



Parallized State Estimation

ME 766: HPSC

Aryan Agal - 16D170004

Chitrangna Bhatt - 16D170015

Siddhesh Pawar 17D170011

Under guidance of Prof. S. Gopalakrishnan

Model Equations

$$\frac{dC_m}{dt} = \frac{F(C_{min} - C_m)}{V} - (k_p + k_{fm})C_mP_0$$

$$\frac{dC_I}{dt} = \frac{(F_IC_{Iin} - FC_I)}{V} - k_IC_I$$

$$\frac{dD_0}{dt} = (0.5k_{tc} + k_{td})P_0^2 + k_{fm}C_mP_0 - \frac{FD_0}{V}$$

$$\frac{dD_1}{dt} = M_m(k_p + k_{fm})C_mP_0 - \frac{FD_1}{V}$$

$$\frac{dT}{dt} = \frac{F(T_{in} - T)}{V} + \frac{(-\Delta H)k_pC_mP_0}{\rho C_p} - \frac{UA(T - T_j)}{\rho C_p V}$$

$$\frac{dT_j}{dt} = \frac{F_{cw}(T_{w0} - T_j)}{V_0} + \frac{UA(T - T_j)}{\rho_w C_{pw} V_0}$$

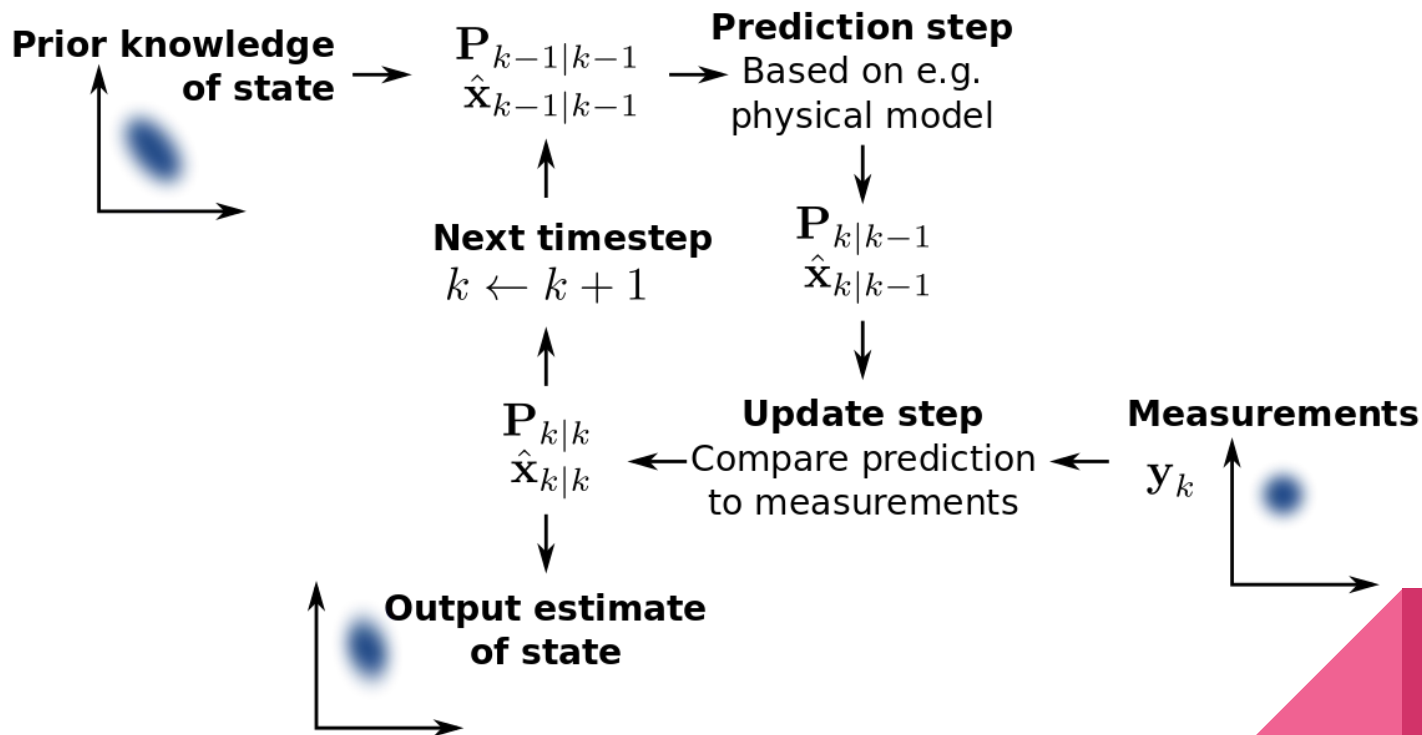
Non-linearities

$$k_s = A_s e^{-E_s/RT}$$

p, fm, I, td, tc

$$P_0 = (2f'C_I k_I / (k_{td} + k_{tc}))^{1/2}$$

Kalman Filtering



Kalman Filtering: Model Linearization

$$\frac{d\mathbf{X}}{dt} = \mathbf{f}(\mathbf{X}, \mathbf{U}, \mathbf{D}) \quad (1)$$

$$\frac{d\mathbf{X}}{dt} \approx \mathbf{f}(\mathbf{X}^*, \mathbf{U}^*, \mathbf{D}^*) + \mathbf{A}(\mathbf{X} - \mathbf{X}^*) + \mathbf{B}(\mathbf{U} - \mathbf{U}^*) + \mathbf{H}(\mathbf{D} - \mathbf{D}^*) \quad (2)$$

