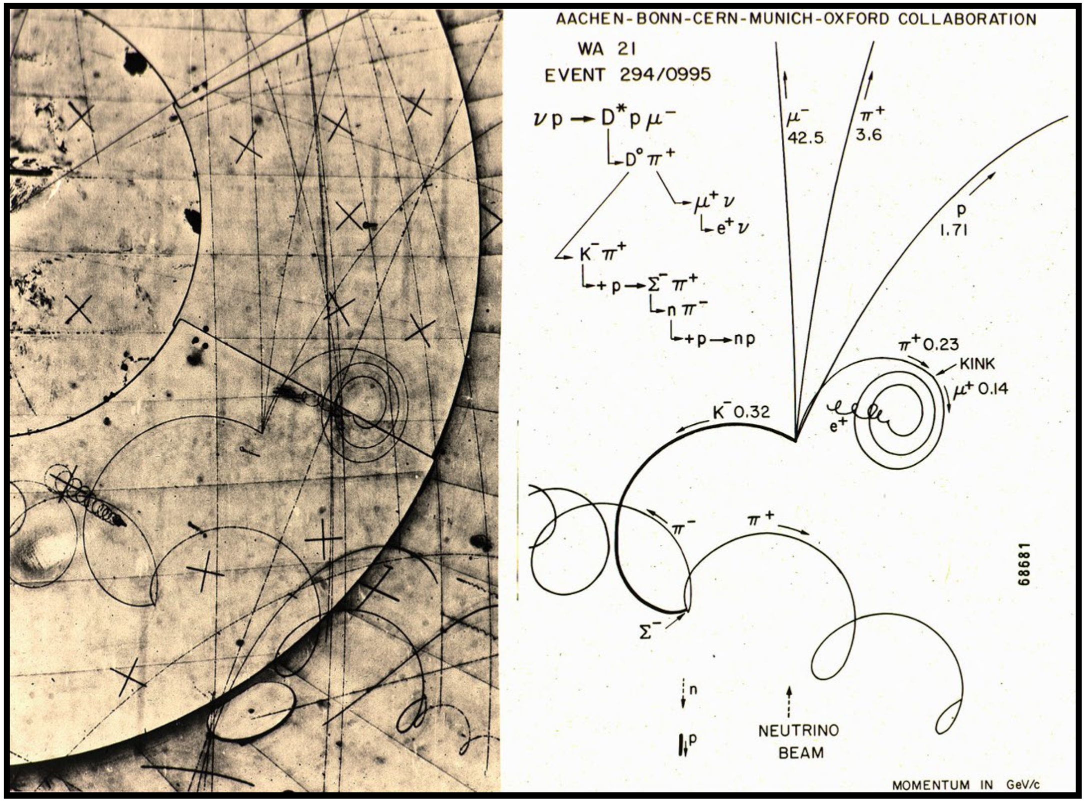
粒子物理学

第 7 章： 对称性 与 夸克模型



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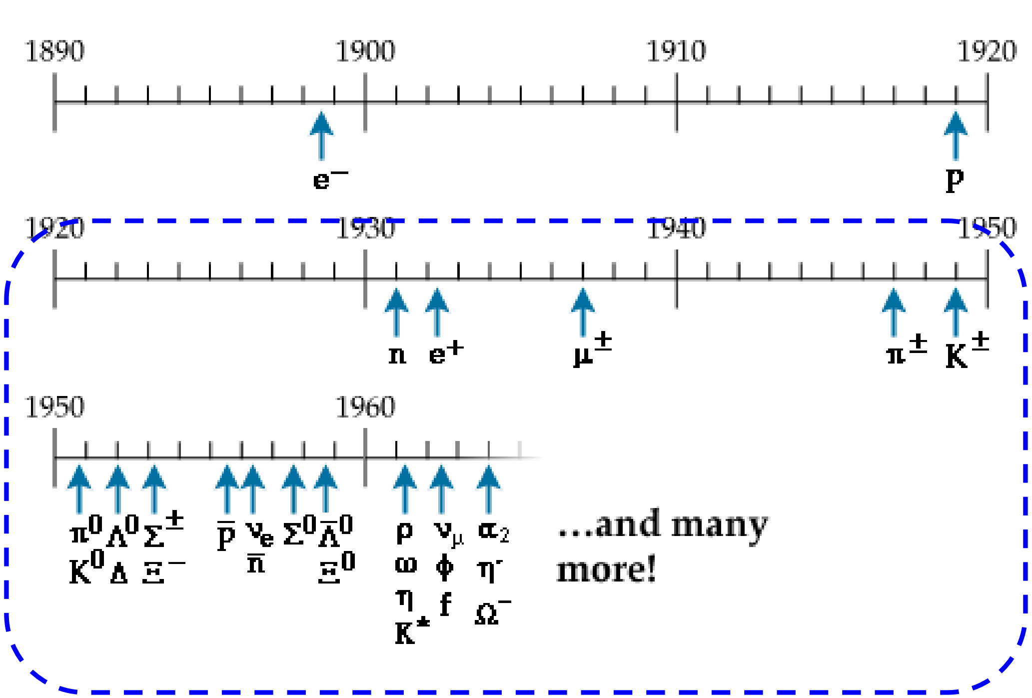
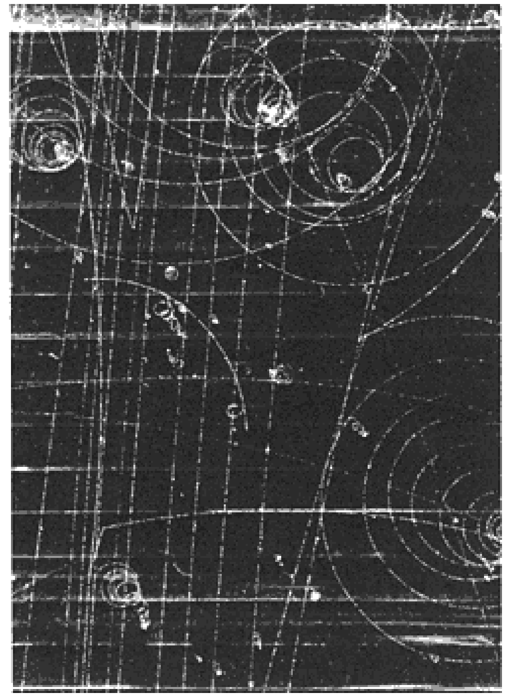
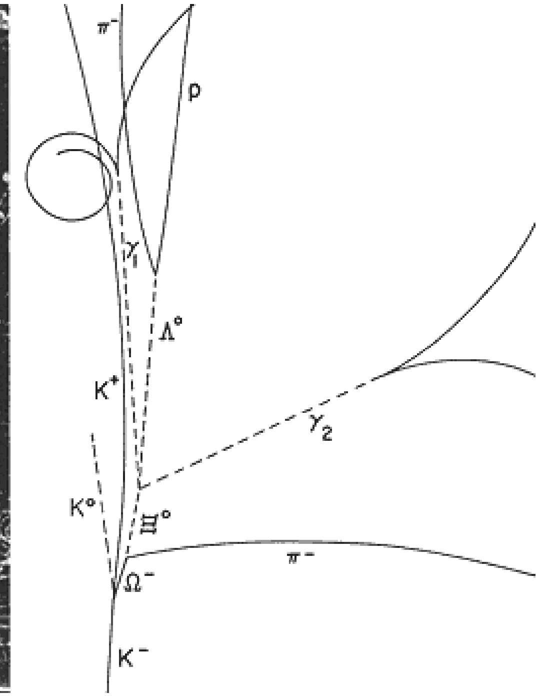
Based on M. Thomson’s notes

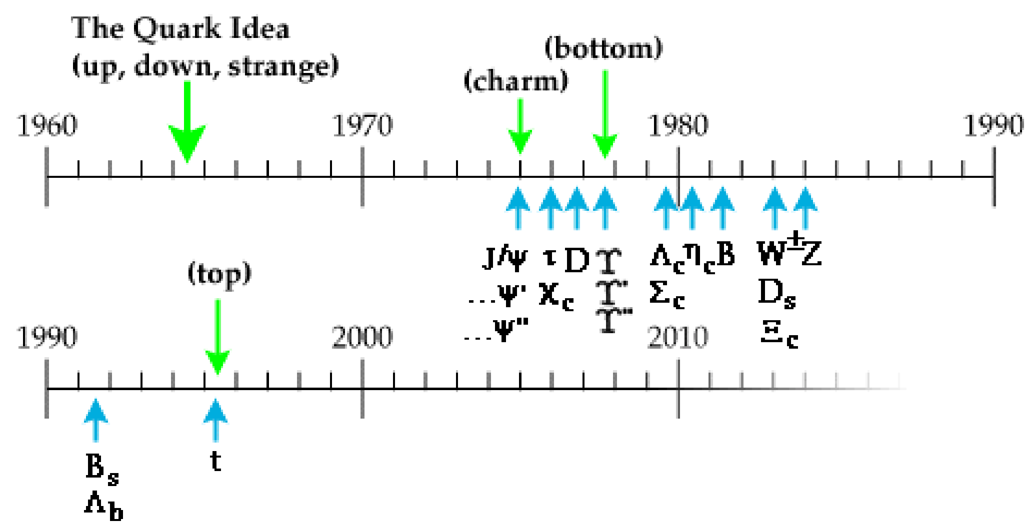
Particle Zoo



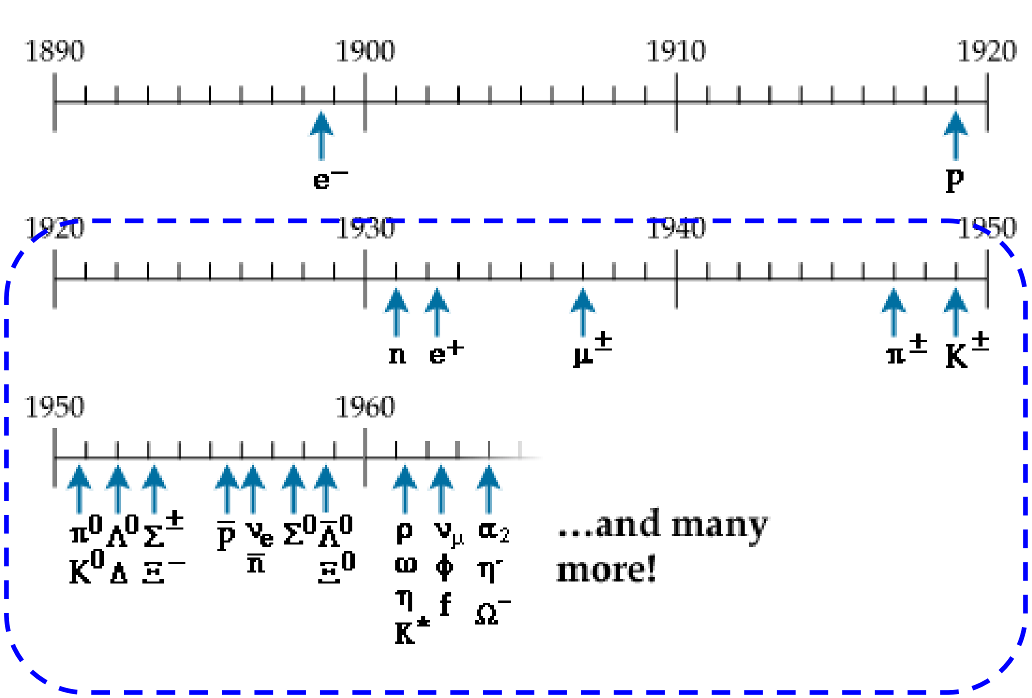


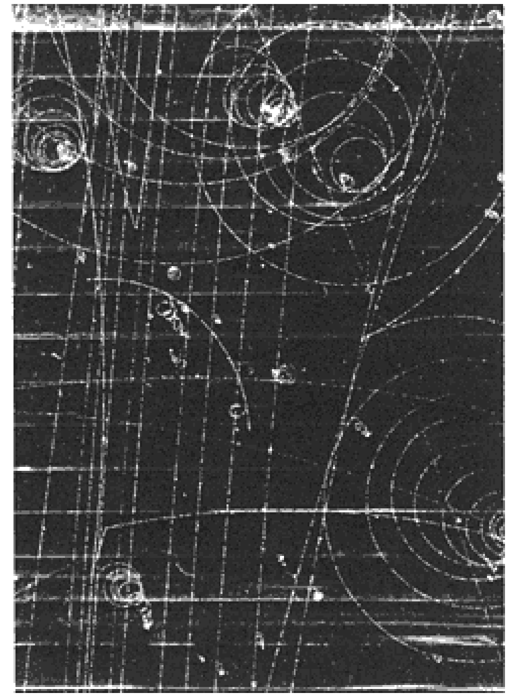


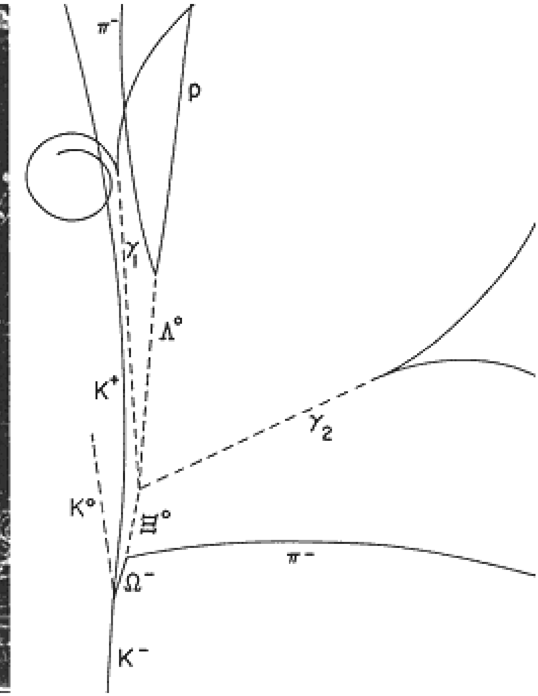
- **1950-60s，加速器诞生后，很多新粒子被发现**
  - 总数超过了元素周期表的元素数，呈现结构
  - 不可能都是基本粒子！



Strangeness







- kaons or 超子hyperons Σ and Λ, 在粒子对撞中成对产生（大量地），但是衰变远比预期缓慢（考虑到其质量和产生截面）
  - 推断存在一种新的守恒量“奇异数(strangeness)”，在产生时守恒，但是衰变过程中不守恒
- 奇异数在强相互作用和电磁相互作用中守恒，但是在弱相互作用中不守恒
  - 因此，最轻的含奇异数的粒子不能通过强作用衰变，且必须通过更缓慢的弱作用衰变

Hadronic cross section Ration

$$R_{\text{had}}(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

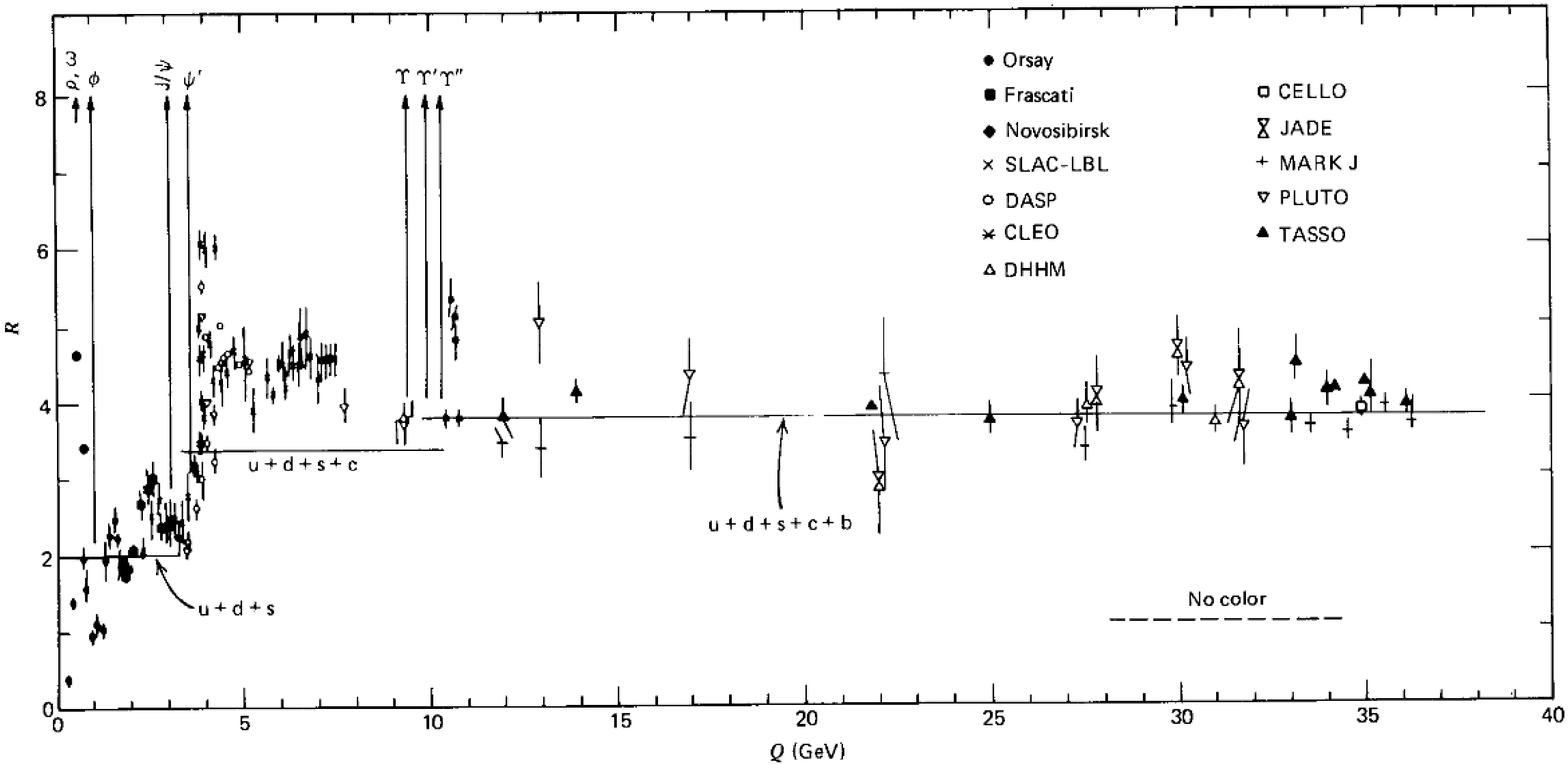
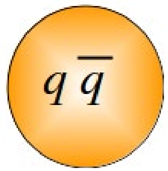


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrov 1^- resonances just below or near the flavor thresholds.)

