Heterogenous Beliefs about Stock Returns and Wealth Inequality

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Abstract

This paper investigates how differences in subjective beliefs about stock returns contribute to wealth inequality through portfolio choice. I argue that this channel significantly contributes to wealth inequality in the US. Using the Michigan Survey of Consumers, I find that (1) subjective beliefs about future stock returns are widely dispersed, (2) optimistic households are more likely to participate in the stock market and invest more in stock, and (3) subjective beliefs are persistent over time. Motivated by these findings, I develop and calibrate a heterogenous-agent model that matches data on income inequality, beliefs distribution and portfolio choice. Compared to a model without dispersion in beliefs, the model with belief heterogeneity generates an additional 0.12 in the Gini coefficient of wealth inequality and 10 percent more wealth owned by the top 15% of the households. This suggests that the dispersion in subjective beliefs about stock returns could be an important factor in explaining the size of wealth inequality in the US.

Keywords: Subjective Beliefs, Wealth Inequality, Portfolio Choice, Heterogeneous-Agent

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1 Introduction

Wealth is distributed more unequally than income in the United States, with the Gini coefficient of wealth being 0.85 compared to 0.43 for income. This makes standard incomplete market models, which feature only uninsurable income risk and a risk-free asset, such as Aiyagari (1994), unable to match the large wealth inequality observed in the data.

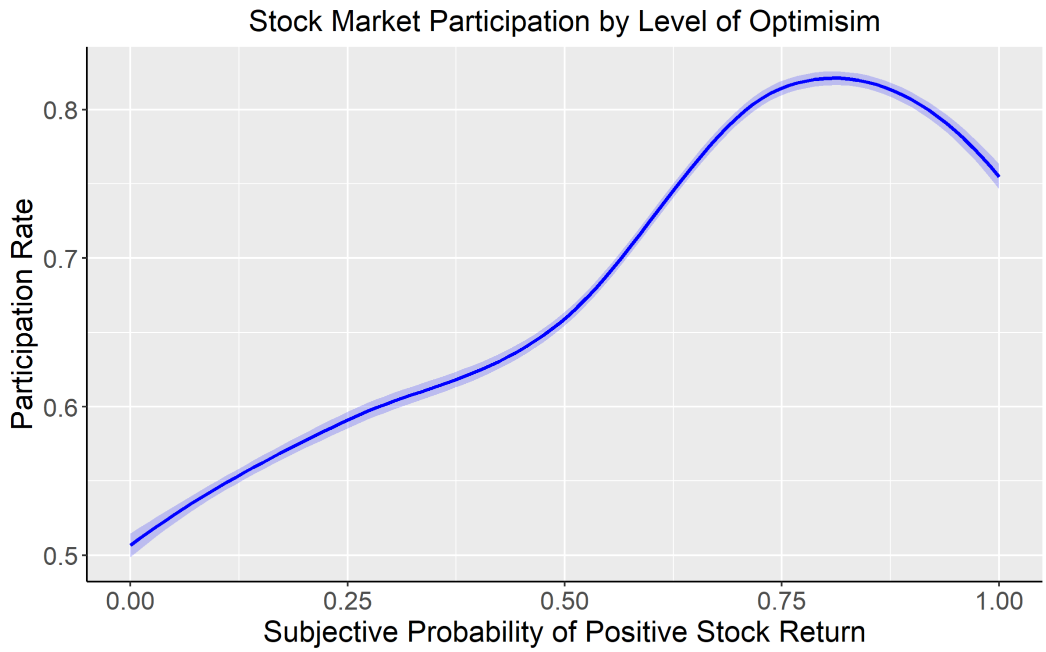
Differences in portfolio returns between wealthy and less wealthy households play a critical role in driving wealth inequality. Wealthy households invest more in high-return assets such as equity and accumulate wealth faster than less wealthy households. Hubmer *et al.* (2021) incorporate evidence documented from Bach *et al.* (2020) that wealthier households experience higher portfolio returns, and find that their model can generate wealth inequality large enough to match the data.

While there is extensive research on portfolio heterogeneity, most studies assume that households share the same outlook on the stock market. However, differences in portfolio choice may stem from differences in subjective beliefs about future stock returns. A household with an optimistic outlook may invest more in equities than a household with a pessimistic outlook, even when both of them share similar characteristics. Figure 1 shows a positive correlation between stock market participation rates and households' subjective beliefs about future stock returns, with more optimistic households being more likely to participate.

In this paper, I argue that differences in subjective beliefs about stock returns significantly contribute to wealth inequality. I document significant dispersion of subjective beliefs and show that optimistic households are more likely to invest in the stock market. Motivated by the empirical evidence, I then develop and calibrate a heterogeneous-agent model that incorporates endogenous portfolio choice and heterogeneous subjective beliefs. Risk preferences are calibrated to match observed equity shares, while subjective beliefs are disciplined by the empirical dispersion. Compared to a model with homogeneous beliefs, my model generates an additional 0.12 in the Gini coefficient of wealth inequality, accounting for roughly 15% in the Gini coefficient of wealth inequality in the US.

Empirically, I show that subjective beliefs may explain large heterogeneity in investment decisions. Using data from the *Michigan Survey of Consumers*, I measure households' subjective beliefs with their reported subjective probability of a positive market return and analyze how this correlates with their investment decisions. I find that beliefs about future stock returns are widely dispersed, with some households believing future returns to be 16% above or below than the historical average. Households who reported a subjective probability one standard deviation above the mean

Figure 1. Stock Market Participation by Subjective Beliefs about Stock Returns



Note: Data from the *Michigan Survey of Consumers*. Subjective Probability of Positive Stock Return is measured by respondents' prediction of the probability that a diversified stock mutual fund would have a positive return in the next year. Stock Market Participation is measured by the percentage of respondents who own stocks. The solid line is the estimated conditional expectation and the shaded area represents the 95% confidence interval.

are 10 percent more likely to participate in the stock market. This correlation holds even after controlling for education and income. Additionally, subjective beliefs are persistent over time: optimism today predicts optimism in six months. This suggests that subjective beliefs may help explain the long-term heterogeneity in investment decisions.

Quantitatively, I show that dispersion in beliefs about future stock returns significantly increases wealth inequality in a heterogeneous-agent model with portfolio choice. I extend a standard partial equilibrium heterogeneous-agent model, which features uninsurable labour income risk and a single risk-free asset, by incorporating subjective beliefs about the return on the risky asset and allowing for endogenous portfolio choice. Subjective beliefs in the model are disciplined by empirical dispersion, and risk preferences are calibrated to match equity share patterns observed in the *Survey of Consumer Finances*. Optimistic households invest significantly more in the risky asset, which affects wealth inequality in two key ways. First, the distribution exhibits a thick right tail: optimistic households with larger equity exposures may experience consecutive realizations of positive returns, which leads to a fast growth of wealth through compounding. Second, optimistic households accumulate more wealth on average by earning additional risk premium. Together, these factors lead to greater wealth dispersion across households with differing beliefs.