$$\mathbf{x} = \mathbf{X} - \mathbf{X}^*, \quad \mathbf{u} = \mathbf{U} - \mathbf{U}^*, \quad \mathbf{d} = \mathbf{D} - \mathbf{D}^* \quad (3)$$

$$\frac{d\mathbf{x}}{dt} \approx \mathbf{f}(\mathbf{X}^*, \mathbf{U}^*, \mathbf{D}^*) + \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{H}\mathbf{d} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{H}\mathbf{d} \quad (4)$$

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}_u\mathbf{u}(k) + \mathbf{\Gamma}_d\mathbf{d}(k)$$

Kalman Filtering

- **Prediction step**

$$\hat{\mathbf{x}}(k | k-1) = \Phi \hat{\mathbf{x}}(k-1 | k-1) + \Gamma \mathbf{u}(k-1)$$

$$\mathbf{P}(k | k-1) = \Phi \mathbf{P}(k-1 | k-1) \Phi^T + \Gamma_d \mathbf{Q}_d \Gamma_d^T$$

- **Kalman Gain Computation**

$$\mathbf{L}^*(k) = \mathbf{P}(k | k-1) \mathbf{C}^T [\mathbf{C} \mathbf{P}(k | k-1) \mathbf{C}^T + \mathbf{R}]^{-1}$$

- **Update step**

$$\mathbf{e}(k) = [\mathbf{y}(k) - \mathbf{C} \hat{\mathbf{x}}(k | k-1)]$$

$$\hat{\mathbf{x}}(k | k) = \hat{\mathbf{x}}(k | k-1) + \mathbf{L}^*(k) \mathbf{e}(k)$$

$$\mathbf{P}(k | k) = [\mathbf{I} - \mathbf{L}^*(k) \mathbf{C}] \mathbf{P}(k | k-1)$$

