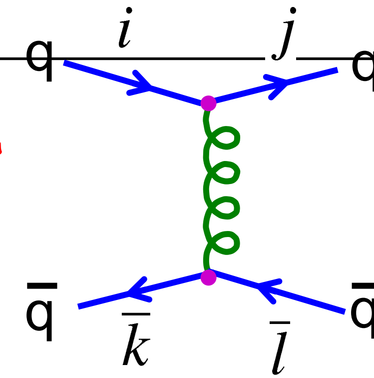
粒子物理学

习题课3

作业8

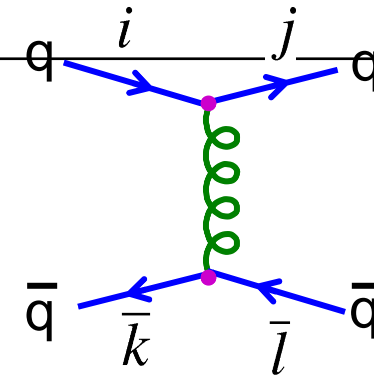


1. 考虑$q\bar{q} \to q\bar{q}$的散射过程，画出费曼图并根据QCD费曼规则写出矩阵元（将颜色部分放在一起）。

$$-iM = \left[\bar{u}_q(p_3)\left(-\frac{1}{2}ig_s\lambda_{ji}^a\gamma^\mu\right)u_q(p_1)\right]\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}\left[\bar{v}_{\bar{q}}(p_2)\left(-\frac{1}{2}ig_s\lambda_{kl}^b\gamma^\nu\right)v_{\bar{q}}(p_4)\right]$$

$$M = -\frac{g_s^2}{4}\lambda_{ji}^a\lambda_{kl}^a\frac{1}{q^2}g_{\mu\nu}[\bar{u}_q(p_3)\gamma^\mu u_q(p_1)][\bar{v}_{\bar{q}}(p_2)\gamma^\nu v_{\bar{q}}(p_4)]$$

作业8



2. 计算以下四种色因子

a) $C(r\bar{r} \to r\bar{r})$

b) $C(b\bar{g} \to b\bar{g})$

c) $C(r\bar{r} \to b\bar{b})$

d) $C(r\bar{g} \to g\bar{b})$

$$M = -\frac{g_s^2}{4}\lambda_{ji}^a\lambda_{kl}^a\frac{1}{q^2}g_{\mu\nu}[\bar{u}_q(p_3)\gamma^\mu u_q(p_1)][\bar{v}_{\bar{q}}(p_2)\gamma^\nu v_{\bar{q}}(p_4)]$$

$$C(i\bar{k} \longrightarrow j\bar{l}) \equiv \frac{1}{4}\sum_{a=1}^{8}\lambda_{ji}^a\lambda_{kl}^a$$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

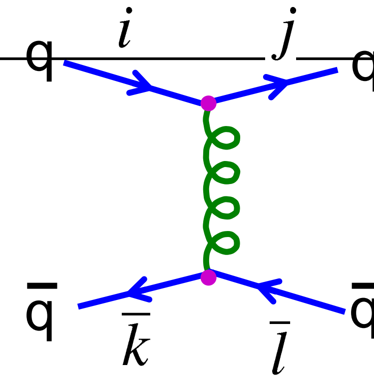
$$C(r\bar{r} \to r\bar{r}) = \frac{1}{4}\sum_{a=1}^{8}\lambda_{11}^a\lambda_{11}^a = \frac{1}{4}(\lambda_{11}^3\lambda_{11}^3 + \lambda_{11}^8\lambda_{11}^8) = \frac{1}{3}$$

$$C(b\bar{g} \to b\bar{g}) = \frac{1}{4}\sum_{a=1}^{8}\lambda_{22}^a\lambda_{33}^a = \frac{1}{4}(\lambda_{22}^8\lambda_{33}^8) = -\frac{1}{6}$$

$$C(r\bar{r} \to b\bar{b}) = \frac{1}{4}\sum_{a=1}^{8}\lambda_{21}^a\lambda_{12}^a = \frac{1}{4}(\lambda_{21}^1\lambda_{12}^1 + \lambda_{21}^2\lambda_{12}^2) = \frac{1}{2}$$

$$C(r\bar{g} \to g\bar{b}) = \frac{1}{4}\sum_{a=1}^{8}\lambda_{31}^a\lambda_{32}^a = 0$$

作业8

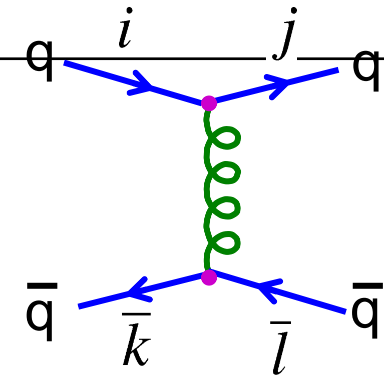


3. 计算出$q\bar{q} \to q\bar{q}$的散射过程的平均色因子$\langle |C|^2 \rangle$

$$\langle |C|^2 \rangle = \frac{1}{9} \sum_{i,j,k,l=1}^{3} |C(ij \to kl)|^2$$

$$\langle |C|^2 \rangle = \frac{1}{9}\left[3 \times \left(\frac{1}{3}\right)^2 + 6 \times \left(-\frac{1}{6}\right)^2 + 6 \times \left(\frac{1}{2}\right)^2\right] = \frac{2}{9}$$

作业8



4. 用Appendix D的方法计算以下四种色因子 $C(ik \to jl) = \frac{1}{2} c_1 c_2$

a) $C(r\bar{r} \to r\bar{r})$

b) $C(b\bar{g} \to b\bar{g})$

c) $C(r\bar{r} \to b\bar{b})$

d) $C(r\bar{g} \to g\bar{b})$

	$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$	$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$	$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$
胶子：	$r\bar{g}, g\bar{r}$	$r\bar{b}, b\bar{r}$	$g\bar{b}, b\bar{g}$	$\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$ $\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$

$$C(r\bar{r} \to r\bar{r}) = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{6}\right) = \frac{1}{3}$$

$\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$

$\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$

$$C(b\bar{g} \to b\bar{g}) = \frac{1}{2}\left(\frac{1}{6}(1 \times -2)\right) = -\frac{1}{6}$$

