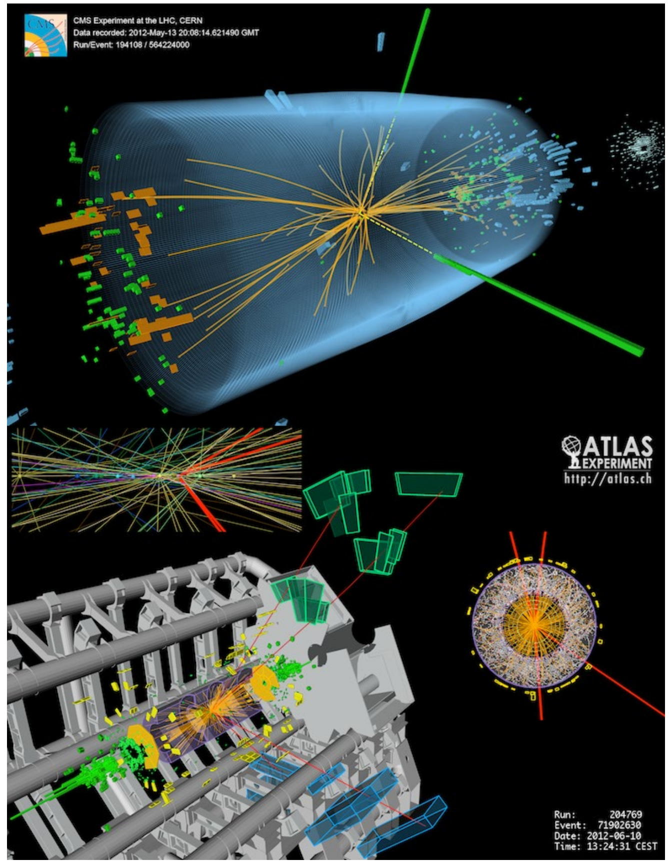
# 粒子物理学

## 第 11 章：希格斯物理

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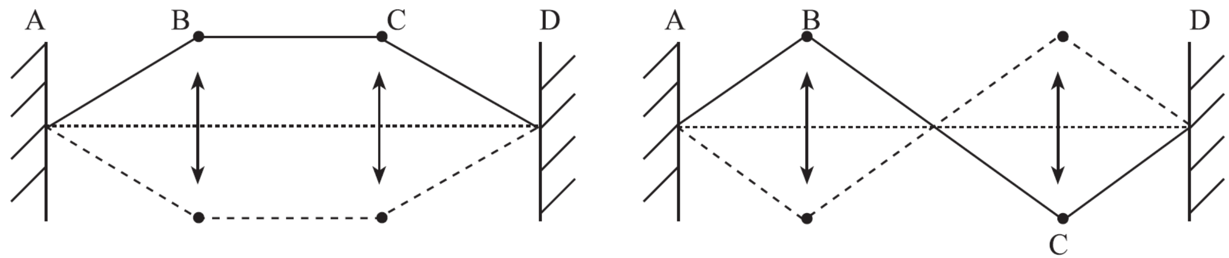
# Classical Lagrangians

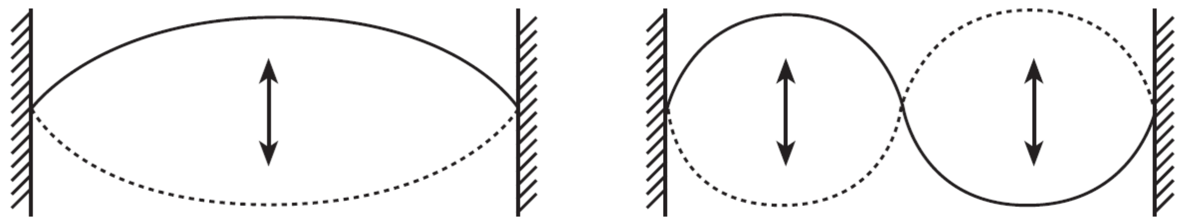
- 在量子力学中，单个粒子被描述为满足特定方程的波函数；但是在量子场论中，粒子被描述为满足适当场方程的量子场的激发

Two normal modes:
(a) frequency $\omega_1$
(b) frequency $\omega_2$

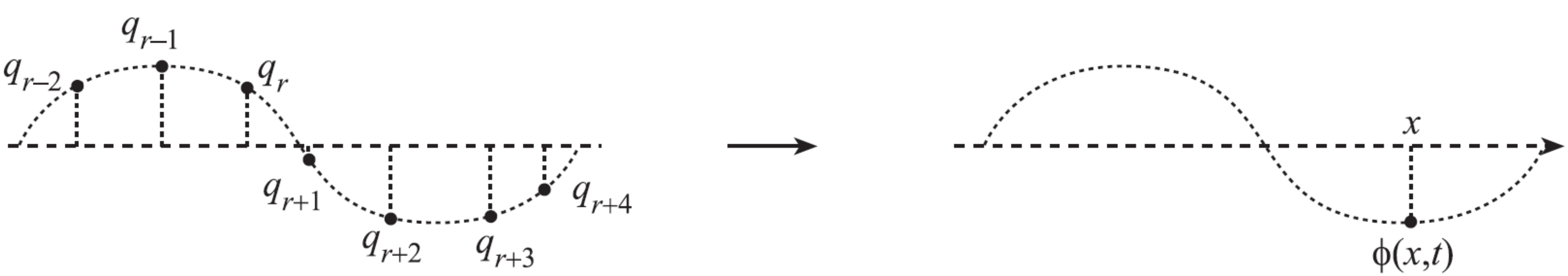



String motion in two normal modes:
(a) r = 1; (b) r =2

$$\phi_r(x,t) = A_r(t)\sin\left(\frac{r\pi x}{\ell}\right)$$



- From a large number of discrete degrees of freedom (mass points) to a continuous degree of freedom (field).



# Classical Lagrangians

- 在量子力学中，单个粒子被描述为满足特定方程的波函数；但是在量子场论中，粒子被描述为满足适当场方程的量子场的激发
- 量子场论的动力学可以用拉格朗日量密度来描述
- （回顾）经典场的拉氏量 $L = T - V$
- 拉氏量$L(q_i, \dot{q}_i)$包含了一组广义坐标及其对时间的导数
- 一旦指定了拉氏量，就可以通过Euler–Lagrange方程得到运动学方程

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0}$$

- 例如，考虑一个沿x轴运动的粒子

$$L = T - V = \frac{1}{2}m\dot{x}^2 - V(x)$$

$$\frac{d}{dt}(m\dot{x}) + \frac{\partial V}{\partial x} = 0 \rightarrow m\ddot{x} = -\frac{\partial V(x)}{\partial x}$$

# Classical Lagrangians

- 对于一个用多个广义坐标$q_i$描述的离散的多粒子系统
  - 只需要将拉氏量替换为拉氏量密度，就可以扩展到一个连续的系统

$$L\left(q_i, \frac{dq_i}{dt}\right) \to \mathcal{L}(\phi_i, \partial_\mu \phi_i)$$

- 场是四维时空坐标的连续函数，拉氏量的定义为 $L = \int \mathcal{L}\, d^3x$
- 在拉氏量密度中，广义坐标$q_i$被替换为了场$\phi_i(t, x, y, z)$
  - 广义坐标对时间的导数$\dot{q}_i$被替换为了场对四维时空坐标的偏导$\partial_\mu \phi_i \equiv \frac{\partial \phi_i}{\partial x^\mu}$
- 使用最小作用量原理，将Euler–Lagrange方程改写为

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0$$

- 场$\phi_i(x^\mu)$表示一个在每一个时空坐标上都有值的连续量
  - 可以是标量：温度函数$T(x,t)$；可以是矢量：电场强度$E(x,t)$；也可以是张量。

# Lagrangians in Quantum Field Theory

- **从经典场拓展到量子场，在量子场论中**
  - 单个粒子的波函数被替换为满足特定场方程的场的激发，场方程可以通过拉氏量密度得到（后面简称为拉氏量）

- **自由实标量场** $\mathcal{L}_s = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0$$

  - 式中没有考虑相互作用项
  - 如果使用Euler–Lagrange方程，可以得到一个我们很熟悉的方程

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi, \qquad \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = \partial^0 \phi, \qquad \frac{\partial \mathcal{L}}{\partial(\partial_k \phi)} = -\partial^k \phi (k = 1,2,3)$$

  - 正是Klein–Gordon方程 $\partial_\mu \partial^\mu \phi + m^2 \phi = 0$

# Lagrangians in Quantum Field Theory

- **自由旋量场** $\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$

$$\boxed{\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\right) - \frac{\partial\mathcal{L}}{\partial\phi_i} = 0}$$

- 类似，对$\psi$和$\bar{\psi}$做偏导，得：

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})} = 0, \qquad \frac{\partial\mathcal{L}}{\partial\bar{\psi}} = i\gamma^\mu\partial_\mu\psi - m\psi$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} = 0, \qquad \frac{\partial\mathcal{L}}{\partial\psi} = i\bar{\psi}\gamma^\mu\partial_\mu - m\bar{\psi}$$

- 分别代入**Euler–Lagrange**方程得到两个方程

$$(i\gamma^\mu\partial_\mu - m)\psi = 0, \qquad i(\partial_\mu\bar{\psi})\gamma^\mu + m\bar{\psi} = 0$$

正是狄拉克方程及其伴随方程

# （回顾）Local Gauge Principle

- **SM中费米子和自旋1的玻色子的相互作用都可以用局域规范不变原理表征**
  - 自由旋量场的拉格朗日量 $\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$
  - **QED**要求在粒子波函数的局域相位变换下物理保持不变

$$\psi \rightarrow \psi' = \psi e^{iq\chi(x)}$$

注：相位依赖于时空坐标 $\chi(t,\vec{x})$

  - 狄拉克方程对应的变换为：

$$\boxed{i\gamma^{\mu}\partial_{\mu}\psi - m\psi = 0} \Rightarrow \boxed{i\gamma^{\mu}(\partial_{\mu} + iq\partial_{\mu}\chi)\psi - m\psi = 0}$$

  - 为使得狄拉克方程在局域相位变换下保持不变，引入无质量的规范玻色场$A_{\mu}$，且狄拉克方程修改为如下形式：

$$\boxed{i\gamma^{\mu}(\partial_{\mu} - qA_{\mu})\psi - m\psi = 0}$$

  - 如果规范场满足下属条件，则修改后的狄拉克方程在局域相位变换下不变：

$$\boxed{A_{\mu} \rightarrow A'_{\mu} = A_{\mu} - \partial_{\mu}\chi}$$

**规范不变性**

# （回顾）Local Gauge Principle

- 为保证物理不变，新场必须满足**规范不变**
  - 即 物理预言在如右变换保持不变 $A_\mu \to A'_\mu = A_\mu - \partial_\mu \chi$

- 因此局域相位不变性完全规定了费米子与规范场（如光子）间的相互作用：

$$i\gamma^\mu(\partial_\mu \psi - \boxed{qA_\mu})\psi - m\psi = 0$$

⇨ 相互作用顶角： $\boxed{i\gamma^\mu q A_\mu}$ ⇨ **QED !**

- QED局域相位变换是**U(1)**幺正变换

$$\psi \to \psi' = \hat{U}\psi \quad \text{i.e.} \quad \psi \to \psi' = \psi e^{iq\chi(x)} \quad \textbf{with} \quad U^\dagger U = 1$$

- 由此可以得到QED的完整拉氏量

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

$$\mathcal{L} = \underbrace{i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi}_{\mathcal{L}_{free}} \underbrace{- q\bar{\psi}\gamma^\mu\psi A_\mu}_{\mathcal{L}_{int}} \underbrace{- \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\mathcal{L}_A}$$

# SU(2) Yang-Mills Theory

- 将刚才的一个旋量场扩充到两个拥有**相同质量**的旋量场 $\psi_1, \psi_2$
  - 没有相互作用的情况下，其拉氏量的形式只是简单的两部分相加

$$\mathcal{L} = \left(i\bar{\psi}_1\gamma^\mu\partial_\mu\psi_1 - m\bar{\psi}_1\psi_1\right) + \left(i\bar{\psi}_2\gamma^\mu\partial_\mu\psi_2 - m\bar{\psi}_2\psi_2\right)$$

  - 定义一个二分量的列矢量 $\Psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}, \qquad \bar{\Psi} = \begin{pmatrix}\bar{\psi}_1 & \bar{\psi}_2\end{pmatrix}$

  - 将拉氏量改写为**紧凑形式** $\mathcal{L} = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi$

  - 和U(1)时一样，考虑SU(2)定域规范变换

$$\hat{U}(\boldsymbol{\alpha}) = e^{i\boldsymbol{\alpha}(x)\cdot\hat{\boldsymbol{T}}} \qquad \Psi(x) \to \hat{U}\Psi$$

  - 仍然，$\mathcal{L}$在这样的变换下是不守恒的，会出现一个额外的项

$$\partial_\mu\Psi \to \hat{U}\partial_\mu\Psi + \left(\partial_\mu\hat{U}\right)\Psi$$

# SU(2) Yang-Mills Theory

- 需要将偏微分$\partial_\mu$变为协变微分

$$\mathcal{D}_\mu \equiv \partial_\mu + ig\hat{\boldsymbol{T}} \cdot \boldsymbol{W}_\mu(x) = \partial_\mu + ig\hat{T}^a W_\mu^a(x)$$

  - 其中$g$是一个类似于电荷数的耦合常数
  - $\boldsymbol{W}_\mu(x) = \begin{pmatrix} W_\mu^1 & W_\mu^2 & W_\mu^3 \end{pmatrix}$是引入的三个矢量场——规范场

- 与$A_\mu$场一样，需要找到一个变换规则，使得$\boldsymbol{W}_\mu$场在规范变化下保持不变
  - 其结果为

$$\hat{\boldsymbol{T}} \cdot \boldsymbol{W}_\mu \to \hat{U}\hat{\boldsymbol{T}} \cdot \boldsymbol{W}_\mu \hat{U}^{-1} + \frac{i}{g}\left(\partial_\mu \hat{U}\right)\hat{U}^{-1}$$

  - 最后一样写出Yang-Mills场的完整拉氏量

$$\mathcal{L}_{Y.M.} = \underbrace{i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi}_{\mathcal{L}_{free}} \underbrace{-\left(g\bar{\Psi}\gamma^\mu\hat{\boldsymbol{T}}\Psi\right) \cdot \boldsymbol{W}_\mu}_{\mathcal{L}_{int}} \underbrace{-\frac{1}{4}\boldsymbol{G}_{\mu\nu} \cdot \boldsymbol{G}^{\mu\nu}}_{\mathcal{L}_W}$$

# Issues on particle mass

- **无论是U(1)还是SU(2)定域规范变换，**
  - 规范玻色子场都必须是无质量的，有质量的规范玻色子会破坏对称性

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu \chi$$

$$\frac{1}{2}m_\gamma^2 A_\mu A^\mu \to \frac{1}{2}m_\gamma^2 \left(A_\mu - \partial_\mu \chi\right)(A^\mu - \partial^\mu \chi) \neq \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

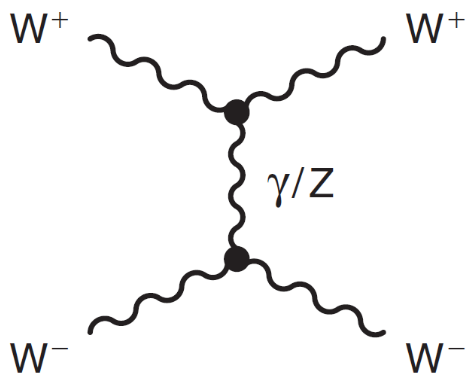
  - 对于任意费米子场$\psi$，质量项可以写为

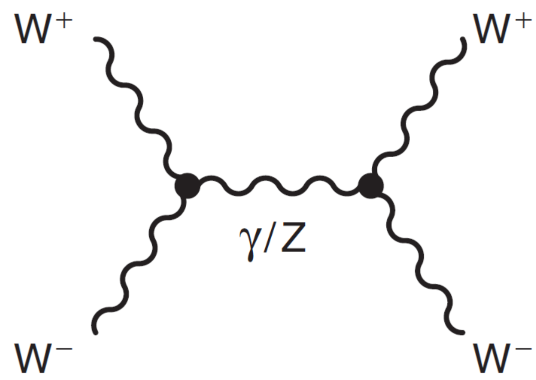
$$-m\bar{\psi}\psi = -m(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R) = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

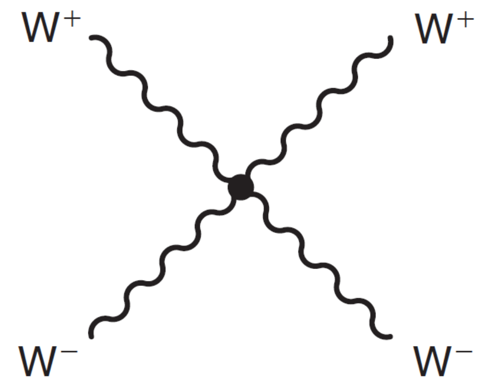
  - 但是在SU(2)定域规范变换下，左手场表现为二重态，右手场表现为单态，两者的组合破坏了规范不变性，因此电弱统一理论中不能如上式一般写出费米子质量项

# Unitarity violation in $W^+_L W^-_L$ scattering

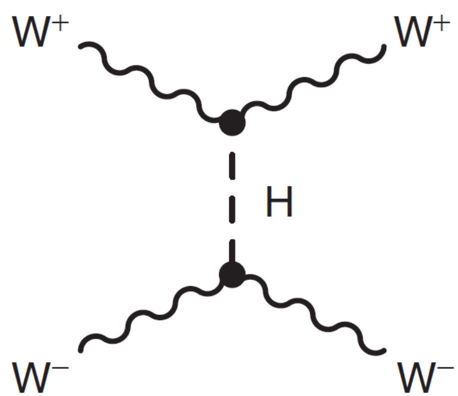
- Unitarity violation of $e^+e^-\to W^+W^-$ resolved by Z boson
  - Similarly, $W^+W^- \to W^+W^-$ violates unitarity at about 1 TeV
  - Originated from $W_LW_L\to W_LW_L$, longitudinal not existed for massless particles

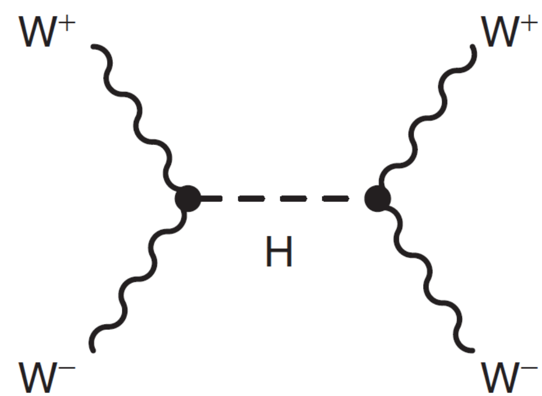



- Unitarity violation can be cancelled by exchanging of a scalar particle
  - if the scalar couplings are related to the electroweak couplings



