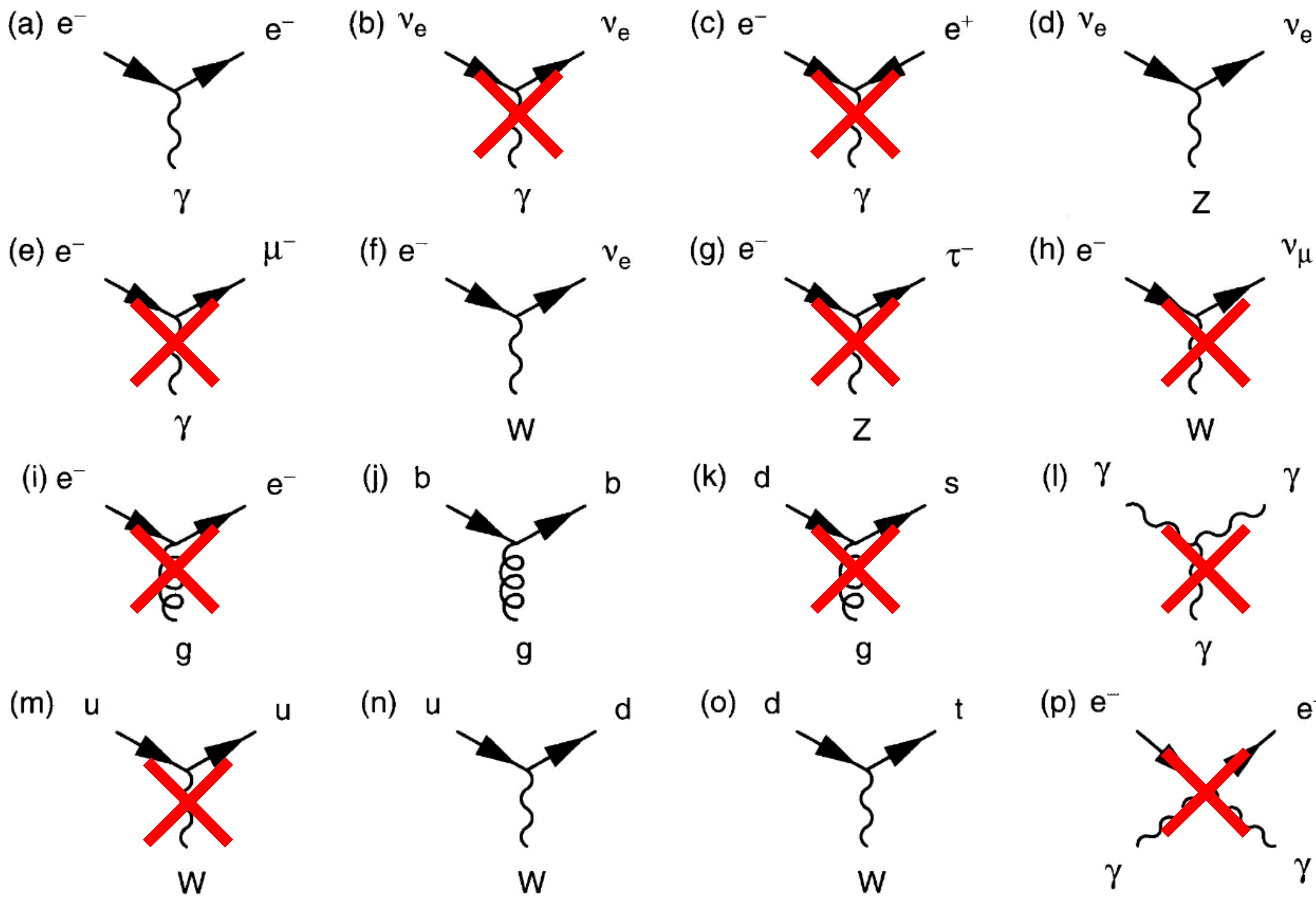
粒子物理学

习题课2

作业4

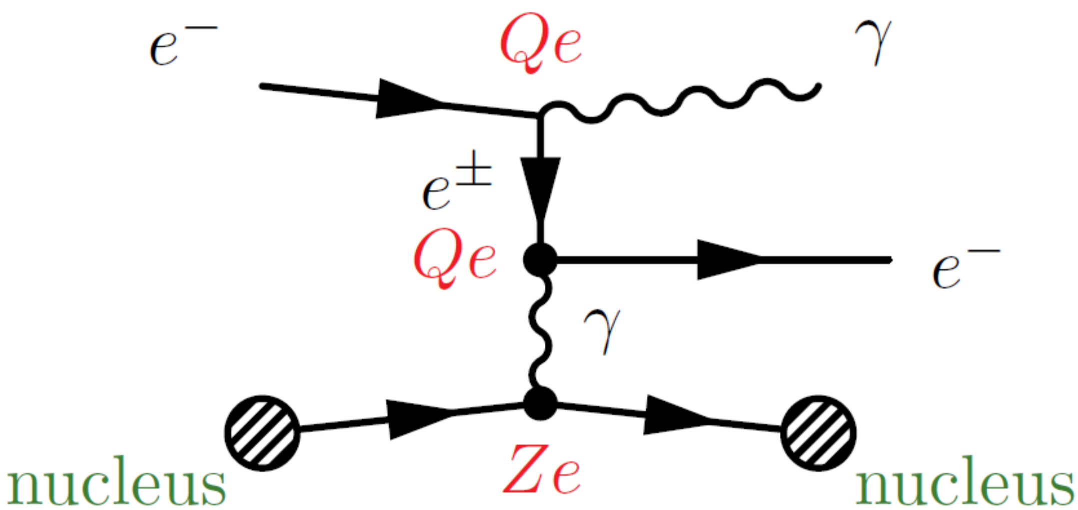
1. 下面这些过程是否是标准模型费曼图？



作业4

2. 画出电子与原子核（核电荷数为Ze）发生电磁相互作用，韧致辐射出光子的最低阶费曼图，并且估计其反应截面

Bremsstrahlung ($e^- \to e^- \gamma$)



$$M \propto Ze^3$$

$$\sigma \propto |M|^2 \propto Z^2 e^6$$

$$\propto (4\pi)^3 Z^2 \alpha^3$$

作业4

3. γ矩阵相关证明题：

a) **利用**$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$**，尝试证明：**

$$\gamma^\mu\gamma_\mu = 4, \gamma^\mu \not{a} \gamma_\mu = -2\not{a}, \gamma^\mu \not{a}\not{b}\gamma_\mu = 4a\cdot b$$

i) $\gamma^\mu\gamma_\mu = g_{\mu\nu}\gamma^\mu\gamma^\nu = g_{\mu\nu}(2g^{\mu\nu} - \gamma^\nu\gamma^\mu) = 8 - g_{\mu\nu}\gamma^\nu\gamma^\mu = 8 - \gamma_\mu\gamma^\mu$

$$\gamma^\mu\gamma_\mu = 4$$

ii) $\gamma^\mu\not{a}\gamma_\mu = \gamma^\mu\gamma^\nu a_\nu\gamma_\mu = (2g^{\mu\nu} - \gamma^\nu\gamma^\mu)a_\nu\gamma_\mu = 2g^{\mu\nu}a_\nu\gamma_\mu - \gamma^\nu\gamma^\mu a_\nu\gamma_\mu = 2\not{a} - 4\not{a} = -2\not{a}$

iii) $\gamma^\mu\not{a}\not{b}\gamma_\mu = \gamma^\mu\gamma^\nu a_\nu\gamma^\sigma b_\sigma\gamma_\mu = \gamma^\mu\gamma^\nu a_\nu b^\sigma\gamma_\sigma\gamma_\mu = (2g^{\mu\nu} - \gamma^\nu\gamma^\mu)a_\nu b^\sigma\left(2g_{\sigma\mu} - \gamma_\mu\gamma_\sigma\right)$

$$= 4g^{\mu\nu}a_\nu b^\sigma g_{\sigma\mu} - 2g_{\sigma\mu}\gamma^\nu\gamma^\mu a_\nu b^\sigma - 2g^{\mu\nu}a_\nu b^\sigma\gamma_\mu\gamma_\sigma + \gamma^\nu\gamma^\mu a_\nu b^\sigma\gamma_\mu\gamma_\sigma$$

$$= 4a\cdot b - 2\not{a}\not{b} - 2\not{a}\not{b} + 4\not{a}\not{b} = 4a\cdot b$$