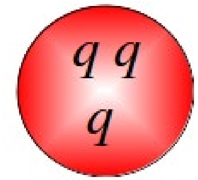
Hadron Wavefunctions

Quarks are always confined in hadrons (i.e. colourless states)

介子Mesons
Spin 0, 1, ...



重子Baryons
Spin 1/2, 3/2, ...



Treat quarks as identical fermions with states labelled with spatial, spin, flavour and colour. $\psi = \psi_{\text{space}}\psi_{\text{flavour}}\psi_{\text{spin}}\psi_{\text{colour}}$

All hadrons are colour singlets, i.e. net colour zero

Mesons $$\psi^{q\bar{q}}_{\text{colour}} = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

Baryons $$\psi^{qqq}_{\text{colour}} = \frac{1}{\sqrt{6}}(rgb + gbr + brg - grb - rbg - bgr)$$

Parity

Parity operator $\hat{P}$ performs spatial inversion $\hat{P}|\psi(\vec{r},t)\rangle = |\psi(-\vec{r},t)\rangle$

- Eigenvalue of $\hat{P}$ called Parity $\hat{P}|\psi\rangle = P|\psi\rangle, \qquad P = \pm 1$
  - Most particles are eigenstates of Parity and in this case P represents intrinsic Parity of a particle/antiparticle.
  - Parity is a useful concept. If the Hamiltonian for an interaction commutes with $\hat{P}$: $[\hat{P}, \hat{H}] = 0$,

then Parity is conserved in the interaction:
Parity conserved in the strong and EM interactions, but not in the weak interaction.

Parity

对于角动量为ℓ的两粒子复合系统: $P = P_1 P_2 (-1)^{\ell}$

- 其中 $P_{1,2}$ 是粒子的本征宇称

量子场论:

- 费米子和反费米子：宇称 相反
- 玻色子和反玻色子：宇称 相同

惯例:

- 夸克和轻子：$P_{q/\ell} = +1$，反夸克和反轻子：$P_{\bar{q}/\bar{l}} = -1$
- 规范玻色子：(γ, g,W, Z) 为矢量场$J^P = 1^-$, $P_\gamma = -1$

Light Mesons

介子是q 和 $\bar{q}$ 的束缚态

- 考虑由轻夸克(u, d, s)组成的基态介子

  $m_u \sim 0.3$ GeV, $m_d \sim 0.3$ GeV, $m_s \sim 0.5$ GeV

- 基态($\ell = 0$): 介子 “自旋” (总角动量)由$q\bar{q}$ 自旋态给出

两个可能的$q\bar{q}$自旋态：S=0,1

- S = 0: pseudoscalar mesons; S = 1: vector mesons

介子宇称：(q and $\bar{q}$ have opposite parity)

- $\mathrm{P} = \mathrm{P}(q)\mathrm{P}(\bar{q})(-1)^{\ell} = (+1)(-1)(-1)^{\ell} = -1$ (for $\ell = 0$)
- Flavour States: $u\bar{d}$, $u\bar{s}$, $d\bar{u}$, $d\bar{s}$, $s\bar{u}$, $s\bar{d}$, and $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ mixtures

预期有:

9个 $J^P = 0^-$ 介子: Pseudoscalar nonet; 9个 $J^P = 1^-$ 介子: Vector nonet

$u\bar{u}$ $d\bar{d}$ $s\bar{s}$ States

$u\bar{u}$ $d\bar{d}$ and $s\bar{s}$ states all have zero flavour quantum numbers and can mix

$J^P = 0^-$

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

$$\eta' = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$J^P = 1^-$

$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$\omega^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$\phi = s\bar{s}$$

混合系数在实验上由介子质量和衰变来确定

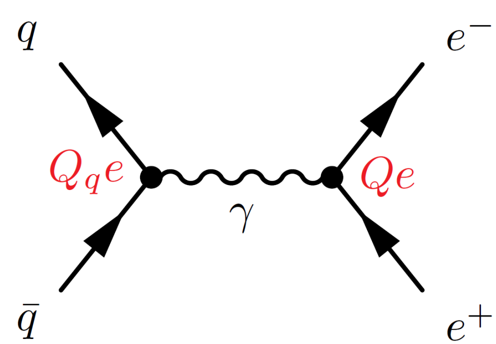
$$M(\rho^0 \to e^+e^-) \sim \frac{e}{q^2}\left[\frac{1}{\sqrt{2}}(Q_u e - Q_d e)\right]$$

举例：矢量介子的轻子衰变

$$\Gamma(\rho^0 \to e^+e^-) \propto \left[\frac{1}{\sqrt{2}}(\frac{2}{3} - (-\frac{1}{3}))\right]^2 = \frac{1}{2}$$

$$\Gamma(\omega^0 \to e^+e^-) \propto \left[\frac{1}{\sqrt{2}}(\frac{2}{3} + (-\frac{1}{3}))\right]^2 = \frac{1}{18}$$

$$\Gamma(\phi \to e^+e^-) \propto \left[\frac{1}{3}\right]^2 = \frac{1}{9}$$



$M \sim Q_q\alpha \quad \Gamma \sim Q_q^2\alpha^2$

Predict: $\Gamma_\rho : \Gamma_\omega : \Gamma_\phi = 9 : 1 : 2$ Experiment: $(8.8 \pm 2.6) : 1 : (1.7 \pm 0.4)$

Meson Masses Spin-spin Interaction

Meson masses are only partly from constituent quark masses:

- $\mathrm{m(K)} > \mathrm{m(\pi)} \Rightarrow$ 说明 $\mathrm{m_s} > \mathrm{m_u}, \mathrm{m_d}$

  $495\ \mathrm{MeV}\ 140\ \mathrm{MeV}$

Not the whole story...

- $\mathrm{m(\rho)} > \mathrm{m(\pi)} \Rightarrow$ although both are $u\bar{d}$

  $770\ \mathrm{MeV}\ 140\ \mathrm{MeV}$

Only difference is the orientation of the quark spins (↑↑ vs ↑↓)

⇒ spin-spin interaction

Meson Masses Spin-spin Interaction

QED: Hyperfine splitting in H_2 $(\ell = 0)$

- Energy shift due to electron spin in magnetic field of proton

$$\Delta E = \vec{\mu}.\vec{B} \;\propto\; \frac{2}{3}\vec{\mu}_e.\vec{\mu}_p \;\propto\; \alpha\frac{\vec{S}_e}{m_e}\frac{\vec{S}_p}{m_p} \quad \text{use } \vec{\mu} = \frac{e}{2m}\vec{S}$$

QCD: Colour Magnetic Interaction

- 夸克和胶子的基本相互作用形式与电子和光子的相同
- 因此，也有色磁相互作用

$$\Delta E \propto \alpha_s \frac{\vec{S}_1}{m_1}\frac{\vec{S}_2}{m_2}$$

Meson Masses Meson Mass Formula (ℓ = 0)

$$M_{q\bar{q}} = m_1 + m_2 + A\frac{\vec{S}_1}{m_1}\frac{\vec{S}_2}{m_2}$$ where A is a constant

For a state of spin $\vec{S} = \vec{S}_1 + \vec{S}_2$ $\vec{S}^2 = \vec{S}_1^2 + \vec{S}_2^2 + 2\vec{S}_1.\vec{S}_2$

$$\vec{S}_1.\vec{S}_2 = \frac{1}{2}\left(\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2\right) \qquad \vec{S}_1^2 = \vec{S}_2^2 = \vec{S}_1(\vec{S}_1 + 1) = \frac{1}{2}\left(\frac{1}{2} + 1\right) = \frac{3}{4}$$

给出 $\vec{S}_1.\vec{S}_2 = \frac{1}{2}\vec{S}^2 - \frac{3}{4}$

$J^P = 0^-$ 介子: $\vec{S}^2 = 0 \quad \Rightarrow \vec{S}_1.\vec{S}_2 = -3/4$

$J^P = 1^-$ 介子: $\vec{S}^2 = S(S+1) = 2 \quad \Rightarrow \vec{S}_1.\vec{S}_2 = +1/4$

给出 (ℓ=0)介子的质量公式:

$$M_{q\bar{q}} = m_1 + m_2 - \frac{3A}{4m_1m_2} \quad \left(J^P = 0^-\right)$$

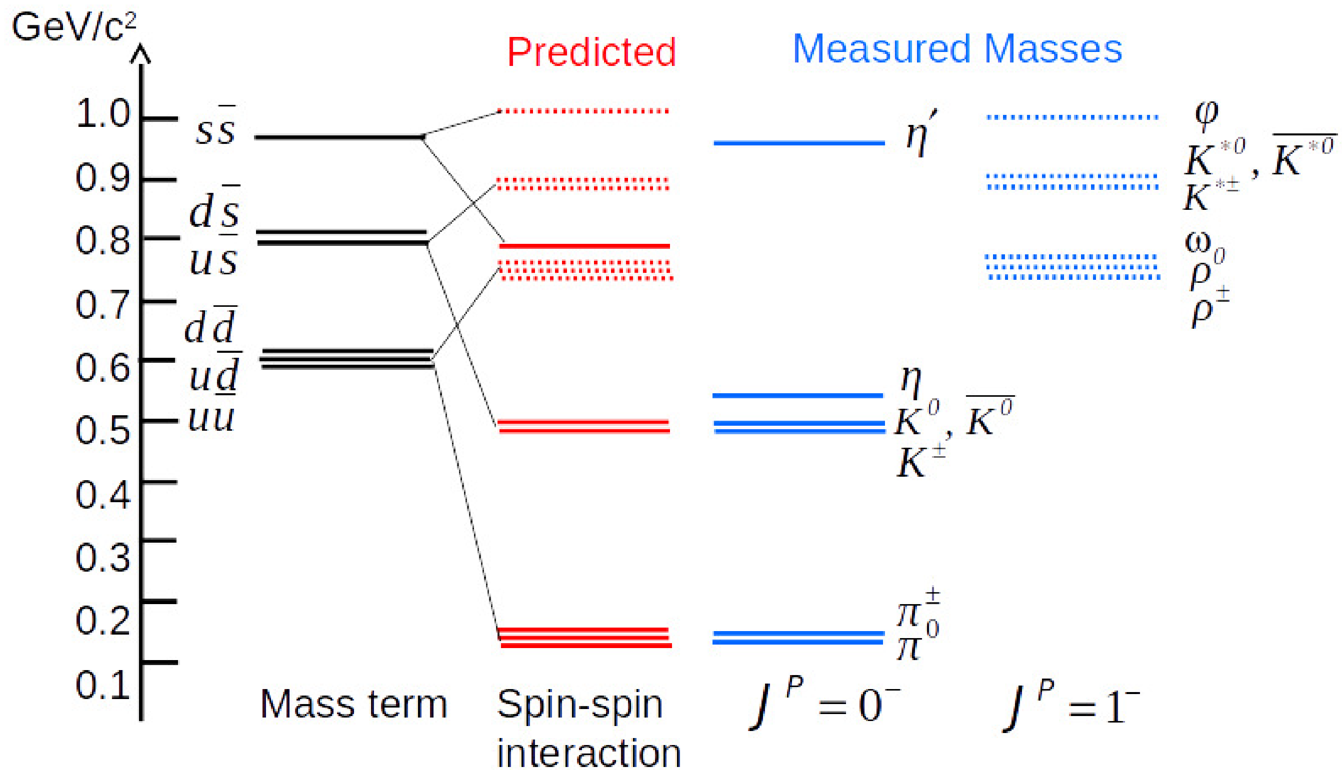
$$M_{q\bar{q}} = m_1 + m_2 + \frac{A}{4m_1m_2} \quad \left(J^P = 1^-\right)$$

因此 $J^P=0^-$ 介子比 $J^P=1^-$ 介子轻

Meson Masses

m_u=0.305 GeV,
m_d=0.308 GeV,
m_s=0.487 GeV,
A=0.06 GeV^3

上述参数可以很好地拟合不同味道组合的质量
$(u\bar{d}, u\bar{s}, d\bar{u}, d\bar{s}, s\bar{u}, s\bar{d})$



η 和 η′ 是混合态，如：$\eta = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$

$$M_\eta = \frac{1}{6}\left(2m_u - \frac{3A}{4m_u^2}\right) + \frac{1}{6}\left(2m_d - \frac{3A}{4m_d^2}\right) + \frac{4}{6}\left(2m_s - \frac{3A}{4m_s^2}\right)$$

Baryons

- 重子由3个不可区分夸克组成(味道为波函数中的另一个量子数)

$$\psi_{\text{baryon}} = \psi_{\text{space}}\, \psi_{\text{flavour}}\, \psi_{\text{spin}}\, \psi_{\text{colour}}$$

ψ_{baryon} 在交换任意2个夸克下，必须是反对称

举例：$\Omega^-(sss)$ 波函数 $(\ell = 0, J = 3/2)$

- $\psi_{\text{spin}}\, \psi_{\text{flavour}} = s\uparrow s\uparrow s\uparrow$ 是 对称的 $\Rightarrow$ 要求 ψ_{colour} 反对称

只考虑基态(ℓ=0)，零轨道角动量 ψ_{space} 对称的

→ 所有强子都是色单态

$$\psi_{\text{colour}} = 1\sqrt{6}(rgb + gbr + brg - grb - rbg - bgr)$$ 反对称的

- 因此，$\psi_{\text{spin}}\, \psi_{\text{flavour}}$ 必须是对称的

Baryon spin wavefunctions (ψspin)

联合 3 个自旋1/2 夸克：总自旋 $J = 1/2 \oplus 1/2 \oplus 1/2 = 1/2$ or $3/2$

- 考虑 J = 3/2，容易写出|3/2, 3/2>态的自旋波函数：|3/2, 3/2> =|↑↑↑>
- 利用阶梯算符$\hat{J}_{-}$产生其他态

$$\hat{J}_{-}\left|\frac{3}{2},\frac{3}{2}\right\rangle = (\hat{J}_{-}\uparrow)\uparrow\uparrow + \uparrow(\hat{J}_{-}\uparrow)\uparrow + \uparrow\uparrow(\hat{J}_{-}\uparrow)$$

$$\sqrt{\frac{3}{2}\frac{5}{2}-\frac{3}{2}\frac{1}{2}}\left|\frac{3}{2},\frac{1}{2}\right\rangle = \downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow)$$

$$\hat{J}_{-}|j,m\rangle = \sqrt{j(j+1)-m(m-1)}\,|j,m-1\rangle$$

给出J = 3/2 态:

All symmetric under interchange of any two spins

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow)$$

$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)$$

$$\left|\frac{3}{2},-\frac{3}{2}\right\rangle = \downarrow\downarrow\downarrow$$

Baryon Masses Baryon Mass Formula ($\ell = 0$)

$$M_{qqq} = m_1 + m_2 + m_3 + A'\left(\frac{\vec{S}_1}{m_1}.\frac{\vec{S}_2}{m_2} + \frac{\vec{S}_1}{m_1}.\frac{\vec{S}_3}{m_3} + \frac{\vec{S}_2}{m_2}.\frac{\vec{S}_3}{m_3}\right)$$

其中 A′ 是常数

- 举例：所有夸克的质量相同 $\mathrm{m_1 = m_2 = m_3 = m_q}$

$$M_{qqq} = 3m_q + A'\sum_{i<j}\frac{\vec{S}_i.\vec{S}_j}{m_q^2}$$

$$\vec{S}^2 = \left(\vec{S}_1 + \vec{S}_2 + \vec{S}_3\right)^2 = \vec{S}_1^2 + \vec{S}_2^2 + \vec{S}_3^2 + 2\sum_{i<j}\vec{S}_i.\vec{S}_j$$

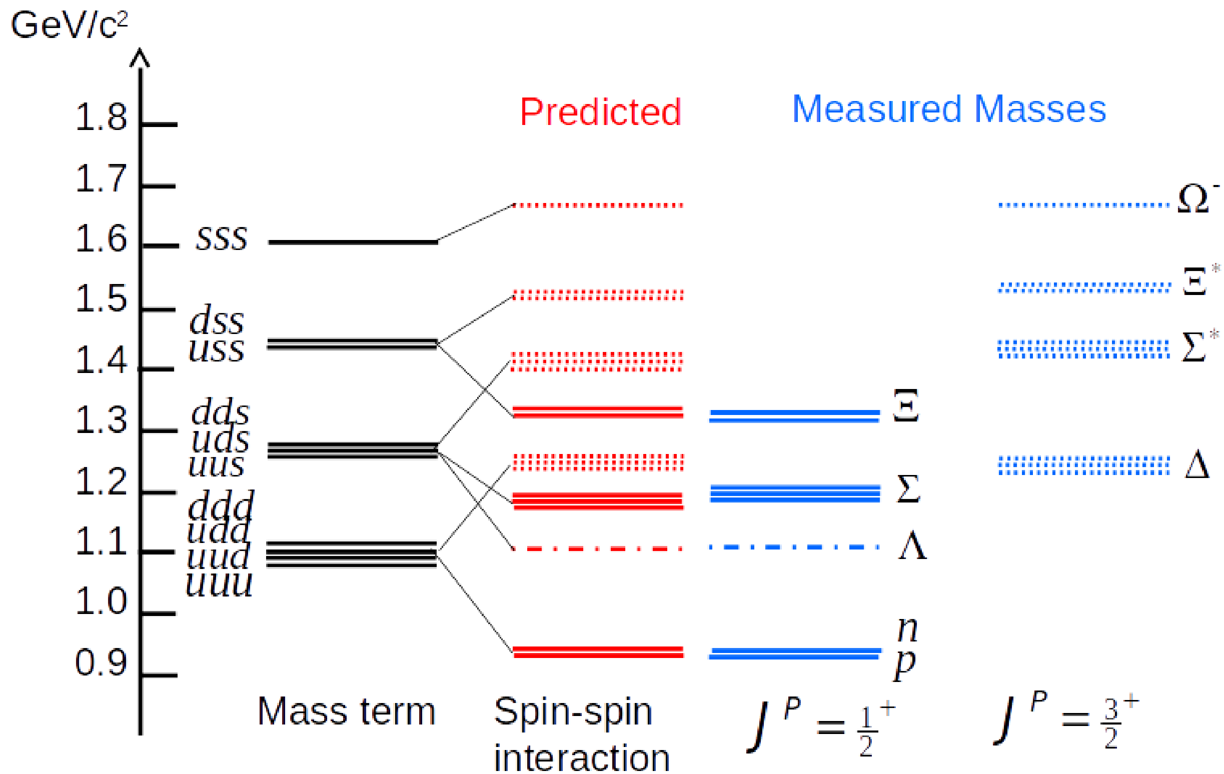
$$2\sum_{i<j}\vec{S}_i.\vec{S}_j = S(S+1) - 3\frac{1}{2}(\frac{1}{2}+1) = S(S+1) - \frac{9}{4}$$

$$\sum_{i<j}\vec{S}_i.\vec{S}_j = -\frac{3}{4}\ \left(J = \frac{1}{2}\right) \qquad \sum_{i<j}\vec{S}_i.\vec{S}_j = +\frac{3}{4}\ \left(J = \frac{3}{2}\right)$$

如，质子(uud) 对比Δ (uud) – 同样的夸克成分

$$M_p = 3m_u - \frac{3A'}{4m_u^2} \qquad M_\Delta = 3m_u + \frac{3A'}{4m_u^2}$$

Baryon Masses



Colour factor of 2

Excellent agreement using

- m_u=0.362 GeV, m_d=0.366 GeV, m_s=0.537 GeV, A′=0.026 GeV3 ~ A/2

组分夸克 质量依赖于强子波函数，且包含了胶子云和$q\bar{q}$ 对

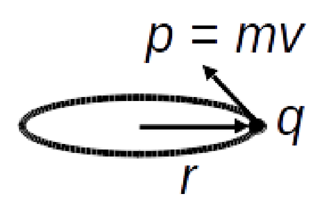
- ⇒ 对于介子和重子，取值略有不同

Baryon Magnetic Moments

磁偶极矩来自：带电夸克的轨道运动；夸克的自旋相关的内秉磁矩

轨道运动：经典的电流环

$$\mu = IA = \frac{qv}{2\pi r}\pi r^2 = \frac{qpr}{2m} = \frac{q}{2m}L_z$$



量子力学得到相同结果 $\hat{\mu} = g_\ell \frac{q}{2m}\hat{L}_z$

g_ℓ: "g-factor"
= 1 带电粒子，= 0 中性粒子

内秉自旋

粒子内秉自旋的磁矩算符 $\hat{\mu} = g_s \frac{q}{2m}\hat{S}_z$

g_s: "spin g-factor"= 2
对于自旋1/2的点状狄拉克粒子

Baryon Magnetic Moments

狄拉克费米子夸克的磁矩算符 $\hat{\boldsymbol{\mu}} = Q\frac{e}{m}\hat{\mathbf{S}}$ and $\hat{\mu}_z = Q\frac{e}{m}\hat{S}_z$

Spin-up (m_s = +1/2)

$$\mu_{\mathrm{u}} = \langle \mathrm{u}\uparrow|\hat{\mu}_z|\mathrm{u}\uparrow\rangle = \left(+\tfrac{2}{3}\right)\frac{e\hbar}{2m_{\mathrm{u}}} = +\frac{2m_{\mathrm{p}}}{3m_{\mathrm{u}}}\mu_N$$

$$\mu_{\mathrm{d}} = \langle \mathrm{d}\uparrow|\hat{\mu}_z|\mathrm{d}\uparrow\rangle = \left(-\tfrac{1}{3}\right)\frac{e\hbar}{2m_{\mathrm{d}}} = -\frac{m_{\mathrm{p}}}{3m_{\mathrm{d}}}\mu_N$$

Spin-down (m_s = -1/2) $\langle \mathrm{d}\downarrow|\hat{\mu}_z|\mathrm{d}\downarrow\rangle = -\mu_{\mathrm{d}}$ $\langle \mathrm{u}\downarrow|\hat{\mu}_z|\mathrm{u}\downarrow\rangle = -\mu_{\mathrm{u}}$

重子总磁矩：$\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}}^{(1)} + \hat{\boldsymbol{\mu}}^{(2)} + \hat{\boldsymbol{\mu}}^{(3)}$

- 三个组分夸克磁矩的矢量和

质子磁矩 $\mu_{\mathrm{p}} = \langle\hat{\mu}_z\rangle = \langle \mathrm{p}\uparrow|\hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)}|\mathrm{p}\uparrow\rangle$

Baryon Magnetic Moments

质子波函数： $|p\uparrow\rangle = \frac{1}{\sqrt{6}}(2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow)$

与磁矩：

$$\mu_p = \frac{1}{6}\langle(2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow)|\hat{\mu}_z|(2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow)\rangle$$

考虑味道和自旋态的正交性 $\langle u\uparrow u\uparrow d\downarrow | u\downarrow u\uparrow d\uparrow\rangle = 0$，质子磁矩表达式可以简化为：

$$\mu_p = \frac{4}{6}\langle u\uparrow u\uparrow d\downarrow|\hat{\mu}_z|u\uparrow u\uparrow d\downarrow\rangle + \frac{1}{6}\langle u\uparrow u\downarrow d\uparrow|\hat{\mu}_z|u\uparrow u\downarrow d\uparrow\rangle + \frac{1}{6}\langle u\downarrow u\uparrow d\uparrow|\hat{\mu}_z|u\downarrow u\uparrow d\uparrow\rangle .$$

利用下式估算

$$\hat{\mu}_z|u\uparrow\rangle = +\mu_u\,|u\uparrow\rangle \quad \text{and} \quad \hat{\mu}_z|u\downarrow\rangle = -\mu_u\,|u\downarrow\rangle$$
$$\hat{\mu}_z|d\uparrow\rangle = +\mu_d\,|d\uparrow\rangle \quad \text{and} \quad \hat{\mu}_z|d\downarrow\rangle = -\mu_d\,|d\downarrow\rangle$$

得到 $\mu_p = \frac{4}{6}(\mu_u + \mu_u - \mu_d) + \frac{1}{6}(\mu_u - \mu_u + \mu_d) + \frac{1}{6}(-\mu_u + \mu_u + \mu_d)$

Baryon Magnetic Moments

因此，夸克模型预言质子的磁矩为：$\mu_{\mathrm{p}} = \frac{4}{3}\mu_{\mathrm{u}} - \frac{1}{3}\mu_{\mathrm{d}}$

中子磁矩由u、d夸克互换得到 $\mu_{\mathrm{n}} = \frac{4}{3}\mu_{\mathrm{d}} - \frac{1}{3}\mu_{\mathrm{u}}$

假设$m_{\mathrm{u}} \approx m_{\mathrm{d}}$，意味着$\mu_{\mathrm{u}} = -2\mu_{\mathrm{d}}$
质子中子的磁矩比为：

$$\frac{\mu_{\mathrm{p}}}{\mu_{\mathrm{n}}} = \frac{4\mu_{\mathrm{u}} - \mu_{\mathrm{d}}}{4\mu_{\mathrm{d}} - \mu_{\mathrm{u}}} = -\frac{3}{2}$$

- m_{u} =0.338 GeV, m_{d} =0.322 GeV and m_{s} =0.510 GeV

Baryon Magnetic Moments in Quark Model

对于其他$\ell=0$重子也可以得到，预言 $\mu_n/\mu_p = -2/3$

- 对比实验值 −0.685

Baryon	μ_B in Quark Model	Predicted [μ_N]	Observed [μ_N]
p (uud)	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	$+2.79$	$+2.793$
n (ddu)	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	-1.86	-1.913
Λ (uds)	μ_s	-0.61	-0.614 ± 0.005
Σ^+ (uus)	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	$+2.68$	$+2.46 \pm 0.01$
Ξ^0 (ssu)	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	-1.44	-1.25 ± 0.014
Ξ^- (ssd)	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	-0.51	-0.65 ± 0.01
Ω^- (sss)	$3\mu_s$	-1.84	-2.02 ± 0.05

如下参数可以与数据较好地符合：$m_u = m_d = 0.336$ GeV, $m_s \sim 0.509$ GeV

Hadron Decays

强子可通过强相互作用衰变到轻质量态。前提：

- 能量运行，即，母粒子质量大于子粒子质量
- 强相互作用中，角动量和宇称必须守恒

举例：

$\rho^0 \to \pi^+\pi^-$

$m(\rho0) > m(\pi+) + m(\pi-)$

769　　140　　140 MeV

$\Delta^{++} \to p\pi^+$

$m(\Delta++) > m(p) + m(\pi+)$

1231　　938　　140 MeV

还需要检查末态中 全同粒子，举例：

$\omega^0 \to \pi^0\pi^0$

$m(\omega^0) > m(\pi0) + m(\pi0)$

782　　135　　135 MeV

$\omega^0 \to \pi^+\pi^-\pi^0$

$m(\omega^0) > m(\pi^+) + m(\pi^-) + m(\pi0)$

782　　140　　140　　135 MeV

Look at isospin, $\rho=|1,0\rangle\rho=|1,0\rangle$ and $\pi 0=|1,0\rangle\pi 0=|1,0\rangle$.

SU(2) isospin is a good symmetry in strong interactions, it must be conserved.

Looking at the isospin of the final state:

$|1,0\rangle\otimes|1,0\rangle=\sqrt{2/3}|2,0\rangle+0|1,0\rangle-\sqrt{1/3}|0,0\rangle$

There is no $|1,0\rangle|1,0\rangle$ component in the final state, and therefore the process is not allowed by SU(2) isospin symmetry.

https://physics.stackexchange.com/questions/88593/why-is-the-decay-of-a-neutral-rho-meson-into-two-neutral-pions-forbidden

Hadron Decays

强子可通过电磁相互作用衰变

举例： $\rho^0 \to \pi^0\gamma$

$m(\rho^0) > m(\pi^0) + m(\gamma)$

769 135 MeV

$\Sigma^0 \to \Lambda^0\gamma$

$m(\Sigma^0) > m(\Lambda^0) + m(\gamma)$

1193 1116 MeV

质量最轻的态 (p, $K^\pm$, K^0, $\overline{K^0}$, Λ, n) 要求衰变过程中改变夸克味道

- 因此，通过弱相互作用衰变 (see later).

Summary of light (uds) hadrons

Baryons and mesons are composite particles (complicated).

- However, naive Quark Model can be used for masses/magnetic moments.
- Reasonably consistent values for the constituent quark masses:

	$m_{u/d}$	m_s
Meson Masses	307 MeV	487 MeV
Baryon Masses	364 MeV	537 MeV
Baryon Magnetic Moments	336 MeV	509 MeV

$$m_u \sim m_d \sim 335\ \mathrm{MeV},\ m_s \sim 510\ \mathrm{MeV}$$

Hadrons decay

- Via strong interaction to lighter mass states if energetically feasible.
- Can also via EM interaction.
- Lightest mass states require a change of quark flavour to decay
  - Therefore decay via the weak interaction (see later).

Introduction/Aims

- 对称性是粒子物理学的一个核心
  - 粒子物理学研究的其中一个目标：发现自然界的基本对称性

- 本节课将对称性运用到夸克模型，将得到：
  - 推导强子波函数
  - 引出概念："色"和QCD（下节课）
  - 最终解释强子为什么只存在介子$(q\bar{q})$，重子(qqq)或反重子$(\overline{q}\overline{q}\overline{q})$

- 引入SU(2) 和 SU(3) 对称群，以及他们在粒子物理中重要作用
- 分立对称性在弱相互作用后讲

Symmetries and Conservation Laws

- 设如下变换后物理结果不变 $\psi \to \psi' = \hat{U}\psi$ 如 坐标轴的转动

- 概率归一条件要求 $\langle \psi | \psi \rangle = \langle \psi' | \psi' \rangle = \langle \hat{U}\psi | \hat{U}\psi \rangle = \langle \psi | \hat{U}^{\dagger}\hat{U} | \psi \rangle$

$\Rightarrow$ $\boxed{\hat{U}^{\dagger}\hat{U} = 1}$ i.e. $\hat{U}$ has to be unitary

- 在对称变换下，物理结果要保持不变需要所有的QM矩阵元不变

$$\boxed{\langle \psi | \hat{H} | \psi \rangle = \langle \psi' | \hat{H} | \psi' \rangle} = \langle \psi | \hat{U}^{\dagger}\hat{H}\hat{U} | \psi \rangle$$

i.e. require $\hat{U}^{\dagger}\hat{H}\hat{U} = \hat{H} \xrightarrow{\times \hat{U}} \hat{U}\hat{U}^{\dagger}\hat{H}\hat{U} = \hat{U}\hat{H} \Rightarrow \hat{H}\hat{U} = \hat{U}\hat{H}$

therefore $\boxed{[\hat{H}, \hat{U}] = 0}$ $\boxed{\hat{U} \text{ commutes with Hamiltonian}}$

- 现在考虑无穷小变换（ε为小量）

$$\hat{U} = 1 + i\varepsilon\hat{G}$$ ($\hat{G}$: Generator transformation)

Symmetries and Conservation Laws

- $\hat{U}$的幺正性 $\hat{U}\hat{U}^{\dagger} = (1 + i\varepsilon\hat{G})(1 - i\varepsilon\hat{G}^{\dagger}) = 1 + i\varepsilon(\hat{G} - \hat{G}^{\dagger}) + O(\varepsilon^2)$

忽略ε^2项 $UU^{\dagger} = 1 \Rightarrow \boxed{\hat{G} = \hat{G}^{\dagger}}$

即$\hat{G}$是厄米，因此对应一个可观测量G!