# Spontaneous symmetry breaking

- 考虑一个有自相互作用的经典实标量场$\phi$，给出系统的拉氏量

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

  - $\mathrm{T} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)$是动能项；$\mathrm{V}(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$是势能项，要求$\lambda > 0$保证势能有下限；表达式中没有$\phi^3$项，这使得整个系统的拉氏量在如下变换下保持不变

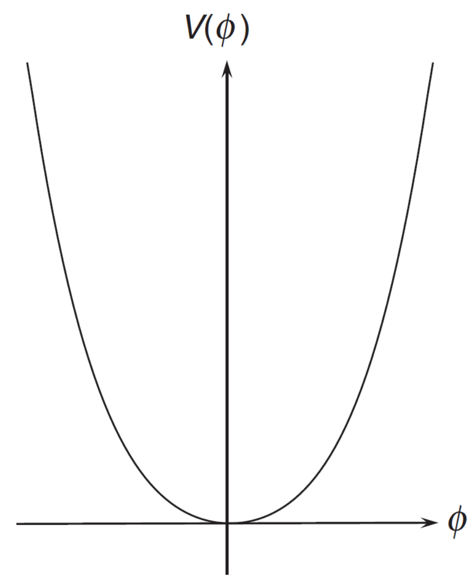
$$\phi(x) \rightarrow \phi'(x) = -\phi(x)$$

- $\phi$取何值时系统的能量最低：$\dfrac{\partial V(\phi)}{\partial\phi} = 0$

**$\mu^2 > 0$**

- 此时极小值出现在原点处，真空态只有一个，不会发生自发对称破缺

$$\phi_{min} = 0$$



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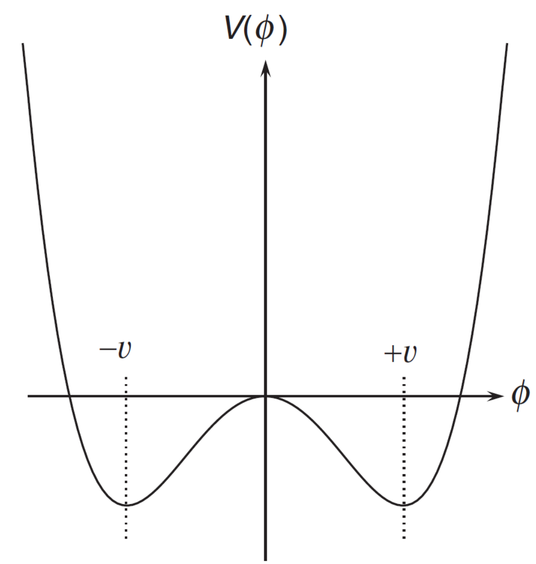


$\mu^2 < 0$

- 此时$V(\phi)$有两个极小值 $\phi_{min} = \pm\sqrt{\dfrac{-\mu^2}{\lambda}} \equiv \nu$
- 对应两个简并的真空态$\nu$或者$-\nu$
- 由于$\frac{1}{2}\mu^2\phi^2 < 0$，该项不能被解释为质量项

# SSM in Quantum Field

- **将经典场推广到量子场**
  - 仍然记为$\phi$，量子场的真空期望值（VEV）是经典场
  - 于是当$\mu^2 < 0$时，量子场$\phi$的VEV就是上式中的任意一个
  - 这里取$\phi_{min} = +v$为例：$\langle 0|\phi|0\rangle = +v$

- **量子场算符作用在真空态上必须为0**

$\langle 0|\eta|0\rangle = 0$

  - 需要对$\phi$场作平移：$\phi(x) = \eta(x) + v$
  - 新的场$\eta$是量子场，用它表示之前的拉氏量

$$\mathcal{L}_\eta = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + C$$

  - 很明显，新的场存在质量项：$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$
  - 并且出现了$\eta^3$项，失去了原有的对称性，这被称为自发的对称性破缺。

# Goldstone Boson

## ➢ 推广到连续变换

- 之前讨论的是$\phi \to -\phi$分立变换，现考虑U(1)整体变换下的对称性自发破缺
- 写出一个复标量场的拉氏量：$\mathcal{L} = \left(\partial_\mu \phi\right)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2$
- 其满足U(1)整体变换：$\phi(x) \to \phi'(x) = e^{i\alpha}\phi(x)$
- 使用相似的方法寻找系统极小值时的$\phi$，$\mu^2 > 0$不会发生自发破缺

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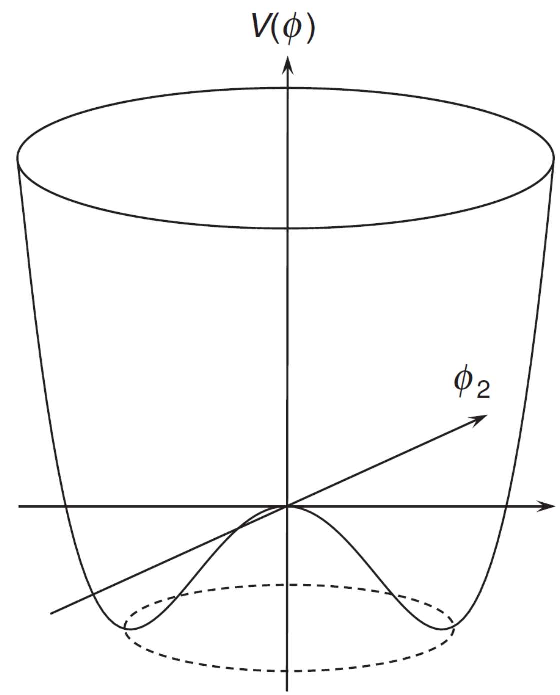
- 使用相似的方法寻找系统极小值时的$\phi$，$\mu^2 > 0$不会发生自发破缺

## ➢ 关注$\mu^2 < 0$，当系统能量极小时，$\phi$可以写成

$$|\phi|_{min} = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{\nu}{\sqrt{2}} \qquad \phi_{min} = |\phi|_{min} e^{i\gamma}$$




- 能量极小值出现在$\phi$复平面的一个半径为$\frac{\nu}{\sqrt{2}}$的圆周上
- 系统有无穷多个简并的真空态。

# Goldstone Boson

- 将经典场视为量子场的VEV，不失一般性地选取$\gamma = 0$的态 $\langle 0|\phi|0\rangle = \dfrac{\nu}{\sqrt{2}}$
  - 仍然需要对$\phi$场作平移使其可以被解释为量子场

$$\phi(x) = \frac{1}{\sqrt{2}}[\eta(x) + \nu + i\xi(x)] \qquad \langle 0|\eta|0\rangle = 0, \qquad \langle 0|\xi|0\rangle = 0$$

- 代入系统拉氏量，得

$$\mathcal{L}_{\eta,\xi} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial_\mu\xi\partial^\mu\xi - \lambda\nu^2\eta^2 - \lambda\nu(\eta^2 + \xi^2)\eta - \frac{\lambda}{4}(\eta^2 + \xi^2)^2 + C$$

# Goldstone Boson

- **将经典场视为量子场的VEV，不失一般性地选取$\gamma = 0$的态** $\langle 0|\phi|0\rangle = \dfrac{v}{\sqrt{2}}$
  - 仍然需要对$\phi$场作平移使其可以被解释为量子场

$$\phi(x) = \frac{1}{\sqrt{2}}[\eta(x) + v + i\xi(x)] \qquad \langle 0|\eta|0\rangle = 0, \qquad \langle 0|\xi|0\rangle = 0$$

- **代入系统拉氏量，得**

$$\mathcal{L}_{\eta,\xi} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial_\mu\xi\partial^\mu\xi - \lambda v^2\eta^2 - \lambda v(\eta^2 + \xi^2)\eta - \frac{\lambda}{4}(\eta^2 + \xi^2)^2 + C$$

  - 上式失去了U(1)的整体变换下的不变性（实际上对称性仍然存在，只是隐藏了）
  - 式中包含了$\eta$场的质量项：$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$

- **$\xi$场仍然无质量，称为Goldstone玻色子场**：U(1)整体对称性自发破缺的必然结果

- **扩展到$N = n^2 - 1$个生成元的SU(n)群：**
  - 当SU(n)的整体对称发生自发破缺时，会出现$n^2 - 1$个无质量的Goldstone玻色子

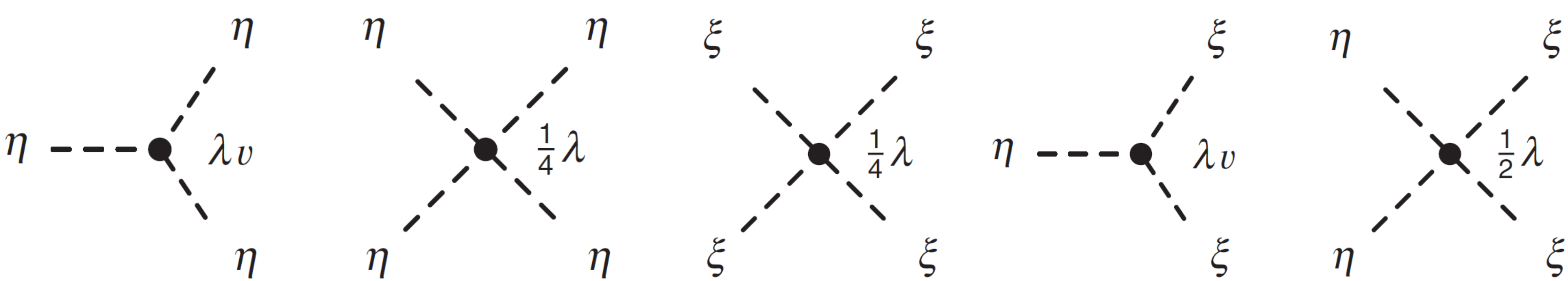
# Goldstone Boson

- **将经典场视为量子场的VEV，不失一般性地选取$\gamma = 0$的态** $\langle 0|\phi|0\rangle = \dfrac{\nu}{\sqrt{2}}$
  - 仍然需要对$\phi$场作平移使其可以被解释为量子场

$$\phi(x) = \frac{1}{\sqrt{2}}[\eta(x) + \nu + i\xi(x)] \qquad \langle 0|\eta|0\rangle = 0, \qquad \langle 0|\xi|0\rangle = 0$$

- **代入系统拉氏量，得**

$$\mathcal{L}_{\eta,\xi} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial_\mu\xi\partial^\mu\xi - \lambda\nu^2\eta^2 - \lambda\nu(\eta^2+\xi^2)\eta - \frac{\lambda}{4}(\eta^2+\xi^2)^2 + C$$



# Higgs Mechanism

- 定域规范不变性导致无质量规范玻色子，自发对称性破缺导致无质量Goldstone玻色子
- 如果将这两者结合起来？
  - 使得：（未观测到的）无质量的Goldstone玻色子消失，无质量规范玻色子获得质量
- 仍然考虑自由复标量场，但将U(1)整体变换推广到定域规范变换

$$\mathcal{L}_0 = \left(\partial_\mu \phi\right)^* (\partial^\mu \phi) + \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \qquad \phi(x) \to \phi'(x) = e^{i\alpha(x)} \phi(x)$$

  - 为了保证$\mathcal{L}_0$在变换下保持不变，需要作替换

$$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + iqA_\mu(x) \qquad A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x)$$

  - 写出变换后的拉氏量

$$\mathcal{L} = \left(\partial_\mu \phi\right)^* (\partial^\mu \phi) + \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ -qi \underbrace{(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)}_{j^\mu} A_\mu + q^2 A_\mu A^\mu \phi^* \phi$$

# Higgs Mechanism

$\phi(x) = \frac{1}{\sqrt{2}}[\eta(x) + \nu + i\xi(x)]$

- 设$\mu^2 < 0$，发生自发对称性破缺，拉氏量进一步改写为
  - 将之前定义过的$\eta$场和$\xi$场代入原拉氏量，得到

$$\mathcal{L}_{\eta,\xi} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial_\mu\xi\partial^\mu\xi - \lambda\nu^2\eta^2 - \lambda\nu(\eta^2 + \xi^2)\eta - \frac{\lambda}{4}(\eta^2 + \xi^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+q(\eta\partial^\mu\xi - \xi\partial^\mu\eta)A_\mu + q\nu A_\mu\partial^\mu\xi$$
$$+\frac{1}{2}q^2A_\mu A^\mu(\eta^2 + \xi^2 + 2\nu\eta) + \frac{1}{2}q^2\nu^2A_\mu A^\mu$$

# Higgs Mechanism

$\phi(x) = \frac{1}{\sqrt{2}}[\eta(x) + v + i\xi(x)]$

- 设$\mu^2 < 0$，发生自发对称性破缺，拉氏量进一步改写为
  - 将之前定义过的$\eta$场和$\xi$场代入原拉氏量，得到

$$\mathcal{L}_{\eta,\xi} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial_\mu\xi\partial^\mu\xi - \lambda v^2\eta^2 - \lambda v(\eta^2 + \xi^2)\eta - \frac{\lambda}{4}(\eta^2 + \xi^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+q(\eta\partial^\mu\xi - \xi\partial^\mu\eta)A_\mu + qvA_\mu\partial^\mu\xi$$
$$+\frac{1}{2}q^2A_\mu A^\mu(\eta^2 + \xi^2 + 2v\eta) + \frac{1}{2}q^2v^2A_\mu A^\mu$$

1. 标量场$\eta(x)$的质量$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$，标量场$\xi(x)$没有质量
2. 矢量规范玻色子场$A_\mu$获得质量$m_A = qv$
3. 出现了非物理项$qvA_\mu\partial^\mu\xi$，表示没有外场情况下$A_\mu \leftrightarrow \partial^\mu\xi$的直接转换
   - 这在理论上是难以理解的，需要消去。

# Higgs Mechanism

$$\phi(x)=\frac{1}{\sqrt{2}}[\eta(x)+v+i\xi(x)]$$

- 设$\mu^2<0$，发生自发对称性破缺，拉氏量进一步改写为
  - 将之前定义过的$\eta$场和$\xi$场代入原拉氏量，得到

$$\mathcal{L}_{\eta,\xi}=\frac{1}{2}\partial_\mu\eta\partial^\mu\eta+\frac{1}{2}\partial_\mu\xi\partial^\mu\xi-\lambda v^2\eta^2-\lambda v(\eta^2+\xi^2)\eta-\frac{\lambda}{4}(\eta^2+\xi^2)^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+q(\eta\partial^\mu\xi-\xi\partial^\mu\eta)A_\mu+qvA_\mu\partial^\mu\xi$$
$$+\frac{1}{2}q^2A_\mu A^\mu(\eta^2+\xi^2+2v\eta)+\frac{1}{2}q^2v^2A_\mu A^\mu$$

1. 标量场$\eta(x)$的质量$m_\eta=\sqrt{2\lambda v^2}=\sqrt{-2\mu^2}$，标量场$\xi(x)$没有质量
2. 矢量规范玻色子场$A_\mu$获得质量$m_A=qv$
3. 出现了非物理项$qvA_\mu\partial^\mu\xi$，表示没有外场情况下$A_\mu\leftrightarrow\partial^\mu\xi$的直接转换
   - 这在理论上是难以理解的，需要消去。

- 检查一下系统的自由度

破缺前

| | |
|---|---|
| $2$ | 无质量的矢量规范玻色子场 |
| $+\ 2$ | 复标量场$\phi(x)$ |
| $4$ | |

破缺后

| | |
|---|---|
| $3$ | 有质量的矢量规范玻色子场 |
| $+\ 1$ | 实标量场$\eta(x)$ |
| $+\ 1$ | 实标量场$\xi(x)$ |
| $5$ | |

实验上没有观测到
Goldstone玻色子，
需要想方法让$\xi$场消失

# Higgs Mechanism

$$\phi(x) = \frac{1}{\sqrt{2}}[\eta(x) + v + i\xi(x)]$$

- **改变$\phi(x)$的参数形式** $\phi(x) = \frac{1}{\sqrt{2}} e^{i\frac{\xi(x)}{v}}[\eta(x) + v]$
  - 注： $e^{i\frac{\xi(x)}{v}}$ 作线性展开，取前两项即可回到原参数形式
  - U(1)的定域规范写为 $\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}} e^{i\alpha(x)} e^{i\frac{\xi(x)}{v}}[\eta(x) + v]$

    $$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x)$$

  - 进一步选择 $\alpha(x) = -\frac{\xi(x)}{v}$，将变换改写为

    $$\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}}[\eta(x) + v] \qquad A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{v}\partial_\mu \xi(x)$$

# Higgs Mechanism

$$\phi(x) = \frac{1}{\sqrt{2}}[\eta(x) + v + i\xi(x)]$$

- **改变$\phi(x)$的参数形式**　$\phi(x) = \frac{1}{\sqrt{2}} e^{i\frac{\xi(x)}{v}} [\eta(x) + v]$

- 注：$e^{i\frac{\xi(x)}{v}}$ 作线性展开，取前两项即可回到原参数形式

- U(1)的定域规范写为 $\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}} e^{i\alpha(x)} e^{i\frac{\xi(x)}{v}} [\eta(x) + v]$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x)$$

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) + \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$-qi(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) A_\mu + q^2 A_\mu A^\mu \phi^* \phi$$

- 进一步选择 $\alpha(x) = -\frac{\xi(x)}{v}$，将变

$$\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}}[\eta(x) + v] \qquad A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{v}\partial_\mu \xi(x)$$

**练习：推导新的拉矢量**

$$\mathcal{L}_{\eta,\xi} = \frac{1}{2}\partial_\mu \eta \partial^\mu \eta + \frac{1}{2}\partial_\mu \xi \partial^\mu \xi - \lambda v^2 \eta^2 - \lambda v(\eta^2 + \xi^2)\eta - \frac{\lambda}{4}(\eta^2 + \xi^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+q(\eta \partial^\mu \xi - \xi \partial^\mu \eta) A_\mu + q v A_\mu \partial^\mu \xi$$
$$+\frac{1}{2} q^2 A_\mu A^\mu (\eta^2 + \xi^2 + 2v\eta) + \frac{1}{2} q^2 v^2 A_\mu A^\mu$$

# Higgs Mechanism

$$\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}}[\eta(x) + v]$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{v}\partial_\mu \xi(x)$$

- 写出现在规范变换下的拉氏量

$$\mathcal{L}_\eta = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}q^2A_\mu A^\mu\eta^2 + q^2 vA_\mu A^\mu\eta^2 + \frac{1}{2}q^2v^2A_\mu A^\mu$$