作业4

3. γ矩阵相关证明题：

b) 由手征算符$P_R = \frac{1}{2}(1+\gamma^5), P_L = \frac{1}{2}(1-\gamma^5)$，证明：

$$P_L + P_R = 1, \qquad P_R P_R = P_R, \qquad P_L P_L = P_L, \qquad P_L P_R = 0$$

i) $P_L + P_R = \frac{1}{2}(1+\gamma^5) + \frac{1}{2}(1-\gamma^5) = 1$

ii)$P_R P_R = \frac{1}{4}(1+\gamma^5)(1+\gamma^5) = \frac{1}{4}(1+2\gamma^5+1) = P_R$

iii) $P_L P_L = \frac{1}{4}(1-\gamma^5)(1-\gamma^5) = \frac{1}{4}(1-2\gamma^5+1) = P_L$

iv) $P_L P_R = \frac{1}{4}(1-\gamma^5)(1+\gamma^5) = \frac{1}{4}(1+\gamma^5-\gamma^5-1) = 0$

c) $\left[\bar{\psi}\gamma^\mu\gamma^5\phi\right]^\dagger = \bar{\phi}\gamma^\mu\gamma^5\psi$

$$[\bar{\psi}\gamma^\mu\gamma^5\phi]^\dagger = \left[\psi^\dagger\gamma^0\gamma^\mu\gamma^5\phi\right]^\dagger = \phi^\dagger\gamma^5(\gamma^\mu)^\dagger\gamma^0\psi = \phi^\dagger\gamma^0\gamma^0\gamma^5\gamma^0\gamma^\mu\gamma^0\gamma^0\psi$$
$$= \bar{\phi}\gamma^0\gamma^5\gamma^0\gamma^\mu\psi = -\bar{\phi}\gamma^0\gamma^0\gamma^5\gamma^\mu\psi = \bar{\phi}\gamma^\mu\gamma^5\psi$$

作业4

4. 证明在极端相对论近似下：

$$\frac{(\boldsymbol{p_1} \cdot \boldsymbol{p_3})^2 + (\boldsymbol{p_1} \cdot \boldsymbol{p_4})^2}{(\boldsymbol{p_1} \cdot \boldsymbol{p_2})^2} = \frac{\boldsymbol{t}^2 + \boldsymbol{u}^2}{\boldsymbol{s}^2}$$

$$t = (p_1 - p_3)^2 \approx -2p_1p_3, \qquad s \approx 2p_1p_2 = 2p_3p_4, \qquad u \approx 2p_1p_4 = 2p_2p_3$$

作业4

5. 模仿课件中计算muon current的做法，计算electron current。

$$u_\uparrow = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}, u_\downarrow = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}, v_\uparrow = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}, v_\downarrow = -\sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

$$\begin{aligned}
\overline{\psi}\gamma^0\phi &= \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \\
\overline{\psi}\gamma^1\phi &= \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \\
\overline{\psi}\gamma^2\phi &= \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) \\
\overline{\psi}\gamma^3\phi &= \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2
\end{aligned}$$

$$j_e^\mu = \bar{v}(p_2)\gamma^\mu u(p_1)$$

$$\bar{v}_\uparrow(p_2)\gamma^\mu u_\uparrow(p_1) = (0,0,0,0)$$

$$\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) = 2E(0,-1,i,0)$$

$$\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1) = 2E(0,-1,-i,0)$$

$$\bar{v}_\downarrow(p_2)\gamma^\mu u_\downarrow(p_1) = (0,0,0,0)$$

作业4

6. 阅读Thomson书本6.5节，尝试使用trace的技巧，计算$e^-\mu^- \to e^-\mu^-$的矩阵元的平方：

$$-iM = \bar{u}(p_3 s_3) ie\gamma^\mu u(p_1 s_1) \frac{-ig_{\mu\nu}}{q^2} \bar{u}(p_4 s_4) ie\gamma^\mu u(p_2 s_2)$$

$$\downarrow$$

$$\langle |M_{fi}|^2 \rangle = 2e^4 \frac{s^2 + u^2}{t^2}$$

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Thomson书——6.5节

作业5

1. 证明一些γ矩阵迹的性质：

a) $\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu \equiv 2g^{\mu\nu}I,$$

$$\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right) + \mathrm{Tr}\left(\gamma^\nu\gamma^\mu\right) = 2g^{\mu\nu}\,\mathrm{Tr}\left(I\right),$$

$$\boxed{\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right) = 4g^{\mu\nu}.}$$

b) Tr(奇数个 γ矩阵) = 0

$$\begin{aligned}\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\right) &= \mathrm{Tr}\left(\gamma^5\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\right) \\ &= \mathrm{Tr}\left(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^5\right) && \text{(traces are cyclical)} \\ &= -\mathrm{Tr}\left(\gamma^5\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\right) && \text{(since } \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5\text{)}\end{aligned}$$

$$\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\right) = -\,\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\right),$$

$$\boxed{\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\right) = 0.}$$

作业5

1. 证明一些γ矩阵迹的性质：

c) $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4g^{\mu\nu}g^{\rho\sigma} - 4g^{\mu\rho}g^{\nu\sigma} + 4g^{\mu\sigma}g^{\nu\rho}$

$$\begin{aligned}
\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma &= 2g^{\mu\nu}\gamma^\rho\gamma^\sigma - \gamma^\nu\gamma^\mu\gamma^\rho\gamma^\sigma \\
&= 2g^{\mu\nu}\gamma^\rho\gamma^\sigma - 2g^{\mu\rho}\gamma^\nu\gamma^\sigma + \gamma^\nu\gamma^\rho\gamma^\mu\gamma^\sigma \\
&= 2g^{\mu\nu}\gamma^\rho\gamma^\sigma - 2g^{\mu\rho}\gamma^\nu\gamma^\sigma + 2g^{\mu\sigma}\gamma^\nu\gamma^\rho - \gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\mu \\
\Rightarrow \quad \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma + \gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\mu &= 2g^{\mu\nu}\gamma^\rho\gamma^\sigma - 2g^{\mu\rho}\gamma^\nu\gamma^\sigma + 2g^{\mu\sigma}\gamma^\nu\gamma^\rho. \qquad (6.57)
\end{aligned}$$

$$2\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right) = 2g^{\mu\nu}\,\mathrm{Tr}\left(\gamma^\rho\gamma^\sigma\right) - 2g^{\mu\rho}\,\mathrm{Tr}\left(\gamma^\nu\gamma^\sigma\right) + 2g^{\mu\sigma}\,\mathrm{Tr}\left(\gamma^\nu\gamma^\rho\right),$$

$$\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right) = 4g^{\mu\nu}g^{\rho\sigma} - 4g^{\mu\rho}g^{\nu\sigma} + 4g^{\mu\sigma}g^{\nu\rho}.$$

