一、 单选

1. 以下四组国际单位制-自然单位制的换算，正确的有：

$$c = \text{speed of light} = 2.9979 \times 10^{8}\ \text{m/s}$$
$$\hbar = \text{reduced Planck constant} = 1.0546 \times 10^{-34}\ \text{J s}$$

①$1\text{GeV} = 1.8 \times 10^{-27}\text{kg}$

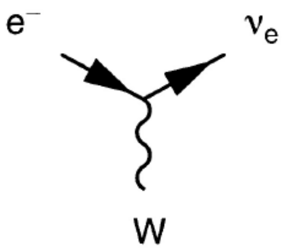
②$1\text{GeV}^2 = 8.19 \times 10^{-5}\text{N}$

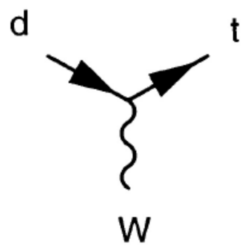
③$1\text{GeV} = 5.39 \times 10^{-19}\text{kg} \cdot \text{m/s}$

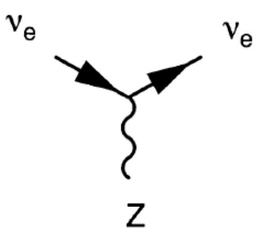
④$1\text{GeV}^{-1} = 6.58 \times 10^{-28}\text{s}$

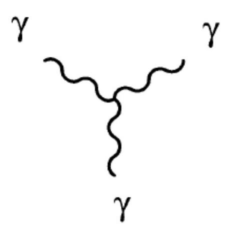
A.①④　　B.①③　　C.②④　　D.①②③

2. 以下哪个选项不是标准模型顶点：

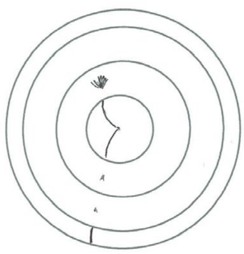
A. 

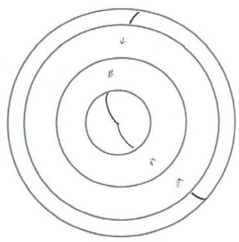
B. 

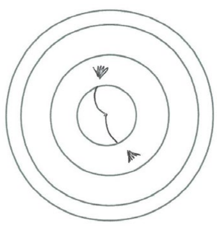
C. 

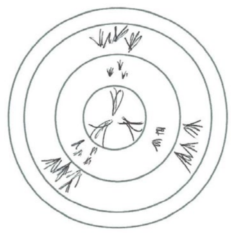
D. 

3. 以下选项中，哪项最可能是$e^+e^- \to Z \to qq$留下的径迹：

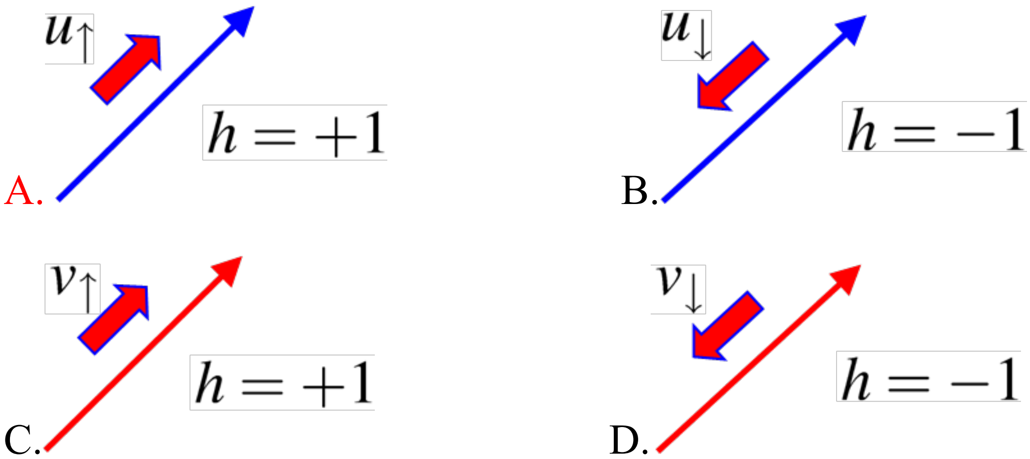
A. 