$\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$

$$C(r\bar{r} \to b\bar{b}) = \frac{1}{2}(1) = \frac{1}{2}$$

$r\bar{b}$

$$C(r\bar{g} \to g\bar{b}) = 0$$

作业9

1.试着解释为什么存在$\rho^0 \to \pi^-\pi^+$的衰变过程，但是不存在$\rho^0 \to \pi^0\pi^0$的衰变过程。
（提示：可以从角动量，宇称，波函数的对称性等角度思考）

先考虑几个粒子的自旋与宇称：

$$J^P(\rho^0) = 1^-, J^P(\pi^0) = J^P(\pi^+) = J^P(\pi^-) = 0^-$$

对于$\rho^0 \to \pi\pi$的过程，宇称守恒：

$$P_{\rho^0} = P_\pi P_\pi (-1)^L$$

$$(-1) = (-1)(-1)(-1)^L$$

$$L\text{必须为奇数}$$

若衰变到$\pi^-\pi^+$， $L = 1$并无不妥；

但若衰变到$\pi^0\pi^0$，末态为全同的玻色子，波函数交换对称，要求L为偶数，矛盾。

作业9

1.试着解释为什么存在$\rho^0 \to \pi^-\pi^+$的衰变过程，但是不存在$\rho^0 \to \pi^0\pi^0$的衰变过程。（提示：可以从角动量，宇称，波函数的对称性等角度思考）

$j_1 = 1,\ j_2 = 1$

$\pi^+ = |1,1\rangle, \quad \pi^0 = |1,0\rangle, \quad \pi^- = |1,-1\rangle$

$m = 2$

m_1, m_2 \ j	2
1, 1	1

$m = 1$

m_1, m_2 \ j	2	1
1, 0	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
0, 1	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$m = 0$

m_1, m_2 \ j	2	1	0
1, −1	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$
0, 0	$\sqrt{\frac{2}{3}}$	0	$-\sqrt{\frac{1}{3}}$
−1, 1	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$

$\rho^0 \to \pi^-\pi^+$

$\rho^0 \to \pi^0\pi^0$

作业9

2.轻子的代数可以通过W玻色子的总衰变宽度估计。标准模型预测 $W^- \to e^-\bar{\nu}_e$的衰变宽度写作：

$$\Gamma(W^- \to e^-\bar{\nu}_e) = \frac{G_F}{\sqrt{2}}\frac{m_W^3}{6\pi}$$

已知W玻色子的质量$m_W = 80.385\mathrm{GeV}$，总衰变宽度$\Gamma_W = 2.085\mathrm{GeV}$，试着由此估计轻子的代数。（$G_F = 1.2 \times 10^{-5}\mathrm{GeV}^{-2}$）

$$\Gamma_W = (2 \times 3 + n)\Gamma(W^- \to e^-\bar{\nu}_e)$$

2代夸克　色因子　n代轻子

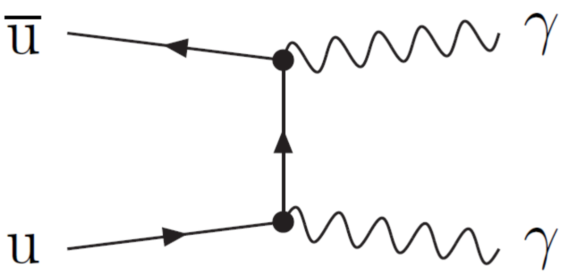
$$n = 2.9 \approx 3$$

作业9

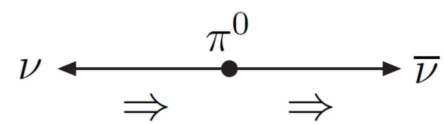
3. 考虑以下几组衰变。在每一组中，在标准模型范围内考虑每一个衰变是否可以发生，如果可以发生，画出对应的费曼图，如果不能，给出理由；在后面两组中，如果有多个过程可以发生，按照衰变率排名。

a) $\pi^0 \to \gamma\gamma$, $\pi^0 \to \pi^- e^+ \nu_e$, $\pi^0 \to \nu\bar{\nu}$

$$J(\pi^0) = 0$$



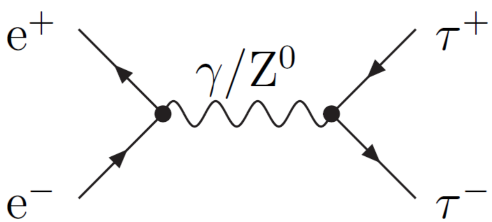
$$m_{\pi^\pm} > m_{\pi^0}$$

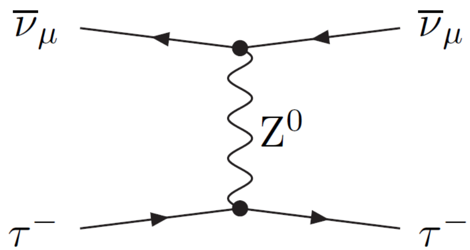


作业9

3. 考虑以下几组衰变。在每一组中，在标准模型范围内考虑每一个衰变是否可以发生，如果可以发生，画出对应的费曼图，如果不能，给出理由；在后面两组中，如果有多个过程可以发生，按照衰变率排名。

b) $e^+e^- \to \tau^+\tau^-$, $\bar{\nu}_\mu + \tau^- \to \bar{\nu}_\mu + \tau^-$, $\nu_\tau + p \to \tau^+ + n$





轻子数不守恒

是散射，题意不明

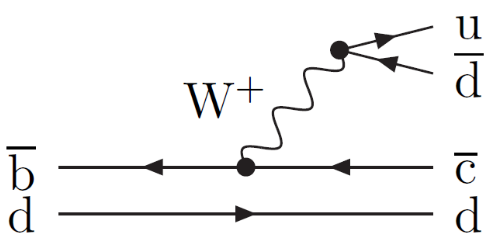
作业9

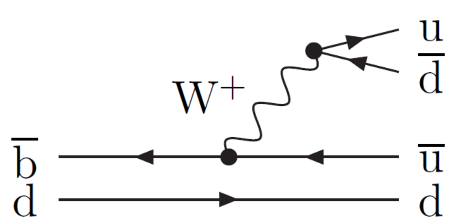
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} \cos\theta_C & \sin\theta_C & \sin^3\theta_C \\ -\sin\theta_C & \cos\theta_C & \sin^2\theta_C \\ \sin^3\theta_C & -\sin^2\theta_C & 1 \end{pmatrix}$$