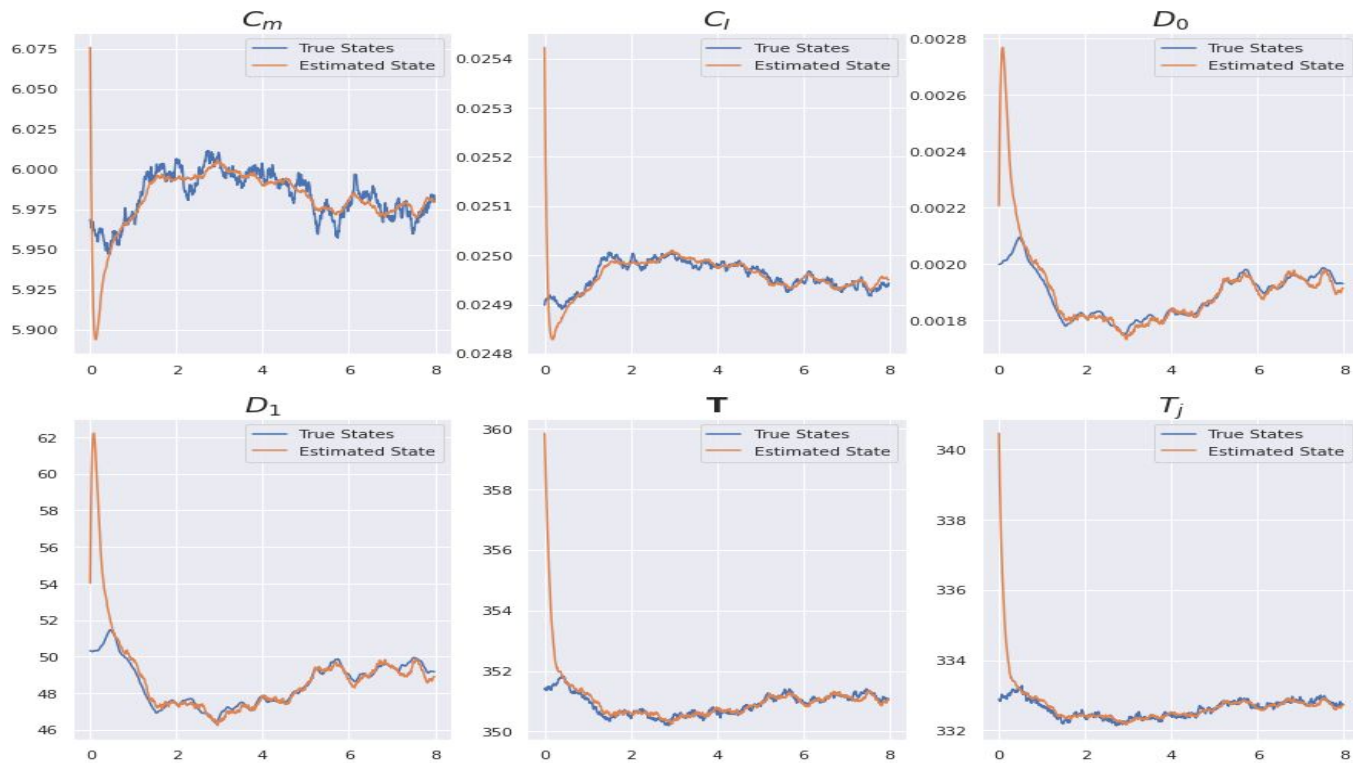
Generating States & Measurements

- Measurements are available for T and Tj
- Sampling interval of 28.8 seconds (=0.008 hrs)
- Expected simulation time: 1000 instants = 8 hrs
- Simulation parameters^[1]

$$\mathbf{x}(0) = [5.9655 \ 0.0249 \ 0.0020 \ 50.3287 \ 351.4013 \ 332.90774]^T$$
$$Q = \text{diag}\{3.2028 \times 10^{-6}; 6.2 \times 10^{-12}; 25 \times 10^{-14}; 25.28 \times 10^{-6}; 12.34 \times 10^{-4}; 11.08 \times 10^{-4}\}$$
$$R = \text{diag}\{2.5 \times 10^{-1} \ 2.5 \times 10^{-1}\} \quad \hat{\mathbf{x}}_{0|0} = 1.025 \times \mathbf{x}_{0|0} \quad P_{0|0} = 10 \times Q$$