B. 

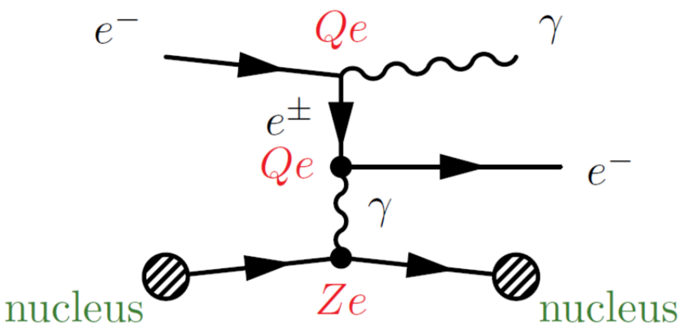
C. 

D. 

4. 选出正粒子的右手螺旋度本征态：



5. 根据图像估算并选出轫致辐射的矩阵元量级（原子核核电荷数为Z）：



A.$M \propto e^3$　　B.$M \propto Z^2e^4$　　C.$M \propto Ze^3$　　D.$M \propto Z^2e^2$

6. 以下手征算符作用结果错误的是：

A.$P_R u_R = u_R$　　B. $P_L u_R = 0$

C.$P_L v_R = v_L$　　D. $P_R v_L = 0$

7. $\rho^0 \to e^+e^-$的衰变宽度与$\omega^0 \to e^+e^-$的衰变宽度之比大约等于？其中，

π^0,ρ^0组分为$(u\bar{u} - d\bar{d})/\sqrt{2}$，$\omega^0$为$(u\bar{u} + d\bar{d})/\sqrt{2}$

A.9　　B.81　　C.3　　D.$\frac{1}{3}$

二、 判断

1. 考虑一个二体散射过程，若为弹性碰撞，质心系中算得的衰变截面与质心能量的平方成反比。√
2. 相较电磁簇射，强子簇射虽然过程更加复杂，但是相互作用强度反而更短。×
3. 强子簇射过程中也会发生电磁簇射。√
4. 因为反粒子沿着时间反向传播，所以费曼图上反粒子的箭头总是与时间相反。×
5. t-channel 的传播子总是离壳的。√
6. 深度非弹性碰撞中，微分散射截面公式中的电磁结构函数和纯磁结构函数互相独立。×
7. 如果夸克自旋为 0，深度非弹性碰撞微分散射截面公式中的纯磁结构函数也为 0. √
8. 强作用和电磁作用过程中同位旋守恒。×
9. 自然界存在色波函数为$\frac{1}{\sqrt{3}}\left(r\bar{r}+g\bar{g}+b\bar{b}\right)$的胶子。×
10. 与 QED 的跑动耦合常数不同，QCD 的跑动耦合常数随着Q^2下降。√

三、 简答与计算

1. 考虑一个二体衰变过程$X\rightarrow 1+2$，计算末态的不变质量。

$$M_X^2=m_1^2+m_2^2+2\left(E_1E_2-|\vec{p}_1||\vec{p}_2|\cos\theta\right)$$

2. 考虑$1+2\rightarrow 3+4$的过程，画出 s-channel 和 t-channel 的图像，并写出 s 和 t 的定义式。

$$s=(p_1+p_2)^2,\quad t=(p_1-p_3)^2$$

3. 推导对撞机和固定靶的质心能量。计算要达到质子-质子 200GeV 质心

能量对撞，固定靶实验需要的束流能量是多少（答案: E=20000GeV）。

- **Fixed Target Collision**:

$$s = m_1^2 + m_2^2 + 2E_1 m_2$$

$\overrightarrow{\mathbf{p}_1^{\mu}(\mathbf{E},\bar{\mathbf{p}})}$ $\bullet\ \mathbf{p}_2^{\mu}(\mathbf{m}_2,\mathbf{0})$