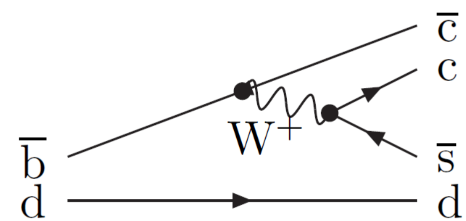
3. 考虑以下几组衰变。在每一组中，在标准模型范围内考虑每一个衰变是否可以发生，如果可以发生，画出对应的费曼图，如果不能，给出理由；在后面两组中，如果有多个过程可以发生，按照衰变率排名。

c) $B^0(\bar{b}d) \to D^-(\bar{c}d)\pi^+$, $B^0 \to \pi^+\pi^-$, $B^0 \to J/\psi K^0$

2.52×10^{-3} 5.12×10^{-6} 8.68×10^{-4}







$V_{bc}V_{ud}$ $V_{bu}V_{ud}$ $V_{bc}V_{cs}$

$1 > 3 > 2$

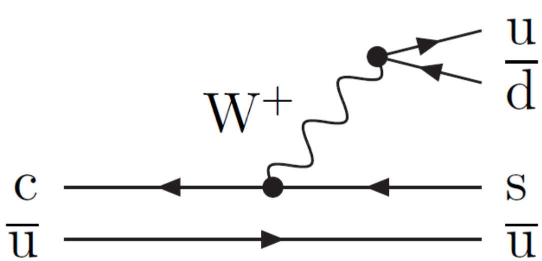
作业9

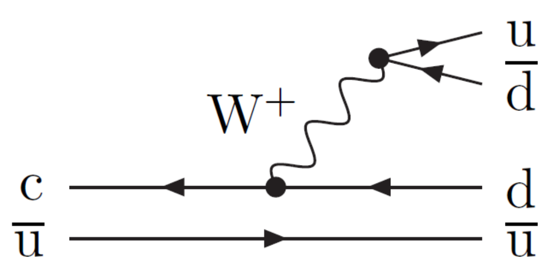
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} \cos\theta_C & \sin\theta_C & \sin^3\theta_C \\ -\sin\theta_C & \cos\theta_C & \sin^2\theta_C \\ \sin^3\theta_C & -\sin^2\theta_C & 1 \end{pmatrix}$$

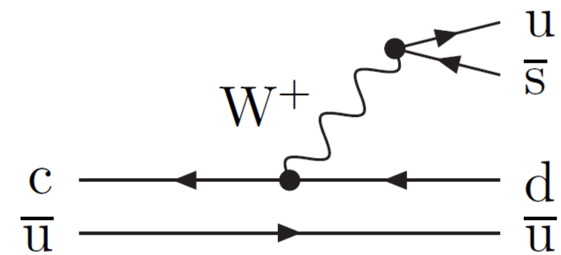
3. 考虑以下几组衰变。在每一组中，在标准模型范围内考虑每一个衰变是否可以发生，如果可以发生，画出对应的费曼图，如果不能，给出理由；在后面两组中，如果有多个过程可以发生，按照衰变率排名。

d) $D^0(c\bar{u}) \to K^-\pi^+$, $D^0 \to \pi^+\pi^-$, $D^0 \to K^+\pi^-$

3.95×10^{-2} 1.455×10^{-3} 1.5×10^{-4}







$V_{ud}V_{cs}$ $V_{ud}V_{cd}$ $V_{us}V_{cd}$

$$1 > 2 > 3$$

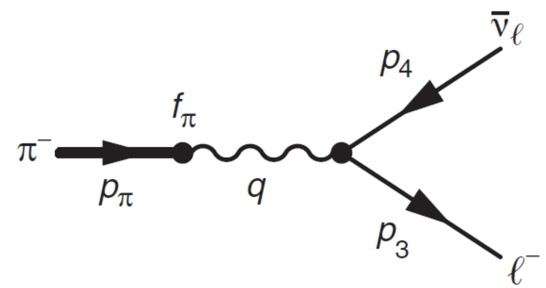
作业9

4.π介子衰变计算：课上我们学到，实验测量得到的$\frac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_e)}$值与预期并不相同，其原因是衰变到电子的过程被螺旋度压低了。之后课上进行了一部分的计算，但没有最终完成，在这里将其补完：

a) **考虑π介子静止系，先写出轻子流（注意手征算符）；至于夸克流，写成正比于$f_\pi p_\pi^\mu$的形式（参考书本11.6）**

$$j_\ell^\nu = \frac{g_W}{\sqrt{2}} \bar{u}(p_3) \frac{1}{2} \gamma^\nu (1 - \gamma^5) v(p_4)$$

$$j_\pi^\mu = \frac{g_W}{\sqrt{2}} \frac{1}{2} f_\pi p_\pi^\mu$$

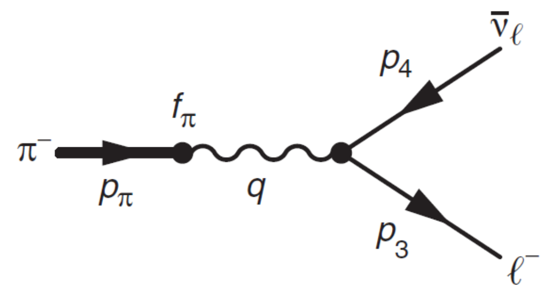


作业9

4. (b) 写出矩阵元M_{fi}的具体形式（考虑$q^2 = m_\pi^2 \ll m_W^2$）

$$M_{fi} = \left[\frac{g_W}{\sqrt{2}}\frac{1}{2}f_\pi p_\pi^\mu\right] \times \left[\frac{g_{\mu\nu}}{m_W^2}\right] \times \left[\frac{g_W}{\sqrt{2}}\bar{u}(p_3)\frac{1}{2}\gamma^\nu(1-\gamma^5)v(p_4)\right]$$
$$= \frac{g_W^2}{4m_W^2} g_{\mu\nu} f_\pi p_\pi^\mu \bar{u}(p_3)\frac{1}{2}\gamma^\nu(1-\gamma^5)v(p_4)$$

在π^-的静止系中，只有零分量不为零，$\bar{u}\gamma^0 = u^\dagger$

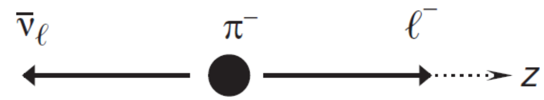


$$M_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3)\frac{1}{2}(1-\gamma^5)v(p_4)$$
$$= \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3) v_\uparrow(p_4)$$