- 此外， $[\hat{H}, \hat{U}] = 0 \quad \Rightarrow [\hat{H}, 1 + i\varepsilon\,\hat{G}] = 0 \quad \Rightarrow [\hat{H}, \hat{G}] = 0$

而量子力学中 $\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{G}\rangle = i\langle[\hat{H}, \hat{G}]\rangle = 0$ 即，G 是一个 守恒 量

对称性 ⟺ 守恒律

自然界每种对称性都有一个守恒的可观测量

Symmetries and Conservation Laws

举例: **Infinitesimal spatial translation** $x \to x+\varepsilon$

即 预期物理规律在如下变换保持不变 $\psi(x) \to \psi' = \psi(x+\varepsilon)$

$$\psi'(x) = \psi(x+\varepsilon) = \psi(x) + \frac{\partial \psi}{\partial x}\varepsilon = \left(1 + \varepsilon \frac{\partial}{\partial x}\right)\psi(x)$$

but $\hat{p}_x = -i\frac{\partial}{\partial x}$ ➡ $\psi'(x) = (1 + i\varepsilon \hat{p}_x)\psi(x)$

对称性变换的产生子：$\hat{p}_x$ ➡ p_x 守恒

- 物理规律的平移不变性 意味着 动量守恒！
- 通常对称性操作可能依赖多个参数 $\hat{U} = 1 + i\vec{\varepsilon}.\vec{G}$

举例：三维平移的无穷小变换 $\vec{r} \to \vec{r} + \vec{\varepsilon}$, $\vec{p} = (\hat{p}_x, \hat{p}_y, \hat{p}_z)$ ➡ $\hat{U} = 1 + i\vec{\varepsilon}.\vec{p}$

Symmetries and Conservation Laws

- 有限的变换可以表达成一系列无穷小变换

$$\hat{U}(\vec{\alpha}) = \lim_{n\to\infty}\left(1 + i\frac{\vec{\alpha}}{n}.\vec{G}\right)^n = e^{i\vec{\alpha}.\vec{G}}$$

举例: 有限的一维空间平移：$x \to x + x_0$ 其中 $\hat{U}(x_0) = e^{ix_0\hat{p}_x}$

$$\begin{aligned}\psi'(x) = \psi(x+x_0) &= \hat{U}\psi(x) = \exp\left(x_0\frac{\mathrm{d}}{\mathrm{d}x}\right)\psi(x) \\ &= \left(1 + x_0\frac{\mathrm{d}}{\mathrm{d}x} + \frac{x_0^2}{2!}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \ldots\right)\psi(x) \\ &= \psi(x) + x_0\frac{\mathrm{d}\psi}{\mathrm{d}x} + \frac{x_0^2}{2}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \ldots\end{aligned}$$

$$\left(p_x = -i\frac{\partial}{\partial x}\right)$$

i.e. obtain the expected Taylor expansion

Symmetries in Particle Physics : Isospin

- 质子和中子的质量相似。核力近似与电荷无关，即 $V_{pp} \approx V_{np} \approx V_{nn}$
- 为反映次对称性，海森堡 (1932) 提出：如果可以“关闭”质子的电荷

则质子和中子完全相同

- 提出将质子和中子看作一个物体(核子)的两个态 $p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- 类比自旋1/2粒子的自旋朝上/朝下，引入同位旋 **ISOSPIN**
- 预期物理规律在同位旋空间的转动下保持不变
  - 中子和质子形成总同位旋的二重态，其总同位旋$I = ½$，第三分量 $I_3 = \pm ½$

将此想法扩展到夸克：假设强相互作用中所有味道的夸克都一样（确实！）

本课件中“同位旋对称性”等同于“味道对称性”

Flavour Symmetry of Strong Interaction

因为$m_u \approx m_d$：

- 强相互作用有近似的味道对称性，即上夸克和下夸克互换不改变物理
  - 选择基矢为 $u = \begin{pmatrix}1\\0\end{pmatrix} \quad d = \begin{pmatrix}0\\1\end{pmatrix}$
- 强相互作用u↔d交换不变性表达为抽象的同位旋空间的"旋转"的不变性

$$\begin{pmatrix}u'\\d'\end{pmatrix} = \hat{U}\begin{pmatrix}u\\d\end{pmatrix} = \begin{pmatrix}U_{11} & U_{12}\\U_{21} & U_{22}\end{pmatrix}\begin{pmatrix}u\\d\end{pmatrix}$$

  - 2x2 幺正矩阵：依赖4个复数(即8个实参数)，和4个约束 $\hat{U}^\dagger\hat{U} = 1$

➡ 8 – 4 = 4 个独立的矩阵 • 群论中，四个矩阵形成了 U(2) 群

- 其中一个矩阵对应只有相位变化，而没有味道变换，这里可以忽略 $\hat{U}_1 = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}e^{i\phi}$

Flavour Symmetry of Strong Interaction

- 剩下的三个矩阵形成一个SU(2)群 (special unitary)，其中 $\boxed{\det U = 1}$
- 用厄米产生子$\hat{G}$ 表示对于无穷小变换 $\hat{U} = 1 + i\varepsilon\hat{G}$

$$\det U = 1 \quad \Rightarrow \quad Tr(\hat{G}) = 0$$

- 泡利自旋矩阵是$\hat{G}$的一个线性选择

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

味道对称性与自旋有相同的变换性质！

- 定义同位旋： $\vec{T} = \frac{1}{2}\vec{\sigma} \qquad \hat{U} = e^{i\vec{\alpha}.\vec{T}}$
- 对于无穷小变换

$$\hat{U} = 1 + \frac{1}{2}i\vec{\varepsilon}.\vec{\sigma} \quad = \quad 1 + \frac{i}{2}(\varepsilon_1\sigma_1 + \varepsilon_2\sigma_2 + \varepsilon_3\sigma_3) = \begin{pmatrix} 1 + \frac{1}{2}i\varepsilon_3 & \frac{1}{2}i(\varepsilon_1 - i\varepsilon_2) \\ \frac{1}{2}i(\varepsilon_1 + i\varepsilon_2) & 1 - \frac{1}{2}i\varepsilon_3 \end{pmatrix}$$

由幺正性要求，得其行列式 $U^\dagger U = I + O(\varepsilon^2) \qquad \det U = 1 + O(\varepsilon^2)$

Properties of Isopin

➢ **性质与自旋相同** $[T_1, T_2] = iT_3 \quad [T_2, T_3] = iT_1 \quad [T_3, T_1] = iT_2$

$$[T^2, T_3] = 0 \qquad T^2 = T_1^2 + T_2^2 + T_3^2$$

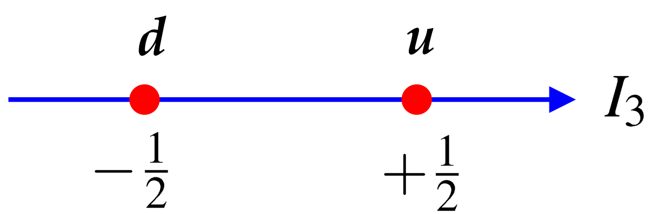
类似自旋，有三个非对易算符 T_1, T_2, T_3，其对应的观测量不能同时确定。因此，通过**总同位旋** I 和同位旋第三分量 I_3 来标记状态

注：同位旋与自旋没有任何关系 – 只是数学相同

• **本征态：**类比角动量的本征态 $|s, m\rangle \longrightarrow |I, I_3\rangle$

其中 $T^2|I, I_3\rangle = I(I+1)|I, I_3\rangle \qquad T_3|I, I_3\rangle = I_3|I, I_3\rangle$

• **根据同位旋：** $u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\tfrac{1}{2}, +\tfrac{1}{2}\rangle \qquad d = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

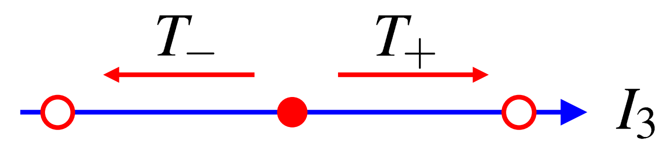


$\boxed{I = \frac{1}{2}, \quad I_3 = \pm\frac{1}{2}}$

• **一般而言** $I_3 = \frac{1}{2}(N_u - N_d)$

Properties of Isopin

- **定义同位旋阶梯算符** u ↔ d $\boxed{T_- \equiv T_1 - iT_2}$
  - 类比自旋阶梯算符



$\boxed{T_+ \equiv T_1 + iT_2}$

d → u

$$T_+|I,I_3\rangle = \sqrt{I(I+1)-I_3(I_3+1)}\,|I,I_3+1\rangle$$
$$T_-|I,I_3\rangle = \sqrt{I(I+1)-I_3(I_3-1)}\,|I,I_3-1\rangle$$

Step up/down in I_3 until reach end of multiplet $T_+|I,+I\rangle = 0 \quad T_-|I,-I\rangle = 0$

$$\boxed{T_+u = 0 \qquad T_+d = u \qquad T_-u = d \qquad T_-d = 0}$$

- **阶梯算符：产生 u→d 和 d→u 的转变**
- **同位旋组合：如双底夸克系统的同位旋是多少，准确地类比自旋组合(即角动量)**

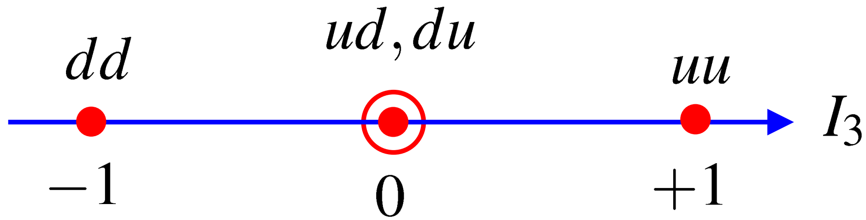
$$|I^{(1)},I_3^{(1)}\rangle|I^{(2)},I_3^{(2)}\rangle \rightarrow |I,I_3\rangle$$

  - I_3 相加性：$I_3 = I_3^{(1)} + I_3^{(2)}$
  - I 矢量相加的整数：从 $|I^{(1)} - I^{(2)}|$ 到 $|I^{(1)} + I^{(2)}|$

- **强相互作用中同位旋变换对称性意味着守恒量的存在**
  - 强相互作用中 I_3 和 I是守恒的，类比J_z 和J 的角动量守恒

Combining Quarks: derive proton wave-function

- 首先合并两个夸克，然后在第三个：利用费米波函数反对称的要求
  - 同位旋可用于定义多夸克态，如 两夸克：这里我们有四种组合：



注： 代表两个态有相同 I_3

  - 立刻能确定极值 (I_3 相加性)

$$uu \equiv |\tfrac{1}{2},\tfrac{1}{2}\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle = |1,+1\rangle \qquad dd \equiv |\tfrac{1}{2},-\tfrac{1}{2}\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle = |1,-1\rangle$$

  - 使用阶梯算符以得到 |1, 0>

$$T_-|1,+1\rangle = \sqrt{2}|1,0\rangle = T_-(uu) = ud + du \Rightarrow |1,0\rangle = \frac{1}{\sqrt{2}}(ud+du)$$

  - 发现末态 |0, 0>与 |1, 0>正交 $\Rightarrow |0,0\rangle = \frac{1}{\sqrt{2}}(ud-du)$

- 总同位旋不同的态在物理上也不同
  - 在交换1-2夸克下，同位旋为1的态是对称的，而单态则是反对称的

矩阵的直积

$A=(a_{ij})_{m\times n}$和$B=(b_{ij})_{p\times q}$，$A\otimes B=(a_{ij}B)_{mp\times nq}$

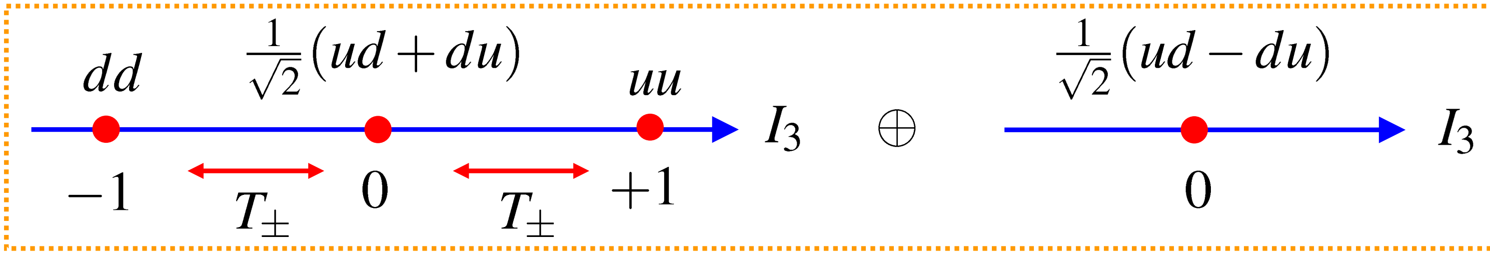
$$A\otimes B=\begin{pmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{pmatrix}_{mp\times nq}$$

- **以电子自旋为例：**
  - 升算符 $T^+=\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$，降算符 $T^-=\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$
  - 两个基态：$|\uparrow\rangle=\begin{pmatrix} 1 \\ 0 \end{pmatrix}, |\downarrow\rangle=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- **现在考虑两个电子，用直积构建Hilbert空间——$S_1\otimes S_2$：**
  - $|\uparrow\uparrow\rangle=\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |\uparrow\downarrow\rangle=\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |\downarrow\uparrow\rangle=\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |\downarrow\downarrow\rangle=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
  - $T_1^+=\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\otimes I_2=\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, T_2^+=I_2\otimes\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
  - $T^+|\downarrow\downarrow\rangle=T_1^+|\downarrow\downarrow\rangle+T_2^+|\downarrow\downarrow\rangle=\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}=|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle$

Combining Quarks: derive proton wave-function

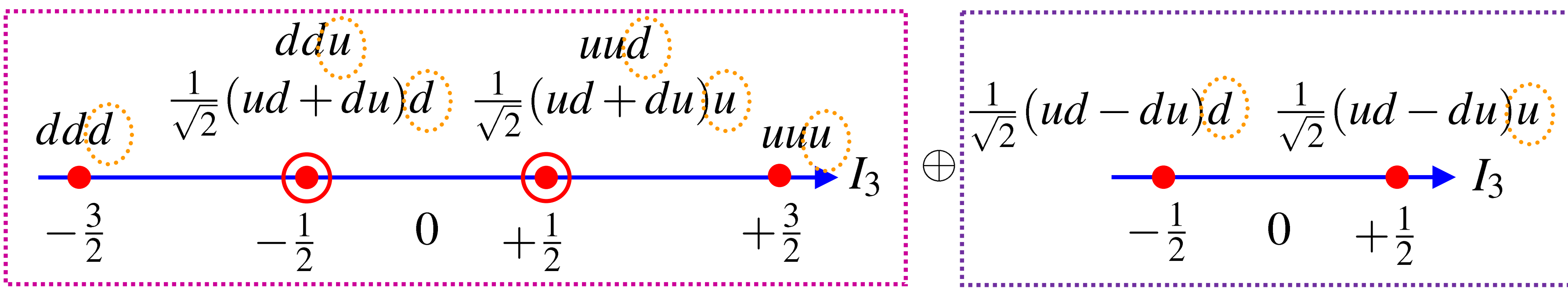
- 从4种可能的同位旋二重态组合得到同位旋为1的三重态和一个同位旋为0的单态

$2 \otimes 2 = 3 \oplus 1$



- 两个I=1/2的态组合，得到I=1的三重态和I=0的单态。
- 如果再增加一个I=1/2的态（一个额外的上或下夸克）
  - 从上面4个态的任意一个，得到两个新同位旋态
  - I=0的态，只能为I=1/2；I=1的态，可以为I=3/2也可以为I=1/2