- 原先无质量的矢量规范玻色子场$A_\mu(x)$获得了质量$m_A = qv$
- 规范场吃掉Goldstone玻色子而产生质量，变为有质量的矢量玻色子场的纵向极化态
- 这就是所谓的Higgs机制

- 此时$\eta(x)$场成为Higgs场，记为$H$，质量为$m_H = m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$

# Higgs Mechanism in EW unification

- **电弱统一使用了$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$规范群，有四个规范场**
  - 如果要用Higgs机制给它们质量，相应也需要四个实标量场
- **考虑到四个规范场两个带电两个电中性**
  - 自然地引入一个复数形式的SU(2)弱同位旋二重态 $\Phi(\mathrm{x}) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$

| 粒子（场） | $I_\mathrm{W}$ | $I_\mathrm{W}^3$ | $Q$ | $Y$ |
|---|---|---|---|---|
| $L_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}: \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $0$<br>$-1$ | $-1$ |
| 没有右手中微子<br>$\ell_{iR}: e_R, \mu_R, \tau_R$ | –<br>$0$ | –<br>$0$ | –<br>$-1$ | –<br>$-2$ |
| $Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}: \begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $+\frac{2}{3}$<br>$-\frac{1}{3}$ | $\frac{1}{3}$ |
| $u_{iR}: u_R, c_R, t_R$<br>$d_{iR}: d_R, s_R, b_R$ | $0$<br>$0$ | $0$<br>$0$ | $+\frac{2}{3}$<br>$-\frac{1}{3}$ | $\frac{4}{3}$<br>$-\frac{2}{3}$ |
| $\Phi(\mathrm{x}) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $1$<br>$0$ | $1$ |

# Higgs Mechanism in EW unification

- 在这里展示一下电弱理论完整的拉氏量

$$\mathcal{L}_{EW} = i\sum_{i=e,\mu,\tau}\bar{L}'_{iL}\gamma^{\mu}\mathcal{D}_{\mu}L'_{iL} + i\sum_{i=1,2,3}\bar{Q}'_{iL}\gamma^{\mu}\mathcal{D}_{\mu}Q'_{iL}$$

$$+i\sum_{i=e,\mu,\tau}\bar{\ell}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}\ell'_{iR} + i\sum_{i=d,s,b}\bar{d}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}d'_{iR} + i\sum_{i=u,c,t}\bar{u}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}u'_{iR}$$

费米子动能项及费米子场与规范场的相互作用$\mathcal{L}_{F,F-G}$

- 其中协变微分的定义为$\mathcal{D}_{\mu} \equiv \partial_{\mu} + ig\frac{\sigma^a}{2}W_{\mu}^a + ig'\frac{Y}{2}B_{\mu}$
- 撇号意味着没有确定的质量

# Higgs Mechanism in EW unification

- 在这里展示一下电弱理论完整的拉氏量

$$\mathcal{L}_{EW} = i\sum_{i=e,\mu,\tau} \bar{L}'_{iL}\gamma^{\mu}\mathcal{D}_{\mu}L'_{iL} + i\sum_{i=1,2,3} \bar{Q}'_{iL}\gamma^{\mu}\mathcal{D}_{\mu}Q'_{iL}$$

$$+i\sum_{i=e,\mu,\tau} \bar{\ell}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}\ell'_{iR} + i\sum_{i=d,s,b} \bar{d}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}d'_{iR} + i\sum_{i=u,c,t} \bar{u}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}u'_{iR}$$

费米子动能项及费米子场与规范场的相互作用$\mathcal{L}_{F,F-G}$

$$-\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

规范场自耦合$\mathcal{L}_G$

- 其中协变微分的定义为$\mathcal{D}_{\mu} \equiv \partial_{\mu} + ig\frac{\sigma^a}{2}W_{\mu}^{a} + ig'\frac{Y}{2}B_{\mu}$
- 撇号意味着没有确定的质量

# Higgs Mechanism in EW unification

➢ 在这里展示一下电弱理论完整的拉氏量

$$
\begin{aligned}
\mathcal{L}_{EW} = & \; i\sum_{i=e,\mu,\tau} \bar{L}'_{iL}\gamma^{\mu}\mathcal{D}_{\mu}L'_{iL} + i\sum_{i=1,2,3} \bar{Q}'_{iL}\gamma^{\mu}\mathcal{D}_{\mu}Q'_{iL} \\
& + i\sum_{i=e,\mu,\tau} \bar{\ell}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}\ell'_{iR} + i\sum_{i=d,s,b} \bar{d}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}d'_{iR} + i\sum_{i=u,c,t} \bar{u}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}u'_{iR} \\
& - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& + (\mathcal{D}_{\mu}\Phi)^{\dagger}(\mathcal{D}_{\mu}\Phi) - \mu^{2}\Phi^{\dagger}\Phi - \lambda(\Phi^{\dagger}\Phi)^{2}
\end{aligned}
$$

费米子动能项及费米子场与规范场的相互作用$\mathcal{L}_{F,F-G}$

规范场自耦合$\mathcal{L}_{G}$

标量场贡献$\mathcal{L}_{S}$

- 其中协变微分的定义为$\mathcal{D}_{\mu} \equiv \partial_{\mu} + ig\frac{\sigma^{a}}{2}W_{\mu}^{a} + ig'\frac{Y}{2}B_{\mu}$
- 撇号意味着没有确定的质量

# Higgs Mechanism in EW unification

➢ 在这里展示一下电弱理论完整的拉氏量

$$\mathcal{L}_{EW} = i\sum_{i=e,\mu,\tau}\bar{L}'_{iL}\gamma^\mu\mathcal{D}_\mu L'_{iL} + i\sum_{i=1,2,3}\bar{Q}'_{iL}\gamma^\mu\mathcal{D}_\mu Q'_{iL}$$

$$+i\sum_{i=e,\mu,\tau}\bar{\ell}'_{iR}\gamma^\mu\mathcal{D}_\mu \ell'_{iR} + i\sum_{i=d,s,b}\bar{d}'_{iR}\gamma^\mu\mathcal{D}_\mu d'_{iR} + i\sum_{i=u,c,t}\bar{u}'_{iR}\gamma^\mu\mathcal{D}_\mu u'_{iR}$$

费米子动能项及费米子场与规范场的相互作用$\mathcal{L}_{F,F-G}$

$$-\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

规范场自耦合$\mathcal{L}_G$

$$+(\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}_\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

标量场贡献$\mathcal{L}_S$

$$-\sum_{i,j=e,\mu,\tau}\left(Y'^{\ell}_{ij}\bar{L}^i_{iL}\Phi\ell'_{jR} + Y'^{\ell *}_{ij}\bar{\ell}'_{jR}\Phi^\dagger L'_{iL}\right)$$

$$-\sum_{i=1,2,3}\sum_{j=d,s,b}\left(Y'^{d}_{ij}\bar{Q}^i_{iL}\Phi d'_{jR} + Y'^{d *}_{ij}\bar{d}'_{jR}\Phi^\dagger Q'_{iL}\right)$$

$$-\sum_{i=1,2,3}\sum_{j=u,c,t}\left(Y'^{u}_{ij}\bar{Q}^i_{iL}\tilde{\Phi} u'_{jR} + Y'^{u *}_{ij}\bar{u}'_{jR}\tilde{\Phi}^\dagger Q'_{iL}\right)$$

费米子场与标量场的Yukawa耦合$\mathcal{L}_{F-S}$

- 其中协变微分的定义为$\mathcal{D}_\mu \equiv \partial_\mu + ig\frac{\sigma^a}{2}W^a_\mu + ig'\frac{Y}{2}B_\mu$
- 撇号意味着没有确定的质量

# Mass of the Gauge Field

- **通过$\mathcal{L}_S$来研究规范玻色子场的质量** $\mathcal{L}_S = \left(\mathcal{D}_\mu\Phi\right)^\dagger\left(\mathcal{D}_\mu\Phi\right) - \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$
  - 系统势能的极小值发生在$\left(\Phi^\dagger\Phi\right)_{\min} = \frac{-\mu^2}{2\lambda} \equiv \frac{\nu^2}{2}$，有无穷多个真空态
  - 考虑到真空是电中性的，复标量场$\Phi$中带电分量$\phi^+$的最小值必须为零
  - 中性分量$\phi^0$的最小值可以不为零，选择真空态为

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu\end{pmatrix}$$

  - 同样的，引入实标量场作为为量子场

$$\Phi(\mathrm{x}) = \frac{1}{\sqrt{2}}e^{i\frac{\sigma}{2}\cdot\frac{\xi(x)}{\nu}}\begin{pmatrix}0\\ \nu + H(x)\end{pmatrix}$$

  - 仍然选择幺正规范消除非物理项 $\alpha(x) = -\frac{\xi(x)}{\nu}$
    - 使$\Phi$的定域规范变换为

$$\Phi(\mathrm{x}) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu + H(x)\end{pmatrix}$$

# Mass of the Gauge Field

- 接下来将变换后的Φ场代入$\mathcal{L}_S$，注意到Φ的弱超荷$Y = +1$

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

- 先计算 $\mathcal{D}_\mu(x)\Phi(x) = \left(\partial_\mu + ig\frac{\sigma^a}{2}W_\mu^a + ig'\frac{1}{2}B_\mu\right)\Phi(x)$

$$= \frac{1}{2\sqrt{2}}\begin{pmatrix} 2\partial_\mu + ig_W W_\mu^{(3)} + ig' B_\mu & ig_W[W_\mu^{(1)} - iW_\mu^{(2)}] \\ ig_W[W_\mu^{(1)} + iW_\mu^{(2)}] & 2\partial_\mu - ig_W W_\mu^{(3)} + ig' B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \quad = \frac{1}{\sqrt{2}}\begin{pmatrix} i\frac{g}{\sqrt{2}}W_\mu(x)[v + H(x)] \\ \partial_\mu H(x) - i\frac{g}{2\cos\theta_W}Z_\mu(x)[v + H(x)] \end{pmatrix}$$

**练习：自己尝试化简**

$$\mathcal{L}_S = \left(\mathcal{D}_\mu\Phi\right)^\dagger\left(\mathcal{D}_\mu\Phi\right) + \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$$

# Mass of the Gauge Field

$$\mathcal{L}_S = (\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}_\mu\Phi) + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

$$\Phi(\mathrm{x}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu + H(x) \end{pmatrix}$$

- 接下来将变换后的Φ场代入$\mathcal{L}_S$，注意到Φ的弱超荷Y = +1
- 先计算 $\mathcal{D}_\mu(x)\Phi(x) = \left(\partial_\mu + ig\frac{\sigma^a}{2}W_\mu^a + ig'\frac{1}{2}B_\mu\right)\Phi(x)$

$$= \frac{1}{2\sqrt{2}}\begin{pmatrix} 2\partial_\mu + ig_{\mathrm{W}}W_\mu^{(3)} + ig'B_\mu & ig_{\mathrm{W}}[W_\mu^{(1)} - iW_\mu^{(2)}] \\ ig_{\mathrm{W}}[W_\mu^{(1)} + iW_\mu^{(2)}] & 2\partial_\mu - ig_{\mathrm{W}}W_\mu^{(3)} + ig'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \quad = \frac{1}{\sqrt{2}}\begin{pmatrix} i\frac{g}{\sqrt{2}}W_\mu(x)[\nu + H(x)] \\ \partial_\mu H(x) - i\frac{g}{2\cos\theta_W}Z_\mu(x)[\nu + H(x)] \end{pmatrix}$$

  - 现在可以写出幺正规范下的复标量场

练习：自己尝试化简

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - \lambda\nu^2H^2 - \lambda\nu H^3 - \frac{1}{4}\lambda H^4$$
$$+\frac{\nu^2g^2}{4}W_\mu W^{\dagger\mu} + \frac{\nu^2g^2}{8\cos^2\theta_W}Z_\mu Z^\mu$$
$$+\frac{\nu g^2}{2}W_\mu W^{\dagger\mu}H + \frac{\nu g^2}{4\cos^2\theta_W}Z_\mu Z^\mu H + \frac{g^2}{4}W_\mu W^{\dagger\mu}H^2 + \frac{g^2}{8\cos^2\theta_W}Z_\mu Z^\mu H^2$$

  - 所有的Goldstone实标量场都消失了，而$H(x)$就是Higgs场

➢ **式中没有电磁场$A_\mu$相关项**

  - 说明在电弱统一理论中电磁场仍然没有质量，且不与Higgs场直接耦合

# Mass of the Gauge Field

➢ **式中第一行：**

- 第一项为Higgs场动能项
- 第二项给出了Higgs场的质量

$$m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

- 第三、四项分别是三个Higgs场和四个Higgs场的作用顶点

$$HHH, \qquad HHHH$$

$$\begin{aligned}\mathcal{L}_S = &\frac{1}{2}\left(\partial_\mu H\right)(\partial^\mu H) - \lambda v^2 H^2 - \lambda v H^3 - \frac{1}{4}\lambda H^4 \\ &+ \frac{v^2 g^2}{4} W_\mu W^{\dagger\mu} + \frac{v^2 g^2}{8\cos^2\theta_W} Z_\mu Z^\mu \\ &+ \frac{v g^2}{2} W_\mu W^{\dagger\mu} H + \frac{v g^2}{4\cos^2\theta_W} Z_\mu Z^\mu H + \frac{g^2}{4} W_\mu W^{\dagger\mu} H^2 + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu H^2\end{aligned}$$

# Mass of the Gauge Field

- **式中第二行：**
  - 第一项给出了带电规范场$W$的质量 $m_W = \frac{vg}{2}$
  - 第二项给出了中性规范场$Z$的质量 $m_Z = \frac{vg}{2\cos\theta_W}$
  - 将两式相除，可以发现$m_W < m_Z$

$$
\begin{aligned}
\mathcal{L}_S = {} & \frac{1}{2}\left(\partial_\mu H\right)(\partial^\mu H) - \lambda v^2 H^2 - \lambda v H^3 - \frac{1}{4}\lambda H^4 \\
& + \frac{v^2 g^2}{4} W_\mu W^{\dagger\mu} + \frac{v^2 g^2}{8\cos^2\theta_W} Z_\mu Z^\mu \\
& + \frac{v g^2}{2} W_\mu W^{\dagger\mu} H + \frac{v g^2}{4\cos^2\theta_W} Z_\mu Z^\mu H + \frac{g^2}{4} W_\mu W^{\dagger\mu} H^2 + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu H^2
\end{aligned}
$$

# Mass of the Gauge Field

- **式中第三行：**
  - 包含了$(\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}_\mu\Phi)$的相互作用，即包含Higgs玻色子场与$W_\mu$，$Z_\mu$之间的作用顶点

$$WWH, \quad ZZH, \quad WWHH, \quad ZZHH$$

$$\begin{aligned}\mathcal{L}_S = &\frac{1}{2}(\partial_\mu H)(\partial^\mu H) - \lambda v^2 H^2 - \lambda v H^3 - \frac{1}{4}\lambda H^4 \\ &+ \frac{v^2 g^2}{4} W_\mu W^{\dagger\mu} + \frac{v^2 g^2}{8\cos^2\theta_W} Z_\mu Z^\mu \\ &+ \frac{v g^2}{2} W_\mu W^{\dagger\mu} H + \frac{v g^2}{4\cos^2\theta_W} Z_\mu Z^\mu H + \frac{g^2}{4} W_\mu W^{\dagger\mu} H^2 + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu H^2\end{aligned}$$

# Mass of the Fermions

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu + H(x) \end{pmatrix}$$

- **标准模型中费米子的质量被称为汤川耦合（Yukawa coupling）**
  - 标准模型中不存在右手中微子，根据$\mathcal{L}_{F-S}$，中微子没有质量，只需要考虑带电轻子
  - 写出轻子场和复标量场的Yukawa拉氏量，$\overline{L}^i_{iL}\ell'_{jR}$的弱超荷乘积为−1，Φ的弱超荷为1

$$\mathcal{L}_{F-S,L} = -\sum_{i,j=e,\mu,\tau} Y'^{\ell}_{ij}\overline{L}^i_{iL}\Phi\ell'_{jR} + \mathrm{h.c.}$$

  - 仍然使用幺正规范下的Φ二重态

$$\mathcal{L}_{F-S,L} = -\frac{\nu + H}{\sqrt{2}}\sum_{i,j=e,\mu,\tau} Y'^{\ell}_{ij}\overline{\ell}^i_{iL}\ell'_{jR} + \mathrm{h.c.}$$

  - 正比$\nu$的项给出带电轻子的质量
  - 正比$H$的项给出带电轻子和Higgs场的三线耦合

- **一般情况下矩阵$Y'^{\ell}$是非对角的，导致$e', \mu', \tau'$场没有确定的质量**
  - 寻找适当的3 × 3幺正矩阵$V^{\ell}_L, V^{\ell}_R$使得矩阵$Y'^{\ell}$对角化，得到

$$\mathcal{L}_{F-S,L} = -\frac{\nu + H}{\sqrt{2}}\overline{\ell}_L Y^{\ell}\ell_R + \mathrm{h.c.}$$

$$\ell_L = V^{\ell\dagger}_L\ell'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}, \ell_R = V^{\ell\dagger}_R\ell'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

# 练习：推导带电轻子的质量项

$$\mathcal{L}_{F-S,L} = -\sum_{i,j=e,\mu,\tau} Y_{ij}^{\prime\ell} \overline{L}_{iL}^{i} \Phi \ell_{jR}^{\prime} + \text{h.c.}$$

$$\Phi(\mathrm{x}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu + H(x) \end{pmatrix}$$

| 粒子（场） | $I_{\mathrm{w}}$ | $I_{\mathrm{w}}^{3}$ | $Q$ | $Y$ |
|---|---|---|---|---|
| $L_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}: \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $0$<br>$-1$ | $-1$ |
| 没有右手中微子<br>$\ell_{iR}: e_R, \mu_R, \tau_R$ | $-$<br>$0$ | $-$<br>$0$ | $-$<br>$-1$ | $-$<br>$-2$ |
| $Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}: \begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $+\frac{2}{3}$<br>$-\frac{1}{3}$ | $\frac{1}{3}$ |
| $u_{iR}: u_R, c_R, t_R$<br>$d_{iR}: d_R, s_R, b_R$ | $0$<br>$0$ | $0$<br>$0$ | $+\frac{2}{3}$<br>$-\frac{1}{3}$ | $\frac{4}{3}$<br>$-\frac{2}{3}$ |
| $\Phi(\mathrm{x}) = \begin{pmatrix} \phi^{+}(x) \\ \phi^{0}(x) \end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $1$<br>$0$ | $1$ |

# Mass of the Fermions

- **写出最终的Yukawa拉氏量** $\mathcal{L}_{F-S,L} = -\sum_{i=e,\mu,\tau} \frac{y_i^{\ell} v}{\sqrt{2}} \bar{\ell}_i \ell_i - \sum_{i=e,\mu,\tau} \frac{y_i^{\ell}}{\sqrt{2}} \bar{\ell}_i \ell_i H$

