b', 6, 5, 8]  [1, 2, 'b', 7, 4, 3, 6, 5, 8]  [1, 'b', 2, 7, 4, 3, 6, 5, 8]  [1, 4, 2, 7, 'b', 3, 6, 5, 8]  [1, 4, 2, 'b', 7, 3, 6, 5, 8]  ['b', 4, 2, 1, 7, 3, 6, 5, 8]  [4, 'b', 2, 1, 7, 3, 6, 5, 8]  [4, 2, 'b', 1, 7, 3, 6, 5, 8]  [4, 2, 3, 1, 7, 'b', 6, 5, 8]  [4, 2, 3, 1, 'b', 7, 6, 5, 8]  [4, 2, 3, 1, 5, 7, 6, 'b', 8]  steps: 22  Initial state: [6,2,3,8,5,7,4,'b',1]</p>	<p>Initial State1: [4,2,3,1,5,7,6,'b',8]</p> <p>Final solution path:  [1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 'b', 7, 8, 6]  [1, 2, 3, 4, 'b', 5, 7, 8, 6]  [1, 2, 3, 4, 8, 5, 7, 'b', 6]  [1, 2, 3, 4, 8, 5, 7, 6, 'b']  [1, 2, 3, 4, 8, 'b', 7, 6, 5]  [1, 2, 3, 4, 'b', 8, 7, 6, 5]  [1, 2, 3, 'b', 4, 8, 7, 6, 5]  [1, 2, 3, 7, 4, 8, 'b', 6, 5]  [1, 2, 3, 7, 4, 8, 6, 'b', 5]  [1, 2, 3, 7, 4, 8, 6, 5, 'b']  [1, 2, 3, 7, 4, 'b', 6, 5, 8]  [1, 2, 'b', 7, 4, 3, 6, 5, 8]  [1, 'b', 2, 7, 4, 3, 6, 5, 8]  [1, 4, 2, 7, 'b', 3, 6, 5, 8]  [1, 4, 2, 'b', 7, 3, 6, 5, 8]  ['b', 4, 2, 1, 7, 3, 6, 5, 8]  [4, 'b', 2, 1, 7, 3, 6, 5, 8]  [4, 2, 'b', 1, 7, 3, 6, 5, 8]  [4, 2, 3, 1, 7, 'b', 6, 5, 8]  [4, 2, 3, 1, 'b', 7, 6, 5, 8]  [4, 2, 3, 1, 5, 7, 6, 'b', 8]  steps: 22  Initial state: [6,2,3,8,5,7,4,'b',1]</p>
<p>[1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 6, 7, 'b', 8]  [1, 2, 3, 4, 'b', 6, 7, 5, 8]  [1, 2, 3, 4, 6, 'b', 7, 5, 8]  [1, 2, 3, 4, 6, 8, 7, 5, 'b']  [1, 2, 3, 4, 6, 8, 7, 'b', 5]  [1, 2, 3, 4, 6, 8, 'b', 7, 5]  [1, 2, 3, 'b', 6, 8, 4, 7, 5]  [1, 2, 3, 6, 'b', 8, 4, 7, 5]  [1, 2, 3, 6, 8, 'b', 4, 7, 5]  [1, 2, 'b', 6, 8, 3, 4, 7, 5]  [1, 'b', 2, 6, 8, 3, 4, 7, 5]  ['b', 1, 2, 6, 8, 3, 4, 7, 5]  [6, 1, 2, 'b', 8, 3, 4, 7, 5]  [6, 1, 2, 8, 'b', 3, 4, 7, 5]  [6, 'b', 2, 8, 1, 3, 4, 7, 5]  [6, 2, 'b', 8, 1, 3, 4, 7, 5]</p>	<p>[1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 6, 7, 'b', 8]  [1, 2, 3, 4, 'b', 6, 7, 5, 8]  [1, 2, 3, 4, 6, 'b', 7, 5, 8]  [1, 2, 3, 4, 6, 8, 7, 5, 'b']  [1, 2, 3, 4, 6, 8, 7, 'b', 5]  [1, 2, 3, 4, 6, 8, 'b', 7, 5]  [1, 2, 3, 'b', 6, 8, 4, 7, 5]  [1, 2, 3, 6, 'b', 8, 4, 7, 5]  [1, 2, 3, 6, 8, 'b', 4, 7, 5]  [1, 2, 'b', 6, 8, 3, 4, 7, 5]  [1, 'b', 2, 6, 8, 3, 4, 7, 5]  ['b', 1, 2, 6, 8, 3, 4, 7, 5]  [6, 1, 2, 'b', 8, 3, 4, 7, 5]  [6, 1, 2, 8, 'b', 3, 4, 7, 5]  [6, 'b', 2, 8, 1, 3, 4, 7, 5]  [6, 2, 'b', 8, 1, 3, 4, 7, 5]</p>	<p>[1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 6, 7, 'b', 8]  [1, 2, 3, 4, 'b', 6, 7, 5, 8]  [1, 2, 3, 4, 6, 'b', 7, 5, 8]  [1, 2, 3, 4, 6, 8, 7, 5, 'b']  [1, 2, 3, 4, 6, 8, 7, 'b', 5]  [1, 2, 3, 4, 6, 8, 'b', 7, 5]  [1, 2, 3, 'b', 6, 8, 4, 7, 5]  [1, 2, 3, 6, 'b', 8, 4, 7, 5]  [1, 2, 3, 6, 8, 'b', 4, 7, 5]  [1, 2, 'b', 6, 8, 3, 4, 7, 5]  [1, 'b', 2, 6, 8, 3, 4, 7, 5]  ['b', 1, 2, 6, 8, 3, 4, 7, 5]  [6, 1, 2, 'b', 8, 3, 4, 7, 5]  [6, 1, 2, 8, 'b', 3, 4, 7, 5]  [6, 'b', 2, 8, 1, 3, 4, 7, 5]  [6, 2, 'b', 8, 1, 3, 4, 7, 5]</p>