6

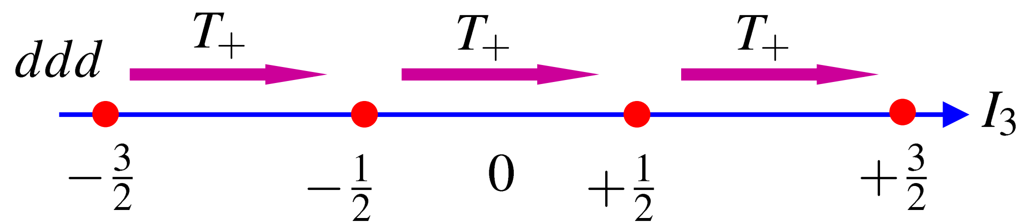


2

- 使用阶梯算符和正交性，把态分类到多态：例如，从ddd开始提升

Combining Quarks: derive proton wave-function

➢ 从ddd=| 3/2, -3/2>导出
所有 I=3/2的态



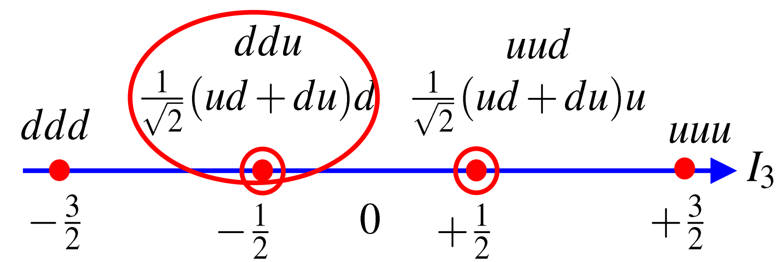
$$
\begin{aligned}
T_+|\tfrac{3}{2},-\tfrac{3}{2}\rangle &= T_+(ddd) = (T_+d)dd + d(T_+d)d + dd(T_+)d \\
\sqrt{3}|\tfrac{3}{2},-\tfrac{1}{2}\rangle &= udd + dud + ddu \\
\boxed{|\tfrac{3}{2},-\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}(udd + dud + ddu)} & \\
T_+|\tfrac{3}{2},-\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}T_+(udd + dud + ddu) \\
2|\tfrac{3}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}(uud + udu + uud + duu + udu + duu) \\
\boxed{|\tfrac{3}{2},+\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}(uud + udu + duu)} & \\
T_+|\tfrac{3}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}T_+(uud + udu + duu) \\
\sqrt{3}|\tfrac{3}{2},+\tfrac{3}{2}\rangle &= \tfrac{1}{\sqrt{3}}(uuu + uuu + uuu) \\
\boxed{|\tfrac{3}{2},+\tfrac{3}{2}\rangle = uuu} &
\end{aligned}
$$

$$
\begin{aligned}
T_+|I,I_3\rangle &= \sqrt{I(I+1) - I_3(I_3+1)}\,|I,I_3+1\rangle \\
T_-|I,I_3\rangle &= \sqrt{I(I+1) - I_3(I_3-1)}\,|I,I_3-1\rangle
\end{aligned}
$$

- 从前一页的[6]重态，利用正交性找到|1/2, ±1/2>态
- 从前一页的[2]重态给出另外一个|1/2, ±1/2>二重态

两个二重态的导出

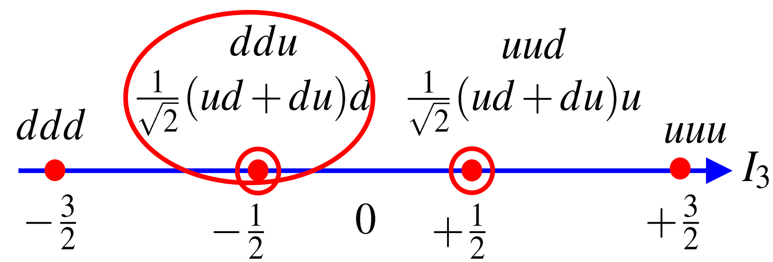
- 从前一页的[6]重态，利用正交性找到|1/2, ±1/2>态
  - $\left|\frac{1}{2},-\frac{1}{2}\right\rangle$应该与$\left|\frac{3}{2},-\frac{1}{2}\right\rangle=\frac{1}{\sqrt{3}}(udd+dud+ddu)$正交
    - 又$I_3=-\frac{1}{2}$且属于6重态，所以必须是ddu和$\frac{1}{\sqrt{2}}(ud+du)d$的线性组合
  - $\left|\frac{1}{2},-\frac{1}{2}\right\rangle=A\cdot ddu+B\cdot\left(\frac{1}{\sqrt{2}}(ud+du)d\right)$
    - $\left\langle\frac{3}{2},-\frac{1}{2}\middle|\frac{1}{2},-\frac{1}{2}\right\rangle=A+\frac{2B}{\sqrt{2}}=0$
    - $\left\langle\frac{1}{2},-\frac{1}{2}\middle|\frac{1}{2},-\frac{1}{2}\right\rangle=A^2+B^2=1$
  - $\left|\frac{1}{2},-\frac{1}{2}\right\rangle=-\frac{1}{\sqrt{6}}(2ddu-udd-dud)$



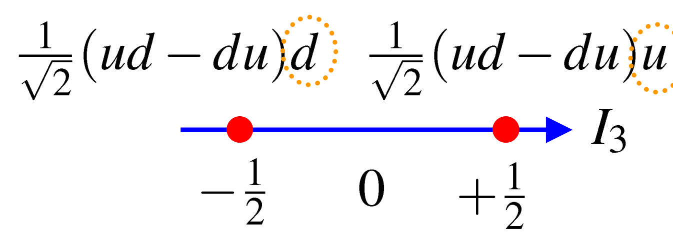
✓ **课堂练习：请同样方法得到** $\left|\frac{1}{2},+\frac{1}{2}\right\rangle=?$

两个二重态的导出

- 从前一页的[6]重态，利用正交性找到|1/2, ±1/2>态
  - $\left|\frac{1}{2},-\frac{1}{2}\right\rangle$应该与$\left|\frac{3}{2},-\frac{1}{2}\right\rangle=\frac{1}{\sqrt{3}}(udd+dud+ddu)$正交
    - 又$I_3=-\frac{1}{2}$且属于6重态，所以必须是ddu和$\frac{1}{\sqrt{2}}(ud+du)d$的线性组合
  - $\left|\frac{1}{2},-\frac{1}{2}\right\rangle=A\cdot ddu+B\cdot\left(\frac{1}{\sqrt{2}}(ud+du)d\right)$
    - $\left\langle\frac{3}{2},-\frac{1}{2}\middle|\frac{1}{2},-\frac{1}{2}\right\rangle=A+\frac{2B}{\sqrt{2}}=0$
    - $\left\langle\frac{1}{2},-\frac{1}{2}\middle|\frac{1}{2},-\frac{1}{2}\right\rangle=A^2+B^2=1$
  - $\left|\frac{1}{2},-\frac{1}{2}\right\rangle=-\frac{1}{\sqrt{6}}(2ddu-udd-dud)$
  - 同样方法得到$\left|\frac{1}{2},+\frac{1}{2}\right\rangle=\frac{1}{\sqrt{6}}(2uud-udu-duu)$



- 从前一页的[2]重态给出另外一个|1/2, ±1/2>二重态
  - $\left|\frac{1}{2},-\frac{1}{2}\right\rangle=\frac{1}{\sqrt{2}}(udd-dud)$
  - $\left|\frac{1}{2},+\frac{1}{2}\right\rangle=\frac{1}{\sqrt{2}}(udu-duu)$



Combining Quarks: derive proton wave-function

★ 八个态 $uuu, uud, udu, udd, duu, dud, ddu, ddd$

分成1个 同位旋四重态 和 2个 同位旋 二重态

$$2 \otimes 2 \otimes 2 = 2 \otimes (3 \oplus 1) = (2 \otimes 3) \oplus (2 \otimes 1) = 4 \oplus 2 \oplus 2$$

- 不同的多重态有不同的对称性特征

四重态在交换任意两个夸克对称

$$\boxed{\mathbf{S}} \left\{ \begin{aligned} \left|\tfrac{3}{2}, +\tfrac{3}{2}\right\rangle &= uuu \\ \left|\tfrac{3}{2}, +\tfrac{1}{2}\right\rangle &= \tfrac{1}{\sqrt{3}}(uud + udu + duu) \\ \left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle &= \tfrac{1}{\sqrt{3}}(ddu + dud + udd) \\ \left|\tfrac{3}{2}, -\tfrac{3}{2}\right\rangle &= ddd \end{aligned} \right.$$

混合对称：1-2交换对称

$$\boxed{\mathbf{M_S}} \left\{ \begin{aligned} \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle &= -\tfrac{1}{\sqrt{6}}(2ddu - udd - dud) \\ \left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle &= \tfrac{1}{\sqrt{6}}(2uud - udu - duu) \end{aligned} \right.$$

混合对称：1-2交换反对称

$$\boxed{\mathbf{M_A}} \left\{ \begin{aligned} \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle &= \tfrac{1}{\sqrt{2}}(udd - dud) \\ \left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle &= \tfrac{1}{\sqrt{2}}(udu - duu) \end{aligned} \right.$$

★ 混合对称态对于，如交换1-3夸克，没有明确的对称性

Combining Spin

- 运用同样的数学来确定3个自旋1/2粒子组合的可能自旋波函数

四重态在交换任意两个夸克对称

S
$$\begin{cases} \left|\frac{3}{2},+\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow \\ \left|\frac{3}{2},+\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \\ \left|\frac{3}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow) \\ \left|\frac{3}{2},-\frac{3}{2}\right\rangle = \downarrow\downarrow\downarrow \end{cases}$$

混合对称：1-2交换对称

$\mathbf{M_S}$
$$\begin{cases} \left|\frac{1}{2},-\frac{1}{2}\right\rangle = -\frac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow) \\ \left|\frac{1}{2},+\frac{1}{2}\right\rangle = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \end{cases}$$

混合对称：1-2交换反对称

$\mathbf{M_A}$
$$\begin{cases} \left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow) \\ \left|\frac{1}{2},+\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \end{cases}$$

现在可以构建三个夸克组合的总波函数

Baryon Wave-functions (ud)

- **夸克是费米子，因此要求：**总波函数在交换任意两个夸克下为反对称
  - 总波函数表达为：$\psi = \phi_{\text{flavour}}\chi_{\text{spin}}\xi_{\text{colour}}\eta_{\text{space}}$
- **对于全部qqq束缚态色波函数反对称（“色单态”见后续QCD章节）**
  - 此处仅考虑最低质量态，无轨道角动量的基态重子
  - 对于L=0，空间波函数对称 (-1)^L.

$$\left.\begin{array}{lll} \Rightarrow & \xi_{\text{colour}}\eta_{\text{space}} & \boxed{\text{anti-symmetric}} \\ \Rightarrow & \phi_{\text{flavour}}\chi_{\text{spin}} & \boxed{\text{symmetric}} \end{array}\right\} \boxed{\text{全局反对称}}$$

- **两种方式构建自旋-同位旋的完全对称波函数**

❶ 合并总的自旋和同位旋的对称波函数 $\phi(S)\chi(S)$

ddd	$\frac{1}{\sqrt{3}}(ddu+dud+udd)$		$\frac{1}{\sqrt{3}}(uud+udu+duu)$	uuu
Δ^-	Δ^0		Δ^+	Δ^{++}
$-\frac{3}{2}$	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{3}{2}$

I_3

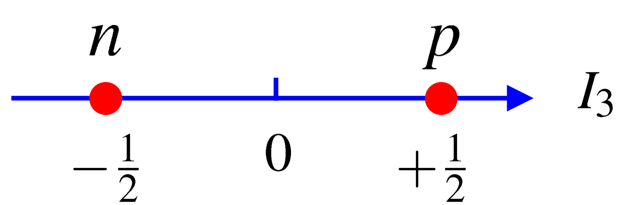
Spin 3/2
Isospin 3/2

Baryon Wave-functions (ud)

❷ 合并自旋和同位旋的混合对称波函数

- $\phi(M_S)\chi(M_S)$ 和 $\phi(M_A)\chi(M_A)$ 在交换1–2夸克下是对称的
- 不足：这些组合对于1–3交换，没有明确的对称性
- 但是，（归一化）线性组合 $\frac{1}{\sqrt{2}}\phi(M_S)\chi(M_S)+\frac{1}{\sqrt{2}}\phi(M_A)\chi(M_A)$ 总体对称

（即 对于交换1–2, 1–3, 2–3 下）



自旋 1/2，同位旋 1/2

- 自旋向上的质子波函数为：

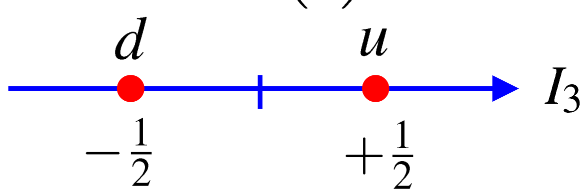
$$|p\uparrow\rangle=\frac{1}{6\sqrt{2}}(2uud-udu-duu)(2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)+\frac{1}{2\sqrt{2}}(udu-duu)(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$$

$$\Rightarrow \boxed{\begin{aligned}|p\uparrow\rangle=\frac{1}{\sqrt{18}}(\ & 2u\uparrow u\uparrow d\downarrow-u\uparrow u\downarrow d\uparrow-u\downarrow u\uparrow d\uparrow+\\ & 2u\uparrow d\downarrow u\uparrow-u\uparrow d\uparrow u\downarrow-u\downarrow d\uparrow u\uparrow+\\ & 2d\downarrow u\uparrow u\uparrow-d\uparrow u\downarrow u\uparrow-d\uparrow u\uparrow u\uparrow)\end{aligned}}$$

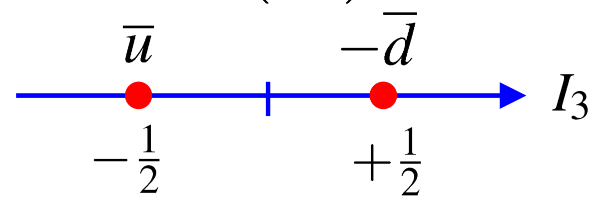
Anti-quarks and Mesons (u and d)

- u, d 夸克 和 $\bar{u}, \bar{d}$反夸克表示为同位旋二重态

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$



$$\bar{q} = \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$



$$\bar{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \bar{d} = -\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- 要点：反夸克二重态的排序和符号保证反夸克和夸克以相同方式变换（见附录I）
  - 这使得物理预言在u-d 夸克和 $\bar{u}$–$\bar{d}$交换下不变

- 考虑阶梯算符作用在反夸克同位旋态

如 $T_+\bar{u} = T_+\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -\bar{d}$

- 效应为：

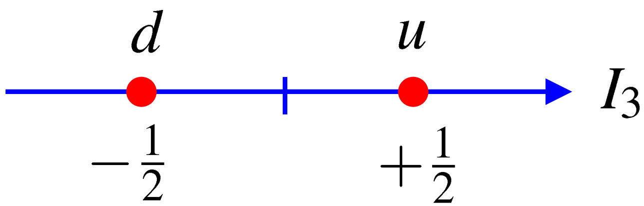
$$\boxed{T_+\bar{u} = -\bar{d} \qquad T_+\bar{d} = 0 \qquad T_-\bar{u} = 0 \qquad T_-\bar{d} = -\bar{u}}$$

