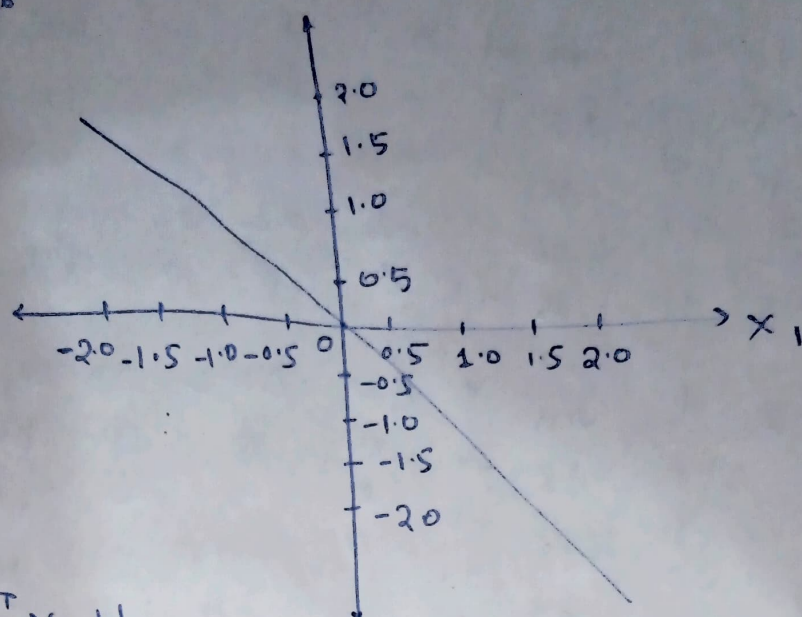Assume weight vector of initial decision boundary $w^T x = 0$

$w = [1, 1]$

a) $x_1 + x_2 = 0$

$b = 0$

$y_{in} = w_i^T x_i + b = w_1 x_1 + w_2 x_2 + b$

$x_1 + x_2 = 0$

Assume learning rate as 1

$$y = \begin{cases} 1 & \text{if } y_{in} > 0 \\ 0 & \text{if } y_{in} = 0 \\ -1 & \text{if } y_{in} < 0 0 \end{cases}$$

$\Delta w_1 = \alpha t x_1$ $\Delta b = \alpha t$

$\Delta w_2 = \alpha t x_2$

I)

x_1	x_2	class(t)	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
					0	0	0	1	1	0
1	1	+1	2	+1	0	0	0	1	1	0
-1	-1	-1	-2	-1	0	-0.5	-1	1	0.5	-1
0	0.5	-1	0.5	+1	0	0	0	1	0.5	-1
0.1	0.5	-1	-0.65	-1	0	0.2	0.2	1	1.2	0.7
0.2	0.2	+1	0.7	-1	0.2	0	0	1.2	0.7	0
0.9	0.5	+1	1.43	+1	0		1.2	0.7	0	

x_1	x_2	t	$\dot{y}_{in}$	$\dot{y}$	$\Delta\dot{w}_1$	$\Delta\dot{w}_2$	Δb	$\dot{w}_1$	$\dot{w}_2$	b
1	1	+1	1.9	1	0	0	0	1.2	0.7	0
-1	-1	-1	-1.9	-1	0	0	0	1.2	0.7	0
0	0.5	-1	0.35	1	0	-0.5	-1	1.2	0.2	-1
0.1	0.5	-1	-0.78	-1	0	0	0	1.2	0.2	-1
0.2	0.2	+1	-0.72	-1	0.2	0.2	1	1.4	0.4	0
0.9	0.5	+1	1.46	+1	0	0	0	1.4	0.4	0

(II)

x_1	x_2	t	$\dot{y}_{in}$	$\dot{y}$	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.8	+1	0	0	0	1.4	0.4	0
-1	-1	-1	-1.8	-1	0	0	0	1.4	0.4	0
0	0.5	-1	-1.8	+1	0	-0.5	-1	1.4	-0.1	-1
0.1	0.5	-1	0.2	-1	0	0	0	1.4	-0.1	-1
0.2	0.2	+1	-0.81	-1	0.2	0.2	1	1.6	0.1	0
0.9	0.5	+1	-0.74 / 1.44	+1	0	0	0	1.6	0.1	0