For $E_1 \gg m_1, m_2$, we have $s = 2E_1 m_2$
The energy in the CM frame $\sqrt{s} = \sqrt{2E_1 m_2}$
e.g. $100\,\mathrm{GeV}$ proton hitting a proton ar rest

$$\sqrt{s} = \sqrt{2E_p m_p} \approx \sqrt{2 \times 100 \times 1} \approx 14\,\mathrm{GeV}$$

- **Collider Experiment** (pp **colliding head on**):

$$s = m_1^2 + m_2^2 + 2(E_1 E_2 - |\mathbf{p}_1||\mathbf{p}_2| \cos\theta)$$

$\overrightarrow{\mathbf{p}_1^{\mu}(\mathbf{E},\bar{\mathbf{p}})}$ $\overleftarrow{\mathbf{p}_2^{\mu}(\mathbf{E},-\bar{\mathbf{p}})}$

If $E \gg m_1, m_2$, then $s = 2(E^2 - E^2 \cos\theta) = 4E^2$
The energy in the CM frame $\sqrt{s} = 2E$
e.g. $100\,\mathrm{GeV}$ proton colliding with a $100\,\mathrm{GeV}$ proton

$$\sqrt{s} = 2E = 2 \times 100 = 200\,\mathrm{GeV}$$

4. γ矩阵相关计算：

a) 定义：$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$，证明：

i. $(\gamma^5)^2 = I$.

ii. $\{\gamma^\mu, \gamma^5\} = 0$.

iii. $(\gamma^5)^\dagger = \gamma^5$.

$$\boldsymbol{(i)}\quad (\gamma^5)^2 =-1 \times \gamma^0\gamma^1\gamma^2\gamma^3\gamma^0\gamma^1\gamma^2\gamma^3 =-1 \times 1 \times -1 \times -1 \times -1 = 1$$

$$\boldsymbol{(ii)}\quad \{\gamma^0, \gamma^5\} = i\gamma^0\gamma^0\gamma^1\gamma^2\gamma^3 + i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^0 = i(\gamma^1\gamma^2\gamma^3 - \gamma^1\gamma^2\gamma^3) = 0$$
$$\{\gamma^1, \gamma^5\} = i\gamma^1\gamma^0\gamma^1\gamma^2\gamma^3 + i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^1 = i(\gamma^0\gamma^2\gamma^3 - \gamma^0\gamma^2\gamma^3) = 0$$
$$\{\gamma^2, \gamma^5\} = i\gamma^2\gamma^0\gamma^1\gamma^2\gamma^3 + i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^2 = i(-\gamma^0\gamma^1\gamma^3 + \gamma^0\gamma^1\gamma^3) = 0$$
$$\{\gamma^3, \gamma^5\} = i\gamma^1\gamma^0\gamma^1\gamma^2\gamma^3 + i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^3 = i(\gamma^0\gamma^1\gamma^2 - \gamma^0\gamma^1\gamma^2) = 0$$

$$\boldsymbol{(iii)}\quad (\gamma^5)^\dagger =-i(\gamma^3)^\dagger(\gamma^2)^\dagger(\gamma^1)^\dagger(\gamma^0)^\dagger = i\gamma^3\gamma^2\gamma^1\gamma^0 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma^5$$

b) 利用$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$，尝试证明：

$$\gamma^\mu\gamma_\mu = 4, \qquad \gamma^\mu \not{a} \gamma_\mu = -2\not{a}, \qquad \gamma^\mu \not{a}\not{b} \gamma_\mu = 4a \cdot b$$