作业9

4. (c)写出狄拉克方程的螺旋度解，并代入矩阵元具体计算，得到矩阵元平方$\left|M_{fi}\right|^2$的具体形式

$$M_{fi}=\frac{g_W^2}{4m_W^2}f_\pi m_\pi u^\dagger(p_3)v_\uparrow(p_4)$$



$$u_\uparrow(p_3)=\sqrt{E_\ell+m_\ell}\begin{pmatrix}1\\0\\\frac{p}{E_\ell+m_\ell}\\0\end{pmatrix},u_\downarrow(p_3)=\sqrt{E_\ell+m_\ell}\begin{pmatrix}1\\0\\0\\-\frac{p}{E_\ell+m_\ell}\end{pmatrix}\qquad v_\uparrow(p_3)=\sqrt{p}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}$$

根据四动量守恒

$$M_{fi}=\frac{g_W^2}{4m_W^2}f_\pi m_\pi\sqrt{E_\ell+m_\ell}\sqrt{p}\left(1-\frac{p}{E_\ell+m_\ell}\right)$$

$$=\frac{g_W^2}{4m_W^2}f_\pi m_\ell\sqrt{m_\pi^2-m_\ell^2}$$

$$E_\ell=\frac{m_\pi^2+m_e^2}{2m_\pi},|\vec{p}|=\frac{m_\pi^2-m_e^2}{2m_\pi}$$

$$\left|M_{fi}\right|^2=2G_F^2f_\pi^2m_\ell^2\left(m_\pi^2-m_\ell^2\right)$$

$$\frac{G_F}{\sqrt{2}}=\frac{g_W^2}{8m_W^2}$$

作业9

4. (d)计算出$\frac{\Gamma(\pi^-\to e^-\overline{\nu}_e)}{\Gamma(\pi^-\to\mu^-\overline{\nu}_e)}$的值

$$E_\ell = \frac{m_\pi^2 + m_e^2}{2m_\pi}, |\vec{p}| = \frac{m_\pi^2 - m_e^2}{2m_\pi}$$

$$\frac{1}{\tau} = \Gamma = \frac{|\vec{p}^*|}{32\pi^2 m_i^2}\int |M_{fi}|^2 \mathrm{d}\Omega$$

$$\left|M_{fi}\right|^2 = 2G_F^2 f_\pi^2 m_\ell^2\left(m_\pi^2 - m_\ell^2\right)$$

$$\Gamma = \frac{4\pi}{32\pi^2 m_\pi^2} p\left|M_{fi}\right|^2 = \frac{G_F^2}{8\pi m_\pi^3} f_\pi^2\left[m_\ell\left(m_\pi^2 - m_\ell^2\right)\right]^2$$

$$\frac{\Gamma(\pi^- \to e^-\bar{\nu}_e)}{\Gamma(\pi^- \to \mu^-\bar{\nu}_e)} = \left[\frac{m_e(m_\pi^2 - m_e^2)}{m_\mu\left(m_\pi^2 - m_\mu^2\right)}\right]^2 = 1.26 \times 10^{-4}$$

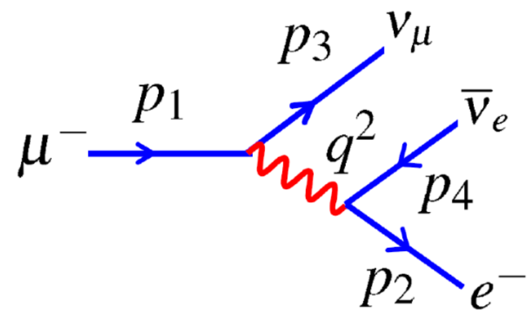
(e)计算$\frac{\Gamma(K^-\to e^-\overline{\nu}_e)}{\Gamma(K^-\to\mu^-\overline{\nu}_e)}$的值

2.55×10^{-5}

作业9

4. μ**子衰变计算：**

(a)写出图示过程的矩阵元（不用化简）



$$-i\mathcal{M}_{fi} = \left[\overline{u}(p_3)\frac{-ig_W}{\sqrt{2}}\gamma^\mu\frac{1-\gamma^5}{2}u(p_1)\right]$$

$$\frac{ig_{\mu\nu}}{m_W^2}$$

$$\left[\overline{u}(p_2)\frac{-ig_W}{\sqrt{2}}\gamma^\nu\frac{1-\gamma^5}{2}v(p_4)\right]$$

$$\mathcal{M}_{fi} = \frac{g_W^2}{8m_W^2}g_{\mu\nu}\left[\overline{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)\right]\left[\overline{u}(p_2)\gamma^\nu(1-\gamma^5)v(p_4)\right]$$

作业9

4. μ子衰变计算：

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu} \qquad P_L P_L = P_L$$

$$\mathrm{Tr}(\text{odd number of } \gamma\text{s}) = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4g^{\mu\nu}g^{\rho\sigma} - 4g^{\mu\rho}g^{\nu\sigma} + 4g^{\mu\sigma}g^{\nu\rho}$$

$$Tr[\gamma^\mu\gamma^\nu\gamma^5] = 0$$

$$Tr[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5] = 4i\epsilon^{\mu\nu\rho\sigma}$$

$$\epsilon^{abcd}\epsilon_{ebfd} = -2\delta_{ae}\delta_{cf} + 2\delta_{af}\delta_{ce}$$

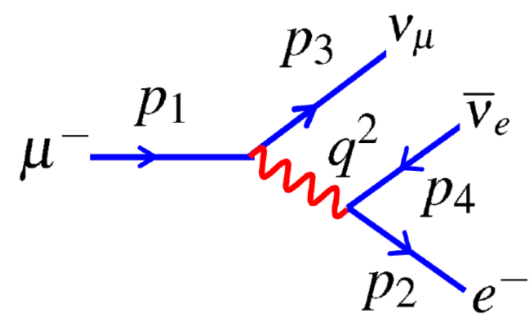
(b)使用之前使用过的求迹技巧进行计算

$$\left|M_{fi}\right|^2 = \frac{G_F^2}{2}\frac{1}{2}L_{\mu\nu}W_{\mu\nu}$$

$$L_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[(\not{p}_1 + m)\gamma^\mu\frac{1-\gamma^5}{2}(\not{p}_3)\gamma^\nu\frac{1-\gamma^5}{2}\right]$$
$$= 2\left[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu - g^{\mu\nu}p_1p_3 + i\epsilon^{\mu\nu\nu\sigma}p_{3\rho}p_{1\sigma}\right]$$

$$W_{\mu\nu} = 2\left[p_{2\mu}p_{4\nu} + p_{2\nu}p_{4\mu} - g_{\mu\nu}p_2p_4 + i\epsilon_{\mu\alpha\nu\beta}p_2^\alpha p_4^\beta\right]$$