[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 6, 7, 'b', 8]  
 [1, 2, 3, 4, 'b', 6, 7, 5, 8]  
 [1, 2, 3, 4, 6, 'b', 7, 5, 8]  
 [1, 2, 3, 4, 6, 8, 7, 5, 'b']  
 [1, 2, 3, 4, 6, 8, 7, 'b', 5]  
 [1, 2, 3, 4, 6, 8, 'b', 7, 5]  
 [1, 2, 3, 'b', 6, 8, 4, 7, 5]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']  
 [1, 2, 3, 4, 5, 6, 7, 'b', 8]  
 [1, 2, 3, 4, 'b', 6, 7, 5, 8]  
 [1, 2, 3, 4, 6, 'b', 7, 5, 8]  
 [1, 2, 3, 4, 6, 8, 7, 5, 'b']  
 [1, 2, 3, 4, 6, 8, 7, 'b', 5]  
 [1, 2, 3, 4, 6, 8, 'b', 7, 5]  
 [1, 2, 3, 'b', 6, 8, 4, 7, 5]

[1, 2, 3, 4, 5, 6, 7, 8, 'b']
[1, 2, 3, 4, 5, 6, 7, 'b', 8]
[1, 2, 3, 4, 'b', 6, 7, 5, 8]
[1, 2, 3, 4, 6, 'b', 7, 5, 8]
[1, 2, 3, 4, 6, 8, 7, 5, 'b']
[1, 2, 3, 4, 6, 8, 7, 'b', 5]
[1, 2, 3, 4, 6, 8, 'b', 7, 5]
[1, 2, 3, 'b', 6, 8, 4, 7, 5]