In the baseline model, I assume that beliefs and labour income are uncorrelated, which provides a lower bound on the impact of belief heterogeneity on wealth inequality. This is because, the data suggests that high-income households tend to be more optimistic about the stock market. If we account for this correlation, differences in subjective beliefs further amplify the wealth inequality between high-income and low-income households. To isolate the effect of belief heterogeneity, I conduct a counterfactual exercise in which all households are assumed to share the median belief about stock returns. Compared to this counterfactual, the model with belief heterogeneity generates an increase of 0.12 in Gini coefficient of wealth inequality and results in 10 percent more wealth owned by the top 15% of households.

My findings point out potential policy implications regarding wealth inequality. Both heterogeneous risk preferences and heterogeneous subjective beliefs could generate heterogeneity in portfolio choice and wealth inequality. However, if differences in portfolio choices are solely driven by differences in risk preferences, there is no role for policy intervention. My findings indicate that portfolio differences could also be driven by heterogeneous subjective beliefs. Policies focused on financial literacy could shape households' beliefs, which potentially reduce wealth inequality.

Finally, I consider two extensions to my model. First, to address the correlation between beliefs and income, I calibrate an alternative model to capture such a correlation and I find that this model generates even higher wealth inequality. Since high-income households are likely stock participants, changing their beliefs about the stock market has a large impact on the equity share. Second, I consider the case where households can update their beliefs. I find that models which generate more belief dispersion between stock market participants and non-participants further amplify the effect of subjective beliefs to wealth inequality[1].

Literature

This paper offers a behavioral perspective on wealth inequality. Macro models with uninsurable income risk, such as those by Aiyagari (1994), Bewley (1977), Huggett (1996) and Krusell and Smith (1998) are widely used to study wealth inequality. Prior literature, including De Nardi (2015) has highlighted a key limitation of these models: households in such frameworks save primarily for precautionary reason. This fails to capture the high growth rate of wealth observed among the wealthy

[1] They are currently work-in-progress. More detail will be posted soon

households. Understanding the motives behind the savings decisions of high-income households is crucial for explaining wealth inequality in the US.

One possible explanation for high savings rates among the wealthy is heterogeneous portfolio returns, which incentivize certain households to save more than others. Benhabib *et al.* (2019) demonstrates that a model with heterogeneity in returns aligns well with wealth inequality data, and Hubmer *et al.* (2021) shows that incorporating empirical return heterogeneity from data such as Bach *et al.* (2020) and Fagereng *et al.* (2020) can generate wealth inequality comparable to the data. However, a critical question remains: what drives the heterogeneity in returns? Kacperczyk *et al.* (2019) shows that heterogeneous information capacity can theoretically produce wealth inequality. They calibrate their model to match aggregate moments. My approach differs by using micro-level survey data to calibrate heterogeneity in subjective beliefs, allowing us to assess how much beliefs can explain wealth inequality.

Empirical literature has shown that subjective beliefs reported in survey data can predict individual investment decisions. Giglio *et al.* (2021) shows that beliefs influence retail investors' portfolio choice. Das *et al.* (2020) uses the same data as this paper to demonstrate that beliefs correlate with socioeconomic status and investment decisions. My work closely relates to the latter. While Das *et al.* (2020) examines average beliefs across households with different socioeconomic backgrounds, I focus on the dispersion of beliefs within households of the same socioeconomic status.

Finally, the paper contributes to the growing literature that uses micro-level survey data on subjective beliefs to calibrate heterogeneous-agent models that depart from full-information rational expectations (e.g. Bhandari *et al.* (2019) Broer *et al.* (2021) and Guerreiro (2022)). My paper differs by focusing on subjective beliefs about stock market returns rather than beliefs about macroeconomic variables.

Organization

In section 2, I describe the data and empirical facts on subjective beliefs and portfolio choice. In section 3, I lay out the quantitative framework for analyzing the effect of subjective beliefs on wealth inequality. In subsection 3.1 and subsection 3.3, I discuss the model and the key calibrations for my result. I present simulation results in section 4. Finally, section 5 concludes.

2 Data and Empirical Facts

In this section, I argue that subjective beliefs about stock returns are widely dispersed and they are correlated with investment decisions of households. I first describe the measurement of subjective beliefs about stock returns in the *Michigan Survey of Consumers*. Then, I show the heterogeneity in subjective beliefs and its relationship with investment decisions. Finally, I discuss the persistence of subjective beliefs over time that would be relevant for the model calibration.

2.1 Measuring Subjective Beliefs

The *Michigan Survey of Consumers* elicits subjective beliefs about the stock market of hosueholds by asking the subjective probability of a positive stock return in the next year. The exact question is as follows:

> *What do you think is the percent chance that a one thousand dollar investment in a diversified stock mutual fund will increase in value in the year ahead, so that it is worth more than one thousand dollars one year from now?*

Responses of this question informs us about households' subjective beliefs about the short-term stock market returns. I consider this subjective probability as an measurement of optimism, the higher the reported probability, the more optimistic the household is about stock returns. This is different from measuring stock-picking skills or risk-aversion, which are both important for the household's investment decision. The question specifically asks for a performance prediction for a "diversified stock mutual fund", which excludes the stock-picking skill of the respondent. The question also asks for a subjective probability of an event, which does not depend on the risk-aversion of the respondent.

2.2 Heterogeneity in Subjective Beliefs

Households disagree on the prospect about the stock market returns. In this section, I show the distribution of subjective beliefs about stock returns and discuss how the dispersion in beliefs could be interpreted as the dispersion in the subjective risk-adjusted return.

As shown in Figure 2, households disagree on the probability of positive stock market return, with considerable shares of respondents reporting 100% or 0% chance of positive stock return. This dispersion is mostly driven by cross-sectional differences. Even after demeaning the data by time average, some hosueholds reported $+50\%$ or -50% probability relative to the average belief at that time. There is a mass of households reporting probability exactly at 50%. One concern is that households may not have a clear idea about the stock market return and they may report 50% as a default answer. Households who answered 50% throughout their presence in the survey constitute around 3% of the sample. This is a small share of the sample and the results are robust to excluding these households[2].

Under additional assumptions, the subjective probability of positive stock return can be interpreted as the subjective risk-adjusted return, which is more accessible for macro-finance models. Suppose households believe that the stock return follows a log-normal distribution, with subjective mean return μ_i and variance $(\sigma_i)^2$. Let R be the annual stock return

$$R = \exp(r) \quad \text{with} \quad r \sim N(\mu_i, (\sigma_i)^2) \tag{1}$$

The subjective probability of positive stock return is a strictly increasing function of the subjective risk-adjusted return defined as μ_i/σ_i, subjective mean return divided by standard derivation of return[3]. The exact expression is given by

$$\underbrace{P_i(R \geq 1)}_{\text{Subjective Probability of Positive Stock Return}} = 1 - \Phi\Big(- \underbrace{\frac{\mu_i}{\sigma_i}}_{\text{Subjective Risk-adjusted Return}} \Big) \tag{2}$$

where $\Phi(.)$ is the cumulative distribution function of a standard normal random variable.