i) $\gamma^\mu\gamma_\mu = g_{\mu\nu}\gamma^\mu\gamma^\nu = g_{\mu\nu}(2g^{\mu\nu} - \gamma^\nu\gamma^\mu) = 8 - g_{\mu\nu}\gamma^\nu\gamma^\mu = 8 - \gamma_\mu\gamma^\mu$

$$\gamma^\mu\gamma_\mu = 4$$

ii) $\gamma^\mu \not{a} \gamma_\mu = \gamma^\mu\gamma^\nu a_\nu \gamma_\mu = (2g^{\mu\nu} - \gamma^\nu\gamma^\mu) a_\nu \gamma_\mu = 2g^{\mu\nu} a_\nu \gamma_\mu - \gamma^\nu\gamma^\mu a_\nu \gamma_\mu = 2\not{a} - 4\not{a} =-2\not{a}$

iii) $\gamma^\mu \not{a}\not{b} \gamma_\mu = \gamma^\mu\gamma^\nu a_\nu \gamma^\sigma b_\sigma \gamma_\mu = \gamma^\mu\gamma^\nu a_\nu b^\sigma \gamma_\sigma\gamma_\mu = (2g^{\mu\nu} - \gamma^\nu\gamma^\mu) a_\nu b^\sigma (2g_{\sigma\mu} - \gamma_\mu\gamma_\sigma)$

$$= 4g^{\mu\nu} a_\nu b^\sigma g_{\sigma\mu} - 2g_{\sigma\mu}\gamma^\nu\gamma^\mu a_\nu b^\sigma - 2g^{\mu\nu} a_\nu b^\sigma \gamma_\mu\gamma_\sigma + \gamma^\nu\gamma^\mu a_\nu b^\sigma \gamma_\mu\gamma_\sigma$$
$$= 4a \cdot b - 2\not{a}\not{b} - 2\not{a}\not{b} + 4\not{a}\not{b} = 4a \cdot b$$

c) 由手征算符$P_R = \frac{1}{2}(1+\gamma^5), P_L = \frac{1}{2}(1-\gamma^5)$，证明：

$$P_L + P_R = 1, \qquad P_R P_R = P_R, \qquad P_L P_L = P_L, \qquad P_L P_R = 0.$$

i) $P_L + P_R = \frac{1}{2}(1+\gamma^5) + \frac{1}{2}(1-\gamma^5) = 1$

ii) $P_R P_R = \frac{1}{4}(1+\gamma^5)(1+\gamma^5) = \frac{1}{4}(1+2\gamma^5+1) = P_R$

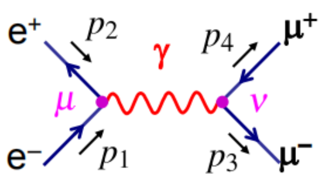
iii) $P_L P_L = \frac{1}{4}(1-\gamma^5)(1-\gamma^5) = \frac{1}{4}(1-2\gamma^5+1) = P_L$

iv) $P_L P_R = \frac{1}{4}(1-\gamma^5)(1+\gamma^5) = \frac{1}{4}(1+\gamma^5-\gamma^5-1) = 0$

5. 只考虑u, d夸克，考虑它们的 SU(2)对称性，用图文并茂的方式给出重子所有可能的同位旋态。

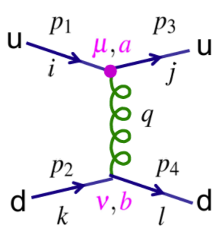
$$\begin{aligned}
|\tfrac{3}{2},+\tfrac{3}{2}\rangle &= uuu \\
|\tfrac{3}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}(uud+udu+duu) \\
|\tfrac{3}{2},-\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}(ddu+dud+udd) \\
|\tfrac{3}{2},-\tfrac{3}{2}\rangle &= ddd \\
|\tfrac{1}{2},-\tfrac{1}{2}\rangle &= -\tfrac{1}{\sqrt{6}}(2ddu-udd-dud) \\
|\tfrac{1}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{6}}(2uud-udu-duu) \\
|\tfrac{1}{2},-\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{2}}(udd-dud) \\
|\tfrac{1}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{2}}(udu-duu)
\end{aligned}$$