作业5

2. 第一课中得到，微分截面满足公式：

$$\frac{d\sigma}{dt}=\frac{1}{64\pi s\left|\vec{p}_i^*\right|^2}\left|M_{fi}\right|^2$$

第四节课中又得到，对$e^-\mu^-\to e^-\mu^-$散射过程，有：

$$\left|M_{fi}\right|^2=2e^4\frac{s^2+u^2}{t^2}$$

从这两个式子证明：

$$\frac{d\sigma}{dq^2}=\frac{2\pi\alpha^2}{q^4}\left[1+\left(1+\frac{q^2}{s}\right)^2\right]$$

$$|\vec{p}_i^*|^2=\frac{1}{4s}[s-(m_1+m_2)^2][s-(m_1-m_2)^2]\approx\frac{s}{4}$$

$$\frac{d\sigma}{dt}=\frac{1}{16\pi s^2}2e^4\frac{s^2+u^2}{t^2}=\frac{e^4}{8\pi t^2}\frac{s^2+u^2}{s^2}$$

$$=\frac{2\pi\alpha^2}{q^4}\left[1+\frac{(s+t)^2}{s^2}\right]$$

$$=\frac{2\pi\alpha^2}{q^4}\left[1+\left(1+\frac{q^2}{s}\right)^2\right]$$

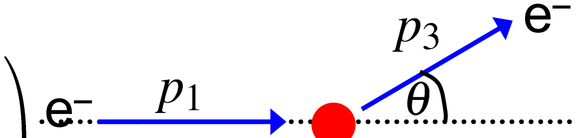
$$t=q^2$$

$$\alpha=\frac{e^2}{4\pi}$$

$$s+t+u=\sum m^2=0$$

作业5

3. 模仿课上对卢瑟福散射的推导，尝试推导莫特散射公式（提示：莫特散射中靶的反弹可忽略且忽略电子质量）



(neglect proton recoil)

$$u_\uparrow(p_1)=N_e\begin{pmatrix}1\\0\\\alpha\\0\end{pmatrix}\quad u_\downarrow(p_1)=N_e\begin{pmatrix}0\\1\\0\\-\alpha\end{pmatrix}\quad u_\uparrow(p_3)=N_e\begin{pmatrix}c\\s\\\alpha c\\\alpha s\end{pmatrix}\quad u_\downarrow(p_3)=N_e\begin{pmatrix}-s\\c\\\alpha s\\-\alpha c\end{pmatrix}$$

$$\bar{u}_\uparrow(p_3)\gamma^\mu u_\uparrow(p_1)=(E+\cancel{m_e})\left[(\alpha^2+1)c,2\alpha s,-2i\alpha s,2\alpha c\right]$$

$$\bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)=(E+\cancel{m_e})\left[(\alpha^2+1)c,2\alpha s,-2i\alpha s,2\alpha c\right]$$

$$\cancel{\bar{u}_\uparrow(p_3)\gamma^\mu u_\downarrow(p_1)=(E+m_e)\left[(1-\alpha^2)s,0,0,0\right]}$$

$$\cancel{\bar{u}_\downarrow(p_3)\gamma^\mu u_\uparrow(p_1)=(E+m_e)\left[(\alpha^2-1)s,0,0,0\right]}$$

极端相对论极限：$\alpha\to 1$

$$u_\uparrow(0)=\sqrt{2M_p}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\quad u_\downarrow(0)=\sqrt{2M_p}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\quad j_{p\uparrow\uparrow}=j_{p\downarrow\downarrow}=2M_p(1,0,0,0)\quad j_{p\uparrow\downarrow}=j_{p\downarrow\uparrow}=0$$

$$\mathcal{M}_{fi}=\frac{e^2}{q^2}j_e\cdot j_p\qquad \left\langle|M_{fi}|^2\right\rangle=\frac{1}{4}\frac{e^4}{q^4}(16E^2M_p^2)\left(4\cos^2\frac{\theta}{2}\right)\qquad q^2=-4E^2\sin^2\frac{\theta}{2}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{1}{64\pi^2M_p^2}|M_{fi}|^2\qquad \left(\frac{d\sigma}{d\Omega}\right)_{Mott}=\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}$$

作业5

4. 在一个e^-p散射的实验中，入射电子的能量$E_1 = 529.5MeV$，出射电子在相对于入射电子角度等于75°的地方被探测，试求出射电子的能量E_3和四动量转移的平方Q^2

$$E_3 = \frac{ME_1}{M + E_1(1 - \cos\theta)} \qquad E_3 = \frac{938 \times 529}{938 + 529(1 - \cos 75^\circ)} = 373\ \text{MeV}$$

$$q^2 = -\frac{2ME_1^2(1 - \cos\theta)}{M + E_1(1 - \cos\theta)} \qquad |q^2| = \frac{2 \times 938 \times 529^2(1 - \cos 75^\circ)}{938 + 529(1 - \cos 75^\circ)} = 29400\text{MeV}^2$$

作业5

5. 在将弹性碰撞过渡到非弹性碰撞时，介绍了四个新的运动学变量：x, y, v, Q^2

a) 写出四个变量的定义式并给出文字描述

b) 将x, y表示为v的形式，将Q^2表示为x, y的形式

$$x \equiv \frac{Q^2}{2p_2.q} \qquad Q^2 \equiv -q^2 \qquad y \equiv \frac{p_2.q}{p_2.p_1} \qquad \nu \equiv \frac{p_2.q}{M}$$

$$x = \frac{Q^2}{2M\nu} \qquad y = \frac{2M}{s-M^2}\nu \qquad Q^2 = (s-M^2)xy$$

作业5

$$y = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$

6.在一个固定靶的e^-p弹性碰撞中，已知：

$$Q^2 = 2M(E_1 - E_3) = 2ME_1 y, \qquad Q^2 = 4E_1E_3\sin^2\frac{\theta}{2}$$

a) 证明：

$$\sin^2\frac{\theta}{2} = \frac{E_1}{E_3}\frac{M^2}{Q^2}y^2, \qquad \frac{E_3}{E_1}\cos^2\frac{\theta}{2} = 1 - y - \frac{M^2y^2}{Q^2}$$

i) $Q^4 = 4M^2E_1^2y^2, Q^2 = 4E_1E_3\sin^2\frac{\theta}{2}$

两式相除：$Q^2 = \frac{4M^2E_1y^2}{4E_3\sin^2\frac{\theta}{2}}$，即 $\sin^2\frac{\theta}{2} = \frac{E_1}{E_3}\frac{M^2}{Q^2}y^2$

ii) $\frac{E_3}{E_1}cos^2\frac{\theta}{2} = \frac{E_3}{E_1}\left(1 - \sin^2\frac{\theta}{2}\right) = \frac{E_3}{E_1} - \frac{M^2}{Q^2}y^2 = 1 - y - \frac{M^2}{Q^2}y^2$