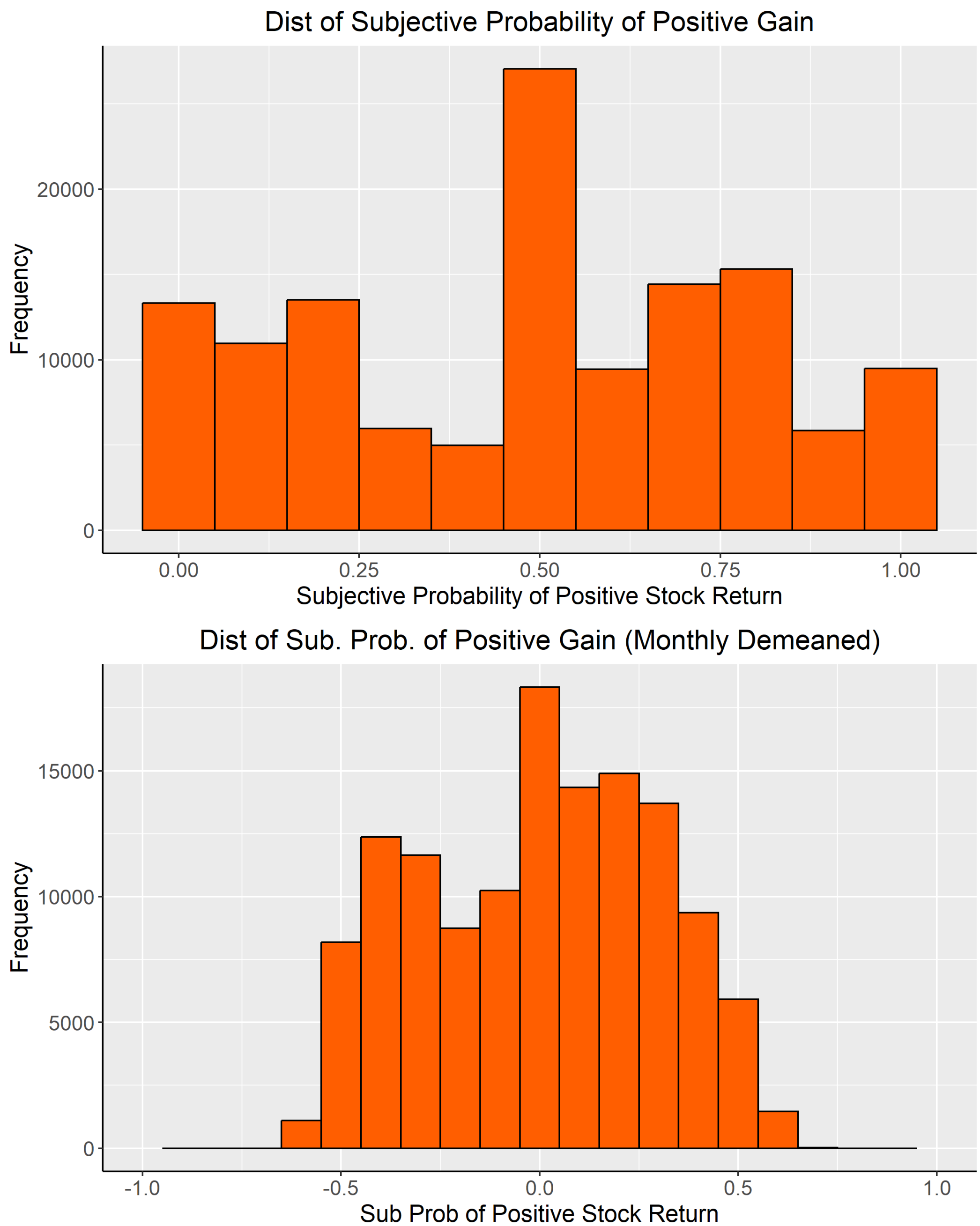
As shown in Figure 3, the risk-adjusted return is also widely dispersed, with some households believing a risk-adjusted return between -1.5 to 1.5. Suppose the subjective standard deviation of return is same as the post-war historical standard derivation of the S&P 500 of 0.15, household's subjective beliefs on the mean annual return ranges from -22.5% to 22.5%, reflecting a wide disagreement on the stock market return. This transformation also facilitate comparisons to historical data. The risk-premium and standard derivation for post-war US market is estimated to be around $E[r - r^f] = 7\%$ and 15% respectively[4]. For a risk-free rate of 1%, the risk-adjusted return according to the historical data is around 0.53 and the probability of positive stock return is around 0.65. Hence, households are slightly more pessimistic than the historical data.

[2] See Appendix C for more robustness checks

[3] Notice that this is different from Sharpe ratio because the numerator is the expected rate of return instead of the excess return

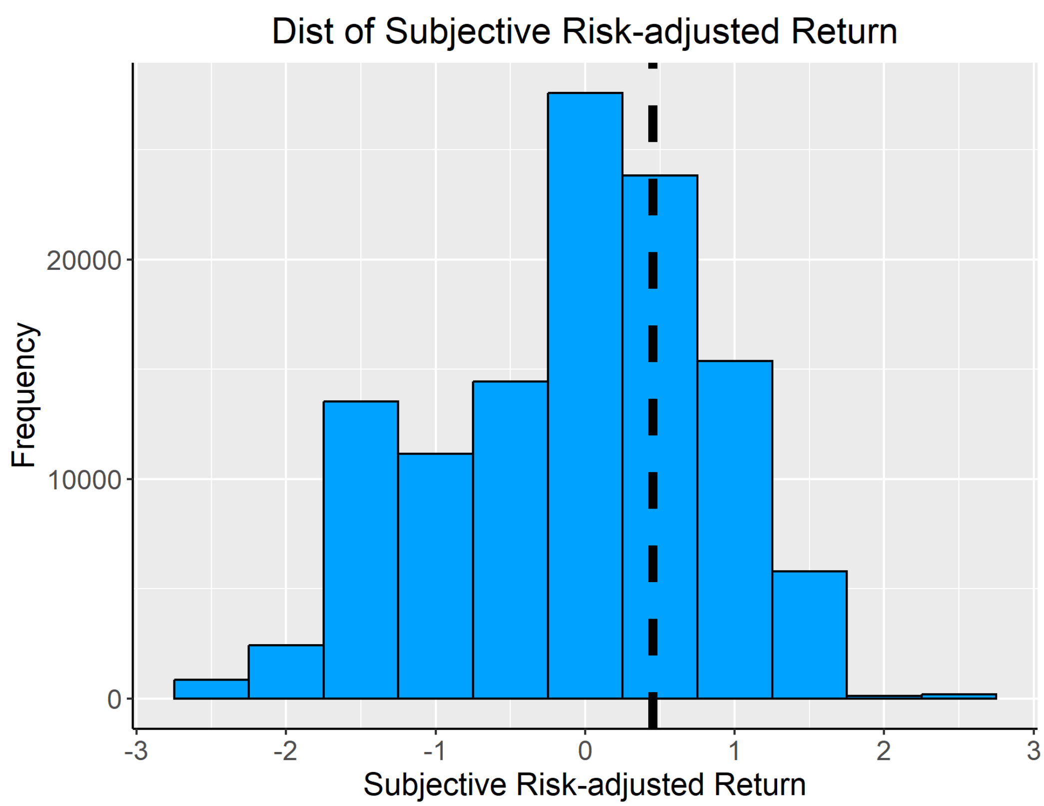
[4] This is in line with Mehra and Prescott (1985)

Figure 2. Distribution of Subjective Beliefs about Stock Returns



Note: Data from *Michigan Survey of Consumers.* Subjective Probability of Positive Stock Return is measured by respondents prediction of the probability that a diversified stock mutual fund would have a positive return in the next year. Raw data (top panel) and Demeaned data (bottom panel) are shown.