对称项与反对称项混合的乘法，可以分别相乘

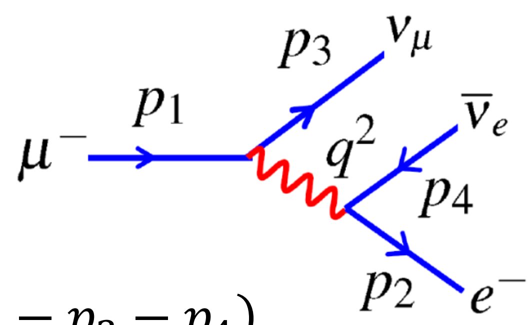


$$\left|M_{fi}\right|^2 = 64G_F^2(p_1\cdot p_4)(p_2\cdot p_3)$$

作业9

4. μ子衰变计算：

(c)计算出最后的衰变宽度Γ



$$d\Gamma = \frac{(2\pi)^4}{2m}\overline{|M|^2}\frac{d^3p_2}{(2\pi)^3 2E_{p2}}\frac{d^3p_3}{(2\pi)^3 2E_{p3}}\frac{d^3p_4}{(2\pi)^3 2E_{p4}}\delta^4(p_1-p_2-p_3-p_4)$$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

作业10

1. 填表

粒子（场）	I_{w}	I_{w}^3	Q	Y
$L_{iL}=\begin{pmatrix}\nu_{iL}\\ \ell_{iL}\end{pmatrix}: \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix}\begin{pmatrix}\nu_{\mu L}\\ \mu_L\end{pmatrix}\begin{pmatrix}\nu_{\tau L}\\ \tau_L\end{pmatrix}$				
没有右手中微子 $\ell_{iR}: e_R, \mu_R, \tau_R$				
$Q_{iL}=\begin{pmatrix}u_{iL}\\ d_{iL}\end{pmatrix}: \begin{pmatrix}u_L\\ d_L\end{pmatrix}\begin{pmatrix}c_L\\ s_L\end{pmatrix}\begin{pmatrix}t_L\\ b_L\end{pmatrix}$				
$u_{iR}: u_R, c_R, t_R$ $d_{iR}: d_R, s_R, b_R$				

费米子	Q	I_w^3	c_L	c_R	c_V	c_A
$\nu_{e,\mu,\tau}$						
e,μ,τ						
u,c,t						
d,s,b						

作业10

1. 填表

粒子（场）	I_w	I_w^3	Q	Y
$L_{iL}=\begin{pmatrix}\nu_{iL}\\ \ell_{iL}\end{pmatrix}:\begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix}\begin{pmatrix}\nu_{\mu L}\\ \mu_L\end{pmatrix}\begin{pmatrix}\nu_{\tau L}\\ \tau_L\end{pmatrix}$	$\frac{1}{2}$	$+\frac{1}{2}$ $-\frac{1}{2}$	0 -1	-1
没有右手中微子 $\ell_{iR}: e_R,\mu_R,\tau_R$	— 0	— 0	— -1	— -2
$Q_{iL}=\begin{pmatrix}u_{iL}\\ d_{iL}\end{pmatrix}:\begin{pmatrix}u_L\\ d_L\end{pmatrix}\begin{pmatrix}c_L\\ s_L\end{pmatrix}\begin{pmatrix}t_L\\ b_L\end{pmatrix}$	$\frac{1}{2}$	$+\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{2}{3}$ $-\frac{1}{3}$	$\frac{1}{3}$
$u_{iR}: u_R, c_R, t_R$ $d_{iR}: d_R, s_R, b_R$	0 0	0 0	$+\frac{2}{3}$ $-\frac{1}{3}$	$\frac{4}{3}$ $-\frac{2}{3}$

费米子	Q	I_w^3	c_L	c_R	c_V	c_A
$\nu_{e,\mu,\tau}$	0	$+\frac{1}{2}/-$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
e,μ,τ	-1	$-\frac{1}{2}/0$	$-\frac{1}{2}+\sin^2\theta_w$	$\sin^2\theta_w$	$-\frac{1}{2}+2\sin^2\theta_w$	$-\frac{1}{2}$
u,c,t	$+\frac{2}{3}$	$+\frac{1}{2}/0$	$\frac{1}{2}-\frac{2}{3}\sin^2\theta_w$	$-\frac{2}{3}\sin^2\theta_w$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_w$	$\frac{1}{2}$
d,s,b	$-\frac{1}{3}$	$-\frac{1}{2}/0$	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta_w$	$\frac{1}{3}\sin^2\theta_w$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

作业10

2. 仿照课件，从$j_\mu^{em}, j_\mu^{W^3}, j_\mu^{Y}$推导出$j_\mu^{Z}$的具体形式

$$j_\mu^{em} = e\overline{\psi}Q_e\gamma_\mu\psi = e\bar{\mathrm{e}}_L Q_{\mathrm{e}}\gamma_\mu \mathrm{e}_L + e\bar{\mathrm{e}}_R Q_e \gamma_\mu \mathrm{e}_R$$

$$j_\mu^{W^3} = -\frac{g_W}{2}\bar{\mathrm{e}}_L\gamma_\mu \mathrm{e}_L$$

$$j_\mu^{Y} = \frac{g'}{2}\overline{\psi}Y_e\gamma_\mu\psi = \frac{g'}{2}\bar{\mathrm{e}}_L Y_{\mathrm{e}_L}\gamma_\mu \mathrm{e}_L + \frac{g'}{2}\bar{\mathrm{e}}_R Y_{\mathrm{e}_R}\gamma_\mu \mathrm{e}_R$$