(IV)

x_1	x_2	t	$\dot{y}_{in}$	$\dot{y}$	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.7	+1	0	0	0	1.6	0.1	0
-1	-1	-1	-1.7	-1	0	0	0	1.6	0.1	0
0	0.5	-1	0.05	+1	0	-0.5	-1	1.6	-0.4	-1
0.1	0.5	-1	-1.04	-1	0	0	0	1.6	-0.4	-1
0.2	0.2	+1	-0.76	-1	0.2	0.2	1	1.8	-0.2	0
0.9	0.5	+1	1.52	+1	0	0	0	1.8	-0.2	0

(V)

| x_1	x_2	t	$\dot{y}_{in}$	$\dot{y}$	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.4	+1	0	0	0	1.9	-0.5	0
-1	-1	-1	-1.4	-1	0	0	0	1.9	-0.5	0
0	0.5	-1	-0.25	-1	0	0	0	1.9	-0.5	0
0.1	0.5	-1	-0.06	-1	0	0	0	1.9	-0.5	0
0.2	0.2	+1	0.28	+1	0	0	0	1.9	-0.5	0
0.9	0.5	+1	1.46	+1	0	0	0	1.9	-0.5	0

The Perceptron learning algorithm converged in 6 steps.
The final weight vector of the decision boundary is

$$w = [1.9, -0.5]$$

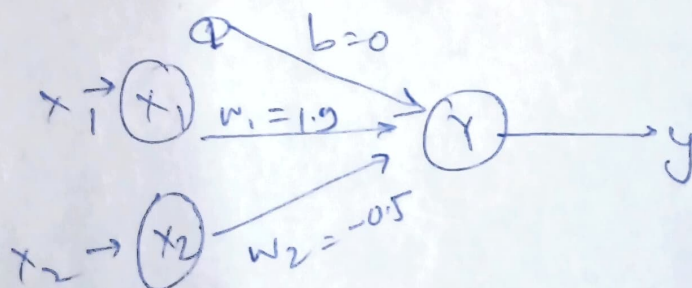$$1.9x_1 + (-0.5)x_2 = 0 \Rightarrow 1.9x_1 - 0.5x_2 = 0$$

Lets plot the final decision boundary

We can see that $1.9x_1 - 0.5x_2 = 0$ line separates the two classes correctly
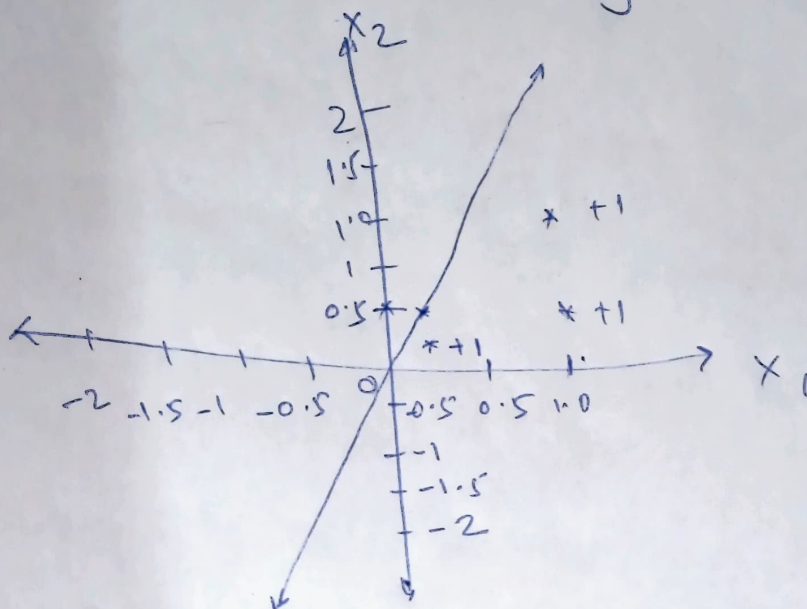
$$1.9x_1 - 0.5x_2 = 0$$

Final decision boundary

Neural network corresponding to the perceptron.