  - 其中$\ell_i = \ell_{iL} + \ell_{iR}$
  - 式中第一项给出带电轻子的质量 $m_i = \frac{y_i^{\ell} v}{\sqrt{2}}, i = e, \mu, \tau$
  - 式中第二项给出带电轻子和Higgs场的耦合，利用质量项可改写为

$$-\sum_{i=e,\mu,\tau} \frac{m_i}{v} \bar{\ell}_i \ell_i H$$

表明该耦合和带电轻子的质量正比

# Higgs Mechanism in EW unification

- 在这里展示一下电弱理论完整的拉氏量

$$
\begin{aligned}
\mathcal{L}_{EW} = {} & i\sum_{i=e,\mu,\tau}\bar{L}'_{iL}\gamma^{\mu}\mathcal{D}_{\mu}L'_{iL} + i\sum_{i=1,2,3}\bar{Q}'_{iL}\gamma^{\mu}\mathcal{D}_{\mu}Q'_{iL} \\
& + i\sum_{i=e,\mu,\tau}\bar{\ell}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}\ell'_{iR} + i\sum_{i=d,s,b}\bar{d}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}d'_{iR} + i\sum_{i=u,c,t}\bar{u}'_{iR}\gamma^{\mu}\mathcal{D}_{\mu}u'_{iR} \\
& - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& + (\mathcal{D}_{\mu}\Phi)^{\dagger}(\mathcal{D}_{\mu}\Phi) - \mu^2\Phi^{\dagger}\Phi - \lambda(\Phi^{\dagger}\Phi)^2 \\
& - \sum_{i,j=e,\mu,\tau}\left(Y'^{\ell}_{ij}\bar{L}^{i}_{iL}\Phi\ell'_{jR} + Y'^{\ell *}_{ij}\bar{\ell}'_{jR}\Phi^{\dagger}L'_{iL}\right) \\
& - \sum_{i=1,2,3}\sum_{j=d,s,b}\left(Y'^{d}_{ij}\bar{Q}^{i}_{iL}\Phi d'_{jR} + Y'^{d *}_{ij}\bar{d}'_{jR}\Phi^{\dagger}Q'_{iL}\right) \\
& - \sum_{i=1,2,3}\sum_{j=u,c,t}\left(Y'^{u}_{ij}\bar{Q}^{i}_{iL}\tilde{\Phi}u'_{jR} + Y'^{u *}_{ij}\bar{u}'_{jR}\tilde{\Phi}^{\dagger}Q'_{iL}\right)
\end{aligned}
$$

费米子动能项及费米子场与规范场的相互作用$\mathcal{L}_{F,F-G}$

规范场自耦合$\mathcal{L}_{G}$

标量场贡献$\mathcal{L}_{S}$

费米子场与标量场的Yukawa耦合$\mathcal{L}_{F-S}$

- 其中协变微分的定义为$\mathcal{D}_{\mu} \equiv \partial_{\mu} + ig\frac{\sigma^a}{2}W^a_{\mu} + ig'\frac{Y}{2}B_{\mu}$
- 撇号意味着没有确定的质量

# Mass of the Fermions

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

- **对于夸克场，其乘积有两种可能的形式**

  1. $\bar{Q}^i_{iL} d'_{jR}$, $i = 1,2,3$, $j = d, s, b$
  2. $\bar{Q}^i_{iL} u'_{jR}$, $i = 1,2,3$, $j = u, c, t$

  - 第一种情况和轻子的讨论类似，$\bar{Q}^i_{iL} d'_{jR}$的弱超荷乘积为−1，$\Phi$的弱超荷为1
  - 拉氏量$-\sum_{i=1,2,3}\sum_{j=d,s,b}\left(Y'^d_{ij}\bar{Q}^i_{iL}\Phi d'_{jR}\right)$在幺正规范下为 $-\frac{v+H}{\sqrt{2}}\sum_{i,j=d,s,b} Y'^d_{ij}\bar{d}^i_{iL} d'_{jR}$

# Mass of the Fermions

$$\Phi(x)=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}$$

- **对于夸克场，其乘积有两种可能的形式**

  1. $\bar{Q}^i_{iL}d'_{jR},\ i=1{,}2{,}3,\ j=d,s,b$
  2. $\bar{Q}^i_{iL}u'_{jR},\ i=1{,}2{,}3,\ j=u,c,t$

  - 第一种情况和轻子的讨论类似，$\bar{Q}^i_{iL}d'_{jR}$的弱超荷乘积为-1，$\Phi$的弱超荷为1
  - 拉氏量$-\sum_{i=1,2,3}\sum_{j=d,s,b}\left(Y'^d_{ij}\bar{Q}^i_{iL}\Phi d'_{jR}\right)$在幺正规范下为 $-\frac{v+H}{\sqrt{2}}\sum_{i,j=d,s,b}Y'^d_{ij}\bar{d}^i_{iL}d'_{jR}$

- **第二种情况，$\bar{Q}^i_{iL}u'_{jR}$的弱超荷乘积为+1**

  - 需要一个弱超荷为-1的复标量二重态，才能得到定域规范不变的拉氏量
  - 通过如下变换得到

$$\tilde{\Phi}=i\sigma_2\Phi^*=\begin{pmatrix}\phi^{0*}\\ -\phi^-\end{pmatrix}$$

  - $\tilde{\Phi}$是Higgs二重态的电荷共轭
  - $\phi^-$是$\phi^+$的复共轭

  - 实际上，在幺正规范下：$\tilde{\Phi}=\frac{1}{\sqrt{2}}\begin{pmatrix}v+H(x)\\ 0\end{pmatrix}$
  - 这样，$-\sum_{i=1,2,3}\sum_{j=u,c,t}\left(Y'^u_{ij}\bar{Q}^i_{iL}\tilde{\Phi}u'_{jR}\right)$可写成 ?

## 练习：推导上型夸克的质量项

$$-\sum_{i=1,2,3}\sum_{j=u,c,t}\left(Y'^{u}_{ij}\bar{Q}^{i}_{iL}\tilde{\Phi}u'_{jR}\right)$$

$$\tilde{\Phi}=\frac{1}{\sqrt{2}}\begin{pmatrix}\nu+H(x)\\0\end{pmatrix}$$

| 粒子（场） | $I_{\mathrm{w}}$ | $I_{\mathrm{w}}^{3}$ | $Q$ | $Y$ |
|---|---|---|---|---|
| $L_{iL}=\begin{pmatrix}\nu_{iL}\\\ell_{iL}\end{pmatrix}:\begin{pmatrix}\nu_{eL}\\e_L\end{pmatrix}\begin{pmatrix}\nu_{\mu L}\\\mu_L\end{pmatrix}\begin{pmatrix}\nu_{\tau L}\\\tau_L\end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $0$<br>$-1$ | $-1$ |
| 没有右手中微子<br>$\ell_{iR}: e_R,\mu_R,\tau_R$ | $-$<br>$0$ | $-$<br>$0$ | $-$<br>$-1$ | $-$<br>$-2$ |
| $Q_{iL}=\begin{pmatrix}u_{iL}\\d_{iL}\end{pmatrix}:\begin{pmatrix}u_L\\d_L\end{pmatrix}\begin{pmatrix}c_L\\s_L\end{pmatrix}\begin{pmatrix}t_L\\b_L\end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $+\frac{2}{3}$<br>$-\frac{1}{3}$ | $\frac{1}{3}$ |
| $u_{iR}: u_R,c_R,t_R$<br>$d_{iR}: d_R,s_R,b_R$ | $0$<br>$0$ | $0$<br>$0$ | $+\frac{2}{3}$<br>$-\frac{1}{3}$ | $\frac{4}{3}$<br>$-\frac{2}{3}$ |
| $\Phi(\mathrm{x})=\begin{pmatrix}\phi^{+}(x)\\\phi^{0}(x)\end{pmatrix}$ | $\frac{1}{2}$ | $+\frac{1}{2}$<br>$-\frac{1}{2}$ | $1$<br>$0$ | $1$ |

# Mass of the Fermions

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

- **对于夸克场，其乘积有两种可能的形式**

  1. $\bar{Q}^i_{iL} d'_{jR},\ i = 1,2,3,\ j = d,s,b$
  2. $\bar{Q}^i_{iL} u'_{jR},\ i = 1,2,3,\ j = u,c,t$

  - 第一种情况和轻子的讨论类似，$\bar{Q}^i_{iL} d'_{jR}$的弱超荷乘积为-1，$\Phi$的弱超荷为1
  - 拉氏量$-\sum_{i=1,2,3}\sum_{j=d,s,b}\left(Y'^d_{ij}\bar{Q}^i_{iL}\Phi d'_{jR}\right)$在幺正规范下为 $-\frac{v+H}{\sqrt{2}}\sum_{i,j=d,s,b} Y'^d_{ij}\bar{d}^i_{iL} d'_{jR}$

- **第二种情况，** $\bar{Q}^i_{iL} u'_{jR}$**的弱超荷乘积为+1**
  - 需要一个弱超荷为-1的复标量二重态，才能得到定域规范不变的拉氏量
  - 通过如下变换得到

    $$\tilde{\Phi} = i\sigma_2\Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

    - $\tilde{\Phi}$是Higgs二重态的电荷共轭
    - $\phi^-$是$\phi^+$的复共轭
  - 实际上，在幺正规范下：$\tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$
  - 这样，$-\sum_{i=1,2,3}\sum_{j=u,c,t}\left(Y'^u_{ij}\bar{Q}^i_{iL}\tilde{\Phi} u'_{jR}\right)$可写成 $-\frac{v+H}{\sqrt{2}}\sum_{i,j=u,c,t} Y'^u_{ij}\bar{u}^i_{iL} u'_{jR}$

# Mass of the Fermions

- 接下来使用与轻子场时一样的方法，将$Y'^{u}_{ij}$和$Y'^{d}_{ij}$对角化，得到最后的Yukawa拉氏量

$$\mathcal{L}_{F-S,Q} = -\sum_{i=d,s,b} \frac{y_i^d v}{\sqrt{2}} \bar{d}_i d_i - \sum_{i=u,c,t} \frac{y_i^u v}{\sqrt{2}} \bar{u}_i u_i - \sum_{i=d,s,b} \frac{y_i^d}{\sqrt{2}} \bar{d}_i d_i\, H - \sum_{i=u,c,t} \frac{y_i^u}{\sqrt{2}} \bar{u}_i u_i\, H$$

- 同样的，前面两项给出了夸克的质量

$$m_i = \frac{y_i^d v}{\sqrt{2}}, i = d, s, b, \qquad m_i = \frac{y_i^u v}{\sqrt{2}}, i = u, c, t$$

- 后面两项给出了夸克和Higgs场的三线耦合，利用质量项写为

$$-\sum_{i=d,s,b} \frac{m_i}{v} \bar{d}_i d_i\, H, \qquad -\sum_{i=u,c,t} \frac{m_i}{v} \bar{u}_i u_i\, H$$

# Gauge Field Self Interaction

- 电弱理论还有规范场动能项 $\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$

  - 其中$G^a_{\mu\nu}, B_{\mu\nu}$是规范场张量 $G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon_{abc} W^b_\mu W^c_\nu$

    $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

**非阿贝尔群**

***a ∗ b ≠ b ∗ a***

  - 如果要用物理场$W_\mu, Z_\mu, A_\mu$表示$\mathcal{L}_G$，则还能得到三个或四个规范场之间的耦合

  - 定义 $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $F_W^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu$, $F_Z^{\mu\nu} = \partial^\mu Z^\nu - \partial^\nu Z^\mu$

  - 并且使用之前的结论 $W^- = \frac{1}{\sqrt{2}}(W^1_\mu - iW^2_\mu)$, $\qquad W^+_\mu = \frac{1}{\sqrt{2}}(W^1_\mu + iW^2_\mu)$

    $A^\mu = \cos\theta_W B_\mu + \sin\theta_W A^3_\mu$, $Z^\mu = -\sin\theta_W B_\mu + \cos\theta_W A^3_\mu$

# Gauge Field Self Interaction

➢ 得到最终的拉氏量

$$
\begin{aligned}
\mathcal{L}_{\mathrm{G}} = &-\frac{1}{2}F_{W,\mu\nu}F_W^{\dagger\mu\nu} - \frac{1}{4}F_{Z,\mu\nu}F_Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&+ ig\cos\theta_W\left[F_{W,\mu\nu}Z^\mu W^{\dagger\nu} - F^\dagger_{W,\mu\nu}Z^\mu W^\nu + F_{Z,\mu\nu}W^{\dagger\mu}W^\nu\right] \\
&+ ie\left[F_{W,\mu\nu}A^\mu W^{\dagger\nu} - F^\dagger_{W,\mu\nu}A^\mu W^\nu + F_{\mu\nu}W^{\dagger\mu}W^\nu\right] \\
&+ g^2\cos^2\theta_W\left[(W_\mu Z^\mu)(W^\dagger_\nu Z^\nu) - (W_\mu W^{\dagger\mu})(Z_\nu Z^\nu)\right] \\
&+ e^2\left[(W_\mu A^\mu)(W^\dagger_\nu A^\nu) - (W_\mu W^{\dagger\mu})(A_\nu A^\nu)\right] \\
&+ eg\cos\theta_W\left[(W_\mu Z^\mu)(W^\dagger_\nu A^\nu) + (W^\dagger_\mu Z^\mu)(W_\nu A^\nu) - 2(W_\mu W^{\dagger\mu})(Z_\nu A^\nu)\right] \\
&+ \frac{1}{2}g^2\left[(W_\mu W^\mu)(W^\dagger_\nu W^{\dagger\nu}) - (W_\mu W^{\dagger\mu})^2\right]
\end{aligned}
$$

- 第一行前两项分别是$W$和$Z$的动能项，最后一项是电磁场动能项
- 第二行到第七行是规范场之间的耦合，分别对应以下几种作用顶点

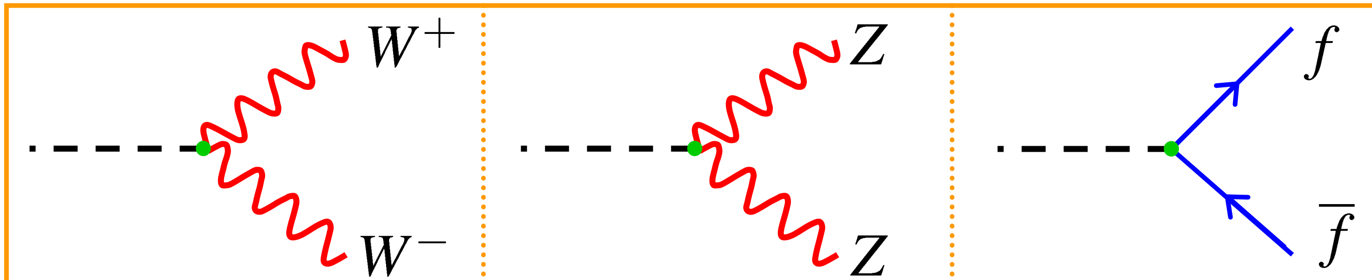
$WWZ$, $WW\gamma$, $WWZZ$, $WW\gamma\gamma$, $WWZ\gamma$, $WWWW$

# Higgs Mechanism

- 希格斯机制直接预言了规范玻色子的质量
- SM中，费米子的质量也可以通过与希格斯场作用而获得
  - 但是，这里不能预言其质量 - 仅是“手动”放入

费曼图

顶角因子：




| | | |
|---|---|---|
| $i g_W m_W g^{\mu\nu}$ | $i g_Z m_Z g^{\mu\nu}$ | $-i\dfrac{g_W}{2m_W}m_f$ |

- 包含希格斯机制的电弱统一标准模型 ⇒ 标准模型参数间的关系

$$\boxed{m_W = \left(\frac{\pi\alpha_{em}}{\sqrt{2}G_F}\right)^{\frac{1}{2}}\frac{1}{\sin\theta_W}}$$

$$\boxed{m_Z = \frac{m_W}{\cos\theta_W}}$$

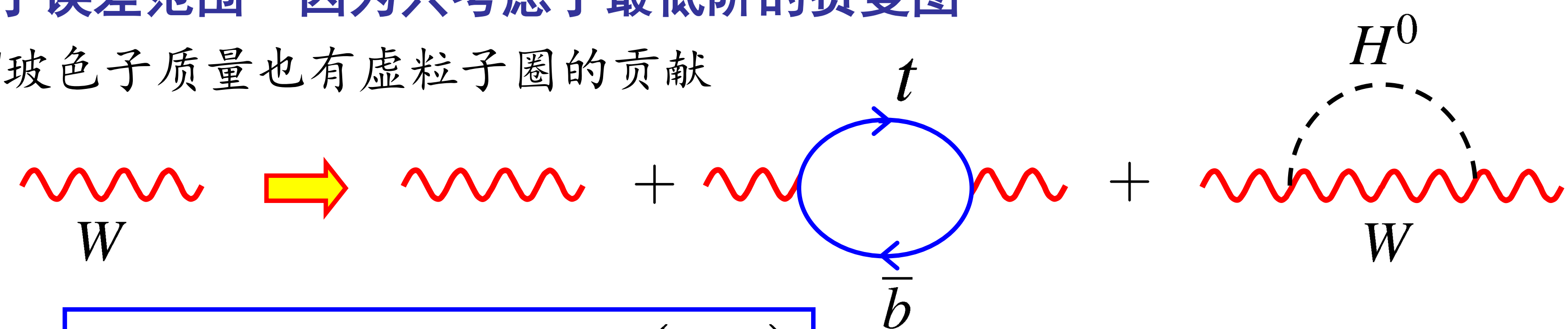
- 因此，知道 $\alpha_{em}, G_F, m_W, m_Z, \sin\theta_W$ 中的任意3个，就可以预言另外2个

# Precision Tests of Standard Model

- LEP等实验的精确测量可以检验标准模型的预言
  - 如预言： $\boxed{m_W = m_Z \cos\theta_W}$ 测量有 $\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}}$ $\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$
  - 可以预期：

$\boxed{m_W = 79.946 \pm 0.008\,\mathrm{GeV}}$ 但是测量得到 $\boxed{m_W = 80.376 \pm 0.033\,\mathrm{GeV}}$

- 超出了误差范围 – 因为只考虑了最低阶的费曼图
  - W玻色子质量也有虚粒子圈的贡献




$$\boxed{m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)}$$

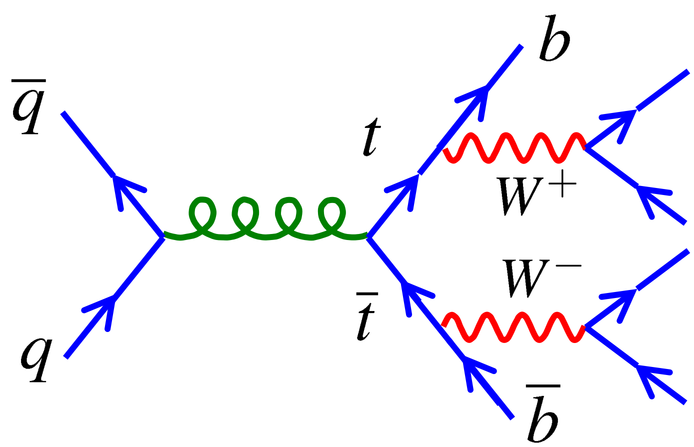
- 上述“差别”由虚粒子圈导致，即，高精度的测量对圈图中的粒子质量敏感！

# Top Quark

- 由圈图修正和精确的LEP测量数据，可以预测顶夸克质量：

$$\boxed{m_t^{\text{loop}} = 173 \pm 11\,\text{GeV}}$$

- 1994年，顶夸克在Fermilab的Tevatron上的质子-反质子对撞实验上被发现
  - With the predicted mass !