Figure 3. Distribution of Subjective Beliefs about Stock Returns



Subjective risk-adjusted return is defined as subjective mean of return divided by subjective standard deviation of return. It is calculated by transforming subjective probability, $\mu_i/\sigma_i = -\Phi^{-1}(1 - P_i(R \geq 1))$ using Equation 2. The histogram shows the distribution of subjective risk-adjusted return and the dashed line shows the risk-adjusted return according to the historical data.

2.3 Subjective Beliefs and Investment Decisions

Beliefs elicited in the survey are not just noise, but they are relevant for the investment decisions of households. Subjective beliefs about stock returns are largely associated with individual investment decisions. Optimistic households are likely stock investors and they invest significantly more in stock. Subjective beliefs seem to have a good explantory power for the investment decisions of households, even after controlling for other household characteristics.

Regression Setup

The following regression extracts the households investment decision associated at each level of subjective beliefs.

$$Y_{it} = \alpha_t + \sum_{i \in g} \alpha_g \times I(i \in g) + \beta \times \text{Belief}_{it} + \epsilon_{it} \tag{3}$$

In this setup, Y_{it} is the outcome investment decision for household i, which is either a binary indicator of *stock market participation* or the *log investment share*, defined as the logarithm of total amount invested in stock divided by current income. Belief_{it} is the subjective probability of positive stock market returns reported by household i at time t. Results are robust to using subjective risk-adjusted returns μ_i/σ_i as the regressor and they are presented in Appendix C[5]. β measures the investment decisions associated with different subjective beliefs. α_t is the time fixed effects to control for movements in aggregate investment and aggregate beliefs over the business cycle. α_g is the group fixed effects to control for the observable characteristics of the households, which include income quintile and whether the respondent has a college degree.

Before presenting the results, it is worth discussing the variations being captured in this regression. After adding time and group fixed effects, the regression captures the cross-sectional variation in investment decisions associated with subjective beliefs, *within* the same time, income quintile and education group. As pointed out by Das *et al.* (2020), optimism is associated with higher income and education attainment. The difference in optimism *across* group may drive the difference in investment decisions across group. Those effects cannot be captured in a regression analysis. In the extension, I consider the correlation between subjective beliefs and other household characteristics in the calibration to account for their effect on wealth inequality.

[5]Response with 0% and 100% for subjective beliefs are removed for the analysis using subjective risk-adjusted returns. This is because they correspond to $-\infty$ and ∞ subjective risk-adjusted returns. A regression could not be performed.

Empirical Findings

On the extensive margin, stock market participation is associated with subjective beliefs about stock returns. In Table 1, the stock market participation rate increases by 33% from the households who believe in 0% chance of positive stock return to the households who believe in 100% chance of positive stock return. The standard deviation of subjective probability is around 0.3. Thus, one standard deviation increase in optimism is associated with $0.3 \times 33\% \approx 10\%$ increase in the stock market participation rate. The association mostly comes from the cross-sectional differences. Once individual fixed effects are added, the association is much less pronounced because all cross-sectional differences are absorbed by the fixed effects. Education and income cannot fully explain the association between subjective beliefs and stock market participation, as the regression coefficients remains significant after controlling for these variables. One standaard deviation increase in optimism still accounts for roughly 5% increase in the stock market participation rate.

On the intensive margin, stock investment conditioned on investing is also associated with subjective beliefs about stock returns. In Table 2, investment in stock increases by 33% from the households who believe in 0% chance of positive stock return to the households who believe in 100% chance of positive stock return. One standard deviation incrase in optiism (≈ 0.3) is associated with 10% increase in the stock investment conditional on investing. The regression coefficient remains large but less significant after controlling for income and education fixed effect. This is because low-income and non-college educated households invest very little in stock. Subjective beliefs have a very small effect on the intensive margin of these households. In Appendix C, the regression coefficients for each income and education group are presented. The investment share is associated strongly with the subjective beliefs for higher income and college-educated households.

The regression analysis so far focuses on the correlation of subjective beliefs and investment decisions *within* the same time, income quintile and education group. High-income college-educated households are more likely to invest in stock and they are more optimistic about the stock market on average. In Figure 4, the differnce in subjective probability between group is about 20 percentage points, while the average stock market participation varies from 25% to 80%. The effect of subjective beliefs on investment decisions could not be analyzed in the regression analysis. I account for this correlation in the calibration in the extension.

Table 1. Stock Market Participation on Beliefs

Dependent Variable:	Participation in Stock Market					
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Sub. Prob of Positive Return	0.331***	0.022***	0.334***	0.182***	0.257**	0.160***
	(0.004)	(0.005)	(0.006)	(0.021)	(0.020)	(0.019)
Individual FE		Yes				
Time FE			Yes	Yes	Yes	Yes
Income Group FE				Yes		Yes
College FE					Yes	Yes
Observations	129,841	129,841	129,841	122,586	129,274	122,172
Adjusted R^2	0.04546	0.75055	0.04940	0.26458	0.12880	0.28105

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Note: Participation in Stock Market is a binary indicator of whether the individual has stock market investment. The subjective probability of positive stock return is the individual's subjective probability of positive stock market return. Income group fixed effects are indicators for income quintiles. Education fixed effects are indicators for college-educated or not.