$$A_\mu = B_\mu\cos\theta_W + W_\mu^3\sin\theta_W$$

$$Z_\mu = -B_\mu\sin\theta_W + W_\mu^3\cos\theta_W$$

$$j_\mu^{em} = j_\mu^{Y}\cos\theta_W + j_\mu^{W^3}\sin\theta_W$$

$$e = g_W\sin\theta_W = g'\cos\theta_W$$

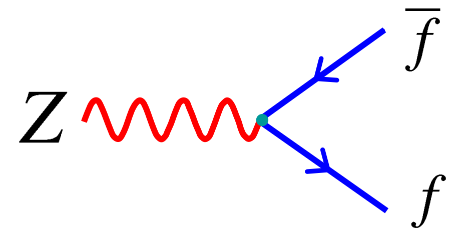
$$\begin{aligned} j_\mu^{Z} &= g_Z(I_W^3 - Q\sin^2\theta_W)[\bar{\mathrm{e}}_L\gamma_\mu \mathrm{e}_L] - g_Z Q\sin^2\theta_W[\mathrm{e}_R\gamma_\mu e_R] \\ &= g_Z c_L[\bar{\mathrm{e}}_L\gamma_\mu \mathrm{e}_L] + g_Z c_R[\mathrm{e}_R\gamma_\mu e_R] \end{aligned} \qquad g_Z = \frac{g_W}{\cos\theta_W}$$

$$c_L = I_W^3 - Q\sin^2\theta_W \qquad\qquad c_R = -Q\sin^2\theta_W$$

作业10

3. 考虑$Z^0 \to f\bar{f}$的过程，其中费米子与Z^0耦合常数为c_V, c_A

(a) 使用费曼规则写出$Z^0 \to f\bar{f}$的矩阵元，并整理成左手部分和右手部分



$$-iM = \epsilon_\mu(p_1)\bar{u}(p_3)\left(-ig_Z\gamma^\mu\frac{1}{2}(c_V - c_A\gamma^5)\right)v(p_4)$$

$$\frac{1}{2}(c_V - c_A\gamma^5) = c_L\frac{1}{2}(1-\gamma^5) + c_R\frac{1}{2}(1+\gamma^5)$$

$$M = g_Z\epsilon_\mu(p_1)\bar{u}(p_3)\gamma^\mu c_L\frac{1}{2}(1-\gamma^5)v(p_4) + g_Z\epsilon_\mu(p_1)\bar{u}(p_3)\gamma^\mu c_R\frac{1}{2}(1+\gamma^5)v(p_4)$$
$$= M_L + M_R$$

作业10

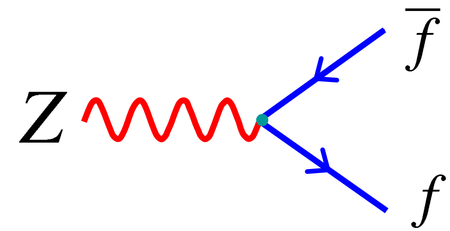
$$M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1)\bar{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_4)$$

3. 考虑$Z^0 \to f\bar{f}$的过程，其中费米子与Z^0耦合常数为c_V, c_A

(b) 仿照课件中计算W玻色子衰变宽度的方法，计算出$Z^0 \to f\bar{f}$的衰变宽度的表达式

$$M_L = g_Z\epsilon_\mu(p_1)\bar{u}(p_3)\gamma^\mu c_L\frac{1}{2}(1-\gamma^5)v(p_4)$$

$$M_R = g_Z\epsilon_\mu(p_1)\bar{u}(p_3)\gamma^\mu c_R\frac{1}{2}(1+\gamma^5)v(p_4)$$



$$\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) = 2E(0, -\cos\theta, i, \sin\theta)$$

$$M_R = g_Z c_R\epsilon_\mu(p_1)\bar{u}_\uparrow(p_3)\gamma^\mu v_\downarrow(p_4)$$

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \quad \varepsilon_L = \frac{1}{m}(p_z,0,0,E) \quad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

$$M_{R-} = \frac{1}{\sqrt{2}}g_Z c_R m_Z(\cos\theta - 1)$$

$$M_{RL} = g_Z c_R m_Z \sin\theta$$

$$M_{R+} = \frac{1}{\sqrt{2}}g_Z c_R m_Z(-\cos\theta - 1)$$

$$\langle|M_R|^2\rangle = \frac{2}{3}g_Z^2 m_Z^2 c_R^2$$

$$\langle|M|^2\rangle = \frac{2}{3}g_Z^2 m_Z^2(c_L^2 + c_R^2)$$

$$c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$$

$$\Gamma(Z \to f\bar{f}) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

作业10

3. 考虑$Z^0 \to f\bar{f}$的过程，其中费米子与Z^0耦合常数为c_V, c_A

（c） $R_\mu = \frac{\Gamma(Z\to\mu^+\mu^-)}{\Gamma(Z\to hadrons)}$

$$\Gamma(Z \to f\bar{f}) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

费米子	Q	I_W^3	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

$$R_\mu = \frac{\Gamma(Z \to \mu^+\mu^-)}{\Gamma(Z \to hadrons)} = \frac{0.04^2 + 0.5^2}{9 \times (0.35^2 + 0.5^2) + 6 \times (0.19^2 + 0.5^2)} = 0.049$$

作业10

4. 在考虑了QED的效应后，$e^+e^- \to \mu^+\mu^-$和$e^+e^- \to hadrons$经过Z共振态的截面为

$$\sigma^0(e^+e^- \to Z \to \mu^+\mu^-) = 1.9993\ nb$$

$$\sigma^0(e^+e^- \to Z \to hadrons) = 41.476\ nb$$

(a) 假设轻子普适性，计算$\frac{\Gamma_{ee}}{\Gamma_Z}$和$\frac{\Gamma_{hadrons}}{\Gamma_Z}$（使用公式$\sigma^0 = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$，记得单位换算）

$$\sigma_\mu^0 = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}^2}{\Gamma_Z^2} \qquad \frac{\Gamma_{ee}}{\Gamma_Z} = \sqrt{\frac{\sigma_\mu^0 m_Z^2}{12\pi}} = 0.0337$$

$$\sigma_{had}^0 = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{hadrons}}{\Gamma_Z^2} \qquad \frac{\Gamma_{hadrons}}{\Gamma_Z} = \frac{\sigma_{had}^0 m_Z^2}{12\pi} / \frac{\Gamma_{ee}}{\Gamma_Z} = 0.6992$$

作业10

4. 在考虑了QED的效应后，$e^+e^- \to \mu^+\mu^-$和$e^+e^- \to hadrons$经过Z共振态的截面为

$$\sigma^0(e^+e^- \to Z \to \mu^+\mu^-) = 1.9993\,nb$$
$$\sigma^0(e^+e^- \to Z \to hadrons) = 41.476\,nb$$