- 顶夸克几乎全部衰变到底夸克，由于 $|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2$
- 复杂的末态事例拓扑：
  $t\bar{t} \to b\bar{b}q\bar{q}q\bar{q} \to 6\ \text{jets}$
  $t\bar{t} \to b\bar{b}q\bar{q}\ell\nu \to 4\ \text{jets} + \ell + \nu$
  $t\bar{t} \to b\bar{b}\ell\nu\ell\nu \to 2\ \text{jets} + 2\ell + 2\nu$

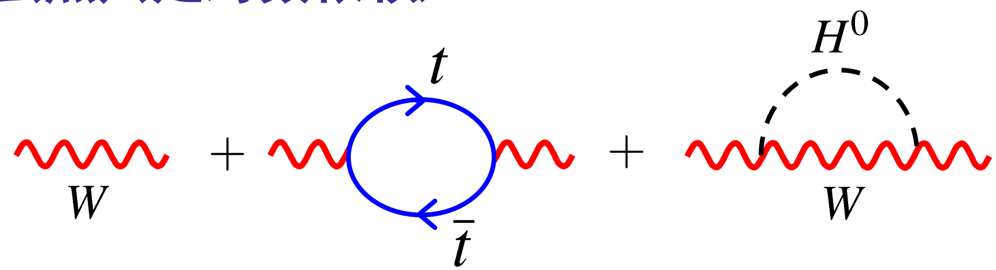
- 质量由直接重建得到（类似W玻色子质量）

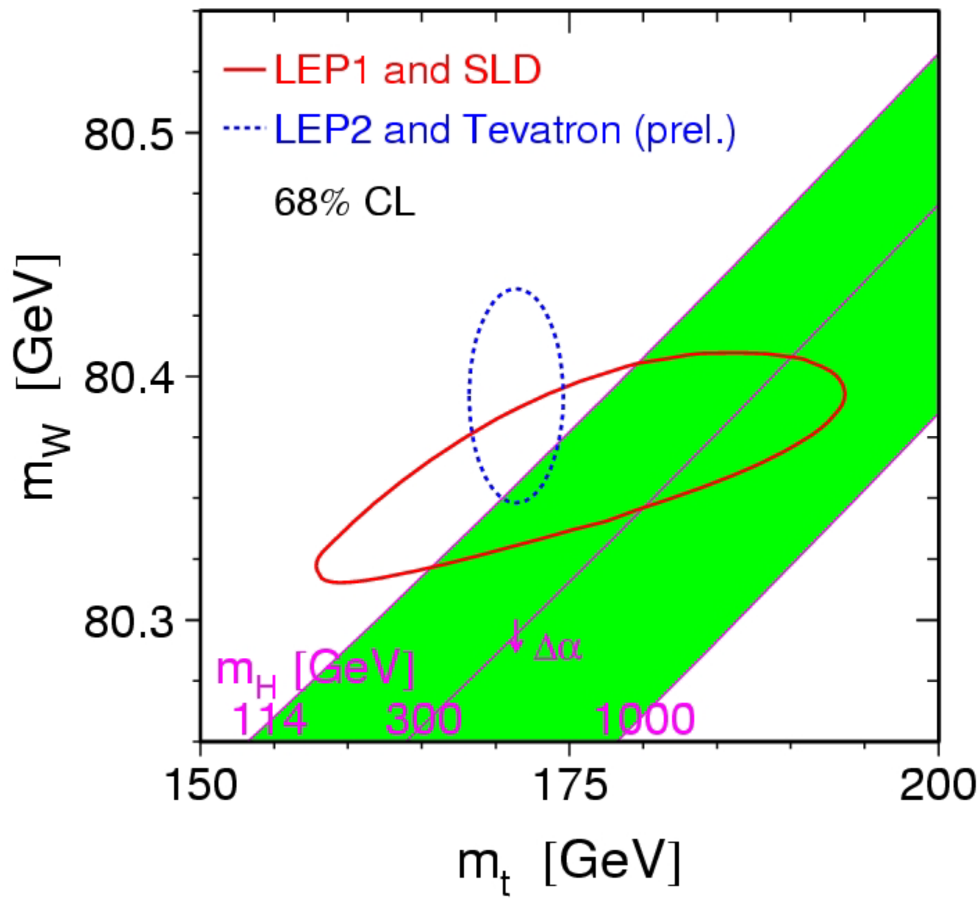
$$\boxed{m_t^{\text{meas}} = 174.2 \pm 3.3\,\text{GeV}}$$

# Top Quark

➢ 但是W的质量也依赖于Higgs的质量(虽然只是对数依赖)

$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$




➢ 测量足够精确到能够约束Higgs质量

- 通过测量的顶夸克和W玻色子质量可以预测Higgs质量

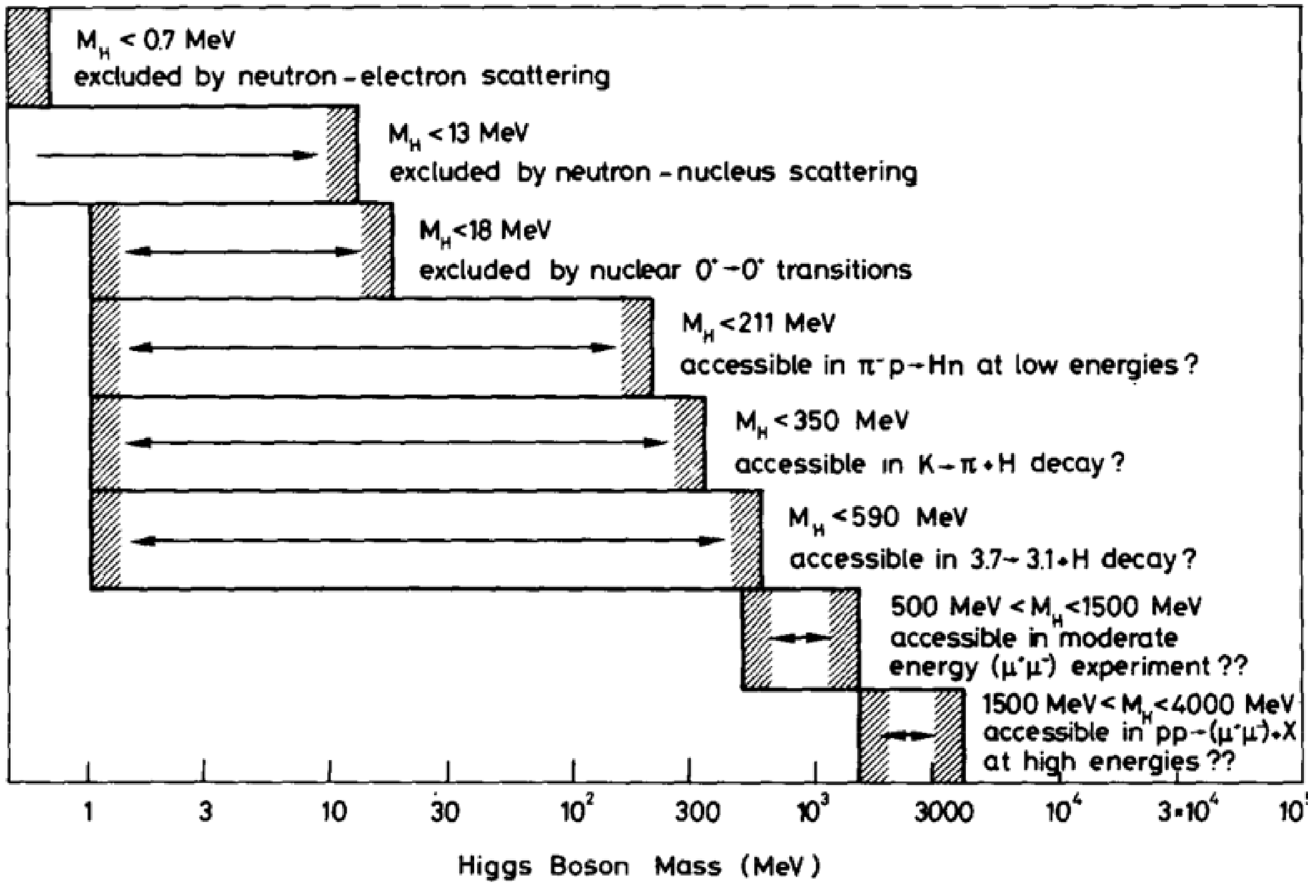
- **Direct:** W and top masses from direct reconstruction
- **Indirect:** from SM interpretation of Z mass, $\theta_W$ etc. and

➢ 数据倾向于“轻的”Higgs:

$$\boxed{m_H < 200\,\mathrm{GeV}}$$

# Higgs below 5 GeV

Ellis, Gaillard, Nanopoulos,1975



# Higgs below 5 GeV

### SINDRUM @ PSI

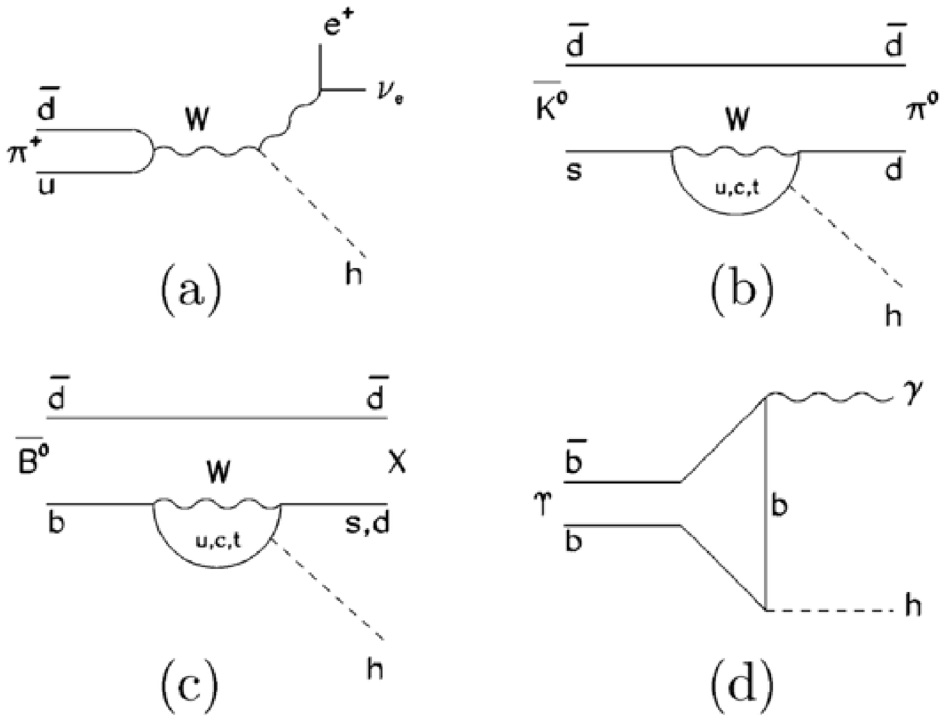
h→ee

$m_h \notin [10, 110]$ MeV

### CERN-Edinburgh-Mainz-Orsay-Pisa-Siegen

h→ee mh<50 MeV

$BR(K^0_L \to \pi^0 H) \times BR(H \to e^+e^-) < 2\times 10^{-8}$




### CLEO

h→ee

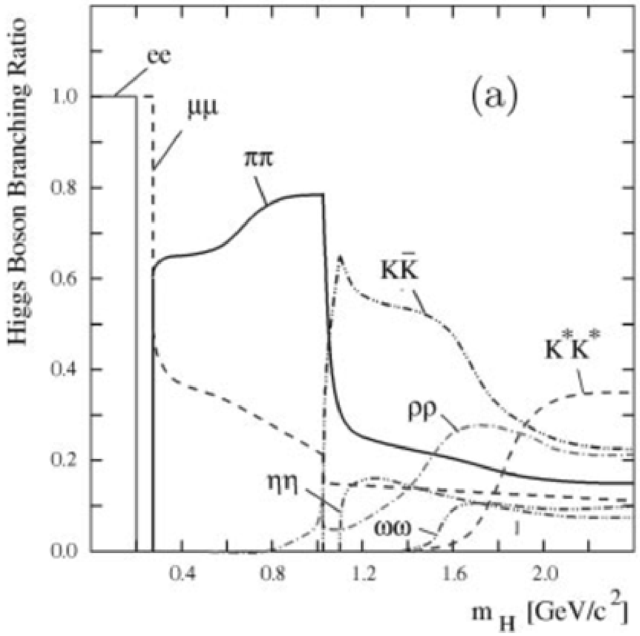
$m_h \notin [0.2, 3.6]$ GeV

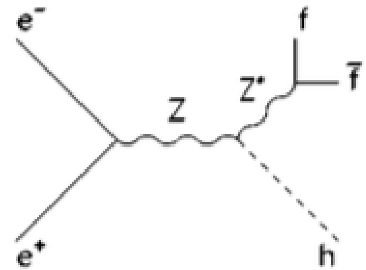
### CUSB

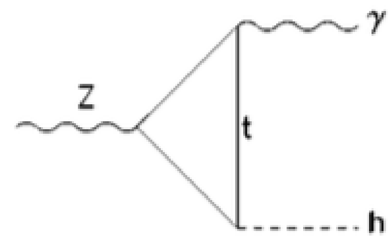
h→ee

$m_h \notin [0.2, 5.0]$ GeV

# LEP 1 searches (1989-1995)



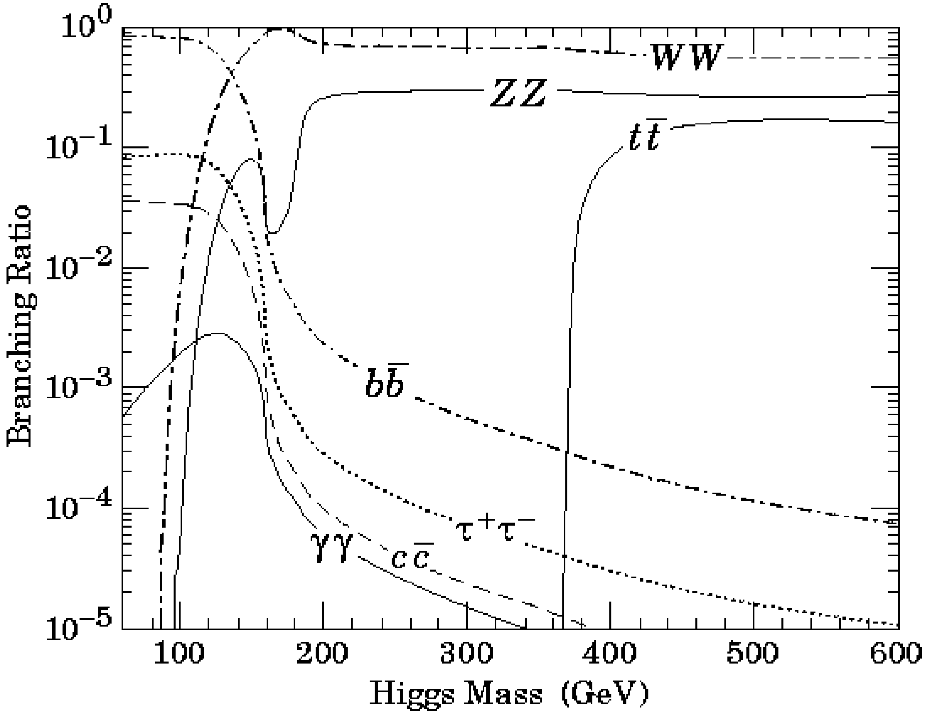



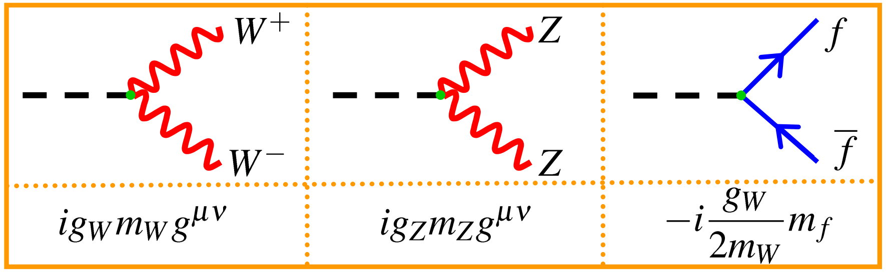

- $2 < m_H < 20$ GeV
  - look for heaviest fermion kinematically allowed
- $m_H > 20$ GeV
  - Higgsstrahlung: $Z \rightarrow \nu\nu$ or $Z \rightarrow e+e-$ $(\mu+\mu-)$ and $H \rightarrow bb$
  - 10 signal events expected in $13 \times 10^6$ events (all other channels swamped by $Z \rightarrow qq$ bkgs)
- LEP 1 limit : $m_H < 65.6$ GeV @ 95% CL

# Hunting Higgs boson

- ★ **The Higgs boson is an essential part of the Standard Model – but does it exist ?**
- ★ **Consider the search at LEP. Need to know how the Higgs decays**




- **Higgs boson couplings proportional to mass**

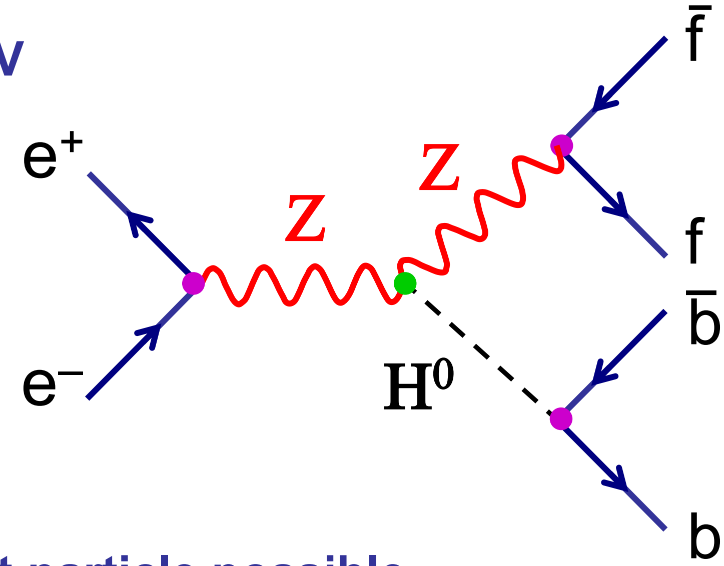



- **Higgs decays predominantly to heaviest particles which are energetically allowed**

| | | |
|---|---|---|
| $m_H < 2m_W$ | **mainly** | $H^0 \to b\bar{b}$ **+ approx 10%** $H^0 \to \tau^+\tau^-$ |
| $2m_W < m_H < 2m_t$ | **almost entirely** | $H^0 \to W^+W^- + H^0 \to ZZ$ |
| $m_H > 2m_t$ | **either** | $H^0 \to W^+W^-,\ H^0 \to ZZ,\ H^0 \to t\bar{t}$ |