<p>[1, 2, 3, 6, 'b', 8, 4, 7, 5]  [1, 2, 3, 6, 8, 'b', 4, 7, 5]  [1, 2, 3, 6, 8, 5, 4, 7, 'b']  [1, 2, 3, 6, 8, 5, 4, 'b', 7]  [1, 2, 3, 6, 8, 5, 'b', 4, 7]  [1, 2, 3, 'b', 8, 5, 6, 4, 7]  [1, 2, 3, 8, 'b', 5, 6, 4, 7]  [1, 'b', 3, 8, 2, 5, 6, 4, 7]  ['b', 1, 3, 8, 2, 5, 6, 4, 7]  [8, 1, 3, 'b', 2, 5, 6, 4, 7]  [8, 1, 3, 2, 'b', 5, 6, 4, 7]  [8, 1, 3, 2, 4, 5, 6, 'b', 7]  [8, 1, 3, 2, 4, 5, 'b', 6, 7]  [8, 1, 3, 'b', 4, 5, 2, 6, 7]  ['b', 1, 3, 8, 4, 5, 2, 6, 7]  steps: 23</p> <p>Initial state:  ['b',1,3,4,5,8,2,6,7]</p> <p>[1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 6, 7, 'b', 8]  [1, 2, 3, 4, 'b', 6, 7, 5, 8]  [1, 'b', 3, 4, 2, 6, 7, 5, 8]  ['b', 1, 3, 4, 2, 6, 7, 5, 8]  [4, 1, 3, 'b', 2, 6, 7, 5, 8]  [4, 1, 3, 2, 'b', 6, 7, 5, 8]  [4, 1, 3, 2, 5, 6, 7, 'b', 8]  [4, 1, 3, 2, 5, 6, 'b', 7, 8]  [4, 1, 3, 'b', 5, 6, 2, 7, 8]  [4, 1, 3, 5, 'b', 6, 2, 7, 8]  [4, 1, 3, 5, 6, 'b', 2, 7, 8]  [4, 1, 3, 5, 6, 8, 2, 7, 'b']  [4, 1, 3, 5, 6, 8, 2, 'b', 7]  [4, 1, 3, 5, 'b', 8, 2, 6, 7]  [4, 1, 3, 'b', 5, 8, 2, 6, 7]  ['b', 1, 3, 4, 5, 8, 2, 6, 7]  steps: 17</p> <p>Average number of steps:  (22+24+22+23+17)/5 = <b>21.6</b></p>	<p>[1, 2, 3, 6, 'b', 8, 4, 7, 5]  [1, 2, 3, 6, 8, 'b', 4, 7, 5]  [1, 2, 3, 6, 8, 5, 4, 7, 'b']  [1, 2, 3, 6, 8, 5, 4, 'b', 7]  [1, 2, 3, 6, 8, 5, 'b', 4, 7]  [1, 2, 3, 'b', 8, 5, 6, 4, 7]  [1, 2, 3, 8, 'b', 5, 6, 4, 7]  [1, 'b', 3, 8, 2, 5, 6, 4, 7]  ['b', 1, 3, 8, 2, 5, 6, 4, 7]  [8, 1, 3, 'b', 2, 5, 6, 4, 7]  [8, 1, 3, 2, 'b', 5, 6, 4, 7]  [8, 1, 3, 2, 4, 5, 6, 'b', 7]  [8, 1, 3, 2, 4, 5, 'b', 6, 7]  [8, 1, 3, 'b', 4, 5, 2, 6, 7]  ['b', 1, 3, 8, 4, 5, 2, 6, 7]  steps: 23</p> <p>Initial state:  ['b',1,3,4,5,8,2,6,7]</p> <p>[1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 6, 7, 'b', 8]  [1, 2, 