作业5

5.在一个固定靶的e^-p弹性碰撞中，已知：

$$Q^2 = 2M(E_1 - E_3) = 2ME_1y, \qquad Q^2 = 4E_1E_3\sin^2\frac{\theta}{2}$$

b) 假设角度对称，并利用$Q^2 = \frac{2ME_1^2(1-\cos\theta)}{M+E_1(1-\cos\theta)}$，证明：

$$\frac{d\sigma}{dQ^2} = \left|\frac{d\Omega}{dQ^2}\right|\frac{d\sigma}{d\Omega} = \frac{\pi}{E_3^2}\frac{d\sigma}{d\Omega}$$

$$d\Omega = \mathrm{d}(\cos\theta)d\phi = 2\pi d(cos\,\theta)$$

$$dQ^2 = \cdots = -2E_3^2 d(cos\,\theta)$$

$$\left|\frac{d\Omega}{dQ^2}\right| = \frac{\pi}{E_3^2}$$

$$\frac{d\sigma}{dQ^2} = \left|\frac{d\Omega}{dQ^2}\right|\frac{d\sigma}{d\Omega} = \frac{\pi}{E_3^2}\frac{d\sigma}{d\Omega}$$

作业5

5.在一个固定靶的e^-p弹性碰撞中，已知：

$$Q^2 = 2M(E_1 - E_3) = 2ME_1 y, \qquad Q^2 = 4E_1E_3\sin^2\frac{\theta}{2}$$

$$\sin^2\frac{\theta}{2} = \frac{E_1}{E_3}\frac{M^2}{Q^2}y^2,$$

$$\frac{E_3}{E_1}\cos^2\frac{\theta}{2} = 1 - y - \frac{M^2y^2}{Q^2}$$

$$\frac{d\sigma}{dQ^2} = \left|\frac{d\Omega}{dQ^2}\right|\frac{d\sigma}{d\Omega} = \frac{\pi}{E_3^2}\frac{d\sigma}{d\Omega}$$

c)利用上面两小问的结果，将Rosenbluth公式

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2}\frac{E_3}{E_1}\left(\frac{G_E^2+\tau G_M^2}{(1+\tau)}\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right)$$

改写为洛伦兹不变的形式：

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\frac{G_E^2+\tau G_M^2}{(1+\tau)}\left(1 - y - \frac{M^2y^2}{Q^2}\right) + \frac{1}{2}y^2G_M^2\right]$$

$$\tau = \frac{Q^2}{4M^2}$$

$$\frac{d\sigma}{dQ^2} = \frac{\pi}{E_3^2}\frac{\alpha^2}{4E_1^2\,sin^4\,\theta/2}\frac{E_3}{E_1}\left(\frac{G_E^2+\tau G_M^2}{(1+\tau)}cos^2\frac{\theta}{2} + 2\tau G_M^2\,sin^2\frac{\theta}{2}\right)$$

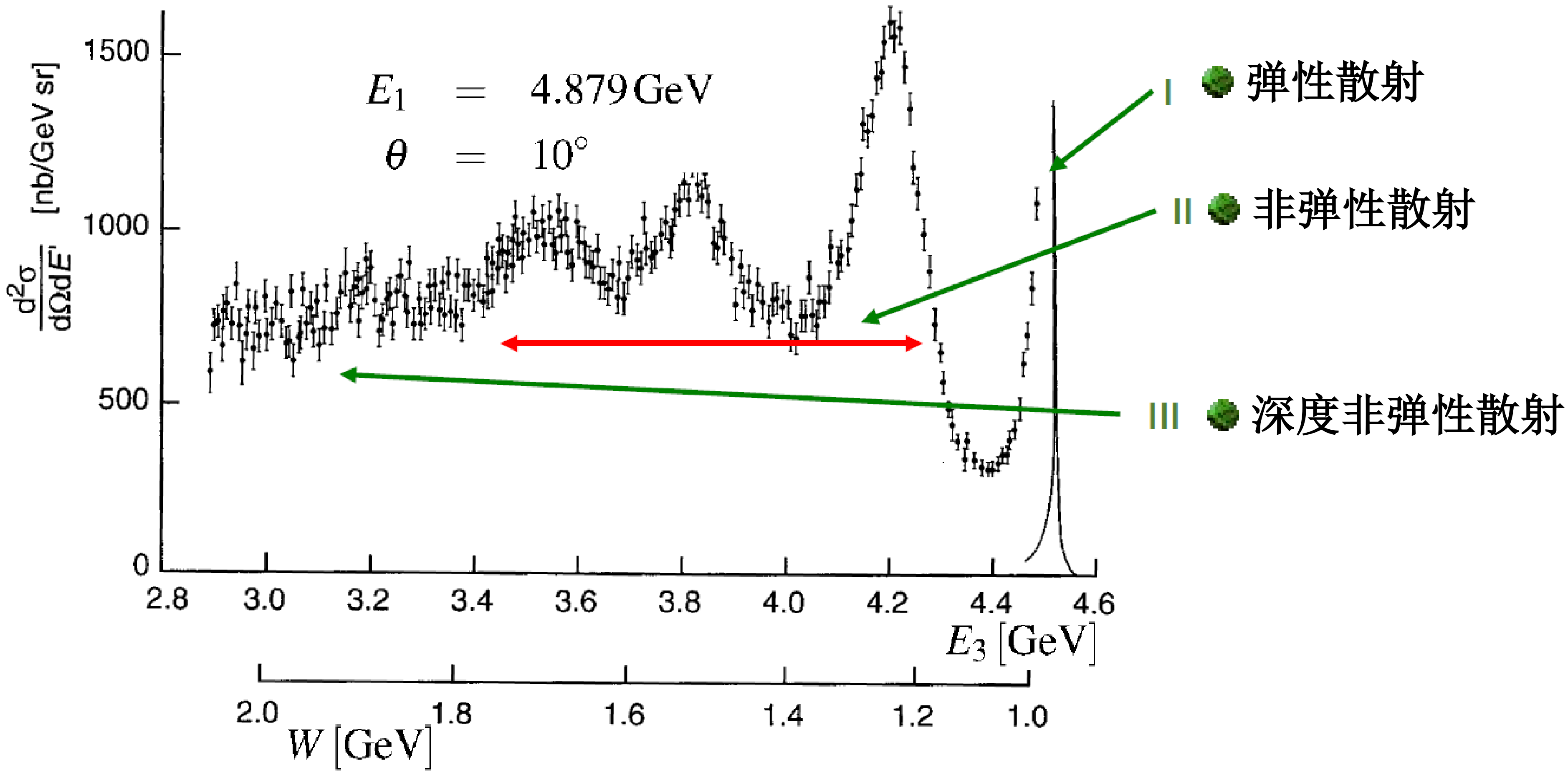
$$= \frac{4\pi\alpha^2}{Q^4}\frac{E_3}{E_1}\left[\frac{G_E^2+\tau G_M^2}{(1+\tau)}\frac{E_1}{E_3}\left(1 - y - \frac{M^2y^2}{Q^2}\right) + 2\frac{Q^2}{4M^2}G_M^2\frac{E_1}{E_3}\frac{M^2}{Q^2}y^2\right]$$

$$= \frac{4\pi\alpha^2}{Q^4}\left[\frac{G_E^2+\tau G_M^2}{(1+\tau)}\left(1 - y - \frac{M^2y^2}{Q^2}\right) + \frac{1}{2}y^2G_M^2\right]$$