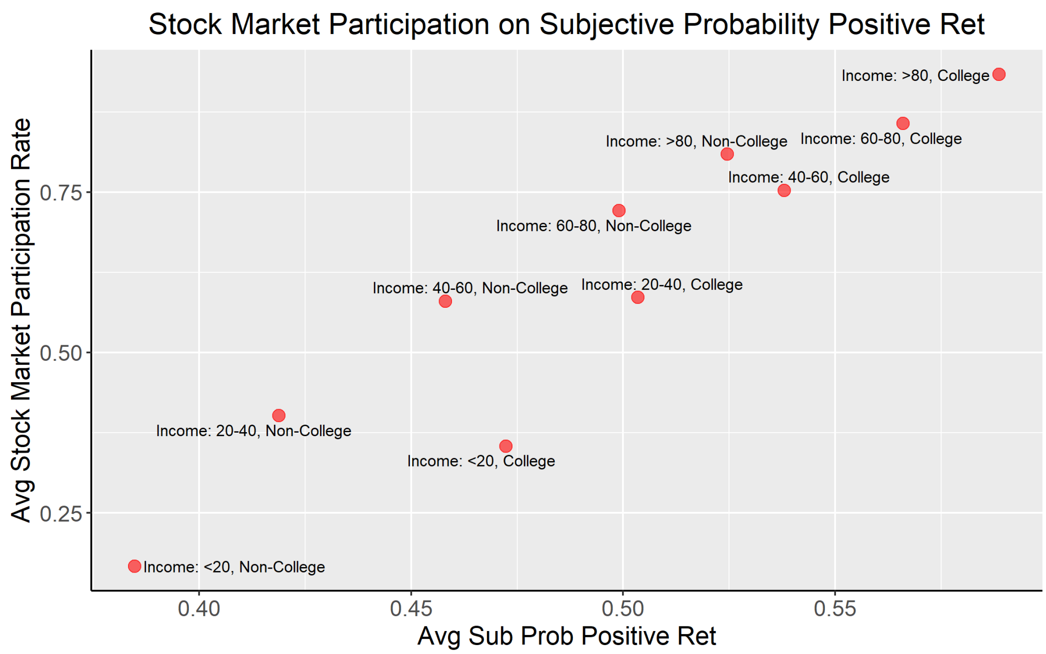
Table 2. Investment Share on Beliefs

Dependent Variable:	Log Investment to Income					
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Sub. Prob of Positive Return	0.335***	0.100***	0.303***	0.263**	0.247	0.227*
	(0.019)	(0.019)	(0.020)	(0.086)	(0.093)	(0.082)
Individual FE		Yes				
Time FE			Yes	Yes	Yes	Yes
Income Group FE				Yes		Yes
College FE					Yes	Yes
Observations	71,845	71,845	71,845	71,845	71,723	71,723
Adjusted R^2	0.00425	0.83148	0.01680	0.02665	0.02445	0.03184

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Note: Invest Share is defined as the logarithm of stock investment amount divided by current income, log(Stock Amt Invested / Income) for households with stock investment only. The subjective probability of positive stock return is the individual's subjective probability of positive stock market return. Income group fixed effects are indicators for income quintiles. Education fixed effects are indicators for college-educated or not.

Figure 4. Stock Market Participation and Subjective Beliefs Accross Groups



Note: Avg Sub PRob Positive Ret is the average subjective probability of positive stock return of the households in the group. Stock Market Participation is the stock market participation rate of the households in the group. The data is from *Michigan Survey of Consumers.*

2.4 Persistent Difference of Subjective Beliefs

Households tend to hold their subjective beliefs about stock returns over time. In this section, I estimate the autocorrelation of the subjective beliefs and show that households who are optimistic today are likely to be optimistic in the future. This suggests that subjective beliefs may explain the long-term differences in portfolio choices across households.

Autoregressive Coefficient

The autoregressive coefficient of the subjective beliefs is estimated by the following regression

$$\text{Belief}_{it} = \alpha_t + \rho \times \text{Belief}_{it-1} + \epsilon_{it} \tag{4}$$

where Belief_{it} is the subjective probability of positive stock return reported by household i at time t. Belief_{it-1} is the subjective probability reported six months before. α_t is the time fixed effects. Because of the time fixed-effect, ρ measures the autoregressive coefficient of the subjective beliefs relative to the average. A positive coefficient indicates that a household who is optimistic today relative to the average is likely to be optimistic relative to the average in the future.

In Table 3, the autoregressive coefficient is estimated to be 0.43. A household who is one standard deviation more optimistic than the average is likely to be 0.43 standard deviation more optimistic than the average in the future. This suggests that the disagreement between households does not resolve instantly. One concern is that this coefficient may be driven by the uninformed households who report 50% probability as a default answer in two consecutive survey periods. Those households contribute to roughly 3% of the sample. The results are robust to excluding these households[6].

Table 3. Beliefs Persistence

Dependent Variable:	Sub. Prob of Positive Return	
Model:	(1)	(2)
Sub. Prob of Positive Return 6m lag	0.433^{***}	0.416^{***}
	(0.004)	(0.005)
Time FE		Yes
Observations	44,941	44,941
Adjusted R^2	0.19212	0.21360

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Note: The subjective probability of positive stock return is the individual's subjective probability of positive stock market return. Sub. Prob of Positive Return 6m lag is the belief reported by the same household six mouths before.

[6] See Appendix C for more robustness checks

3 Quantitative Framework

I argue that the dispersion in subjective beliefs about stock returns could have a significant impact on wealth inequality. In this section, I extend a standard heterogeneous agent model with idiosyncratic income risk, to include subjective beliefs about stock returns and a portfolio choice. The model is calibrated to match the joint process of income and stock returns, empirical dispersion of the subjective beliefs, as well as the equity share in the data. Then I analyze the impact of subjective beliefs by comparing to a counterfactual economy where the households have the same subjective belief about stock returns.

3.1 Model

The model is a partial equilibrium heterogeneous-agent model. Households are different in terms of income, wealth and subjective beliefs about the mean return of the stock. The stock return is set to replicate the performance of *S&P500* in the post-war era. The income process is calibrated to exhibit a countercyclical negative skewness, that Catherine (2022) document to be important for generating realistic equity share for households with different wealth. I use an Epstein and Zin (1989) preference to allow enough flexibility in the model to match data on equity share. Lastly, I discuss how beliefs about stock returns fit into the household's problem.

Aggregate State and Stock Returns

The economy is driven by an iid aggregate state $\gamma_t \in \{\gamma^B, \gamma^G\}$, where $\gamma^B < \gamma^G$. This shock can be seen as fluctations to the economic growth, which affects both risky return and labour income process. Households observe γ_t and its past realizations but γ_{t+k} for $k \geq 0$ is uncertain for the households.

The return of the risky asset from time t to $t+1$ is a random variable given by

$$R_{t,t+1} = \begin{cases} R(\gamma^G) & \text{if} \quad \gamma_{t+1} = \gamma^G \\ R(\gamma^B) & \text{if} \quad \gamma_{t+1} = \gamma^B \end{cases}$$

where $R(\gamma^G) > R(\gamma^B)$. The values of $R(\gamma^G)$ and $R(\gamma^B)$ are calibrated to match the behaviour of *S&P500*. In addition, I assume a small-open economy such that the risk-free rate R^f is exogenously given.

Household

Households are heterogeneous in terms of financial wealth W_{it}, idiosyncratic labour income Y_{it}, and types of subjective belief Π_{it}. Households choose consumption C_{it} and share of risky asset ω_{it} to maximize their value function. This setup is similar to the household problem in standard incomplete market model except that households have access to a risky asset and subjective beliefs.