# A Hint from LEP ?

- ★ LEP operated with a C.o.M. energy upto 207 GeV
- ★ For this energy (assuming the Higgs exists) the main production mechanism would be the "Higgsstrahlung" process

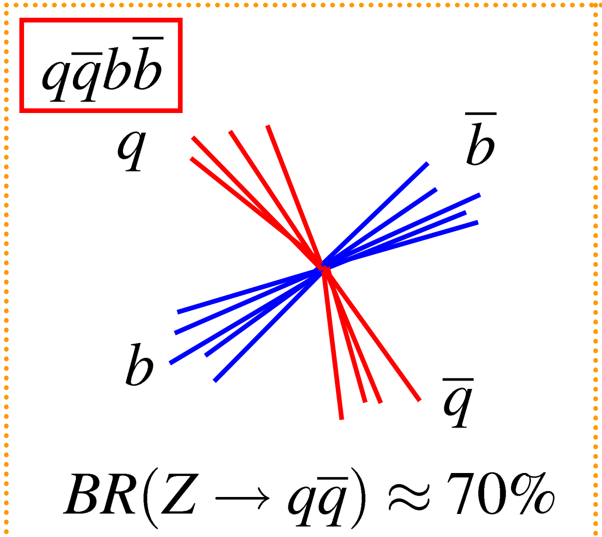


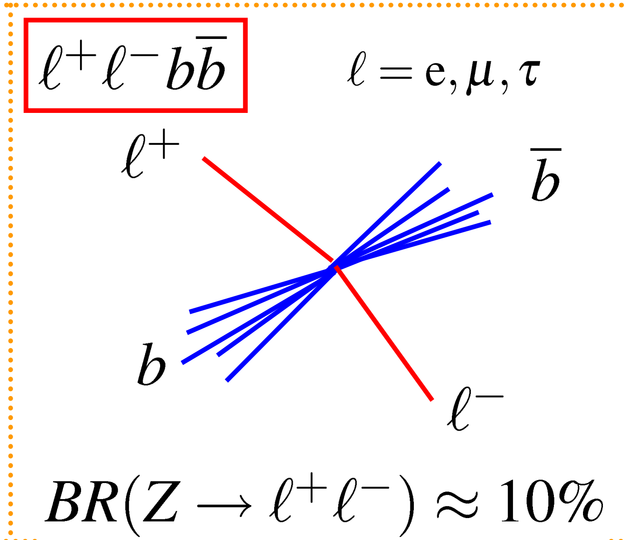

- ★ Need enough energy to make a Z and H; therefore could produce the Higgs boson if

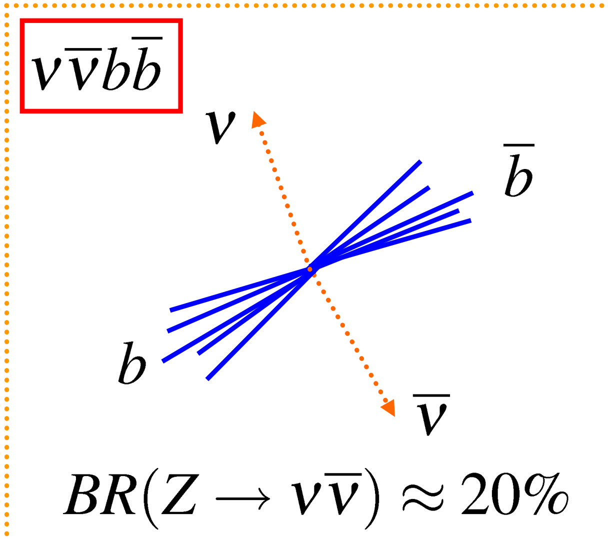
$$m_H < 207\,\mathrm{GeV} - m_Z$$

i.e. if $$m_H < 116\,\mathrm{GeV}$$

- ★ The Higgs predominantly decays to the heaviest particle possible
- ★ For $m_H < 116\,\mathrm{GeV}$ this is the b-quark (not enough mass to decay to WW/ZZ/tt)

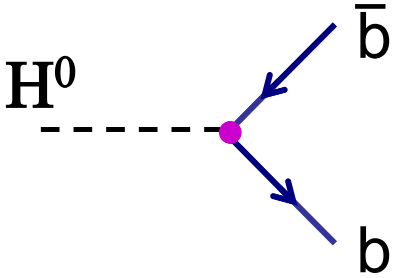


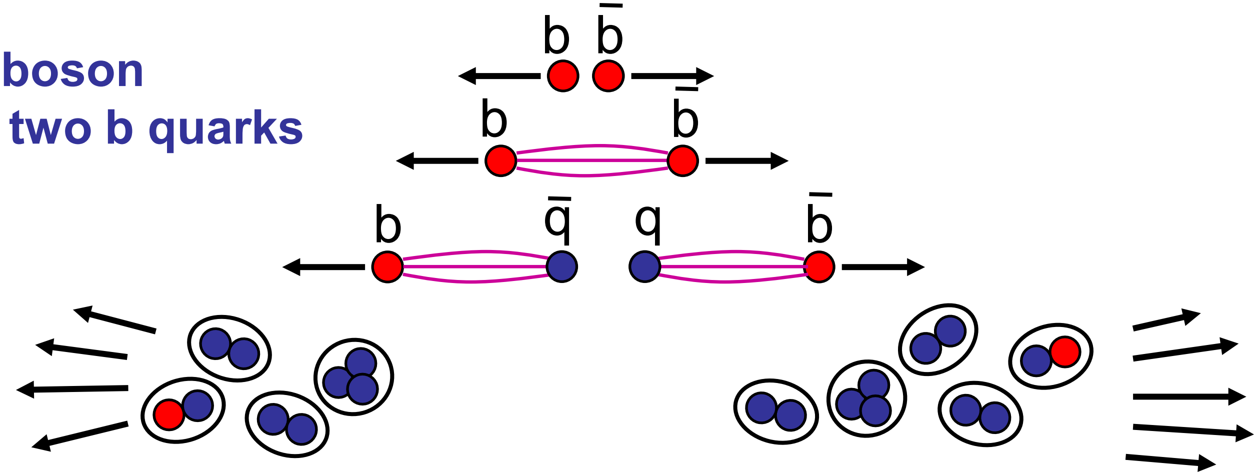



# Tagging Higgs Boson Decays

★ One signature for a Higgs boson decay is the production of two b quarks



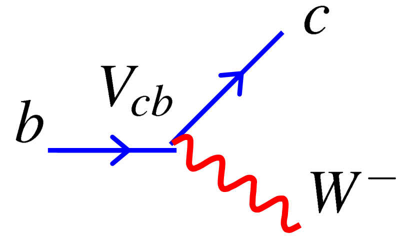



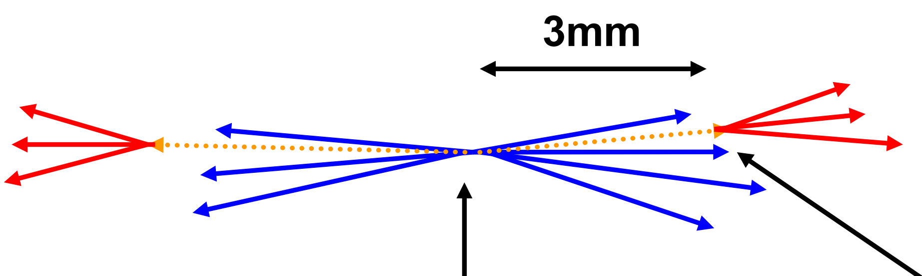

★ Each jet will contain one b-hadron which will decay weakly

★ Because $V_{cb}$ is small $(V_{cb} \approx 0.04)$ hadrons containing b-quarks are relatively long-lived

★ Typical lifetimes of $\tau \sim 1 \times 10^{-12}\,\mathrm{s}$

★ At LEP b-hadrons travel approximately **3mm** before decaying



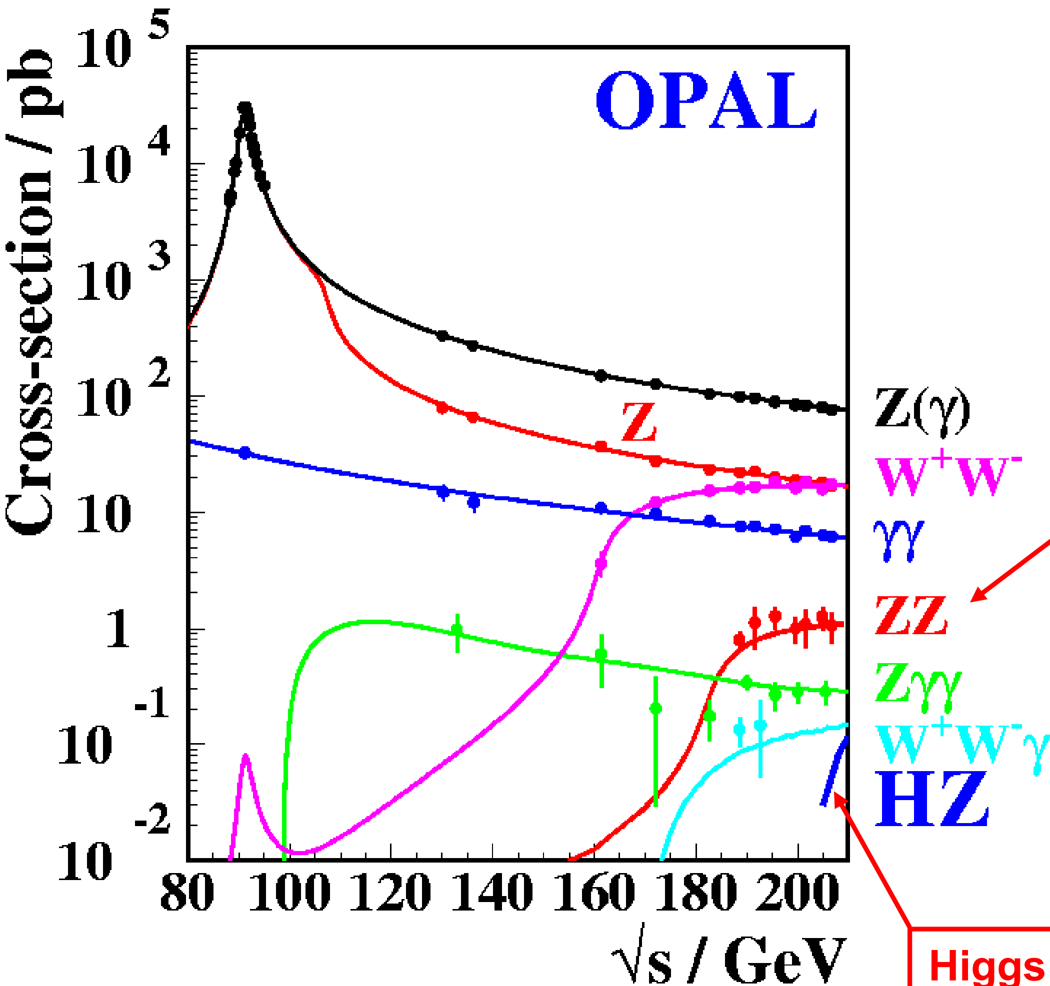

Primary vertex

Displaced Secondary Vertex from decay of B hadron

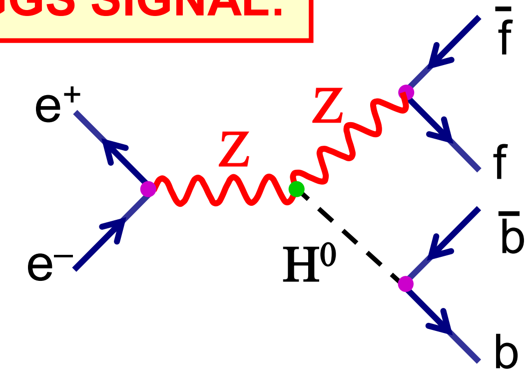
★ Can efficiently identify jets containing b quarks

# Tagging Higgs Boson Decays

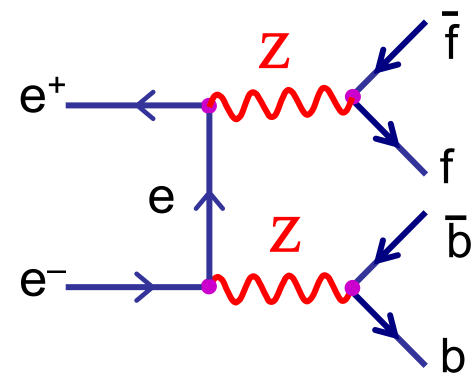
- Clear experimental signature, but small cross section, e.g. for $m_H \approx 115\,\mathrm{GeV}$ would only produce a few tens of $\mathrm{e^+e^-} \rightarrow H^0$ events at LEP
- In addition, there are large "backgrounds"




HIGGS SIGNAL:



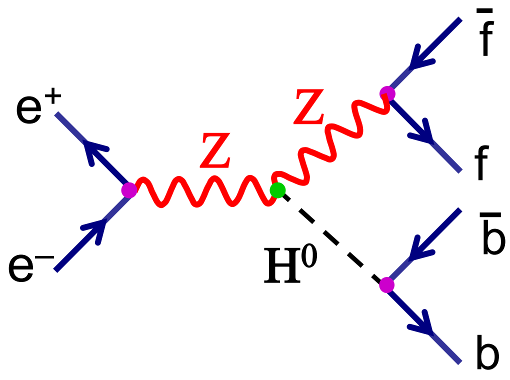

MAIN BACKGROUND:




Higgs production cross section ($m_H$=115 GeV)

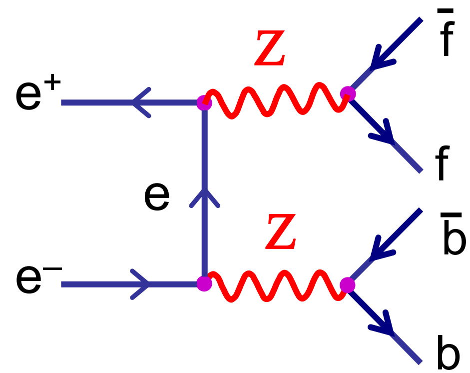
# Tagging Higgs Boson Decays

★ The only way to distinguish




$$m_{b\bar{b}} = m_H$$

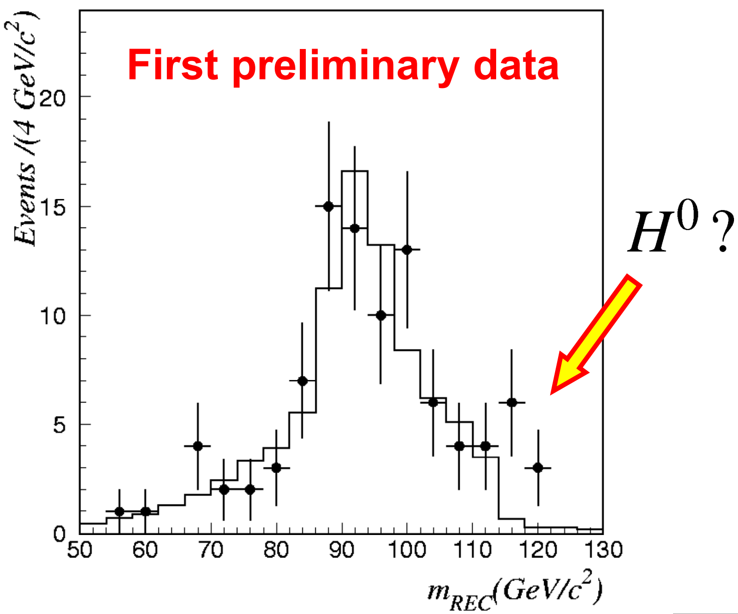
from




$$m_{b\bar{b}} = m_Z$$

is the from the invariant mass of the jets from the boson decays

★ In 2000 (the last year of LEP running) the ALEPH experiment reported an excess of events consistent with being a Higgs boson with mass **115 GeV**