作业6

1.读图答题：

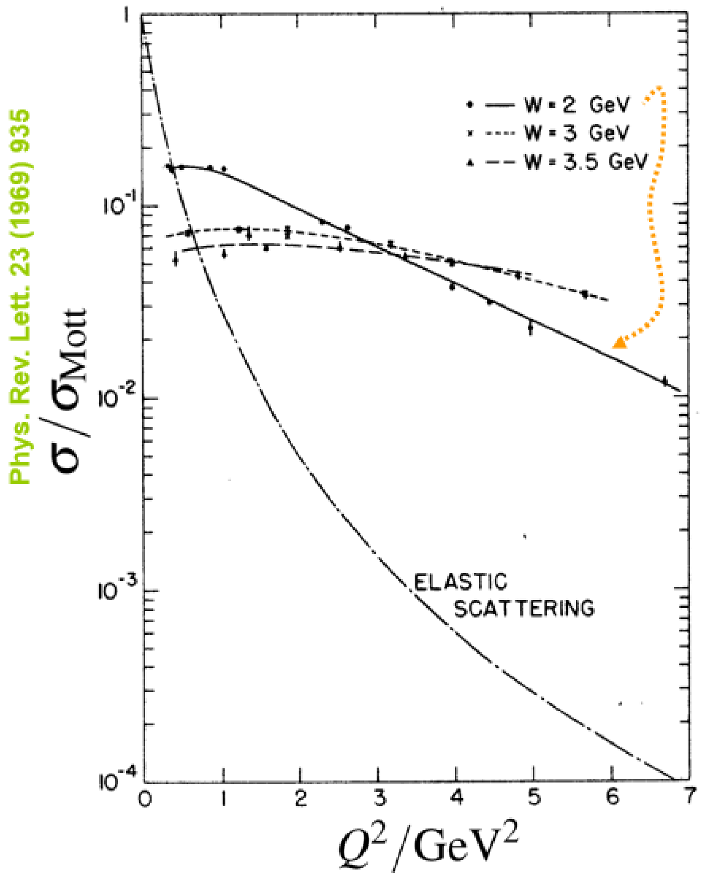
a) 辨别弹性散射、非弹性散射、深度非弹性散射(I,II,III)：



作业6

1.读图答题：

b) 描述弹性散射、深度非弹性散射中散射截面与q^2的关系



- 弹性散射截面随q^2迅速下降，由于质子不是点粒子(即 形状因子)
- 非弹性散射截面随q^2缓慢下降，
- 深度非弹性散射截面几乎与随q^2无关

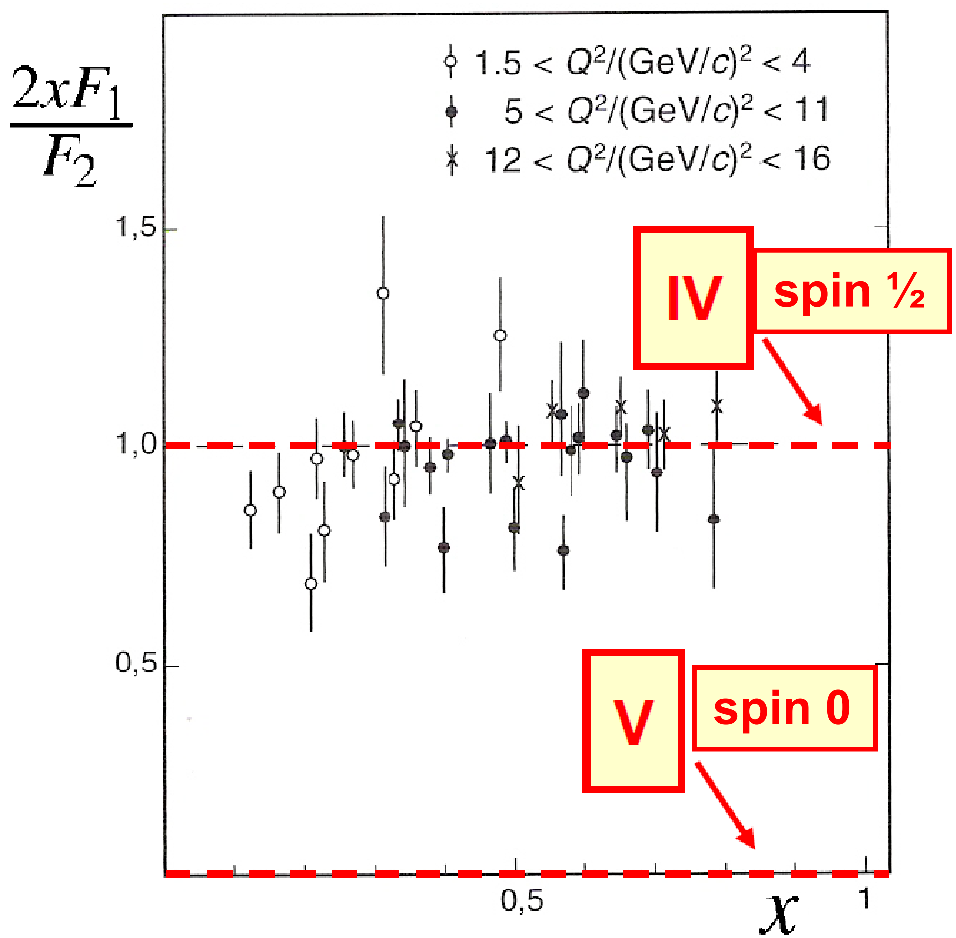
即“形状因子(傅里叶变换)”→1

➡ 质子内部点状对象的散射！

作业6

1.读图答题：

c) 写出Callan-Gross公式并辨认spin-1/2，spin-0 (IV,V)

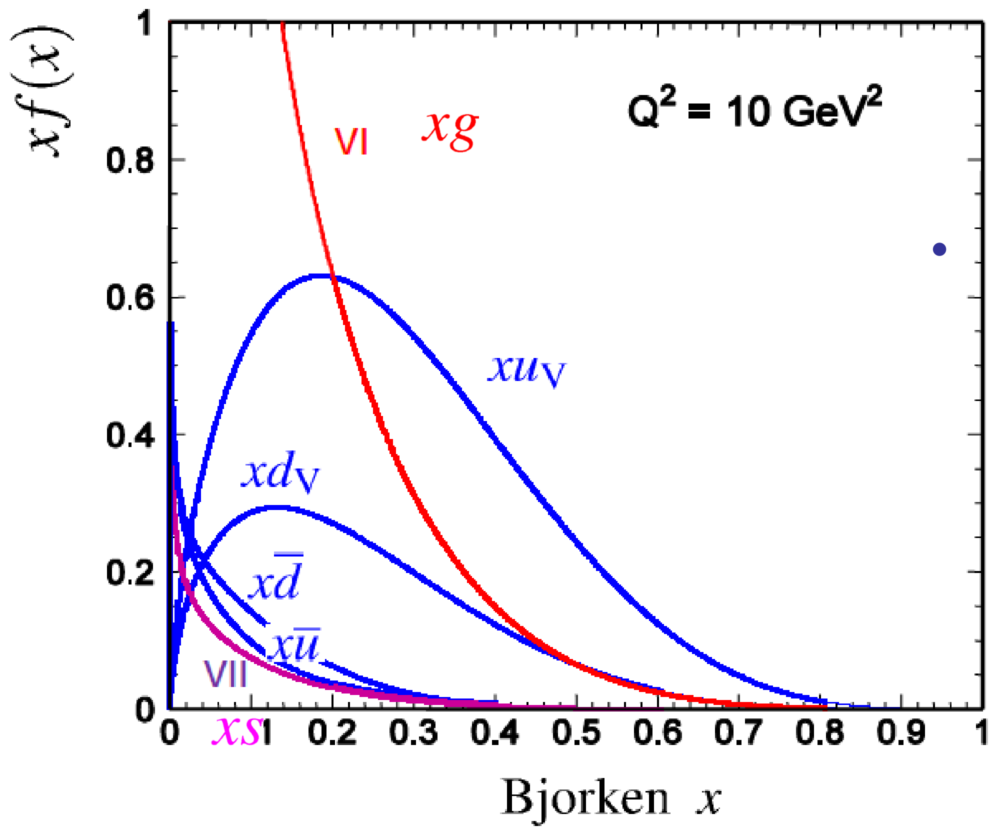


$$F_2(x) = 2xF_1(x)$$

作业6

1.读图答题：

d)辨认胶子和奇异夸克曲线(VI,VII)，并解释为什么在x比较大的时候$u_V(x) \approx 2d_V(x)$



- **分解价夸克和海夸克的贡献：**

$$u(x) = u_V(x) + u_S(x) \qquad d(x) = d_V(x) + d_S(x)$$
$$\bar{u}(x) = \bar{u}_S(x) \qquad \bar{d}(x) = \bar{d}_S(x)$$