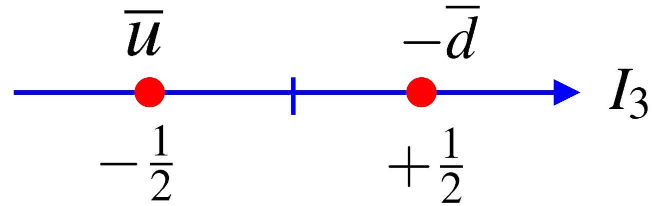
- 对比

$$\boxed{T_+u = 0 \qquad T_+d = u \qquad T_-u = d \qquad T_-d = 0}$$

Light ud Mesons

➢ **现在可以通过上/下夸克组合来构建介子态**





- 考虑 $q\bar{q}$ 组合的同位旋态

Bar：反夸克的同位旋

$$|1,+1\rangle = |\tfrac{1}{2},+\tfrac{1}{2}\rangle\overline{|\tfrac{1}{2},+\tfrac{1}{2}\rangle} = -u\bar{d} \qquad |1,-1\rangle = |\tfrac{1}{2},-\tfrac{1}{2}\rangle\overline{|\tfrac{1}{2},-\tfrac{1}{2}\rangle} = d\bar{u}$$

为得到$I_3=0$ 态，使用阶梯算符和正交性

$$\begin{aligned} T_-|1,+1\rangle &= T_-\left[-u\bar{d}\right] \\ \sqrt{2}|1,0\rangle &= -T_-\left[u\right]\bar{d} - uT_-\left[\bar{d}\right] \\ &= -d\bar{d} + u\bar{u} \end{aligned}$$

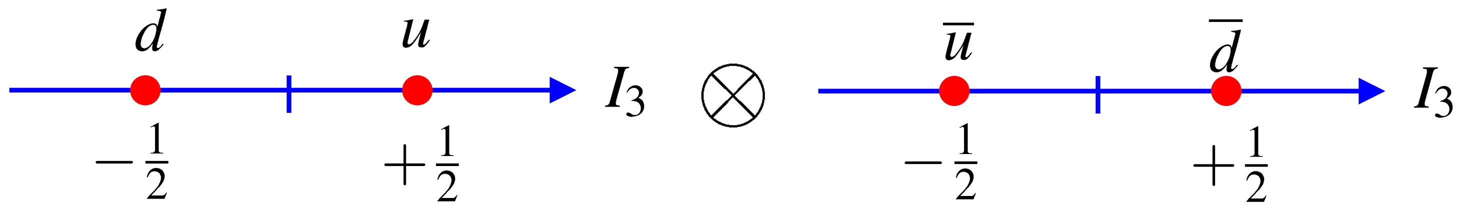
$$\Rightarrow \quad |1,0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

- 正交性给出：

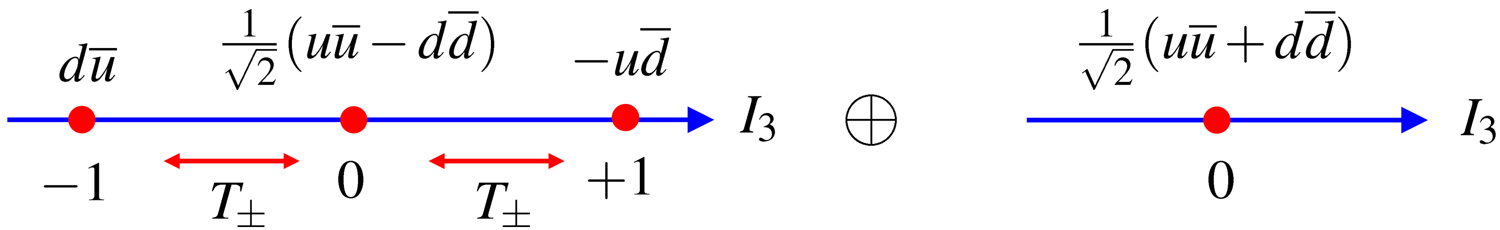
$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$

Light ud Mesons

- **总结：**



$I=1$的三重态 和 $I=0$单态



- **可以记作**

$$2 \otimes \bar{2} = 3 \oplus 1$$

夸克二重态　　反夸克二重态

★ 单态是阶梯算符的“尽头” $T_+|0,0\rangle = T_+\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) = \frac{1}{\sqrt{2}}\left(-u\bar{d}+u\bar{d}\right) = 0$

Light ud Mesons

➤ **总结：**

$$\underset{-\frac{1}{2}}{d}\quad \underset{+\frac{1}{2}}{u}\ \ I_3 \quad \bigotimes \quad \underset{-\frac{1}{2}}{\bar{u}}\quad \underset{+\frac{1}{2}}{\bar{d}}\ \ I_3$$

⇨ **$I = 1$的三重态 和 $I = 0$单态**

$$\underset{-1}{d\bar{u}} \xleftrightarrow{T_\pm} \underset{0}{\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})} \xleftrightarrow{T_\pm} \underset{+1}{-u\bar{d}}\ \ I_3 \quad \bigoplus \quad \underset{0}{\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})}\ \ I_3$$

- **可以记作**

$$2\otimes\bar{2}=3\oplus 1$$

夸克二重态　　反夸克二重态

★ 单态是阶梯算符的“尽头”

$$T_+|0,0\rangle = T_+\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) = \frac{1}{\sqrt{2}}\left(-u\bar{d}+u\bar{d}\right)=0$$

✓ **课堂练习：类似证明** $T_-|0,0\rangle = 0$

SU(3) Flavour

- 将奇异(strange)夸克包括进来
  - 由于$m_s > m_u, m_d$其并非严格的对称性，但m_s 和m_u, m_d差别不是很大
  - 可以认为在强相互作用中(及其产生的强子态)存在s↔u↔d交换对称性

注：任何基于该假设的结果都只是近似的，因为该对称性不是严格的

  - 味道对称性(uds)可表达为 $\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \hat{U}\begin{pmatrix} u \\ d \\ s \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}\begin{pmatrix} u \\ d \\ s \end{pmatrix}$

  - 3x3幺正矩阵依赖于9个复数，即18实参数

$\hat{U}^\dagger \hat{U} = 1$ 给出9个约束

⇒ 18 – 9 = 9 线性独立的矩阵

这9个矩阵形成一个 U(3) 群

  - 类似前述，其中1个矩阵只是单位矩阵乘以一个复相角，与味道对称性无关

- 剩下的 8个矩阵 det U=1 形成一个SU(3) 群 $\vec{T} = \frac{1}{2}\vec{\lambda}$ $\hat{U} = e^{i\vec{\alpha}.\vec{T}}$

(厄米产生子)

SU(3) Flavour

- SU(3)味道对称性
  - 3个夸克态可以表示为

$$u=\begin{pmatrix}1\\0\\0\end{pmatrix}\quad d=\begin{pmatrix}0\\1\\0\end{pmatrix}\quad s=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

- SU(3) uds 味道对称性包含SU(2) ud 对称性
  - 因此前三个矩阵可记为：

$$\lambda_1=\begin{pmatrix}\sigma_1 & \begin{matrix}0\\0\end{matrix}\\ \begin{matrix}0&0\end{matrix} & 0\end{pmatrix}\quad \lambda_2=\begin{pmatrix}\sigma_2 & \begin{matrix}0\\0\end{matrix}\\ \begin{matrix}0&0\end{matrix} & 0\end{pmatrix}\quad \lambda_3=\begin{pmatrix}\sigma_3 & \begin{matrix}0\\0\end{matrix}\\ \begin{matrix}0&0\end{matrix} & 0\end{pmatrix}$$

即 **u ↔ d**

$$\lambda_1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}\quad \lambda_2=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}\quad \lambda_3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$$

- 同位旋第三分量：$\boxed{I_3=\frac{1}{2}\lambda_3}$ 其中 $I_3u=+\frac{1}{2}u\quad I_3d=-\frac{1}{2}d\quad I_3s=0$
- I_3：一个态中“上夸克数 减去 下夸克数”
- 如前，阶梯算符 $T_\pm=\frac{1}{2}(\lambda_1\pm i\lambda_2)$ $\quad d \bullet \xleftrightarrow{T_\pm} \bullet\, u$

SU(3) Flavour

➢ 考虑u↔s和d↔s交换对应的矩阵

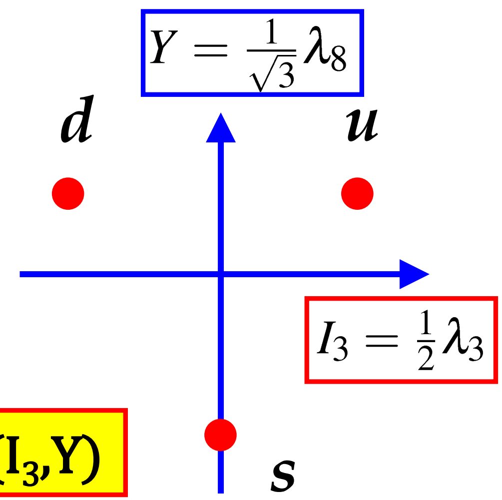
u ↔ s $\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

d ↔ s $\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

- 这样除了 $\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$，还有2个无迹对角矩阵。但这3个对角矩阵并不独立
- 第八个矩阵λ_8的定义：如下线性组合

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

指定2D平面的“垂直位置”



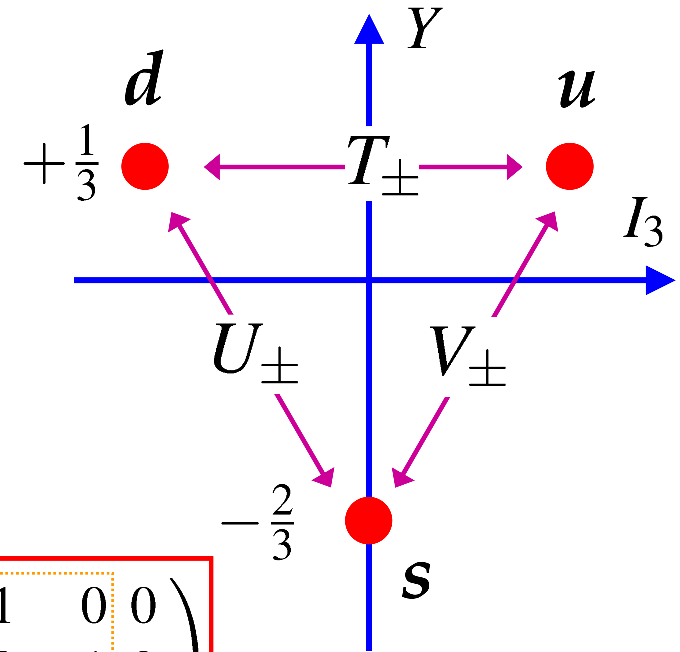
只需要两个坐标轴（量子数）来确定二维平面的一个态：(I_3,Y)

SU(3) Flavour

➢ **另外6个矩阵形成6个阶梯算符，来改变态**

$$\boxed{\begin{aligned} T_\pm &= \tfrac{1}{2}(\lambda_1 \pm i\lambda_2) \\ V_\pm &= \tfrac{1}{2}(\lambda_4 \pm i\lambda_5) \\ U_\pm &= \tfrac{1}{2}(\lambda_6 \pm i\lambda_7) \end{aligned}}$$

其中 $\boxed{I_3 = \frac{1}{2}\lambda_3}$ $\boxed{Y = \frac{1}{\sqrt{3}}\lambda_8}$



以及八个盖尔曼（Gell-Mann）矩阵

u ↔ d

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

u ↔ s

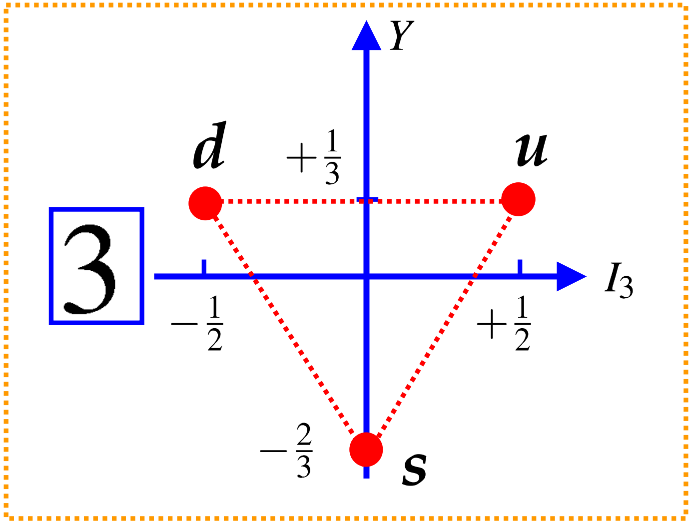
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

d ↔ s

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Quarks and anti-quarks in SU(3) Flavour

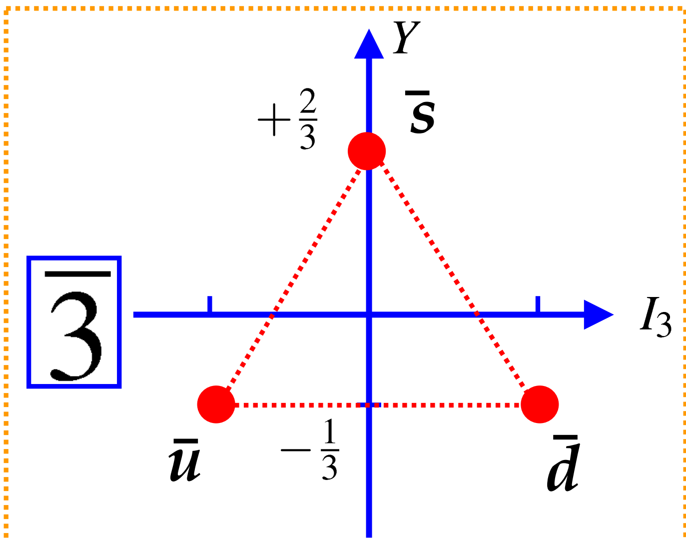


Quarks

$$I_3 u = +\tfrac{1}{2}u; \quad I_3 d = -\tfrac{1}{2}d; \quad I_3 s = 0$$

$$Y u = +\tfrac{1}{3}u; \quad Y d = +\tfrac{1}{3}d; \quad Y s = -\tfrac{2}{3}s$$

- Anti-quarks have opposite SU(3) flavour quantum numbers



Anti-Quarks

$$I_3 \bar{u} = -\tfrac{1}{2}\bar{u}; \quad I_3 \bar{d} = +\tfrac{1}{2}\bar{d}; \quad I_3 \bar{s} = 0$$

$$Y \bar{u} = -\tfrac{1}{3}\bar{u}; \quad Y \bar{d} = -\tfrac{1}{3}\bar{d}; \quad Y \bar{s} = +\tfrac{2}{3}\bar{s}$$

SU(3) Ladder Operators

- SU(3) uds 味道对称性包含ud, us 和 ds的SU(2)对称性
  - 例如，u↔s对称性“V-spin” 和 相应的u↔s阶梯算符

$$V_+ = \frac{1}{2}(\lambda_4 + i\lambda_5) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

其中 $V_+ s = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = +u$

SU(3) 阶梯算符

$$T_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$$

$$V_\pm = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$$

$$U_\pm = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$$

- 6个阶梯算符的效果：

✓ 课堂练习：SU(3)阶梯算符

课堂练习：SU(3)阶梯算符

$$u = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad d = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad s = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$T_{\pm} = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$$
$$V_{\pm} = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$$
$$U_{\pm} = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$$

$$\lambda_1 = \begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix} \quad \lambda_2 = \begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix} \quad \lambda_5 = \begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix} \quad \lambda_7 = \begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}$$

$$T_+ d =$$
$$V_+ s =$$
$$U_+ s =$$
$$T_- u =$$
$$V_- u =$$
$$U_- d =$$

SU(3) Ladder Operators

- SU(3) uds 味道对称性包含ud, us 和 ds的SU(2)对称性
  - 例如，u↔s对称性“V-spin” 和 相应的u↔s阶梯算符

$$V_+ = \frac{1}{2}(\lambda_4 + i\lambda_5) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

