Homework 1

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1. Prove:

$$(A^T)^{-1} = (A^{-1})^T$$

Given:

$A_{m \times n}$ - Invertible matrix

we know that,

$$AA^{-1} = A^{-1}A = I \qquad \Rightarrow AA^{-1} = I$$

Apply Transpose on both sides for the above equation

$$(AA^{-1})^T = (I)^T$$

$$(A^T) \cdot (A^{-1})^T = I$$

Multiply both sides by $(A^T)^{-1}$ to get,

$$(A^T)(A^{-1})^T (A^T)^{-1} = I \cdot (A^T)^{-1}$$

$$(A^{-1})^T \left[(A^T)(A^T)^{-1}\right] = (A^T)^{-1}$$

where $\left[(A^T)(A^T)^{-1}\right] = I$

$$(A^{-1})^T I = (A^T)^{-1}$$

$$\therefore (A^T)^{-1} = (A^{-1})^T$$

Hence Proved.

2. Let A be having a disease

Let B be having tested positive for a disease.

So, the probability of A behaving a disease is,

$$P(A) = \frac{1}{10{,}000} \qquad \therefore P(A) = 0.0001$$

Probability of B be testing positive for a disease

$P(B) = 0.99$

Baye's theorem, $P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$

we know that,

$P(A) = \frac{1}{10,000} = 0.0001$ (the ~~prior~~ probability of having the disease, which is rare)

$P(A^{-1}) = 1 - 0.0001 = 0.999$

$P(B/A) = 0.99$ (the probability of testing positive for disease)

$P(B/A^{-1}) = 1 - P(B/A) = 1 - 0.99 = 0.01$

we need to calculate

$P(B) = P(B/A) * P(A) + P(B/A^{-1}) * P(A^{-1})$

$= 0.99 * 0.0001 + 0.01 * 0.999$

$= 0.009999 + 0.000099$

$= 0.010098$

Now, we need to find the probability of having the disease with the condition of already testing positive

As A has only two possible outcomes as A is a binary variable, so the formula will be,

$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B/A \cdot P(A) + P(B/A^{-1}) \cdot P(A^{-1})}$

$= \frac{(0.99)(0.0001)}{0.010098}$

$$= \frac{0.000099}{0.010098}$$

$$= 0.0098039 2$$

$$\simeq 0.0098$$

$$= 0.98\%$$

Therefore, the chances of having the disease is

0.0098 or 0.98%.

3. The maximum value of 1D Gaussian distribution $P(x)$ occurs at point $x = \mu$

Gaussian equation is :

$$p(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]}$$

where μ = mean σ = variance

Differentiate $P(x)$ w.r.t x to show max & min at x

$$d/dx\,(P(x)) = 0$$

$$d/dx\,(P(x)) = (1/\sigma\sqrt{2\pi})\, e^{-[(x-\mu)^2/2\sigma^2]} \cdot d/dx(-(x-\mu)^2/2\sigma^2)$$

$$= (1/\sigma\sqrt{2\pi})\, e^{[-(x-\mu)^2/2\sigma^2]} \cdot -1/2\sigma^2[2(x-\mu) \cdot d/dx(x-\mu)]$$

$$= \frac{-1}{\sigma^2\sqrt{2\pi}} \cdot e^{[-(x-\mu)^2/2\sigma^2]} (x-\mu)$$

Let $x = \mu$ we get

$$d/dx\,(P(x)) = 0$$

$\therefore$ Min and Max occur at $x = \mu$. Since gaussian curve is convex, it has global maximum at $x = \mu$

4) (i) Let $x =$ no. of heads in 4 coin flips
Given, flips $f = 4$

Probability of getting heads $P = 0.5$
$\therefore$ Expected number $E(x) = f \cdot P$
$= 4 \times 0.5 = 2$

(ii) Let $t =$ no of ~~tosses~~ flips before getting heads
Given, probability of getting heads $P = 0.5$
$\therefore$ Expected number $E(t) = \frac{1}{P} = \frac{1}{0.5} = 2$

5 a) consider the two rooms A and B that Vacuum cleaner has to clean.
Actions: Left, right and suck.

Here ~~are~~ there are 4 cases:

1. Room A is dirty and B is clean then suck ~~square~~ room A and move to right that is to room B.

2. Room B is dirty and A is clean
- Suck room B and move to left that is to room A

3. Room A and B are dirty
- Suck room A and move to right then suck room B.

4. Room A and B are clean
- Nothing will be done on room A and move to right, again nothing will be done on room B.

Below are the points to show Vaccum cleaner is rational:-

* The agent provides information about the action to be performed
(if the room is dirty clean it, else move right)
* The agent will provide an optimum end result (a clean environment)
* The expected execution is better than any other agent.

5 b) Yes, the corresponding agent program needs internal state because each movement costs one point. Hence, it is significant that the agent makes its movements only after checking both ~~areas~~ rooms (A & B)

This will minimise ~~any~~ any losses and save time as well. In our case, the agent begins at room A and preforms a check for clean and dirty ~~or~~ room. If the room is dirty, it will clean. Then it will move right to ~~area B~~. room B. After reaching room B, it will again perform a check for clean & dirty area since it does not retain any data from room A.