3, 4, 'b', 6, 7, 5, 8]  [1, 'b', 3, 4, 2, 6, 7, 5, 8]  ['b', 1, 3, 4, 2, 6, 7, 5, 8]  [4, 1, 3, 'b', 2, 6, 7, 5, 8]  [4, 1, 3, 2, 'b', 6, 7, 5, 8]  [4, 1, 3, 2, 5, 6, 7, 'b', 8]  [4, 1, 3, 2, 5, 6, 'b', 7, 8]  [4, 1, 3, 'b', 5, 6, 2, 7, 8]  [4, 1, 3, 5, 'b', 6, 2, 7, 8]  [4, 1, 3, 5, 6, 'b', 2, 7, 8]  [4, 1, 3, 5, 6, 8, 2, 7, 'b']  [4, 1, 3, 5, 6, 8, 2, 'b', 7]  [4, 1, 3, 5, 'b', 8, 2, 6, 7]  [4, 1, 3, 'b', 5, 8, 2, 6, 7]  ['b', 1, 3, 4, 5, 8, 2, 6, 7]  steps: 17</p> <p>Average number of steps:  (22+24+22+23+17)/5 = <b>21.6</b></p>	<p>[1, 2, 3, 6, 'b', 8, 4, 7, 5]  [1, 2, 3, 6, 8, 'b', 4, 7, 5]  [1, 2, 3, 6, 8, 5, 4, 7, 'b']  [1, 2, 3, 6, 8, 5, 4, 'b', 7]  [1, 2, 3, 6, 8, 5, 'b', 4, 7]  [1, 2, 3, 'b', 8, 5, 6, 4, 7]  [1, 2, 3, 8, 'b', 5, 6, 4, 7]  [1, 'b', 3, 8, 2, 5, 6, 4, 7]  ['b', 1, 3, 8, 2, 5, 6, 4, 7]  [8, 1, 3, 'b', 2, 5, 6, 4, 7]  [8, 1, 3, 2, 'b', 5, 6, 4, 7]  [8, 1, 3, 2, 4, 5, 6, 'b', 7]  [8, 1, 3, 2, 4, 5, 'b', 6, 7]  [8, 1, 3, 'b', 4, 5, 2, 6, 7]  ['b', 1, 3, 8, 4, 5, 2, 6, 7]  steps: 23</p> <p>Initial state:  ['b',1,3,4,5,8,2,6,7]</p> <p>[1, 2, 3, 4, 5, 6, 7, 8, 'b']  [1, 2, 3, 4, 5, 6, 7, 'b', 8]  [1, 2, 3, 4, 'b', 6, 7, 5, 8]  [1, 'b', 3, 4, 2, 6, 7, 5, 8]  ['b', 1, 3, 4, 2, 6, 7, 5, 8]  [4, 1, 3, 'b', 2, 6, 7, 5, 8]  [4, 1, 3, 2, 'b', 6, 7, 5, 8]  [4, 1, 3, 2, 5, 6, 7, 'b', 8]  [4, 1, 3, 2, 5, 6, 'b', 7, 8]  [4, 1, 3, 'b', 5, 6, 2, 7, 8]  [4, 1, 3, 5, 'b', 6, 2, 7, 8]  [4, 1, 3, 5, 6, 'b', 2, 7, 8]  [4, 1, 3, 5, 6, 8, 2, 7, 'b']  [4, 1, 3, 5, 6, 8, 2, 'b', 7]  [4, 1, 3, 5, 'b', 8, 2, 6, 7]  [4, 1, 3, 'b', 5, 8, 2, 6, 7]  ['b', 1, 3, 4, 5, 8, 2, 6, 7]  steps: 17</p> <p>Average number of steps:  (22+24+22+23+17)/5 = <b>21.6</b></p>
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### Summary and Conclusion:

Best-first and A\* search algorithms were implemented with three heuristics – Manhattan Distance, Euclidean Distance and Misplaced tiles. For the Best first search algorithm we obtained best results with Euclidean distance heuristic with average steps taken across 5 inputs to be 40.4. Next was Manhattan distance heuristics averaging 44 steps followed by misplaced tiles at 46 steps. For Best First search since the evaluation function  $f(n) = h(n)$  where  $h(n)$  is the heuristic function, based on the obtained results we can tell that Euclidean distance is the best fit. But, we can expect the data to vary as we increase the input sample size. For A\* algorithm the evaluation function  $f(n) = g(n) + h(n)$  where  $g(n)$  is the cost of reaching the current node from the root node and  $h(n)$  is the heuristic function. So, A\* is wiser in the fact it considers the cost to get to a node from the existing node and like we can see has better results than BFS search. We have an average of 21.6 steps across all heuristics and they have taken a uniform path to get to the final state. From the obtained data we can tell that A\* search is better compared to BFS search.

### 15 puzzle problem:

#### Best-First Search:

<u>Heuristic1: Manhattan Distance</u>	<u>Heuristic2: Euclidean Distance</u>	<u>Heuristic3: Misplaced Tiles</u>
Input: [1, 2, 3, 'b', 5, 6, 8, 4, 9, 10, 7, 11, 13, 14, 15, 12]  Steps: 6	Input: [1, 2, 3, 'b', 5, 6, 8, 4, 9, 10, 7, 11, 13, 14, 15, 12]  Steps: 6	Input: [1, 2, 3, 'b', 5, 6, 8, 4, 9, 10, 7, 11, 13, 14, 15, 12]  Steps: 6
Input: [12, 2, 14, 'b', 5, 6, 8, 11, 9, 10, 7, 4, 13, 3, 15, 1]  Steps: unreachable	Input: [12, 2, 14, 'b', 5, 6, 8, 11, 9, 10, 7, 4, 13, 3, 15, 1]  Steps: unreachable	Input: [12, 2, 14, 'b', 5, 6, 8, 11, 9, 10, 7, 4, 13, 3, 15, 1]  Steps: unreachable
Input: [2, 12, 4, 'b', 5, 6, 8, 1, 9, 10, 7, 14, 13, 3, 15, 11]	Input: [2, 12, 4, 'b', 5, 6, 8, 1, 9, 10, 7, 14, 13, 3, 15, 11]	Input: [2, 12, 4, 'b', 5, 6, 8, 1, 9, 10, 7, 14, 13, 3, 15, 11]

Steps: 68	Steps: 128	Steps: 112
Input: [2,1,4,5,6,8,3,9,10,7,14,12,13,'b',15,11]	Input: [2,1,4,5,6,8,3,9,10,7,14,12,13,'b',15,11]	Input: [2,1,4,5,6,8,3,9,10,7,14,12,13,'b',15,11]
Steps: 149	Steps: 127	Steps: Unreachable

#### A\* Search:

<u>Heuristic1: Manhattan Distance</u>	<u>Heuristic2: Euclidean Distance</u>	<u>Heuristic3: Misplaced Tiles</u>
Input: [1, 2, 3, 'b', 5, 6, 8, 4, 9, 10, 7, 11, 13, 14, 15, 12] Steps: 6	Input: [1, 2, 3, 'b', 5, 6, 8, 4, 9, 10, 7, 11, 13, 14, 15, 12] Steps: 6	Input: [1, 2, 3, 'b', 5, 6, 8, 4, 9, 10, 7, 11, 13, 14, 15, 12] Steps: 6
Input: [12,2,14,'b',5,6,8,11,9,10,7,4,13,3,15,1]	Input: [12,2,14,'b',5,6,8,11,9,10,7,4,13,3,15,1]	Input: [12,2,14,'b',5,6,8,11,9,10,7,4,13,3,15,1]
Steps: unreachable	Steps: unreachable	Steps: unreachable

Input: [2,12,4,'b',5,6,8,1,9,10,7,14,13,3,15,11]  Steps: Unreachable	Input: [2,12,4,'b',5,6,8,1,9,10,7,14,13,3,15,11]  Steps: Unreachable	Input: [2,12,4,'b',5,6,8,1,9,10,7,14,13,3,15,11]  Steps: Unreachable
Input: [2,1,4,5,6,8,3,9,10,7,14,12,13,'b',15,11]  Steps: Unreachable	Input: [2,1,4,5,6,8,3,9,10,7,14,12,13,'b',15,11]  Steps: Unreachable	Input: [2,1,4,5,6,8,3,9,10,7,14,12,13,'b',15,11]  Steps: Unreachable

For the 15 puzzle problem based on the results obtained we see that the Best First Search performs better than A\* algorithm. Thus further analysis is needed to try more samples and understand this better.