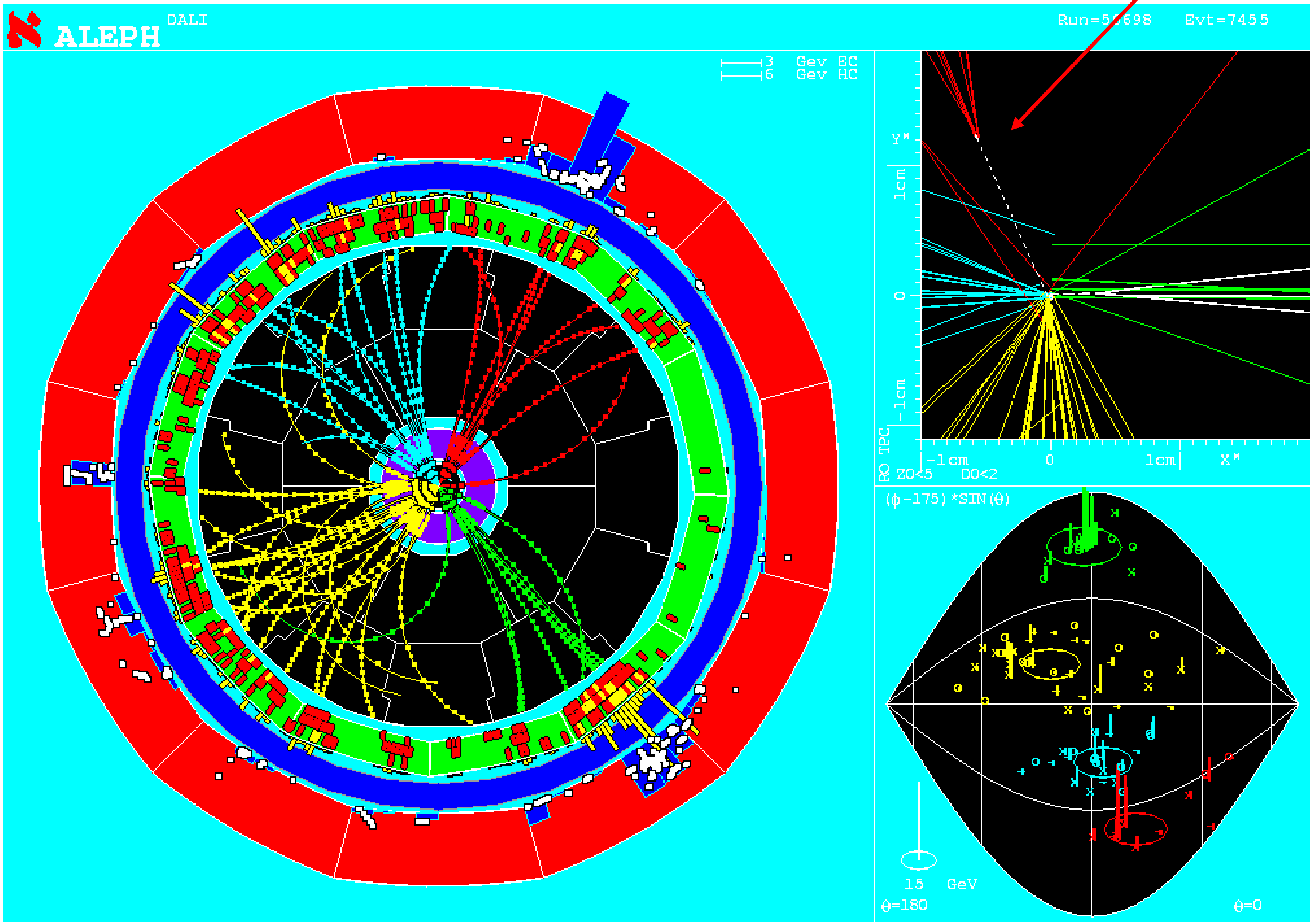



- ALEPH found 3 events which were high relative probability of being signal
- L3 found 1 event with high relative probability of being signal
- OPAL and DELPHI found none

# Tagging Higgs Boson Decays

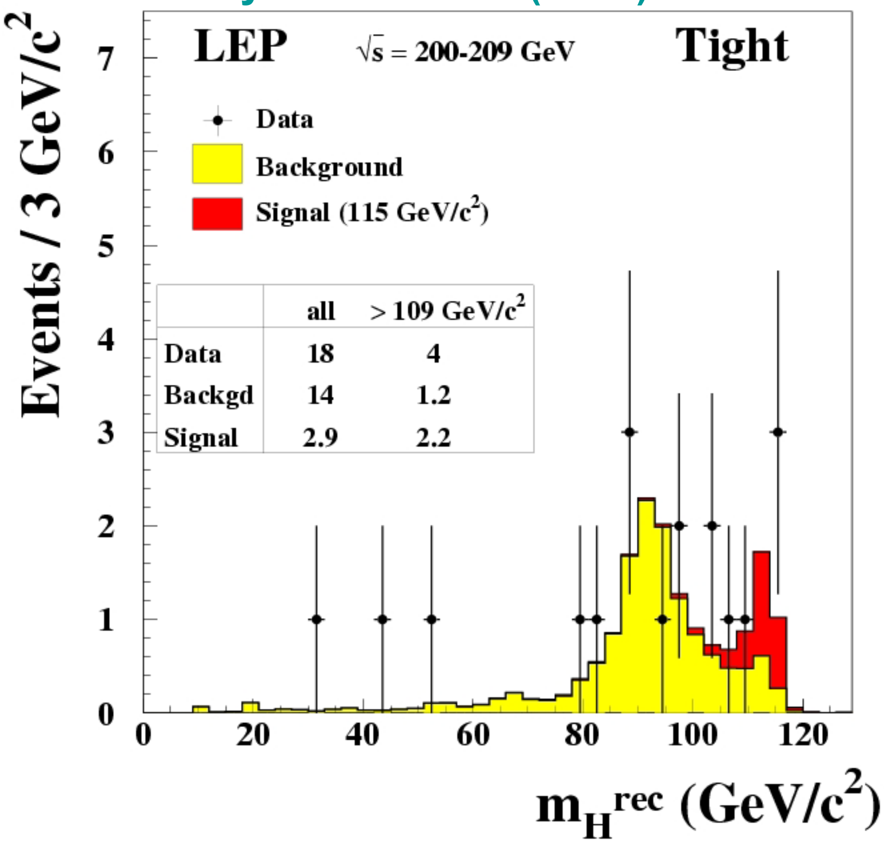
**Example event:**

Displaced vertex from b-decay



# Combined LEP Results

Phys. Lett. B565 (2003) 61-75




- ★ Final combined LEP results fairly inconclusive
- ★ A hint rather than strong evidence…
- ★ All that can be concluded:

$$\boxed{m_H > 114\,\mathrm{GeV}}$$

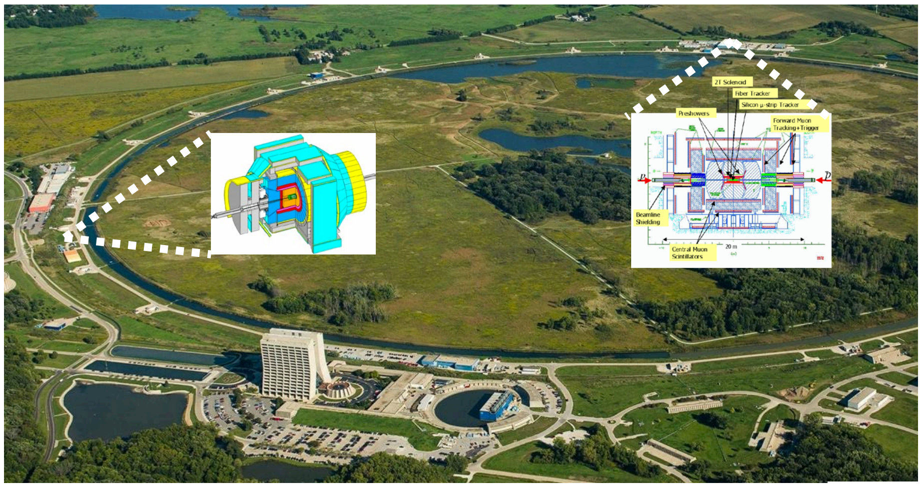
**The Higgs boson remains the missing link in the Standard Model**

- ★ The LHC will take first physics data in early **2010**
- ★ If the Higgs exists it will be found ! (although may take a few years)

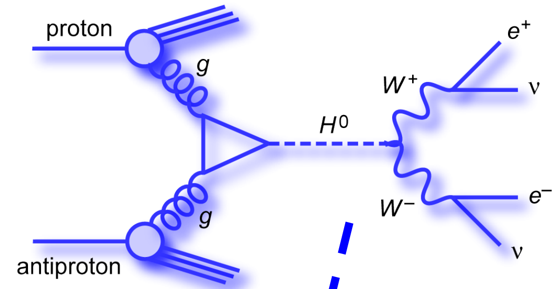
★ The SM will then be complete…

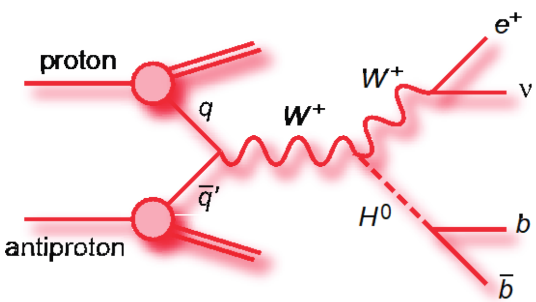
# Dawn of LHC: Direct search at Tevatron

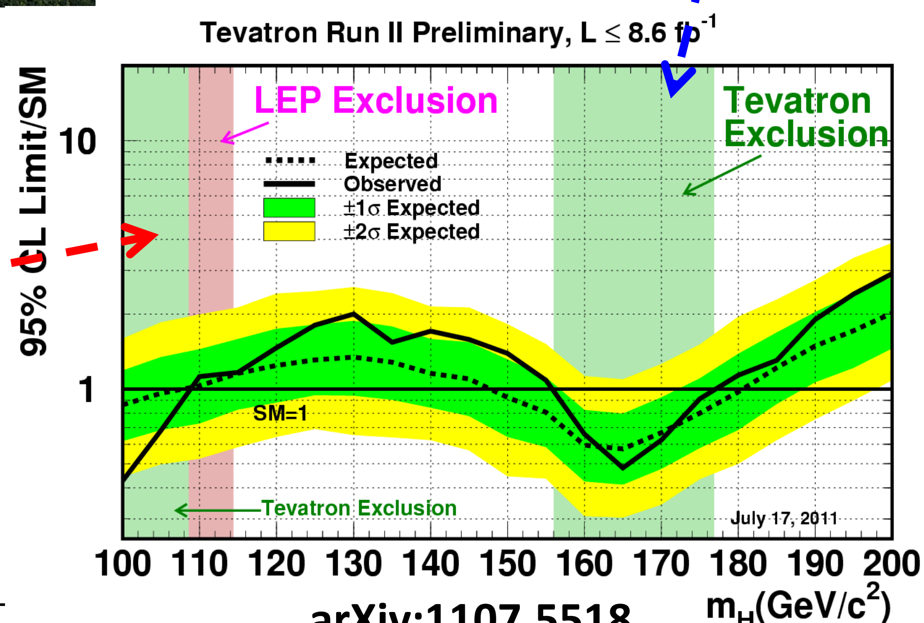
**Tevatron** (1987–2011) at Fermilab, US



- Proton&anti-proton collider
- Sqrt(s)~1.96 TeV
- Up to ~10 $fb^{-1}$ in the end




arXiv:1107.5518

# Dawn of LHC: Put everything together

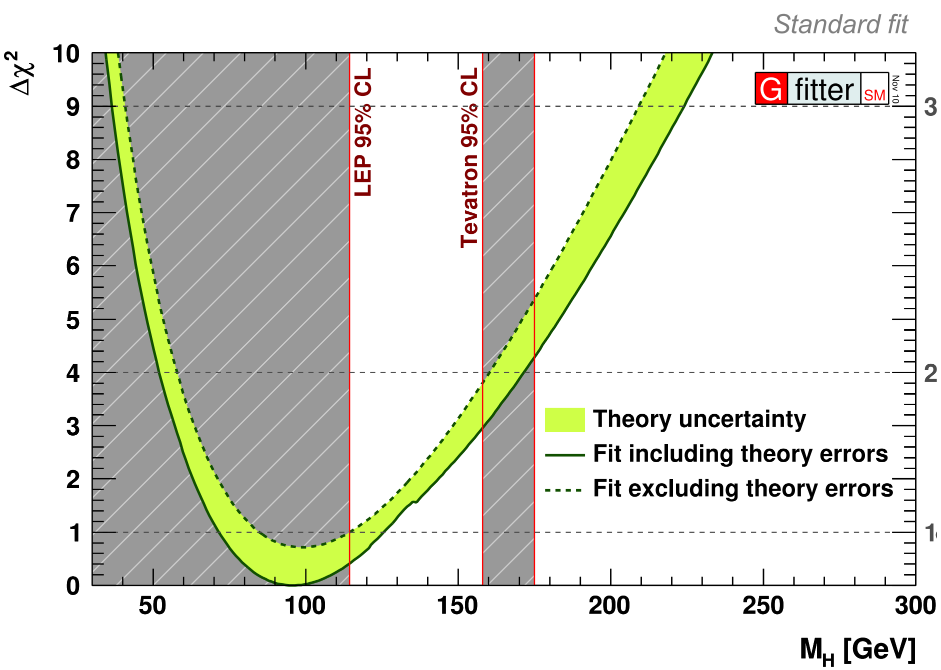
## $M_H$ from *Standard fit*:

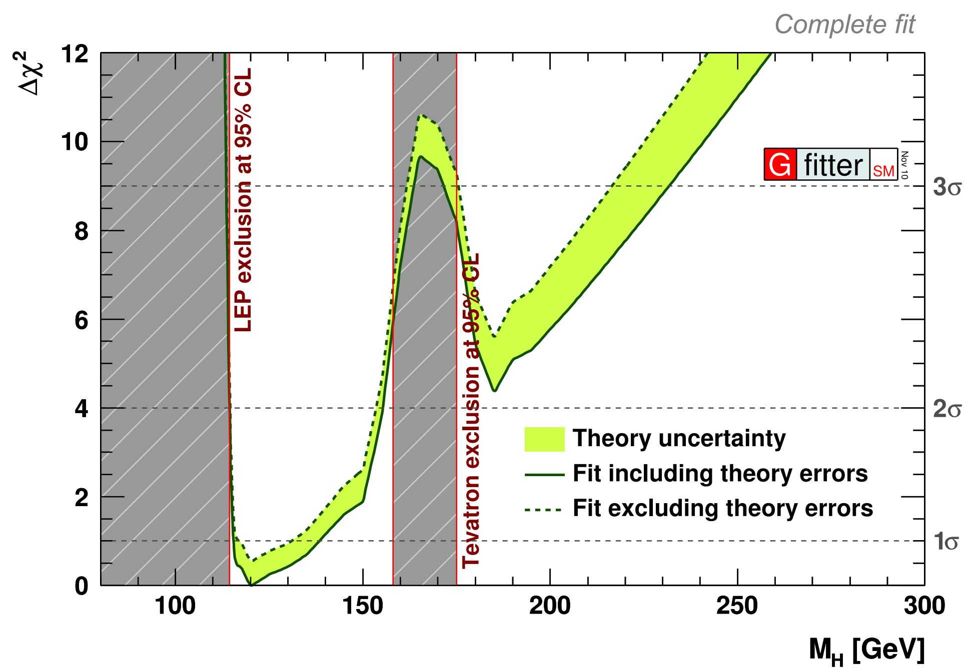
- Central value $\pm 1\sigma$: $M_H = 96^{+30}_{-25}$ GeV
- 95% CL upper limit: 170 GeV

Green band due to *R*fit treatment of theory errors, fixed errors lead to larger $\chi^2_{min}$

## $M_H$ from ***Complete fit***:

- Central value $\pm 1\sigma$: $M_H = 120^{+18}_{-5}$ GeV
- 95% CL upper limit: 155 GeV



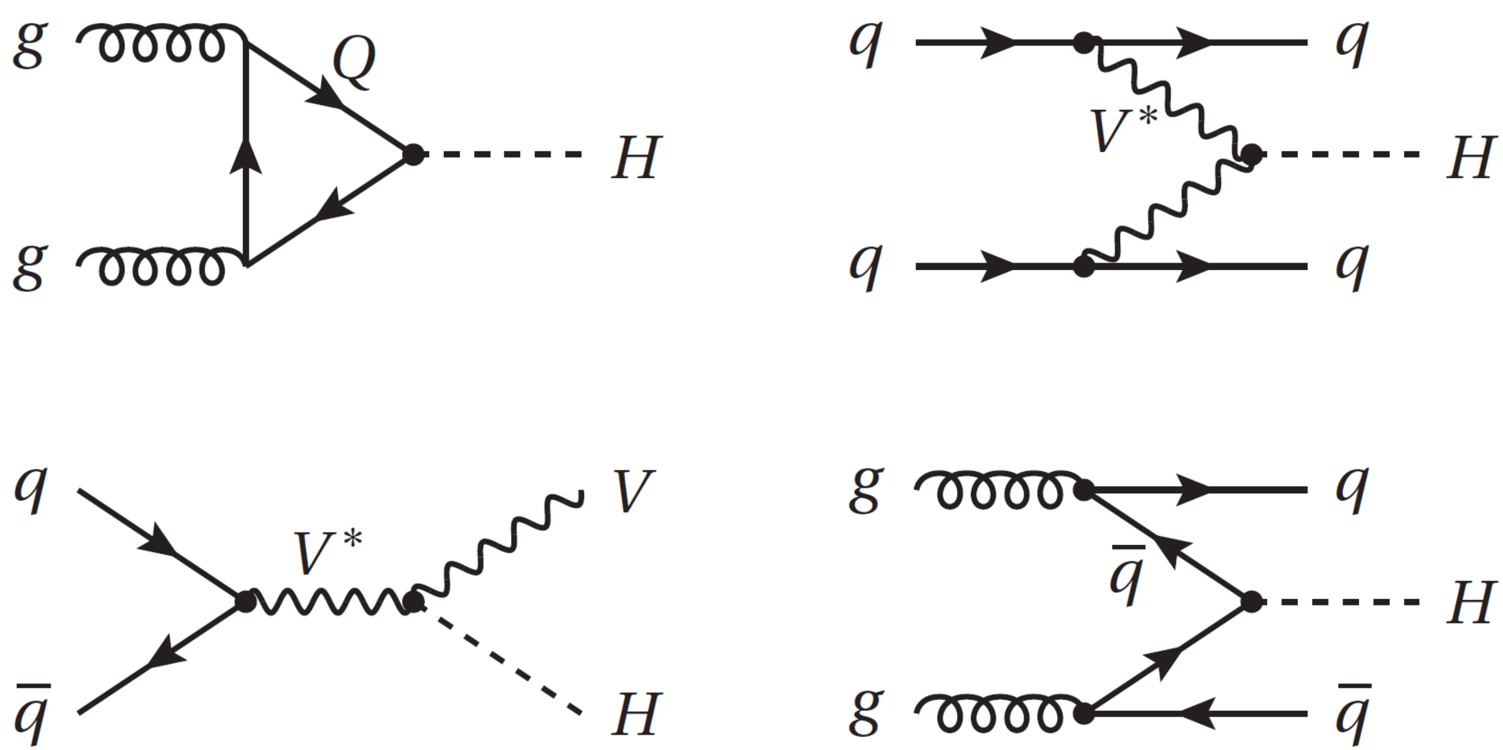



# Discovery of Higgs

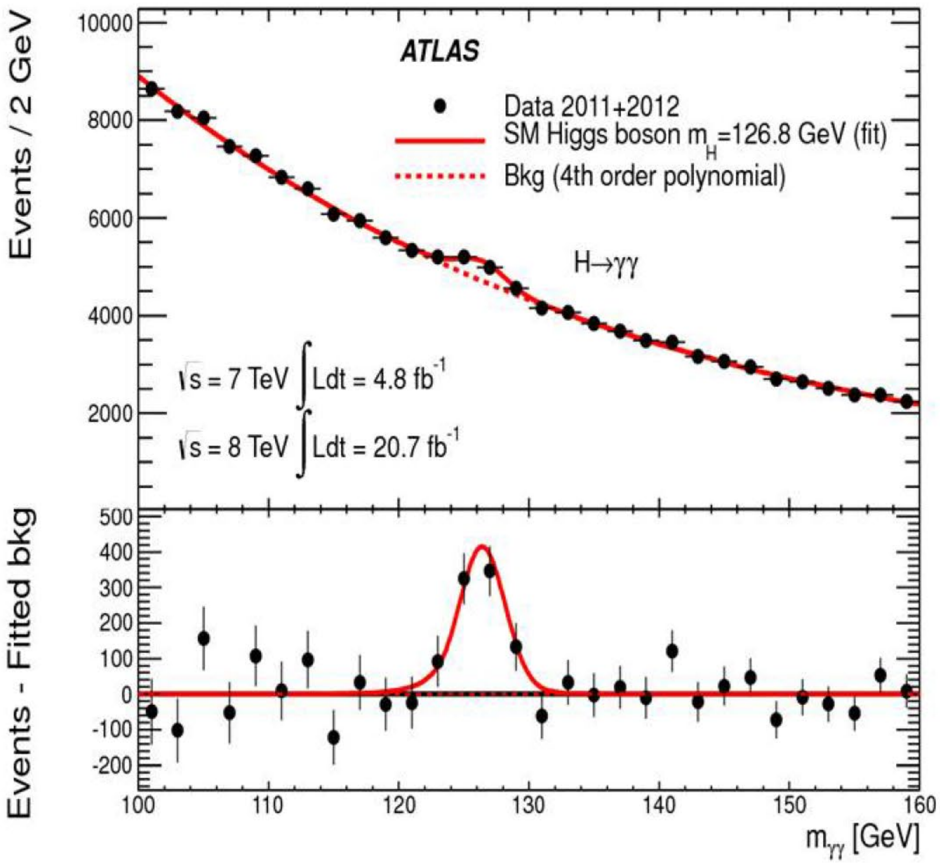
- 2012年，ATLAS和CMS合作组同时宣布在LHC实验中寻找到了Higgs粒子