So the agent requires internal ~~acti~~ state to remember its activities and keep track of it so that it will not repeat the same task again that is if both room A and B are clean then it won't repeatedly clean the same rooms.

This helps agent to perform well, maintain efficiency and it will save the time as well.

5 c) The agent should be able to learn geography of the environment especially when its unknown. If room A and B both are clean the agent might waste its time doing the same things repeatedly. This will affect the efficiency, time and performance.

yes, It makes sense for the agent to learn from its experience because if agent can recall its geography of the environment then it will prevent it from taking unnecessary moves and wasting time.

6 a) False

Agent is smart enough to learn the state and provide effective results even if the agent has partial information about the state that is clean or dirty

b) True

In a partially observable environment, a pure reflex agent ignores the previous percepts and cannot obtain the optimal state estimate.

c) True

consider an environment with a single state, in which all actions have the same reward, irrespective of the action taken. In general, any environment which is reward-invariant under permutations of actions satisfies this property

d) False

The agent program accepts the current percept sequence as the input. The agent function accepts all the percept arrangements as input.

e) False:

same operator capacities cannot be actualited. For eg: the specialist work need to illuminate a issue in a study time it will fail.

6 f) True

It works for an environments where all activities are given same reward.

g) True

Consider an environment in which we can arbitrarily modify the parts that are unreachable by any optimal policy as long as they are unreachable

h) False

Because agent is considered rational if it has all the activities in its memory even if its unobservable environment

I) False

Even if the agent is an exceptional player, winning and losing is dependent on sheer luck and the cards that are dealt.

7) Playing soccer:

Performance: Winning, losing
Environment: Soccer field
Actuators: Hands, legs
Sensors: Eyes, ears
Characteristics of the environment: Partially observable, dynamic, continuous, sequential

Exploring the Subsurface Oceans of Titan:
Performance: Safe & good quality equipment, capture image/videos
Environment: Subsurface of the Oceans
Actuators: accelerating the submarine, Steering wheel & breaks
Sensors: Cameras, navigators

characteristics of the environment: Partially observable, dynamic, continuous, sequential, single agent

Shopping for used AI books on the Internet:
Performance : Cost of book, quality of book, review of book
Environment : Internet, websites
Actuators : Mobile applications / browsers
Sensors : Monitors, keyboard, mouse
Characteristics of the environment: partially observable, static, discrete, sequential, single agent.

Playing a tennis match:
Performance: winning, losing
Environment: Tennis court
Actuators : Legs, arms, ball, racquet
Sensors : Eyes, ears
Characteristics of the environment: fully observable, dynamic, continuous, multiagent, unknown

Practicing tennis against a wall:
Performance: improving the performance by practice, getting experience from past matches.
Environment: wall
Actuators: legs, arms, ball, racquet
Sensors : Eys, ears
Characteristics of the environment: fully observable dynamic, Continuous, single agent, unknown, Sequential

Performing a high jump:
Performance: recalling the jump, improving performance
Environment: track field
Actuators : legs
Sensors : Eyes

Characteristics of the environment: fully observable, static, discrete, single agent, unknown, sequential

Knitting a sweater:
Performance: size, quality
Environment: chair
Actuators: hands, needles, yarn
Sensors: eyes
Characteristics of the environment: fully observable static, continuous, single agent, unknown, sequential.

Bidding on an item at an auction
Performance: knowledge on item to be acquired, getting the item
Environment: auction place, online auction
Actuators: Bidding
Sensors: Eyes, ears
Characteristics of the environment: fully observable, static, multi agent, known, sequential.

8) a Yes, consider an agent program and insert null statement to ensure that the output is unaffected

b. Yes, In a static environment the agent function may instruct the agent to print the True if the percept is a ~~Turing~~ turing machine program that terminates, and False otherwise

c. Yes, The architecture and program determine the agent's behavior

d. Architecture with n bits has 2^n possible agents programs

e. The agent function does not change if the machine is speed up. The only change will be the agent will perform better by learning the environment faster

9. Pseudocode:

Goal based agents:

Function: Goal Based-Agent (percept) returns an action
persistent: state, model, goal, Plan, action

State ← UPDATE-STATE (state, action, Percept, model)
if GOAL-ACHIEVED
if plan is empty then
Plan ← PLAN
action ← PLAN
Return Action

Utility Based agents:

Function: UTILITY-BASED-AGENT (Percept) returns action
Persistent: state, model, utility plan, action

State ← UPDATE-STATE (state, action, percept, model)
if plan is empty then
Plan ← PLAN
action ← Plan
Return Action

10. Let us consider a performance measure of 1point for each step the agent takes and 10 points for each time the agent cleans the dirt. Because a simple reflex will not always be able to reach square one, I limited the iterations to 200 to avoid an infinite loop. So, if there is dirt in the square, there is a chance that the agent will not clean all of the dirt in its world. the

I considered the grid of rooms

1	2	3
4	5	6
7	8	9

Consider the performance measures for the different agents and for a fixed number of initial dirt piles (1, 3, 5) are as follows.

Agent type/ Initial dirt piles	1	3	5
(i)	51.2	72.0	92.2
(ii)	34.5	92.6	101.8
(iii)	246.2	277.2	444.8
(iv)	355.6	430.3	461.2