(b) 使用测量值$\Gamma_Z = 2.4952\mathrm{GeV}$和理论值$\Gamma_{\nu\nu} = 167\mathrm{MeV}$，试着估算中微子有几代

$$\Gamma_{\mathrm{Z}} = 3\Gamma_{\ell\ell} + \Gamma_{\mathrm{hadrons}} + N_\nu \Gamma_{\nu\nu}$$

$$N_\nu = \frac{\Gamma_{\mathrm{Z}} - 3\Gamma_{\ell\ell} - \Gamma_{\mathrm{hadrons}}}{\Gamma_{\nu\nu}} = \frac{2.4952 \times (1 - 3 \times 0.0337 - 0.6992)}{0.167} \approx 2.98$$

作业11

1. 缪子前后向不对称性参数的测量值$A_\mu = 0.1456$，试计算$\sin^2\theta$的值。

$$A_f \equiv \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2}$$

费米子	Q	I_W^3	c_L	c_R	c_V	c_A
$\nu_{e,\mu,\tau}$	0	$+\frac{1}{2}/-$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
e,μ,τ	-1	$-\frac{1}{2}/0$	$-\frac{1}{2}+\sin^2\theta_W$	$\sin^2\theta_W$	$-\frac{1}{2}+2\sin^2\theta_W$	$-\frac{1}{2}$
u,c,t	$+\frac{2}{3}$	$+\frac{1}{2}/0$	$\frac{1}{2}-\frac{2}{3}\sin^2\theta_W$	$-\frac{2}{3}\sin^2\theta_W$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
d,s,b	$-\frac{1}{3}$	$-\frac{1}{2}/0$	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

$$\sin^2\theta \approx 0.2317$$

作业11

2. 使用幺正规范下的$\phi(x)$，写出自由复标量场最终的拉氏量，并解释每一项的意义。

$$\mathcal{L} = \left(\partial_\mu \phi\right)^* (\partial^\mu \phi) + \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$-qi(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) A_\mu + q^2 A_\mu A^\mu \phi^* \phi$$

$$\phi(x) = \frac{1}{\sqrt{2}} [\eta(x) + v]$$

$$\mathcal{L}_\eta = \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{\lambda}{4} \eta^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$+ \frac{1}{2} q^2 A_\mu A^\mu \eta^2 + q^2 v A_\mu A^\mu \eta + \frac{1}{2} q^2 v^2 A_\mu A^\mu$$

作业11

3. 尝试化简$\mathcal{L}_S=(\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}_\mu\Phi)+\mu^2\Phi^\dagger\Phi-\lambda(\Phi^\dagger\Phi)^2$，整理成包含物理场的形式，并指出H、W、Z的质量项。

$$\Phi(x)=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}$$

$$Z_\mu=-B_\mu sin\theta_W+W_\mu^3 cos\theta_W$$

$$e=g_W\sin\theta_W=g'\cos\theta_W$$

$$\mathcal{D}_\mu(x)\Phi(x)=\left(\partial_\mu+ig\frac{\sigma^a}{2}W_\mu^a+ig'\frac{1}{2}B_\mu\right)\Phi(x)$$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}\partial_\mu+\frac{ig_W}{2}W_\mu^3+\frac{ig'}{2}B_\mu & \frac{ig_W}{2}[W_\mu^1-iW_\mu^2]\\ \frac{ig_W}{2}[W_\mu^1+iW_\mu^2] & \partial_\mu-\frac{ig_W}{2}W_\mu^3+\frac{ig'}{2}B_\mu\end{pmatrix}\begin{pmatrix}0\\ v+H\end{pmatrix}$$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}\partial_\mu+\frac{ig_W}{2cos\theta_W}Z_\mu & \frac{ig_W}{\sqrt{2}}W_\mu^+\\ \frac{ig_W}{\sqrt{2}}W_\mu^- & \partial_\mu-\frac{ig_W}{2cos\theta_W}Z_\mu\end{pmatrix}\begin{pmatrix}0\\ v+H\end{pmatrix}$$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}i\frac{g_W}{\sqrt{2}}W_\mu^+(x)[v+H(x)]\\ \partial_\mu H(x)-i\frac{g_W}{2\cos\theta_W}Z_\mu(x)[v+H(x)]\end{pmatrix}$$

作业11

3. 尝试化简$\mathcal{L}_S = \left(\mathcal{D}_\mu\Phi\right)^\dagger\left(\mathcal{D}_\mu\Phi\right) + \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$，整理成包含物理场的形式，并指出H、W、Z的质量项。

$$\Phi(\mathrm{x}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu + H(x) \end{pmatrix}$$

$$\mathcal{D}_\mu(x)\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} i\frac{g_W}{\sqrt{2}}W_\mu^+(x)[\nu + H(x)] \\ \partial_\mu H(x) - i\frac{g_W}{2\cos\theta_W}Z_\mu(x)[\nu + H(x)] \end{pmatrix}$$

$$\mathcal{L}_S = \frac{1}{2}\left(\partial_\mu H\right)(\partial^\mu H) - \lambda\nu^2H^2 - \lambda\nu H^3 - \frac{1}{4}\lambda H^4$$
$$+ \frac{\nu^2g^2}{4}W_\mu^+W^{-\mu} + \frac{\nu^2g^2}{8\cos^2\theta_W}Z_\mu Z^\mu$$
$$+ \frac{\nu g^2}{2}W_\mu^+W^{-\mu}H + \frac{\nu g^2}{4\cos^2\theta_W}Z_\mu Z^\mu H + \frac{g^2}{4}W_\mu^+W^{-\mu}H^2 + \frac{g^2}{8\cos^2\theta_W}Z_\mu Z^\mu H^2$$

作业11

4. 尝试化简$\mathcal{L} = -\sum_{i=1,2,3}\sum_{j=u,c,t}\left(Y'^{u}_{ij}\bar{Q}^{i}_{iL}\tilde{\Phi}u'_{jR}\right)$，并指出质量项和相互作用项。

$$\bar{Q}^{i}_{iL} = \begin{pmatrix} \bar{u}'_{iL} \\ \bar{d}'_{iL} \end{pmatrix}^{T} \quad \tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathcal{L} = -\frac{v + H(x)}{\sqrt{2}}\sum_{j=u,c,t}\left(Y'^{u}_{ij}\bar{u}'_{iL}u'_{jR}\right)$$

$$\mathcal{L}_{F-S,Q} = -\frac{v+H}{\sqrt{2}}\left[\sum_{i,j=d,s,b}\left(Y'^{d}_{ij}\bar{d}'_{iL}d'_{jR}\right) + \sum_{i,j=u,c,t}\left(Y'^{u}_{ij}\bar{u}'_{iL}u'_{jR}\right)\right] + h.c.$$