For some of the simulations, I also consider an additional heterogeneity in the permanent type of households Ξ_i to capture the residual wealth inequality unexplained by income inequality and belief heterogeneity. This includes heterogeneity in the discount factor or permanent income. The permanent type is assumed to be fixed over time and uncorrelated with the subjective beliefs.

Maximization Problem

Households maximize a value function $V_t(W_{it}, Y_{it}, \Pi_{it})$ by choosing consumption C_{it} and risky share ω_{it} to solve the following problem

$$V_t(W_{it}, Y_{it}, \Pi_{it}) = \max_{C_{it}, \omega_{it}} \left((1-\beta) C_{it}^{1-1/\psi} + \beta \Big(CE_{it}[V_{it+1}] \Big)^{1-1/\psi} \right)^{1/(1-1/\psi)} \tag{5}$$

where ψ is the coefficient of elasticity of intertemporal substitution (EIS) and β is the discount factor. $CE_{it}[.]$ denotes household i certainty equivalent operator at time t and future value is denoted as V_{it+1}, with $V_{it+1} = V_{t+1}(W_{it+1}, Y_{it+1}, \Pi_{it+1}, \Xi_i)$. The certainty equivalent of future value with a relative risk aversion parameter (RRA) θ is given by

$$CE_{it}(V_{it+1}) = \left(E\left[\Big(V_{t+1}(W_{it+1}, Y_{it+1}, \Pi_{it+1}, \Xi_i) \Big)^{1-\theta} | \Pi_{it}, Y_{it} \right] \right)^{\frac{1}{1-\theta}} \tag{6}$$

The EIS ψ, RRA θ and discount factor β are crucial parameters in the model. The EIS ψ and RRA θ govern the response of saving rate and risky share repsectively to the perceived returns of saving. The discount factor β determines the patience of the households and thus the size of saving[7].

Household solves the maximization problem subject to the budget constraint

$$W_{it+1} = (W_{it} + Y_{it} - C_{it}) \Big(R_{t,t+1}\omega_{it} + R^f(1-\omega_{it}) \Big) \tag{7}$$

[7] In a model with idiosyncratic income risk and risk-free asset, households would want to save infinite amount of asset as long as $\beta R^f \geq 1$ to save out of the borrowing constraint. Since risky asset provides a risk premium, I impose a stricter restriction of $\beta \tilde{E}_{it}[R_{t,t+1}] < 1$ to avoid divergent wealth distribution.

and constraints on borrowing, short-selling and leveraging

$$W_{it+1} \geq 0 \quad \text{and} \quad 0 \leq \omega_{it} \leq 1$$

The borrowing constraint is standard in the literature for this type of model. It generates wealth inequality by encouraging households to save out of the borrowing constraint. The short-sale constraint and no leveraging constraint are to avoid extreme portfolio positions that are rare for households.

Labour Income Process

The idiosyncratic labour income process determines the income inequality among households but also the risk appetite of the households, which is crucial for the portfolio choice. Aggregate labour income does not comove strongly with the asset return in the data. However, as documented by Guvenen *et al.* (2014), left-tail risk in the idiosyncratic risk increases during recession. To capture all these while keeping the model parsimonous, I use an $AR(1)$ process for the idiosyncratic income but with a mixture-normal innovation to capture the tail risk.

The labour income of household i at time t is given by $Y_{it} = \exp(z_{it}y_t)$. z_{it} is the idiosyncratic income shock and y_t is the aggregate income shock, which depends on γ_t. z_{it} follows an $AR(1)$ process

$$z_{it} = \rho z_{it-1} + \eta_{it} \tag{8}$$

with

$$\eta_{it} = \begin{cases} \eta_{it}^{adj} \sim N(\mu^{adj}(\gamma_t), (\sigma^{adj})^2) & \text{with probability } p_{adj} \\ \eta_{it}^{unadj} \sim N(\mu^{unadj}(\gamma_t), (\sigma^{unadj})^2) & \text{with probability } 1 - p_{adj} \end{cases} \tag{9}$$

The probability p_{adj} captures the probability of a major income shock. The mean of innovation conditional of adjustment, $\mu^{adj}(\gamma_t)$ is calibrated to be a negative during recession to capture the event of large income loss events, such as unemployment. The countercyclical negative skewness generates sufficient correlation between labour income and stock return, discouraging low-wealth households from investing in stock.

Beliefs

Households' heterogeneous beliefs are captured by the state variable Π_{it}. In general, households respond to expectations about future, both in terms of income and stock return. I assume that households forecast their labour income with rational expectation to focus only on the portfolio margin[8].

Households' subjective beliefs about stock returns are given by a scaling factor on the objective return. For belief type Π_{it}, the perceived return of the risky asset is given by

$$R_{t,t+1}(\Pi_{it}) = \begin{cases} \exp(\lambda(\Pi_{it})) \times R^G & \text{if} \quad \gamma_{t+1} = \gamma^G \\ \exp(\lambda(\Pi_{it})) \times R^B & \text{if} \quad \gamma_{t+1} = \gamma^B \end{cases} \tag{10}$$

Hence, the belief Π_{it} enters the perceived return as a scaling factor. The factor $\exp(\lambda(\Pi_{it}))$ captures the heterogeneity in subjective beliefs about the expected log return of the stock, as the scaling factor shifts the mean of log return by $\lambda(\Pi_{it})$, $E[\log(R_{t,t+1}(\Pi_{it}))] = \lambda(\Pi_{it}) + E[\log(R_{t,t+1})]$, while holding the perceived risk constant, $Var(\log(R_{t,t+1}(\Pi_{it}))) = Var(\log(R_{t,t+1}))$.

This formation of beliefs aims to capture a log-normal distribution of stock returns. As discussed in the empirical section, the subjective probability of positive stock return is a one-to-one mapping to subjective risk-adjusted return, defined as mean return divided by the standard deviation of the return. In a continuous setting, the perceived return of the risky asset is given by

$$R_{t,t+1} = \exp(r(\Pi_{it})) \quad \text{where} \quad r(\Pi_{it}) \sim N(\mu(\Pi_{it}), \sigma(\Pi_{it})^2)$$

where beliefs heterogeneity on stock returns are captured by the mean return $\mu(\Pi_{it})$ and the variance $\sigma(\Pi_{it})^2$.

The survey data shows the distribution of subjective risk-adjusted stock returns, but it does not provide information exactly on the mean return or the variance. For the baseline simulation, I assume that households have subjective mean return $\mu(\Pi_{it})$ but objective variance $\sigma(\Pi_{it})^2 = \sigma^2$, which is set to be consistent with the actual distribution of risky returns. This choice would not make a substantial difference in the results as the model is calibrated to match the empirical findings and equity share in the data. Both higher subjective mean return or lower subjective variance would lead to same higher equity share for the households.

[8] In the data, households disagree on the future state of the economy as well and their beliefs about labour income could be correlated with beliefs about stock returns. They may affect the consumption-saving decisions of households

Additional Permanent Types

While the model has enhanced portfolio choices and beliefs compared to the standard model, my model still does not capture all the wealth inequality in the data, especially on the bottom and the top wealth distribution. I consider an alternative benchmark where households have different permanent types Ξ_i that are uncorrelated with the subjective beliefs. Following Carroll *et al.* (2017), I assume an additional heterogeneity in the discount factor β. High β households are more patient and save more. Thus, heterogeneity in β can capture the wealth inequality unexplained by income inequality and belief heterogeneity.