其中 $V_+ s = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = +u$

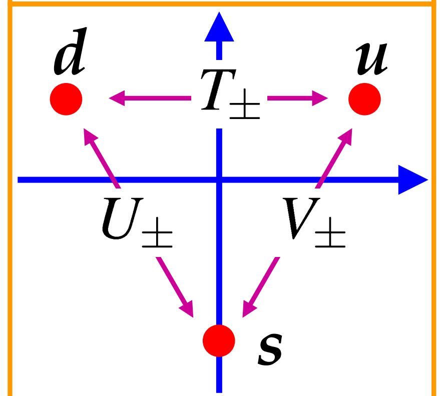
- 6个阶梯算符的效果：

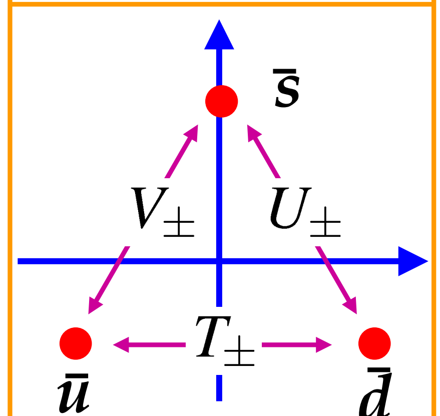
$T_+ d = u; \quad T_- u = d;$	$T_+ \bar{u} = -\bar{d}; \quad T_- \bar{d} = -\bar{u}$
$V_+ s = u; \quad V_- u = s;$	$V_+ \bar{u} = -\bar{s}; \quad V_- \bar{s} = -\bar{u}$
$U_+ s = d; \quad U_- d = s;$	$U_+ \bar{d} = -\bar{s}; \quad U_- \bar{s} = -\bar{d}$

所有其他组合的结果零

SU(3) 阶梯算符

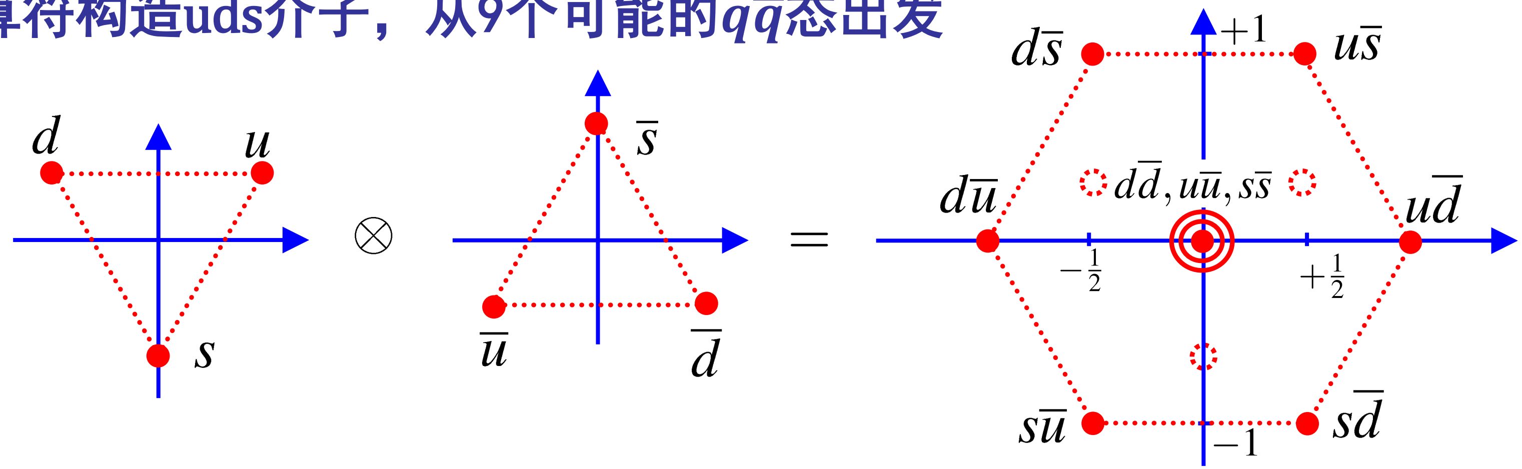
$$T_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$$
$$V_\pm = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$$
$$U_\pm = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$$



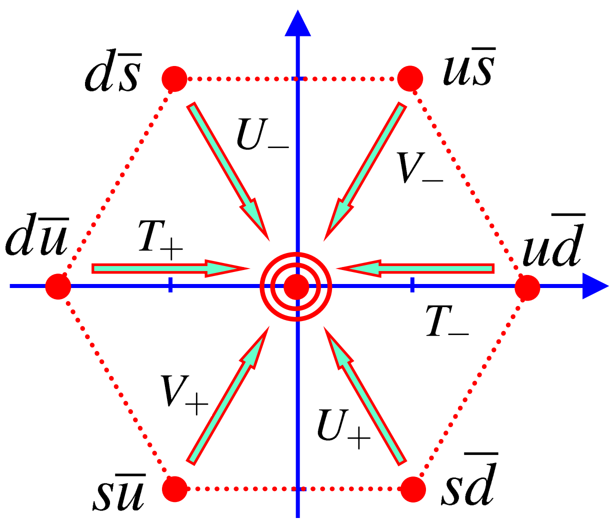


Light (uds) Mesons

- 使用阶梯算符构造uds介子，从9个可能的$q\bar{q}$态出发



- 3个中心态$(Y=1, I_3=0)$的获得，利用阶梯算符和正交性
  - 从外部态出发，6中方法到达中心



$$T_+|d\bar{u}\rangle = |u\bar{u}\rangle - |d\bar{d}\rangle \qquad T_-|u\bar{d}\rangle = |d\bar{d}\rangle - |u\bar{u}\rangle$$
$$V_+|s\bar{u}\rangle = |u\bar{u}\rangle - |s\bar{s}\rangle \qquad V_-|u\bar{s}\rangle = |s\bar{s}\rangle - |u\bar{u}\rangle$$
$$U_+|s\bar{d}\rangle = |d\bar{d}\rangle - |s\bar{s}\rangle \qquad U_-|d\bar{s}\rangle = |s\bar{s}\rangle - |d\bar{d}\rangle$$

- 六个态中只有2个是线性独立的
  - 但 $Y=0, I_3=0$的态有3个
- 因此，其中1个态在不同的多重态中
  - 即，不能通过阶梯算符到达

Light (uds) Mesons

- 从右边3个态构建2个线性独立、正交的态 $\boxed{|u\bar{u}\rangle - |d\bar{d}\rangle}$ $|u\bar{u}\rangle - |s\bar{s}\rangle$ $|d\bar{d}\rangle - |s\bar{s}\rangle$
- 如果味道SU(3)对称性严格成立的话，态的选择就不重要
  - 但由于$m_s > m_{u,d}$，该对称性是近似的
- 实验上，在m~140 MeV观测到三个轻介子: π^+, π^0, π^-
  - 确认同位旋三重态中一个态(π^0): $\boxed{\psi_1 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})}$ (前面推导过)
  - 第二个态通过其他(与π^0正交的)两个态的线性组合得到

$$\psi_2 = \alpha(|u\bar{u}\rangle - |s\bar{s}\rangle) + \beta(|d\bar{d}\rangle - |s\bar{s}\rangle)$$

具有正交归一性: $\langle\psi_1|\psi_2\rangle = 0$, $\langle\psi_2|\psi_2\rangle = 1$ $\Rightarrow$ $\boxed{\psi_2 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})}$

  - 最后一个态（不在同一多重态)可通过与 ψ_1和 ψ_2 的正交性来得到

$$\rightarrow \boxed{\psi_3 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})}$$

SINGLET

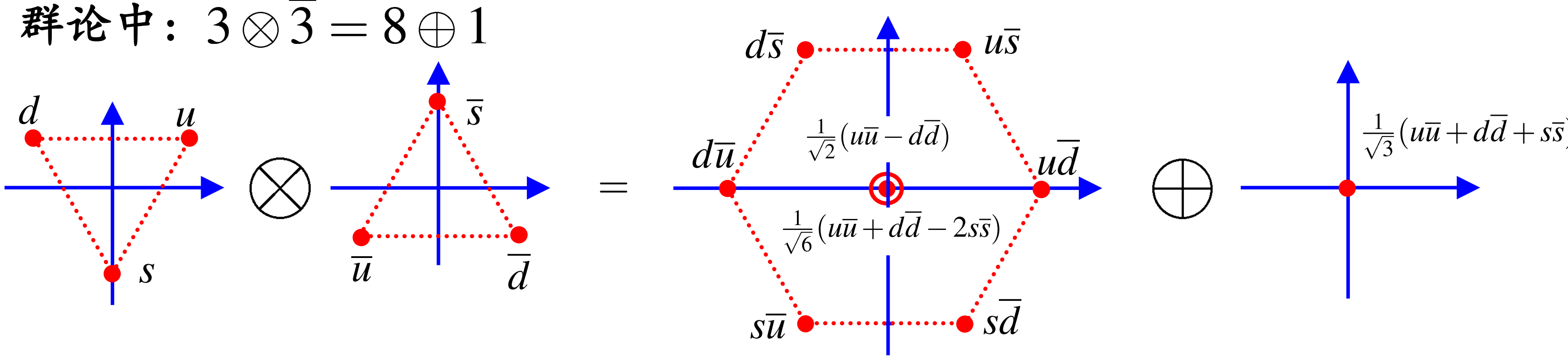
Light (uds) Mesons

- 利用阶梯算符 $T_+\psi_3 = T_-\psi_3 = U_+\psi_3 = U_-\psi_3 = V_+\psi_3 = V_-\psi_3 = 0$

  确认 $\psi_3 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ 是“无味道”的单态

  - 因此夸克和反夸克组合产生9个态，可分为一个八重态和一个单态

  群论中：$3 \otimes \bar{3} = 8 \oplus 1$



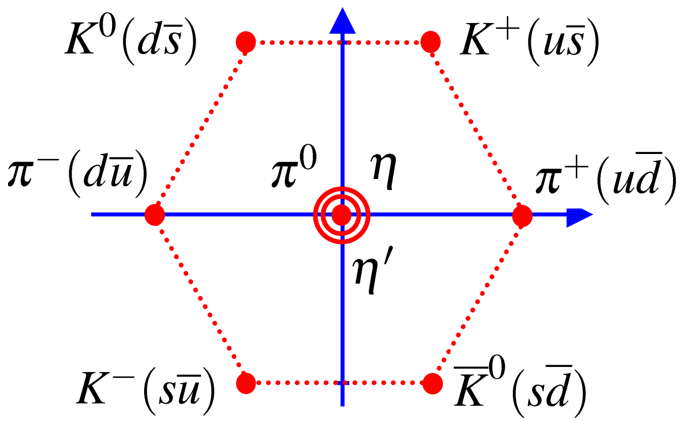
- 对比2个自旋1/2粒子的组合 $2 \otimes 2 = 3 \oplus 1$

  自旋1的三重态 $|1,-1\rangle, |1,0\rangle, |1,+1\rangle$　　自旋0的单态 $|0,0\rangle$

  - 自旋三重态由阶梯算符连接，正如介子八重态由味道阶梯算符连接
  - 单态不携带角动量---对应的SU(3)味道单态是“无味道的”

Light (uds) Mesons

赝标量介子 (L=0, S=0, J=0, P= –1)

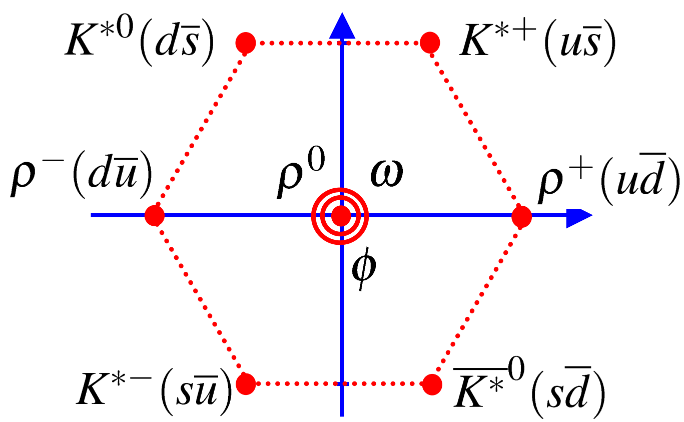


- SU(3)味道群是近似的，$Y=0, I_3=0$的物理态可以是八重态和单态的混合
- 经验性地认为：

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$
$$\eta \approx \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$\eta' \approx \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

单态

矢量介子 (L=0, S=1, J=1, P= –1)



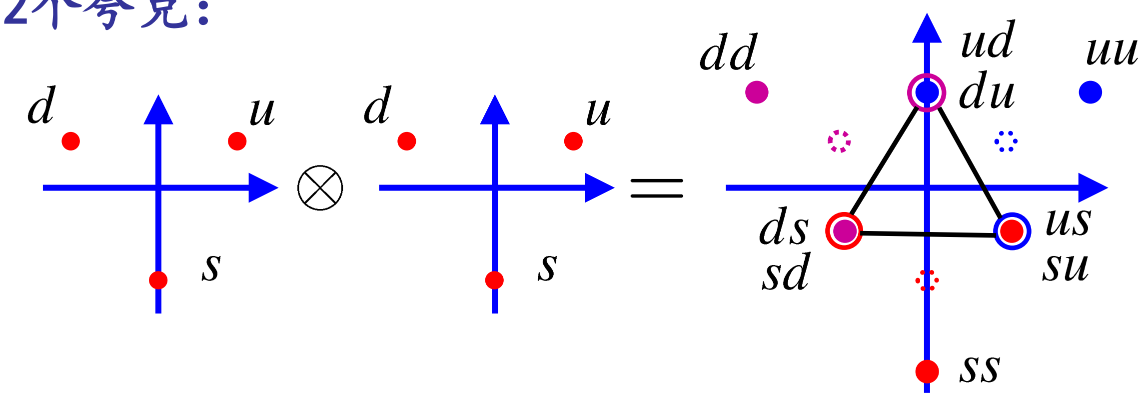
- 对于矢量介子物理态近似“理想混合”：

$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$
$$\omega \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
$$\phi \approx s\bar{s}$$

Combining uds Quarks to form Baryons

- 如质子波函数的推导可以看出，重子态的构造是枯燥的
  - 集中在多重态结构而不是推导完整波函数
  - 首先合并2个夸克：

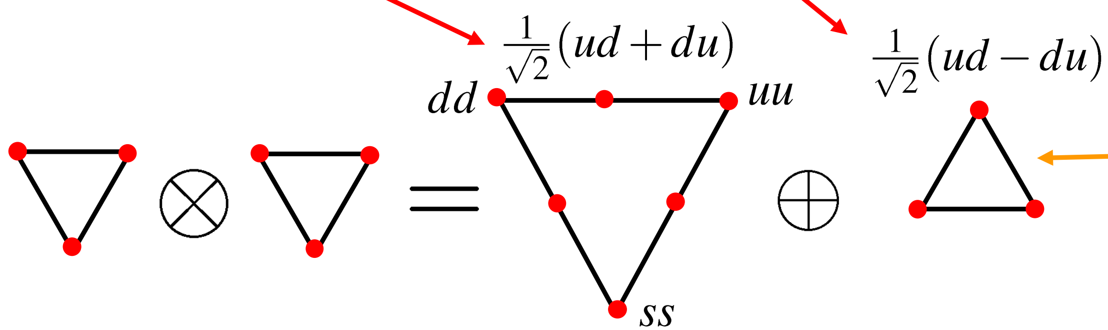
注：此处数学也与色动力学相关



- 产生一个对称的六重态和反对称的三重态：

$$3 \otimes 3 = 6 \oplus \bar{3}$$

与反夸克表达相同“模式”



对称的

反对称的

Combining uds Quarks to form Baryons

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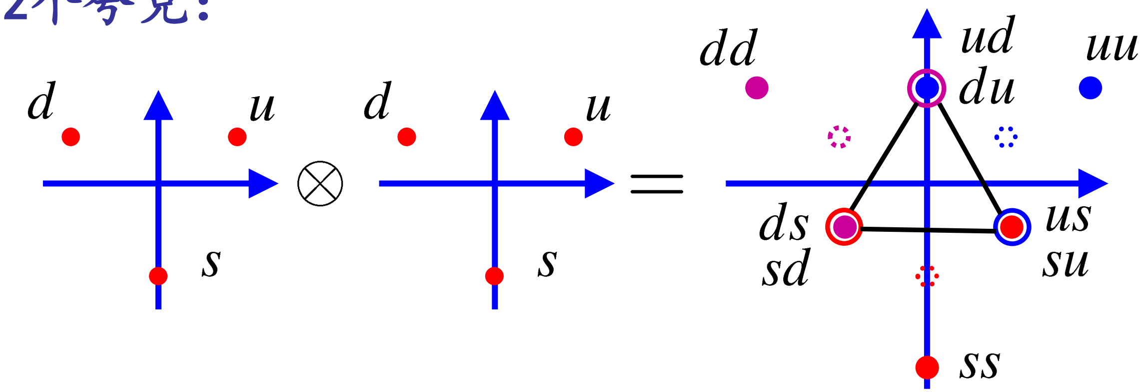