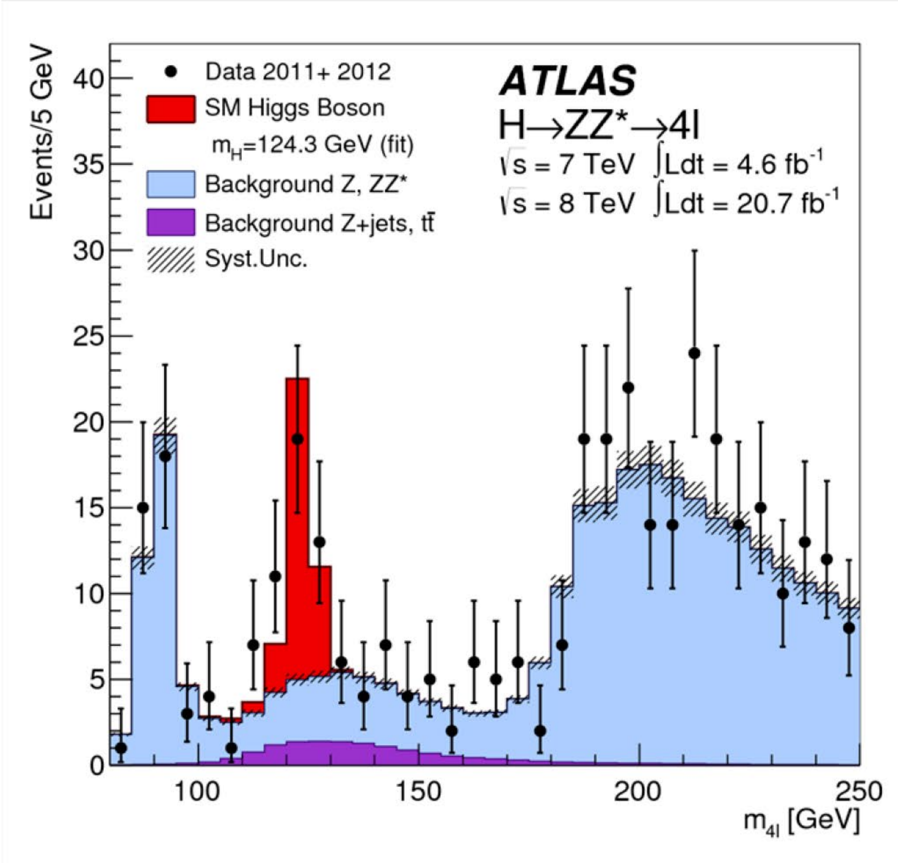
$$m_H \sim 125GeV$$

- 下图展示了LHC上pp对撞产生Higgs玻色子的四种主要模式



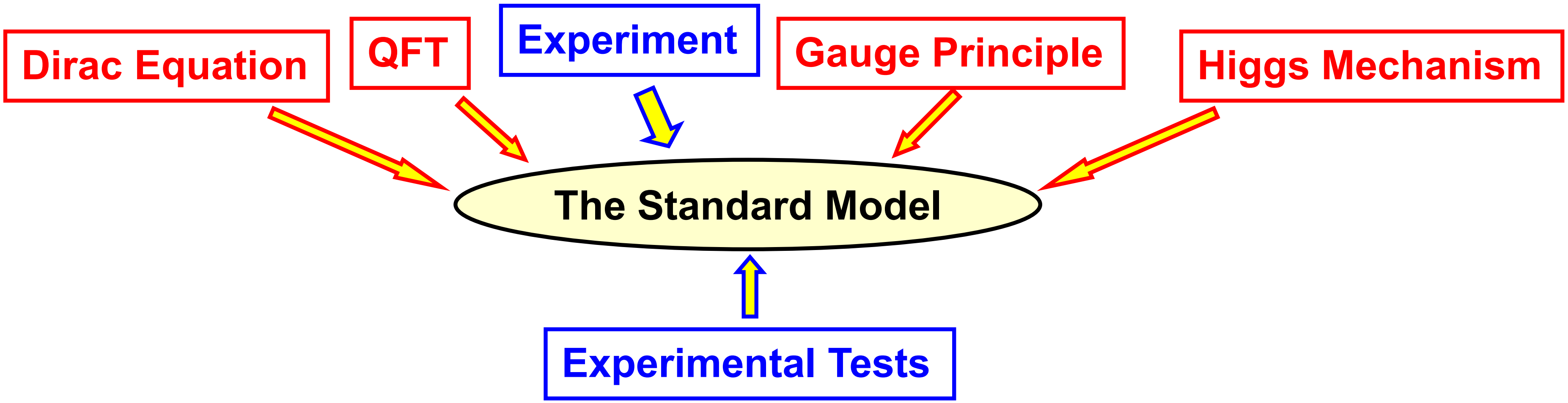
# Discovery of Higgs



# Concluding Remarks

- 粒子物理标准模型是二十世纪后期最伟大的科学成就之一
  - 实验和理论的共同发展起来的




- 现代实验粒子物理学提供了很多精确测量，标准模型**成功地描述了所有实验！**
- 尽管如此成功，我们不能忘记，标准模型只是一个模型
  - 散装了诸多有趣的理论概念来拟合实验数据
  - 但还存在很多问题

# Concluding Remarks

## ➢ Standard Model : Problems/Open Questions

- Standard Model has too many free parameters: $e, G_F, \theta_W, \alpha_S \quad m_H, \theta_{CP}$
$$m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, m_e, m_\mu, m_\tau, m_d, m_s, m_b, m_u, m_c, m_t$$
$$\theta_{12}, \theta_{13}, \theta_{23}, \delta \;\; + \;\; \lambda, A, \rho, \eta$$
- Why three generations ?
- Why $SU(3)_c \times SU(2)_L \times U(1)$ ?
- Unification of the Forces
- Origin of CP violation in early universe ?
- What is Dark Matter ?
- Why is the weak interaction V-A ?
- Why are neutrinos so light ?
- Does the Higgs gives rise to huge cosmological constant ?
- Ultimately need to include gravity

★ **Over the last 25 years particle physics has progressed enormously.**

**In the next 10 years we will almost certainly have answers to some of the above questions – maybe not the ones we expect…**

The difference comes from higher-order diagrams:

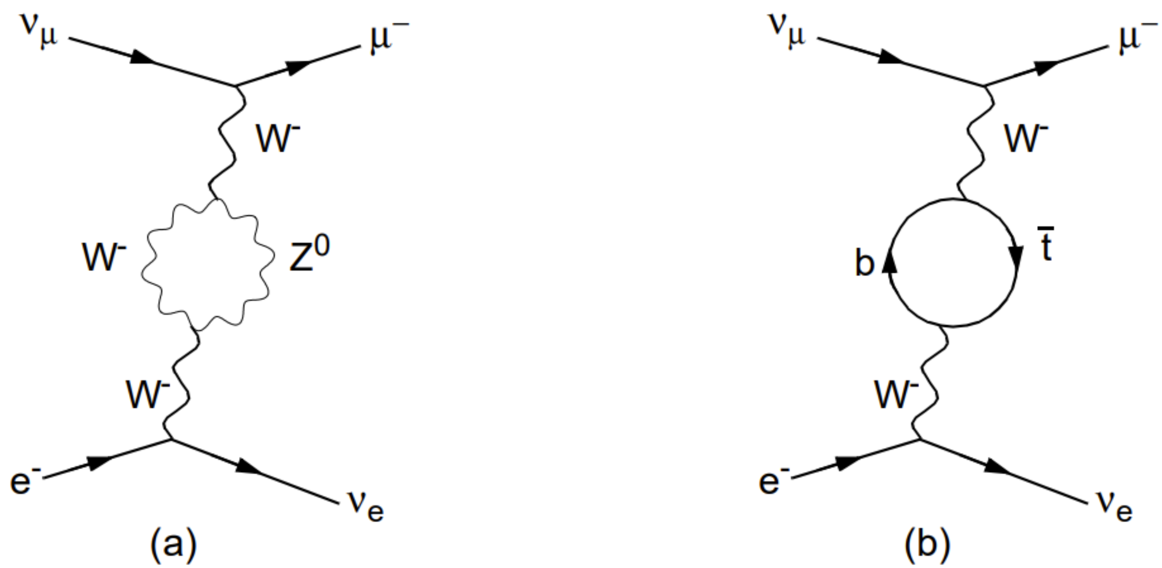



Figure 100: Examples of higher order contributions to inverse muon decay

From higher order corrections, the estimate of top-quark mass is:

$$m_t = 170 \pm 30\ GeV/c^2 \tag{158}$$

Measured value is $m_t = 174 \pm 5\ GeV/c^2$

**The End**

# Appendix I: Non-relativistic Breit-Wigner

★ For energies close to the peak of the resonance, can write $\sqrt{s} = m_Z + \Delta$

$$s = m_Z^2 + 2m_Z\Delta + \Delta^2 \approx m_Z^2 + 2m_Z\Delta \qquad \text{for} \quad \Delta \ll m_Z$$

so with this approximation

$$\begin{aligned}(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2 \approx (2m_Z\Delta)^2 + m_Z^2\Gamma_Z^2 &= 4m_Z^2(\Delta + \tfrac{1}{4}\Gamma_Z^2) \\ &= 4m_Z^2[(\sqrt{s} - m_Z)^2 + \tfrac{1}{4}\Gamma_Z^2]\end{aligned}$$

★ Giving: $\sigma(e^+e^- \to Z \to f\overline{f}) \approx \dfrac{3\pi}{m_Z^4} \dfrac{s}{(\sqrt{s} - m_Z)^2 + \frac{1}{4}\Gamma_Z^2} \Gamma_e \Gamma_f$

★ Which can be written:

$$\sigma(E) = \frac{g\lambda_e^2}{4\pi} \frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \frac{1}{4}\Gamma^2} \qquad \textbf{(3)}$$

$\Gamma_i,\ \Gamma_f$ : are the partial decay widths of the initial and final states

$E,\ E_0$ : are the centre-of-mass energy and the energy of the resonance

$g = \frac{(2J_Z+1)}{(2S_e+1)(2S_e+1)}$ is the spin counting factor $g = \frac{3}{2\times 2}$

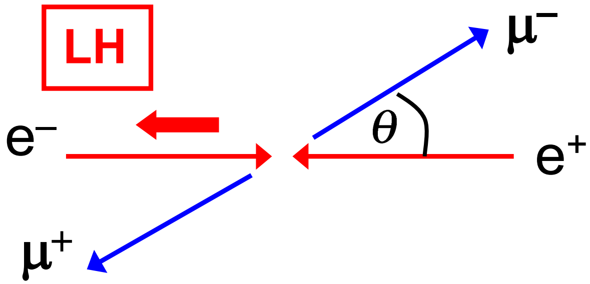
$\lambda_e = \frac{2\pi}{E}$ : is the Compton wavelength (natural units) in the C.o.M of either initial particle

★ This is the **non-relativistic** form of the **Breit-Wigner** distribution first encountered in the part II particle and nuclear physics course.

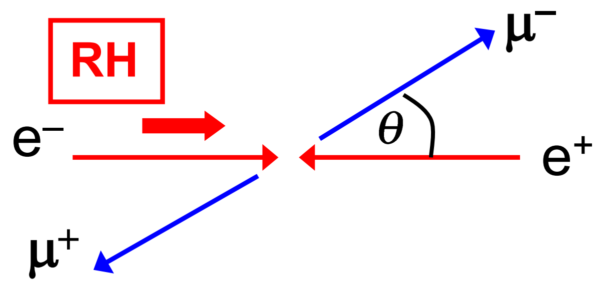
# Appendix II: Left-Right Asymmetry, $A_{LR}$

★ At an $e^+e^-$ linear collider it is possible to produce polarized electron beams e.g. SLC linear collider at SLAC (California), **1989-2000**

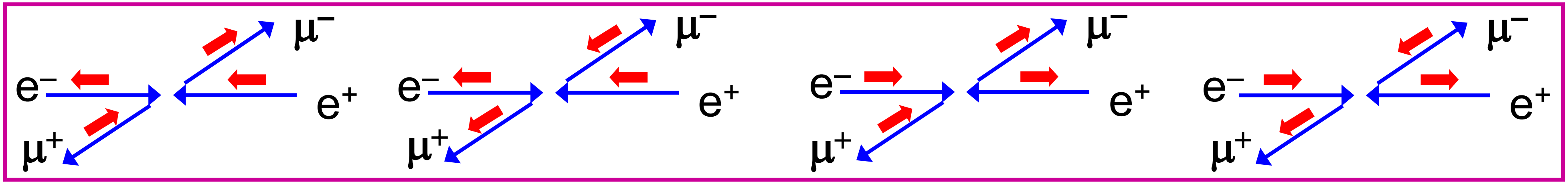
★ Measure cross section for any process for LH and RH electrons separately



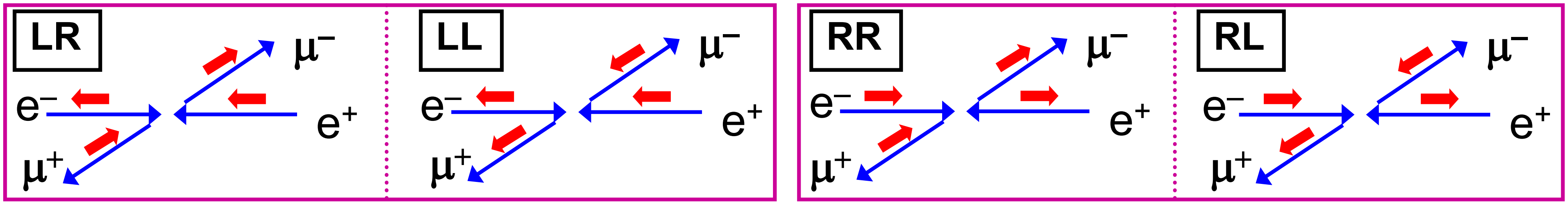

vs.




- At LEP measure total cross section: sum of 4 helicity combinations:




- At SLC, by choosing the polarization of the electron beam are able to measure cross sections **separately** for LH / RH electrons



★ **Averaging over the two possible polarization states of the positron for a given electron polarization:**

$$\langle|M_L|\rangle^2 = \frac{1}{2}(|M_{LL}|^2 + |M_{LR}|^2) \qquad \langle|M_R|\rangle^2 = \frac{1}{2}(|M_{RL}|^2 + |M_{RR}|^2)$$

$$\Rightarrow \quad \boxed{\sigma_L = \frac{1}{2}(\sigma_{LR} + \sigma_{LL})} \qquad \boxed{\sigma_R = \frac{1}{2}(\sigma_{RR} + \sigma_{RL})}$$

★ **Define cross section asymmetry:**

$$\boxed{A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}}$$

★ **Integrating the expressions on page 494 gives:**

$$\sigma_{LL} \propto (c_L^e)^2(c_L^\mu)^2 \quad \sigma_{LR} \propto (c_L^e)^2(c_R^\mu)^2 \quad \sigma_{RL} \propto (c_R^e)^2(c_L^\mu)^2 \quad \sigma_{RR} \propto (c_R^e)^2(c_R^\mu)^2$$

$$\Rightarrow \quad \sigma_L \propto (c_L^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2] \qquad \sigma_R \propto (c_R^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2]$$

$$\boxed{A_{LR} = \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = A_e}$$

★ **Hence the Left-Right asymmetry for any cross section depends only on the couplings of the electron**

# Appendix III: Higgs Mechanism Analog

- **上节课13引入规范对称性和电弱统一，但仅对于无质量的规范玻色子成立**
  - 引入质量将破坏底层的规范对称性
- **希格斯机制提供给于规范玻色子质量的方法**
- **本课介绍希格斯机制的核心思想，类比：**
  - 考虑电磁辐射穿越等离子体，等离子体为可极化的介质，得“能散关系”

$$n^2 = 1 - \frac{n_e e^2}{\varepsilon_0 m_e \omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

**n = refractive index**
**ω = angular frequency**
**$\omega_p$ = plasma frequency**

  - 因为与等离子体作用，波群(wave-groups)只有频率/能量高于某最小值时才能传播 $E > E_0 = \hbar\omega_p$
  - 在此能量以上的波，以群速度$v_g$传播 $v_g = \frac{c^2}{v_p} = nc$

# Appendix III: Higgs Mechanism Analog

- 去掉下标，利用前述n的表达式 $$v^2 = c^2 n^2 = c^2\left(1 - \frac{\hbar^2\omega_p^2}{\hbar^2\omega^2}\right) = c^2\left(1 - \frac{E_0^2}{E^2}\right)$$
- 重新组合后给出

$$\frac{E_0^2}{E^2} = 1 - \frac{v^2}{c^2} \Rightarrow \boxed{E = E_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \gamma mc^2}$$ 其中 $m = E_0/c^2$

- 在等离子体传播的无质量光子，表现为一个在真空中传播的有质量粒子！

> 提出一个标量场(自旋为0)，拥有非零的真空期望值(VEV)

- 在非零Higgs VEV的真空中传播的无质量的规范玻色子，对应于有质量粒子

> 希格斯是电中性但携带弱同位旋1/2

- 光子不与希格斯场耦合，保持无质量；W和Z玻色子耦合到弱超荷，获得质量