To be more precise, households with permanent type Ξ_i maximizes the following objective function

$$V_t(W_{it}, Y_{it}, \Pi_{it}, \Xi_i) = \max_{C_{it}, \omega_{it}} \left((1 - \beta(\Xi_i)) C_{it}^{1-1/\psi} + \beta(\Xi_i) \Big(CE_{it}[V_{it+1}] \Big)^{1-1/\psi} \right)^{1/(1-1/\psi)} \tag{11}$$

where $\beta(\Xi_i)$ specifies a distribution of discount factor β for households with different permanent types Ξ_i. The distribution of $\beta(\Xi_i)$ are calibrated to match wealth shares of the bottom and the top wealth groups in the data. The calibration of the model is discussed in the next section.

3.2 Partial Equilibrium Stationary Distribution

In most of the analysis, I focus on the partial equilibrium stationary distribution of the model given the risk-free rate R^f and the $R_{t,t+1}$. The goal is to understand how changing distribution of $\lambda(\Pi_{it})$ affects the wealth distribution in the stationary distribution.

Given the risk-free return R^f, state variables $(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$, the optimum portfolio choice and saving decision of the households are given by

$$\omega_{it} = \omega(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$$
$$X_{it} = X(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$$

The financial wealth next period is given by their savings times their portfolio returns

$$W_{it+1} = X(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t) \Big(R^f + \omega(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t) \times (R_{t,t+1} - R^f) \Big)$$

which pins down the (stochastic) evolution of the financial wealth. Subjective beliefs Π_{it} affect the wealth distribution through both the portfolio choice and the saving decision.

A distribution $D(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$ specifies the probability mass distribution of households in the state of $(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$. I define the stationary distribution and partial equilibrium as follows

Definition 1. (Stationary Distribution) Given portfolio decision $\omega(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$ and saving decision $X(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$, a distribution $D(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$ is stationary if the distribution satisfies the following conditions

$$\begin{aligned} & D(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t) \\ =& \sum_{\Xi_i} \int_{(W_{it-1}, Y_{it-1}, \Pi_{it-1}, \Xi_i, \gamma_t) | W_t = A_{it}} \int_{Y_{t-1}, \gamma_{t-1}} \left[P(Y_t, \gamma_t | Y_{t-1}, \gamma_{t-1}) \times D(W_{it-1}, Y_{it-1}, \Pi_{it-1}, \Xi_i, \gamma_t) \right] \end{aligned} \tag{12}$$

where A_{it} is the function of realized financial wealth at state γ_t, defined as

$$A_{it} = X(W_{it-1}, Y_{it-1}, \Pi_{it-1}, \Xi_i, \gamma_{t-1})(R^f + \omega(W_{it-1}, Y_{it-1}, \Pi_{it-1}, \Xi_i, \gamma_{t-1}) \times (R(\gamma_t) - R^f))$$

Definition 2. (Partial Equilibrium Stationary Distribution) Given the risk-free rate R^f and the risky return $R_{t,t+1}$, the distribution $D(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$ is in partial equilibrium stationary distribution if the distribution satisfies the following conditions

1. The distribution $D(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$ is stationary
2. The decision rules $\omega(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$ and $X(W_{it}, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$ solve the households' maximization problem

Note that in this definition of stationary distribution, the realized return of the risky asset $R_{t,t+1}$ evolves according to the objective return of the risky asset $R(\gamma_t)$. The subjective beliefs Π_{it} only affect the decision of the households but not the evolution of the risky return. Finally, the wealth distribution $P_W(w)$ in the stationary distribution is defined as

$$P_W(w) = \sum_{\Xi_i} \sum_{\gamma_t} \int_{Y_{it}, \Pi_{it}} D(w, Y_{it}, \Pi_{it}, \Xi_i, \gamma_t)$$

3.3 Calibration

My model is calibrated to replicate the US economy. I calibrate the labour income process to capture the US income dispersion and cyclical properties of labour income. I choose the risk aversion parameter θ to match the top-wealth equity share in the *Survey of Consumer Finance*. Following the calibration strategy in Aiyagari (1994), the discount factor β is chosen to match the aggregate asset to income ratio.

Labour Income Process

The labour income process is calibrated to produce realistic income dispersion and the cyclical properties of labour income change.

The $AR(1)$ process of the log labour income is characterized by the persistence ρ and the distribution of the innovation, which are calibrated to the annual data. The persistence is set to 0.93, corresponding to a quarterly persistence of 0.975 estimated by French (2005)[9]. The variance of the innovation is chosen to match the standard deviation of the log labour income of 0.83 documented by Guvenen *et al.* (2014), which implies a variance of $Var(\eta_{it}) \approx 0.1$. This variance is set to be constant throughout the fluctations of the aggregate state γ_t. This is consistent with Guvenen *et al.* (2014)'s findings that the variance of the innovation is roughly constant over the business cycle.

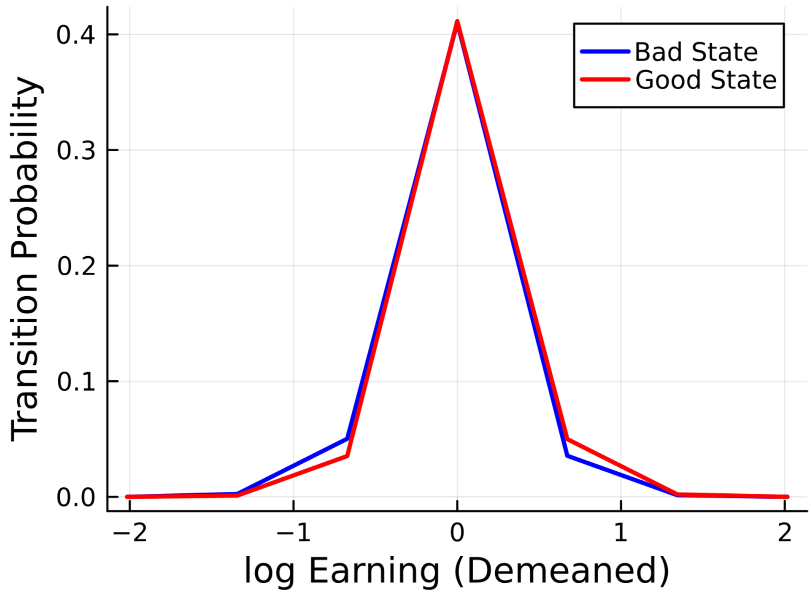
A mixture-normal innovation is defined by the mixture probability p_{adj}, the means and variances of the two normal distributions. The probability of adjustment is set to 0.136, used in Catherine (2022) to match the probability of major income changes. The means and variances of the two normal distributions are chosen to match the mean, variance, 10th, 50th and 90th percentile of log income change along the business cycle.The exact procedure of estimation and discretization using Rouwenhorst Method is described in Appendix F.