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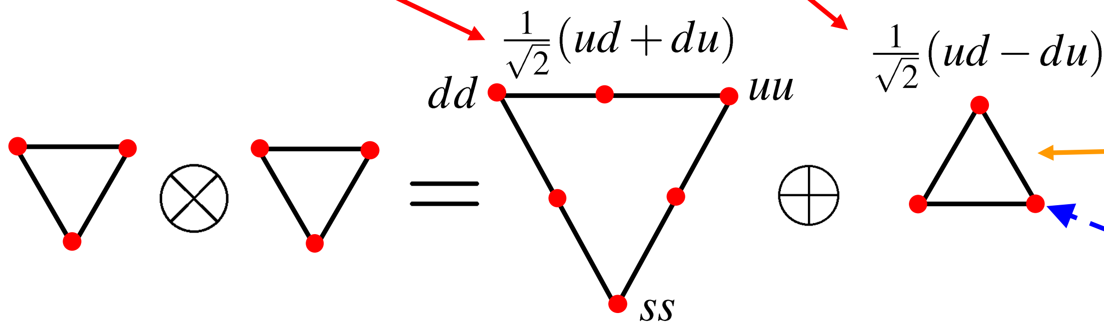
$$3 \otimes 3 = 6 \oplus \overline{3}$$



对称的

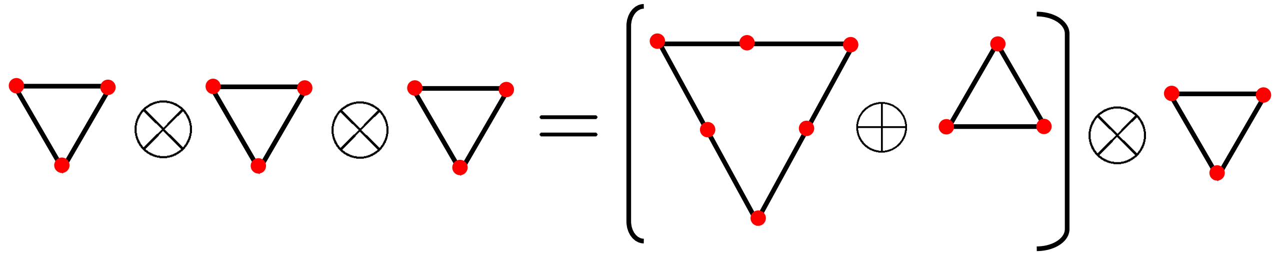
反对称的

与反夸克表达相同“模式”

✓ 课堂练习

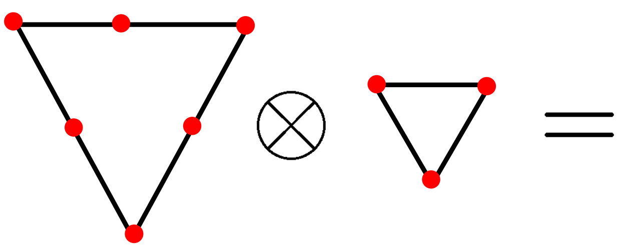
Combining uds Quarks to form Baryons

➢ **现加入第三个夸克：**



- 基于六重态和三重态，分成两部分考虑
  - 再次，集中于多重态结构（对于波函数，参考关于质子波函数的讨论）

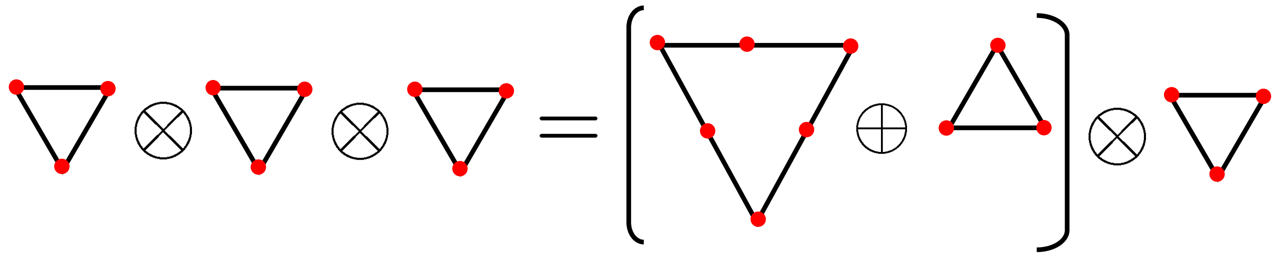
❶ **构建六重态** $3 \otimes 6 = 10 \oplus 8$



✓ **课堂练习**

Combining uds Quarks to form Baryons

➢ **现加入第三个夸克：**



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  - 再次，集中于多重态结构（对于波函数，参考关于质子波函数的讨论）

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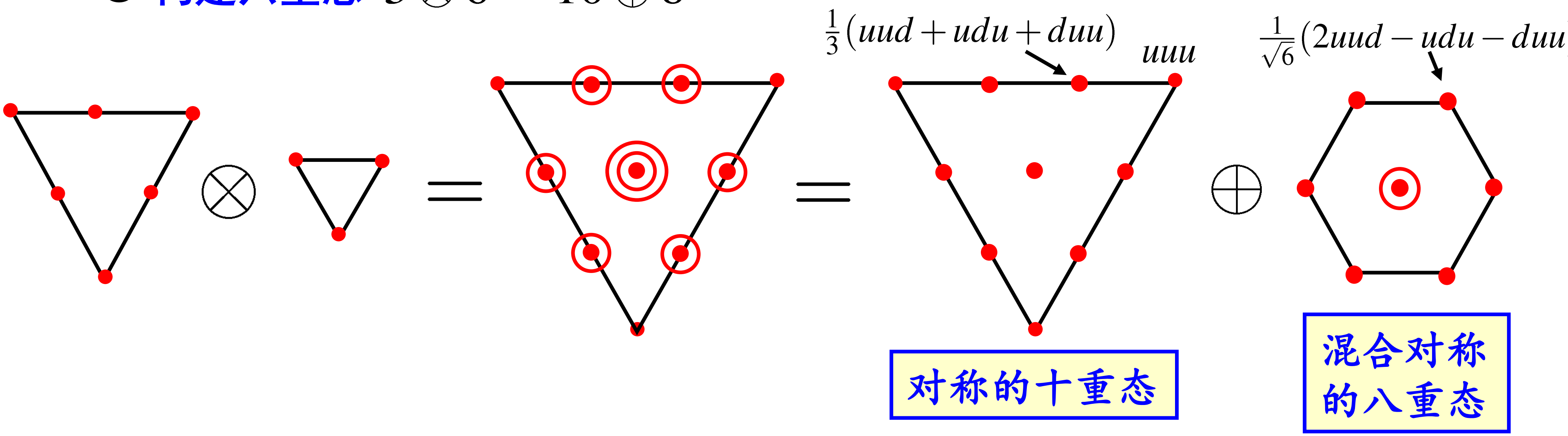
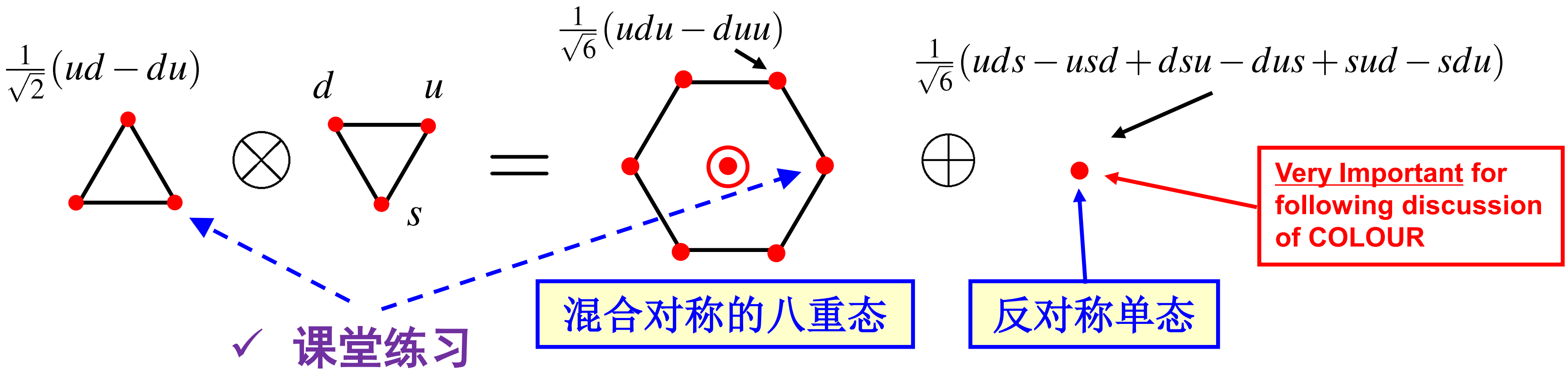


Combining uds Quarks to form Baryons

❷ **构建三重态：**

- 类似uds介子，合并$\bar{3}\times 3$，我们再次得到一个八重态和一个单态



- 利用阶梯算符验证波函数 $\psi_{\text{singlet}}=\frac{1}{\sqrt{6}}(uds-usd+dsu-dus+sud-sdu)$ 是单态，如 $T_{+}\psi_{\text{singlet}}=\frac{1}{\sqrt{6}}(uus-usu+usu-uus+suu-suu)=0$

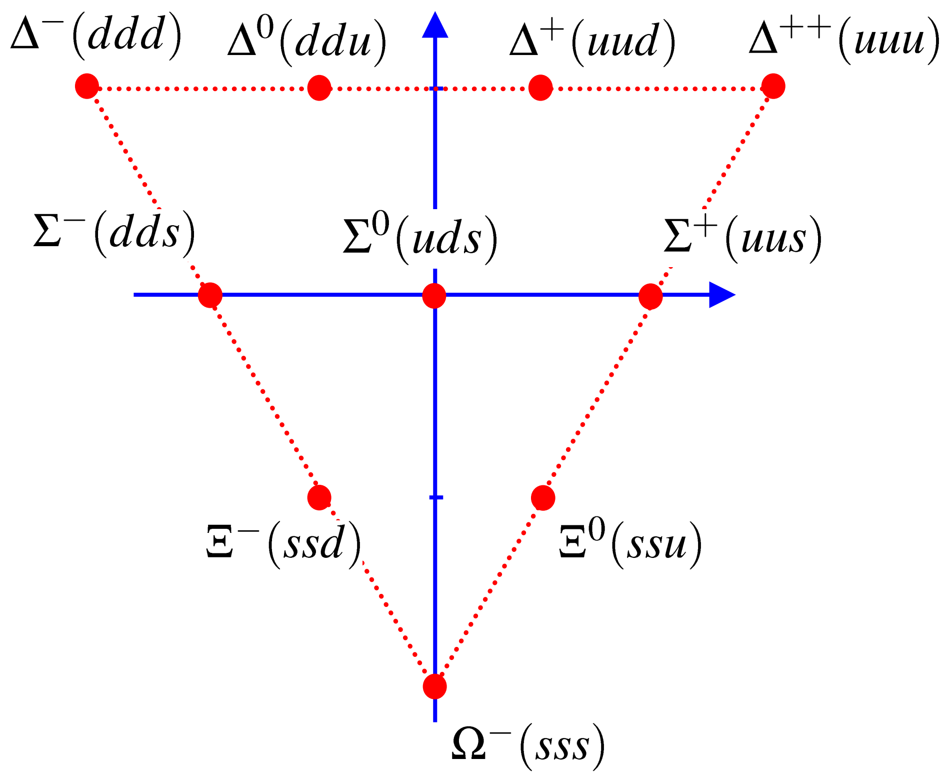
➢ **总之，uds三夸克的组合可以分解为**

$$3\otimes 3\otimes 3=3\otimes(6\oplus\bar{3})=10\oplus 8\oplus 8\oplus 1$$

Baryon Decuplet

- 重子态(L=0):味道对称和自旋对称的自旋3/2十重态波函数 $\phi(S)\chi(S)$

BARYON DECUPLET (L=0, S=3/2, J=3/2, P= +1)



Mass in MeV

Δ(1232)

Σ(1318)

Ξ(1384)

Ω(1672)

- 如果 SU(3)味道对称性是严格对称性，上述所有的质量相同（破坏的对称性）

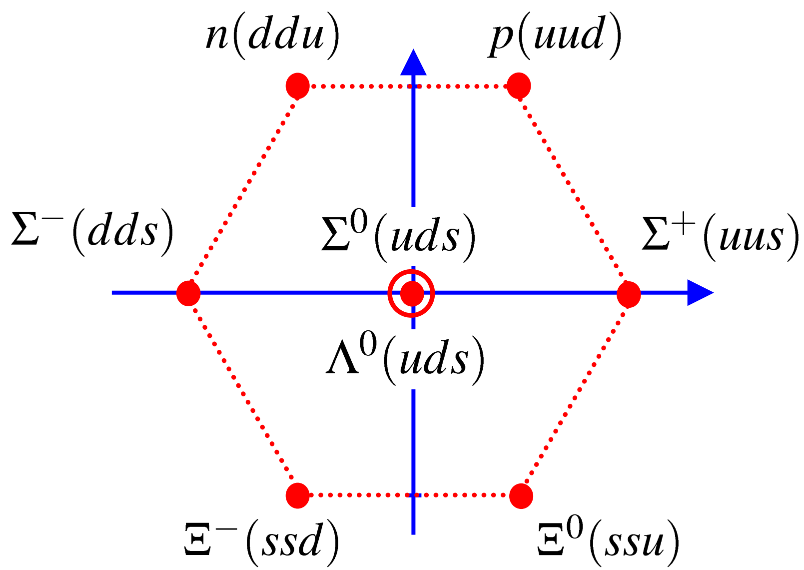
Baryon Octet

➢ 通过味道混合对称和自旋混合对称的波函数构建 自旋1/2八重态

$$\alpha\phi(M_S)\chi(M_S)+\beta\phi(M_A)\chi(M_A)$$

参考前述关于质子的讨论理解如何得到波函数

BARYON OCTET **(L=0, S=1/2, J=1/2, P= +1)**



Mass in MeV

939

Σ(1193)
Λ(1116)

Ξ(1318)

★ 注：没有整体反对称的自旋波函数，因此无法通过反对称味道单态构建整体对称的波函数

Summary

- 讨论了 SU(2) ud 和 SU(3) uds 味道对称性
- 尽管这些味道对称性只是近似成立，仍然可以被用来解释观测到的介子/重子的多重态结构
- SU (3) 对称性的结果，如预言的波函数，应对被谨慎对待
  - 因为 $m_s \neq m_{u,d}$
- 引入单态 的“无自旋”或者“无味道”概念
- 下节课讨论色和量子色动力学QCD

Appendix: SU(2) anti-quark representation

Non-examinable

- 定义反夸克二重态 $\bar{q} = \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} -d^* \\ u^* \end{pmatrix}$

- 夸克二重态 $q = \begin{pmatrix} u \\ d \end{pmatrix}$ 变换规则为 $q' = Uq$

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = U \begin{pmatrix} u \\ d \end{pmatrix} \xrightarrow[\text{conjugate}]{\text{Complex}} \begin{pmatrix} u'^* \\ d'^* \end{pmatrix} = U^* \begin{pmatrix} u^* \\ d^* \end{pmatrix}$$

- 按照反夸克二重态表示 $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \bar{q}' = U \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \bar{q}$

- 因此反夸克变换为

$$\bar{q}' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} U^* \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \bar{q}$$

Appendix: SU(2) anti-quark representation

- 一般地2x2幺正矩阵可以写为： $U = \begin{pmatrix} c_{11} & c_{12} \\ -c_{12}^* & c_{11}^* \end{pmatrix}$

- 给出 $$\begin{aligned} \overline{q}' &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} c_{11}^* & c_{12}^* \\ -c_{12} & c_{11} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \overline{q} \\ &= \begin{pmatrix} c_{11} & c_{12} \\ -c_{12}^* & c_{11}^* \end{pmatrix} \\ &= U\overline{q} \end{aligned}$$

- 因此反夸克二重态 $\overline{q} = \begin{pmatrix} -\overline{d} \\ \overline{u} \end{pmatrix}$

  与夸克二重态以相同的方式变换 $q = \begin{pmatrix} u \\ d \end{pmatrix}$

➢ **注意：这是SU(2)的特殊性质，对于SU(3)则没有类似的反夸克表示**