Results - Kalman Filter Performance

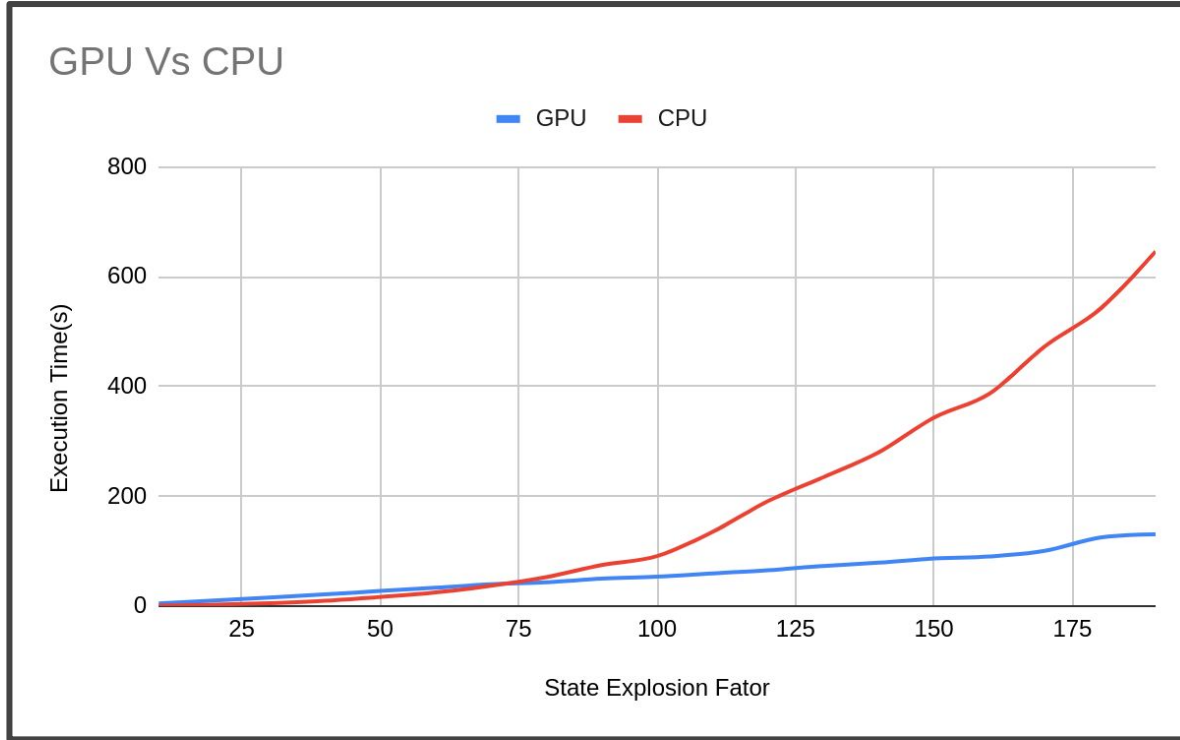
Kalman Filter: True and Estimated States



X axis: Time in hours

State	RMSE
1	0.012
2	0.00003
3	0.0001
4	1.744
5	0.847
6	0.687

Results - Effect of Parallelization



State Explosion Factor= 1, Number of states=6

GPU: 1.013s

CPU: 0.33s

State Explosion Factor	GPU Time(s)	CPU Time(s)
10	4.96	0.9
20	10.17	2.45
30	15.59	5.28
40	21.27	9.48
50	27.48	16.53
60	33.4	24.72
70	39.74	37.05
80	43.41	52.75
90	49.89	74.61

Work Division

- Problem Selection, System Selection: Siddhesh , Chitrangna
- Modelling the System dynamics: Siddhesh
- Understanding Kalman filter: Chitrangna
- True State (data) Generation: Aryan
- KF implementation: Aryan and Siddhesh
- Experiments and Plotting Code: Chitrangna
- Report, PPT: All 3

Conclusion and Future Work

- For small matrices, CPU should be used over GPU and for applications where state space is high dimensional use a GPU
- Using the state estimation algorithm to aid the online Model Predictive Control (MPC) algorithm to control the process
- Predicting adverse phenomena such as Trommsdorff–Norrish effect that occur due to local variations in the viscosity, using a more complicated state space model

Thank you

Supplementary: Meaning of states

C_m concentration of monomer inside the reactor (kg mol/m^3)