需要将Y'^{d}_{ij}和Y'^{u}_{ij}对角化

定义列矩阵

$$\boldsymbol{u}'_{\boldsymbol{L}} = \begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix}, \boldsymbol{u}'_{\boldsymbol{R}} = \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix}, \quad \boldsymbol{d}'_{\boldsymbol{L}} = \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}, \quad \boldsymbol{d}'_{\boldsymbol{R}} = \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}$$

$$\mathcal{L}_{F-S,Q} = -\frac{v+H}{\sqrt{2}}\left[\bar{\boldsymbol{d}}'_{\boldsymbol{L}}Y'^{d}\boldsymbol{d}'_{\boldsymbol{R}} + \bar{\boldsymbol{u}}'_{\boldsymbol{L}}Y'^{u}\boldsymbol{u}'_{\boldsymbol{R}}\right] + h.c.$$

$$V^{d\dagger}_{L}Y'^{d}V^{d}_{R} = Y^{d}, \qquad Y^{d}_{ij} = y^{d}_{i}\delta_{ij}$$

$$V^{u\dagger}_{L}Y'^{u}V^{u}_{R} = Y^{u}, \qquad Y^{u}_{ij} = y^{u}_{i}\delta_{ij}$$

作业11

4. 尝试化简$\mathcal{L} = -\sum_{i=1,2,3}\sum_{j=u,c,t}\left(Y_{ij}'^{u}\bar{Q}_{iL}^{i}\widetilde{\Phi}u_{jR}'\right)$，并指出质量项和相互作用项。

$$\mathcal{L}_{F-S,Q} = -\frac{\nu + H}{\sqrt{2}}\left[\bar{\boldsymbol{d}}_{\boldsymbol{L}}Y^{d}\boldsymbol{d}_{\boldsymbol{R}} + \bar{\boldsymbol{u}}_{\boldsymbol{L}}Y^{u}\boldsymbol{u}_{\boldsymbol{R}}\right] + h.c.$$

$$\boldsymbol{u}_{\boldsymbol{L}} = V_L^{u\dagger}\boldsymbol{u}_{\boldsymbol{L}}' = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}, \boldsymbol{u}_{\boldsymbol{R}} = V_R^{u\dagger}\boldsymbol{u}_{\boldsymbol{R}}' = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$$

$$\boldsymbol{d}_{\boldsymbol{L}} = V_L^{d\dagger}\boldsymbol{d}_{\boldsymbol{L}}' = \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}, \boldsymbol{d}_{\boldsymbol{R}} = V_R^{d\dagger}\boldsymbol{d}_{\boldsymbol{R}}' = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}$$

$$\mathcal{L}_{F-S,Q} = -\frac{\nu + H(x)}{\sqrt{2}}\sum_{j=u,c,t} y_i^u \bar{u}_i u_i = -\sum_{i=u,c,t}\frac{y_i^u \nu}{\sqrt{2}}\bar{u}_i u_i - \sum_{i=u,c,t}\frac{y_i^u}{\sqrt{2}}\bar{u}_i u_i\, H$$

作业11

4. 尝试化简$\mathcal{L} = -\sum_{i=1,2,3}\sum_{j=u,c,t}\left(Y'^{u}_{ij}\bar{Q}^{i}_{iL}\tilde{\Phi}u'_{jR}\right)$，并指出质量项和相互作用项。

$$j^{\mu}_{W,Q} = 2\bar{\boldsymbol{u}}'_{\boldsymbol{L}}\gamma^{\mu}\boldsymbol{d}'_{\boldsymbol{L}}$$

$$j^{\mu}_{W,Q} = 2\bar{\boldsymbol{u}}_{\boldsymbol{L}}V^{u\dagger}_{L}\gamma^{\mu}V^{d}_{L}\boldsymbol{d}_{\boldsymbol{L}} = 2\bar{\boldsymbol{u}}_{\boldsymbol{L}}\gamma^{\mu}V^{u\dagger}_{L}V^{d}_{L}\boldsymbol{d}_{\boldsymbol{L}} = 2\bar{\boldsymbol{u}}_{\boldsymbol{L}}\gamma^{\mu}V\boldsymbol{d}_{\boldsymbol{L}}$$

$$V \equiv V^{u\dagger}_{L}V^{d}_{L}$$

$$\begin{aligned}
j^{\mu}_{Z,Q} &= 2g^{u}_{L}\bar{\boldsymbol{u}}'_{L}\gamma^{\mu}\boldsymbol{u}'_{L} + 2g^{u}_{R}\bar{\boldsymbol{u}}'_{R}\gamma^{\mu}\boldsymbol{u}'_{R} + 2g^{d}_{L}\bar{\boldsymbol{d}}'_{L}\gamma^{\mu}\boldsymbol{d}'_{L} + 2g^{d}_{R}\bar{\boldsymbol{d}}'_{R}\gamma^{\mu}\boldsymbol{d}'_{R} \\
&= 2g^{u}_{L}\bar{\boldsymbol{u}}_{L}V^{u\dagger}_{L}\gamma^{\mu}V^{u}_{L}\boldsymbol{u}_{L} + 2g^{u}_{R}\bar{\boldsymbol{u}}_{R}V^{u\dagger}_{R}\gamma^{\mu}V^{u}_{R}\boldsymbol{u}_{R} \\
&\qquad + 2g^{d}_{L}\bar{\boldsymbol{d}}_{L}V^{d\dagger}_{L}\gamma^{\mu}V^{d}_{L}\boldsymbol{d}_{L} + 2g^{d}_{R}\bar{\boldsymbol{d}}_{R}V^{d\dagger}_{R}\gamma^{\mu}V^{d}_{R}\boldsymbol{d}_{R} \\
&= 2g^{u}_{L}\bar{\boldsymbol{u}}_{L}\gamma^{\mu}\boldsymbol{u}_{L} + 2g^{u}_{R}\bar{\boldsymbol{u}}_{R}\gamma^{\mu}\boldsymbol{u}_{R} + 2g^{d}_{L}\bar{\boldsymbol{d}}_{L}\gamma^{\mu}\boldsymbol{d}_{L} + 2g^{d}_{R}\bar{\boldsymbol{d}}_{R}\gamma^{\mu}\boldsymbol{d}_{R}
\end{aligned}$$

作业11

5. 画出LHC上pp对撞产生Higgs四种主要模式的费曼图