- **质子包含两个上型价夸克和一个下型价夸克**

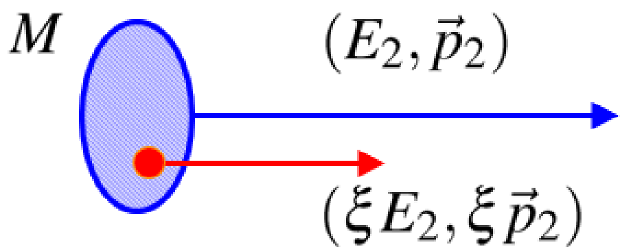
$$\int_0^1 u_V(x)\mathrm{d}x = 2 \qquad \int_0^1 d_V(x)\mathrm{d}x = 1$$

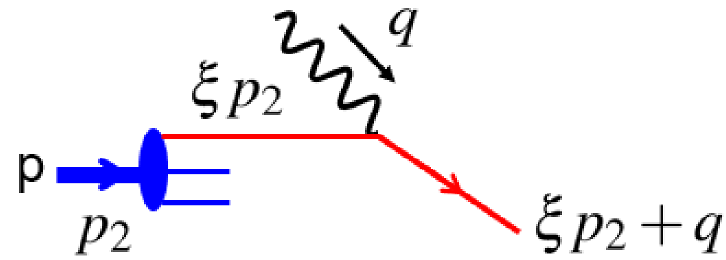
- **高x区域海贡献变小**

作业6

2. 在夸克模型中，证明质子极高能近似下，比约肯变量x可被看作被轰击夸克携带的质子总动量的比分($\xi = x$)

- 设夸克携带的动量占质子总动量的比分为 ξ





- 相互作用后被轰击后的夸克四动量为

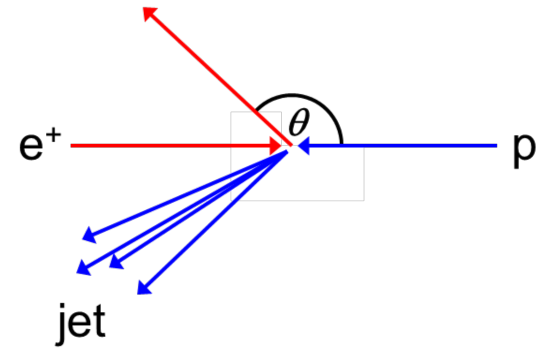
$$(\xi p_2 + q)^2 = m_q^2 \approx 0 \Rightarrow \cancel{\xi^2 p_2^2} + q^2 + 2\xi p_2.q = 0 \Rightarrow \boxed{\xi = \frac{Q^2}{2p_2.q} = x}$$

$$(\xi^2 p_2^2 = m_q^2 \approx 0)$$

作业6

3. 在HERA对撞机上，能量$E_1 = 27.5GeV$的电子与能量$E_2 = 820GeV$的质子对撞，出射的电子能量为$E_3 = 31GeV$，入射、出射电子流夹角为$\theta \approx 45°$，考虑深度非弹性碰撞，证明：

$$x = \frac{E_3}{E_2}\left[\frac{1-\cos\theta}{2-(E_3/E_1)(1+\cos\theta)}\right]$$



并计算x, Q^2的值。

$$E_1 \gg m_e, E_2 \gg m_p$$

$$p_1 = (E_1, 0,0, E_1)$$

$$p_2 = (E_2, 0,0, -E_2)$$

$$p_3 = (E_3, E_3\sin\theta, 0, E_3\cos\theta)$$

$$q = p_1 - p_3 = (E_1 - E_3, -E_3\sin\theta, 0, E_1 - E_3\cos\theta)$$

$$x = \frac{Q^2}{2p_2 \cdot q} = \frac{-(E_1-E_3)^2+\left(E_3^2\sin^2\theta+(E_1-E_3\cos\theta)^2\right)}{2[E_2(E_1-E_3)+E_2(E_1-E_3\cos\theta)]}$$

$$= \frac{E_1E_3(1-\cos\theta)}{2E_1E_2-E_2E_3\cos\theta} = \frac{E_3}{E_2}\left[\frac{1-\cos\theta}{2-(E_3/E_1)(1+\cos\theta)}\right]$$

$$x = 0.146, Q^2 = 499.4GeV^2$$

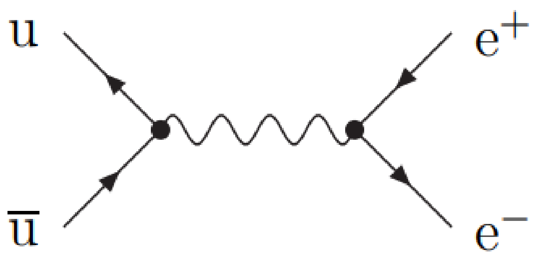
作业6

4. 组分夸克模型：

a) $\rho^0 \to e^+e^-$的衰变宽度为7keV,利用费曼图估算$\omega^0 \to e^+e^-$的衰变宽度

b) $\rho^0 \to \pi^0\gamma$的衰变宽度为77keV,利用费曼图估算$\omega^0 \to \pi^0\gamma$的衰变宽度

(π^0, ρ^0组分为$(u\bar{u} - d\bar{d})/\sqrt{2}$，$\omega^0$为$(u\bar{u} + d\bar{d})/\sqrt{2}$)

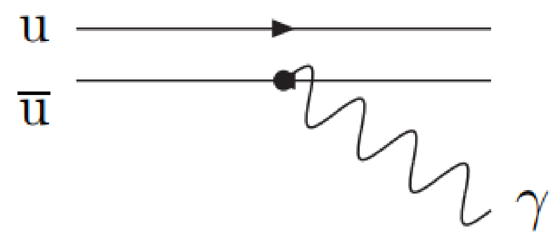


a) $M(\rho^0 \to e^+e^-) \sim \langle u\bar{u} - d\bar{d}|\gamma\rangle \sim \frac{2}{3} - \left(-\frac{1}{3}\right) = 1$

$M(\omega^0 \to e^+e^-) \sim \langle u\bar{u} + d\bar{d}|\gamma\rangle \sim \frac{2}{3} + \left(-\frac{1}{3}\right) = \frac{1}{3}$

$\frac{\Gamma(\rho^0 \to e^+e^-)}{\Gamma(\omega^0 \to e^+e^-)} = \left(\frac{1}{1/3}\right)^2 = 9$