6. 考虑$ee \to \mu\mu$的湮灭产生过程，画出费曼图并根据 QED 费曼规则写出矩阵元



$$-iM = [\bar{v}(p_2) ie\gamma^\mu u(p_1)] \frac{-ig_{\mu\nu}}{q^2} [\bar{u}(p_3) ie\gamma^\nu v(p_4)]$$

7. 考虑$q\bar{q} \to q\bar{q}$的散射过程（t 道），画出费曼图并根据 QCD 费曼规则写出矩阵元（将颜色部分放在一起）。



$$-iM = \left[\bar{u}_u(p_3)\{-\tfrac{1}{2} i g_s \lambda^a_{ji} \gamma^\mu\} u_u(p_1)\right] \frac{-ig_{\mu\nu}}{q^2} \delta^{ab} \left[\bar{u}_d(p_4)\{-\tfrac{1}{2} i g_s \lambda^b_{lk} \gamma^\nu\} u_d(p_2)\right]$$

8. QED 中费米子和光子的基本相互作用顶角因子为$ie\gamma^\mu$，费米子与光子的相互作用可以表示为四矢量流$j^\mu = ie\bar{\psi}\gamma^\mu\phi$.

$$P_R = \frac{1}{2}(1+\gamma^5), \qquad P_L = \frac{1}{2}(1-\gamma^5)$$

$$P_R u_R = u_R, \qquad P_R u_L = 0, \qquad P_L u_R = 0, \qquad P_L u_L = u_L$$

$$P_R v_R = 0, \qquad P_R v_L = v_L, \qquad P_L v_R = v_R, \qquad P_L v_L = 0$$

（a） 证明任意旋量可以写成左手和右手手征分量，并将 QED 中的四矢量流用旋量的左手和右手手征分量展开。

$$\psi = \psi_R + \psi_L = \frac{1}{2}(1+\gamma^5)\psi + \frac{1}{2}(1-\gamma^5)\psi$$

$$j^\mu = \bar{\psi}\gamma^5\phi = \bar{\psi}_R\gamma^5\phi_R + \bar{\psi}_R\gamma^5\phi_L + \bar{\psi}_L\gamma^5\phi_R + \bar{\psi}_L\gamma^5\phi_L$$

（b） 利用简答题 4 中的性质，证明$\bar{\mu}_L\gamma^\mu\mu_R = 0$和$\bar{\nu}_L\gamma^\mu\mu_L = 0$.

$$\bar{u}_L\gamma^\mu u_R = \frac{1}{2}u^\dagger(1-\gamma^5)\gamma^0\gamma^\mu\frac{1}{2}(1+\gamma^5)u = \frac{1}{4}\bar{u}\gamma^\mu(1-\gamma^5)(1+\gamma^5)u$$
$$= 0$$

$$\bar{v}_L\gamma^\mu u_L = \frac{1}{2}v^\dagger(1+\gamma^5)\gamma^0\gamma^\mu\frac{1}{2}(1-\gamma^5)u = \frac{1}{4}\bar{v}\gamma^\mu(1+\gamma^5)(1-\gamma^5)u$$
$$= 0$$

（c） 实际上，弱相互作用中顶角因子不是矢量形式（γ^μ），而是一个特殊的$\gamma^\mu(1-\gamma^5)$的形式，试证明:只有粒子旋量的左手手征分量μ_L和反粒子旋量的右手手征分量ν_R参与带电粒弱相互作用。

$$\frac{1}{2}\bar{\psi}\gamma^\mu(1-\gamma^5)\phi = \bar{\psi}\gamma^\mu\phi_L = \bar{\psi}_L\gamma^\mu\phi_L$$