C_i concentration of monomer inside the reactor

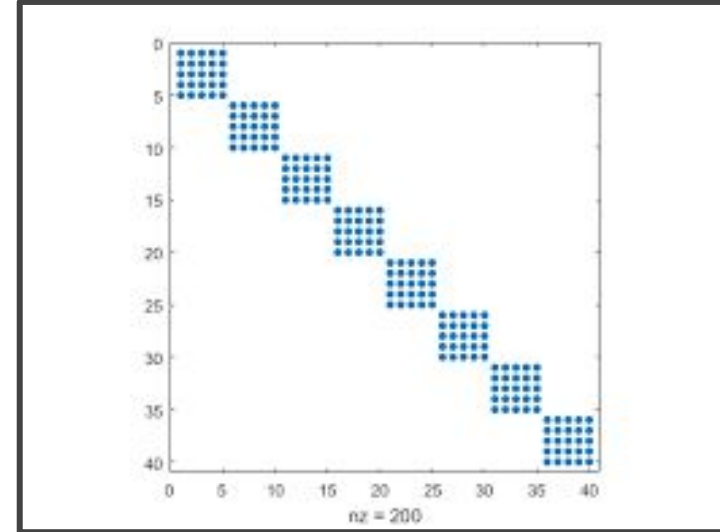
D_0 and D_1 Polymer moments (used to calculate the weight and density of the Polymer)

T Reactor Temperature

T_j Jacket Temperature

Supplementary - Block Diagonal Matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \& \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & 0 & 0 \\ 4 & 5 & 6 & 0 & 0 \\ 7 & 8 & 9 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 & 4 \end{pmatrix}$$



Supplementary - GPU Specifications

NVIDIA-SMI 465.19.01				Driver Version: 460.32.03		CUDA Version: 11.2	
GPU	Name	Persistence-M		Bus-Id	Disp.A	Volatile	Uncorr. ECC
Fan	Temp	Perf	Pwr:Usage/Cap	Memory-Usage		GPU-Util	Compute M.
						MIG M.	
0	Tesla T4		Off	00000000:00:04.0	Off		0
N/A	71C	P0	31W / 70W	11600MiB / 15109MiB		0%	Default
						N/A	

2560 CUDA Cores
Clock speed: 1582 MHz

Supplementary- RAM(VM) Specs GPU

Filesystem	Size	Used	Avail	Use%	Mounted on
overlay	69G	39G	30G	57%	/
tmpfs	64M	0	64M	0%	/dev
tmpfs	6.4G	0	6.4G	0%	/sys/fs/cgroup
shm	5.8G	0	5.8G	0%	/dev/shm
/dev/sda1	75G	41G	35G	54%	/opt/bin
tmpfs	6.4G	24K	6.4G	1%	/var/colab
tmpfs	6.4G	0	6.4G	0%	/proc/acpi
tmpfs	6.4G	0	6.4G	0%	/proc/scsi
tmpfs	6.4G	0	6.4G	0%	/sys/firmware

Supplementary - CPU Specifications(Processor 1)

```
processor      : 0
vendor id     : GenuineIntel
cpu family    : 6
model         : 63
model name    : Intel(R) Xeon(R) CPU @ 2.30GHz
stepping      : 0
microcode     : 0x1
cpu MHz       : 2299.998
cache size    : 46080 KB
physical id   : 0
siblings      : 2
core id       : 0
cpu cores     : 1
apicid        : 0
initial apicid : 0
fpu           : yes
fpu exception : yes
cpuid level   : 13
```


CPU Specs - Continued (Processor 1)

```
wp           : yes
flags        : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov pat pse36 clflush mmx fxsr
sse sse2 ss ht syscall nx pdpe1gb rdtscp lm constant tsc rep good nopl xtopology nonstop tsc cpuid
tsc_known_freq pni pclmulqdq ssse3 fma cx16 pcid sse4_1 sse4_2 x2apic movbe popcnt aes xsave avx f16c
rdrand hypervisor lahf lm abm invpcid single ssbd ibrs ibpb stibp fsgsbase tsc_adjust bmi1 avx2 smep
bmi2 erms invpcid xsaveopt arat md_clear arch_capabilities
bugs         : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass l1tf mds swapgs
bogomips     : 4599.99
clflush size  : 64
cache alignment : 64
address sizes : 46 bits physical, 48 bits virtual
```

CPU Specs continued (Processor 2)

```
processor      : 1
vendor id     : GenuineIntel
cpu family    : 6
model         : 63
model name    : Intel(R) Xeon(R) CPU @ 2.30GHz
stepping      : 0
microcode     : 0x1
cpu MHz       : 2299.998
cache size    : 46080 KB
physical id   : 0
siblings      : 2
core id       : 0
cpu cores     : 1
apicid        : 1
```