$\Gamma(\omega^0 \to e^+e^-) \approx \frac{7}{9} keV$



b) $M(\rho^0 \to \pi^0\gamma) \sim \langle u\bar{u} - d\bar{d}|u\bar{u} - d\bar{d}\rangle \sim \frac{2}{3} + \left(-\frac{1}{3}\right) = \frac{1}{3}$

$M(\omega^0 \to \pi^0\gamma) \sim \langle u\bar{u} + d\bar{d}|u\bar{u} - d\bar{d}\rangle \sim \frac{2}{3} - \left(-\frac{1}{3}\right) = 1$

$\frac{\Gamma(\rho^0 \to e^+e^-)}{\Gamma(\omega^0 \to e^+e^-)} = \left(\frac{1/3}{1}\right)^2 = \frac{1}{9}$

$\Gamma(\omega^0 \to e^+e^-) \approx 77 \times 9 keV = 693 keV$

作业7

1. 仿照课上构造$|p\uparrow\rangle$的方法，组合$\frac{1}{\sqrt{2}}\phi(M_s)\chi(M_s)+\frac{1}{\sqrt{2}}\phi(M_A)\chi(M_A)$，写出自旋向下的中子波函数$|n\downarrow\rangle$

$$|n\downarrow\rangle=\frac{1}{6\sqrt{2}}(2ddu-udd-dud)(2\downarrow\downarrow\uparrow-\uparrow\downarrow\downarrow-\downarrow\uparrow\downarrow)+\frac{1}{2\sqrt{2}}(udd-dud)(\uparrow\downarrow\downarrow-\downarrow\uparrow\downarrow)$$

$$=\frac{1}{\sqrt{18}}\begin{pmatrix}2d\downarrow d\downarrow u\uparrow-d\uparrow d\downarrow u\downarrow-d\downarrow d\uparrow u\downarrow+\\2d\downarrow u\uparrow d\downarrow-d\downarrow u\downarrow d\uparrow-d\uparrow u\downarrow d\downarrow+\\2u\uparrow d\downarrow d\downarrow-u\downarrow d\uparrow d\downarrow-u\downarrow d\downarrow d\uparrow\end{pmatrix}$$

作业7

2.考虑只由u,d夸克及其反夸克组成的介子，尝试从

$$|1,+1\rangle = -u\bar{d}$$

开始，a)用下降算符得到另外两个三重态，b)用正交性给出最后一个单态，c)最后证明单态是阶梯算符的“尽头”。

a) $T_-|1,+1\rangle = \sqrt{2}|1,0\rangle = T_-\left(-u\bar{d}\right) = -d\bar{d} + u\bar{u} \rightarrow |1,0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$

$T_-|1,0\rangle = \sqrt{2}|1,-1\rangle = \frac{1}{\sqrt{2}}T_-\left(u\bar{u} - d\bar{d}\right) = \sqrt{2}d\bar{u} \rightarrow |1,-1\rangle = d\bar{u}$

b) $|0,0\rangle = \alpha(u\bar{u}) + \beta(d\bar{d})$

$\langle 0,0|0,0\rangle = 1$

$\langle 1,0|0,0\rangle = 0$

$|0,0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$

c) $T_-|0,0\rangle = \frac{1}{\sqrt{2}}(d\bar{u} - d\bar{u}) = 0, T_+|0,0\rangle = \frac{1}{\sqrt{2}}\left(-u\bar{d} + u\bar{d}\right) = 0$

作业7

3. 写出八个盖尔曼矩阵及味道SU(3)群中用到的三组阶梯算符，之后在以下关系中任选一组进行证明（六个一组）：

$$\hat{T}_+u=0,\hat{T}_-u=d \qquad \hat{V}_+u=0,\hat{V}_-u=s \qquad \hat{U}_+u=0,\hat{U}_-u=0$$

$$\hat{T}_+d=u,\hat{T}_-d=0 \qquad \hat{V}_+d=0,\hat{V}_-d=0 \qquad \hat{U}_+d=0,\hat{U}_-d=s$$

$$\hat{T}_+s=0,\hat{T}_-s=0 \qquad \hat{V}_+s=u,\hat{V}_-s=0 \qquad \hat{U}_+s=d,\hat{U}_-s=0$$

$$\lambda_1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}\quad \lambda_2=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}\quad \lambda_3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$$

$$\lambda_4=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}\quad \lambda_5=\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}$$

$$\lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

$$\lambda_6=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}\quad \lambda_7=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}$$

$$T_\pm=\frac{1}{2}(\lambda_1\pm i\lambda_2)$$

$$V_\pm=\frac{1}{2}(\lambda_4\pm i\lambda_5)$$

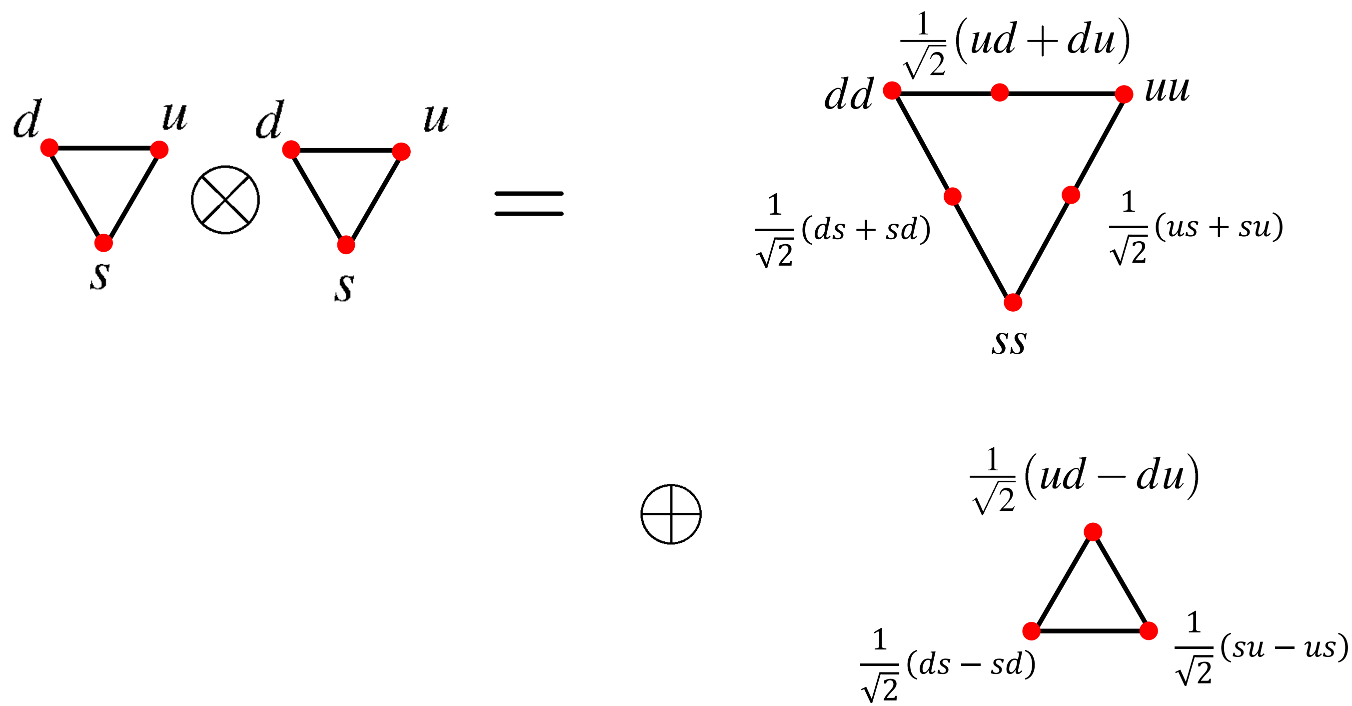
$$U_\pm=\frac{1}{2}(\lambda_6\pm i\lambda_7)$$