CPU Specs Continued(Processor 2)

```
initial apicid   : 1
fpu              : yes
fpu exception    : yes
cpuid level     : 13
wp              : yes
flags           : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov
pat pse36 clflush mmx fxsr sse sse2 ss ht syscall nx pdpe1gb rdtscp lm
constant tsc rep good nopl xtopology nonstop tsc cpuid tsc known freq pni
pclmulqdq ssse3 fma cx16 pcid sse4_1 sse4_2 x2apic movbe popcnt aes xsave
avx f16c rdrand hypervisor lahf_lm abm invpcid_single ssbd ibrs ibpb stibp
fsgsbase tsc adjust bmi1 avx2 smep bmi2 erms invpcid xsaveopt arat md_clear
arch_capabilities
bugs            : cpu_meltdown spectre_v1 spectre_v2 spec_store_bypass lltf mds
swapgs
bogomips       : 4599.99
clflush size    : 64
cache_alignment : 64
address sizes    : 46 bits physical, 48 bits virtual
power management:
```

CPU - VM specs

Filesystem	Size	Used	Avail	Use%	Mounted on
overlay	108G	39G	70G	36%	/
tmpfs	64M	0	64M	0%	/dev
tmpfs	6.4G	0	6.4G	0%	/sys/fs/cgroup
shm	5.9G	0	5.9G	0%	/dev/shm
tmpfs	6.4G	32K	6.4G	1%	/var/colab
/dev/sda1	114G	41G	74G	36%	/etc/hosts
tmpfs	6.4G	0	6.4G	0%	/proc/acpi
tmpfs	6.4G	0	6.4G	0%	/proc/scsi
tmpfs	6.4G	0	6.4G	0%	/sys/firmware

Graphics Processor	
GPU Name:	TU104
GPU Variant:	TU104-895-A1
Architecture:	Turing
Foundry:	TSMC
Process Size:	12 nm
Transistors:	13,600 million
Die Size:	545 mm²

Memory	
Memory Size:	16 GB
Memory Type:	GDDR6
Memory Bus:	256 bit
Bandwidth:	320.0 GB/s

Graphics Features	
DirectX:	12 Ultimate (12_2)
OpenGL:	4.6
OpenCL:	3.0
Vulkan:	1.2
CUDA:	7.5
Shader Model:	6.6

Graphics Card	
Release Date:	Sep 13th, 2018
Generation:	Tesla (Txx)
Production:	Active
Bus Interface:	PCIe 3.0 x16

Clock Speeds	
Base Clock:	585 MHz
Boost Clock:	1590 MHz
Memory Clock:	1250 MHz 10 Gbps effective

Board Design	
Slot Width:	Single-slot
Length:	168 mm 6.6 inches
TDP:	70 W
Suggested PSU:	250 W
Outputs:	No outputs
Power Connectors:	None
Board Number:	PG183 SKU 200

Relative Performance	
Radeon RX Vega 64	85%
GeForce GTX 1080	86%
GeForce RTX 2060	86%
Radeon RX 5700	90%
GeForce RTX 2060 SU...	97%
Tesla T4	100%
GeForce RTX 2070	101%
GeForce RTX 3060	103%
Radeon RX 5700 XT	103%
Radeon VII	104%
TITAN X Pascal	107%

Based on TPU review data: "Performance Summary" at 1920x1080, 4K for 2080 Ti and faster. Performance estimated based on architecture, shader count and clocks.

Render Config	
Shading Units:	2560
TMUs:	160
ROPs:	64
SM Count:	40
Tensor Cores:	320
RT Cores:	40
L1 Cache:	64 KB (per SM)
L2 Cache:	4 MB

Theoretical Performance	
Pixel Rate:	101.8 GPixel/s
Texture Rate:	254.4 GTexel/s
FP16 (half) performance:	65.13 TFLOPS (8:1)
FP32 (float) performance:	8.141 TFLOPS
FP64 (double) performance:	254.4 GFLOPS (1:32)

Tesla
T4