作业7

4. 模仿上课时的方法，用画图的方式画出10重态和8重态的构造过程：

a) $\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \overline{\mathbf{3}}$

计算并标出后面两个三角上各个点所对应的态

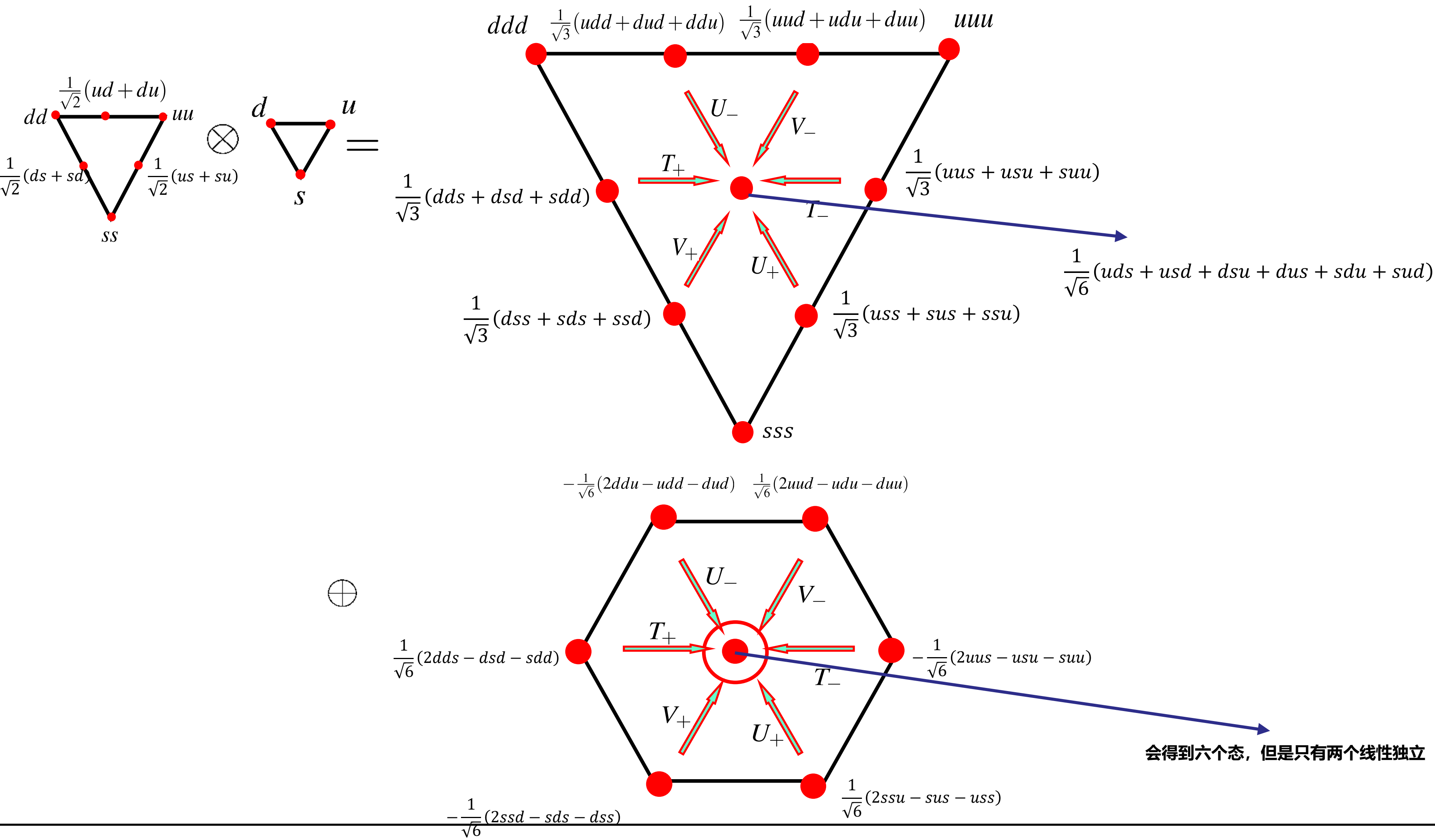


作业7

4. 模仿上课时的方法，用画图的方式画出10重态和8重态的构造过程：

b) $3 \otimes 6 = 10 \oplus 8$

计算并标出最后面两个图形上各个点所对应的态

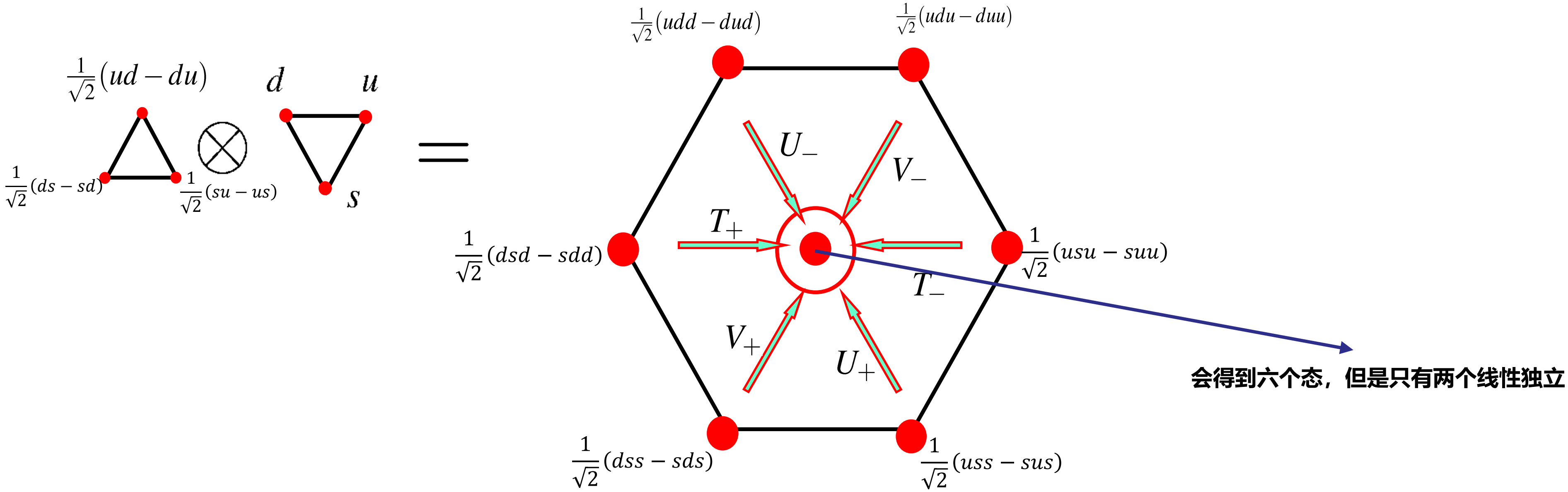


作业7

4. 模仿上课时的方法，用画图的方式画出10重态和8重态的构造过程：

c) $\bar{3} \otimes 3 = 8 \oplus 1$

计算并标出后面两个图形上各个点所对应的态



$$\oplus \quad \bullet \quad \frac{1}{\sqrt{6}}(uds-usd+dsu-dus+sud-sdu)$$