Assets

The risk-free rate is set to 1 percent, a value commonly used in the literature for annual frequency calibration. The risky asset is calibrated to replicate S&P500's returns in Post-war era. The risky premium is chosen to be 7% according to Mehra and Prescott (1985) and the standard deviation of the risky asset is set to 15%, which produce a Sharpe ratio of 0.47.

[9] Guvenen *et al.* (2014) estimate a much higher autocorrelation in the Social Security records. I choose a smaller number instead to ensure that the model remains stationary

Figure 5. Transitional Probability for Median Income Household



Note: The figure shows the transitional probability of the median income household. The probability is calibrated to match the countercyclical negative skewness of income change.

Table 4. Calibrated Parameters for Labour Income Process

Parameter	Description	Value	Sources
ρ	Persistence	0.93	Quarterly persistence of 0.975 in French (2005)
$sd(y_{it})$	Standard Deviation of log income	0.83	Standard Deviation of log wage in Guvenen *et al.* (2014)
p_{adj}	Mixture Probability	0.136	Estimate from Catherine (2022)
$(\mu^{adj}(\gamma^G), \mu^{nonadj}(\gamma^G))$	Conditional Mean in Expansion	$(0.103, 0.01)$	To match Mean, p10, p50, p90 of log wage growth in 05-07
$(\mu^{adj}(\gamma^B), \mu^{nonadj}(\gamma^B))$	Conditional Mean in Recession	$(0.15, -0.05)$	To match Mean, p10, p50, p90 of log wage growth in 08-09

Beliefs

To capture the heterogeneity in subjective beliefs, I assume that there are three types of beliefs about the future stock returns, optimistic, median and pessimistic, with equal weights. In the data, the standard deviation of the subjective risk-adjusted return is around 0.8. The optimistic and the pessimistic groups are assumed to have $\sqrt{3/2} \approx 1.22$ standard derivation above average and below average beliefs, respectively. This ensures that the overall standard deviation of the subjective beliefs is 0.8. The median households report a zero probability of positive return, corresponding to a 0 subjective risk-adjusted return[10].

In the baseline calibration, the beliefs are assumed to be permanent and uncorrelated with other non-belief characteristics. The extension to cover both are discussed in the extension section.

Risk Aversion and Elasticity of Intertemporal Substitution

The risk aversion parameter is calibrated to match the top-wealth equity share in the data. In the data, the top 1% wealthy households holds roughly 30% of their wealth in equity. Since optimistic households are more likely to be wealthy due to their higher equity share, the risk aversion parameter is chosen such that the wealthy optimistic households hold 30% of their wealth in equity. The risky share of the top wealth households can be approximated using a method in Campbell and Viceira (2001), $\omega \approx ((\max_i \mu^{it}) - r^f)/(\theta\sigma^2)$, which implies a risk aversion parameter of $\theta = 18$. The elasticity of intertemporal substitution (EIS) is set to $\psi = 1.5$, a value used in the asset pricing literature. It implies that optimism has a positive effect on the saving rate. This is consistent with my finding that the amount of stock investment is positively correlated with the subjective beliefs about future stock returns.

Table 5. Calibrated Parameters for Asset and Preferences

Parameter	Description	Value	Sources
$\mu - r_f$	Risk Premium	7%	Quarterly persistence of 0.975 in Mehra and Prescott (1985)
σ	Standard Deviation of log return	15%	Standard Deviation of S&P500
θ	Risk Aversion	18	To match the top-wealth equity share of 30% in the data
ψ	Elasticity of Intertemporal Substitution	1.5	Vissing-Jørgensen and Attanasio (2003)

Discount Factor

The discount factor β is calibrated such that the model matches the aggregate asset to income ratio in the data. In 2022, the household net wealth is around $140 trillion dollars and the compensation of

[10]The median is still close to 0 even after removing the households who answer 50 percents in two consecutive sampling period and are likely noisy respondents

employees is around \$14 trillion dollars[11]. The discount factor is chosen to match the asset to income ratio of 10. The calibrated discount factor for the model with heterogeneous belief is $\beta = 0.92$, which is in line with the literature. The discount factors of all single-β models are different under different beliefs to match the same aggregate asset to income ratio. Table 6 shows the estimates. Models with more optimistic households have a smaller discount factor β than the one with pessimistic households. This is because higher perceived returns makes optimistic households more willing to save under an elasticity of intertemporal substitution greater than 1. To match the same aggregate asset to income ratio, the discount factor of the model has to be smaller to discourage them from saving.

Table 6. Calibrated Discount Factor for Single-β Model

Belief Distribution	Value
Heterogeneous Beliefs	0.92
Optimistic Only	0.92
Median Only	0.95
Pessimistic Only	0.95
Rational Expectation Only	0.94

Note: The table shows the calibrated discount factors β for single-β model. The aggregate asset to income ratio is set to 10.

In the alternative calibration with heterogeneous discount factors, the distribution of discount factors is assumed to be independent of other households characteristics and chosen to match the wealth share at different percentiles. I consider three discount factors group to capture the wealth distribution on the top and the bottom. In the *Survey of Consumer Finance* 2022, the bottom 50% of the households own around 2% of the wealth while the top 1% of the households own around 30% of the wealth. Table 7 shows the calibration results. High wealth households are captured by a large discount factor β to match their high wealth share. The average marginal propensity to consume (MPC) is 0.41, which is slightly lower than the annual MPC of 0.68 in HANK literature[12].

In the analysis of the model with heterogeneous discount factors, I consider varying the belief distribution while keeping the discount factors fixed, as opposed to estimating a different distribution of discount factors β. This is because a distribution of discount factors could match any wealth distribution, as demonstrated by Carroll *et al.* (2017). The main goal of this paper is to show that belief heterogeneity still plays an important role in wealth inequality even in the presence of discount

[11] Source: NIPA Table for the total compensation of employees. Flow of Fund Table for households net worth

[12] As discussed in Auclert (2019) and Auclert and Rognlie (2020), the consensus in literature suggests a quarterly MPC of 0.25 so the annual MPC should be around $0.68 \approx 1 - (1 - 0.25)^4$.

Table 7. Calibrated Discount Factor for Dist-β Model

Group Name	Population Weight	Wealth Share	Calibrated β
Bottom	0.5	0.02	0.52
Middle	0.49	0.68	0.93
Top	0.01	0.3	0.95

Note: The table shows the calibration results for multi-β model, including the population weight, wealth share, and calibrated value. The aggregate asset to income ratio is set to 10.

factor heterogeneity[13]. This model provides a good fit of the wealth distribution in the US, as the Gini coefficient of wealth is at 0.82, closely matching the US wealth inequality in the recent years.

[13] An ideal experiment would be matching β to the distribution of MPC.

4 Effect of Subjective Beliefs on Wealth Inequality

In this section, I show how the dispersion in subjective beliefs about stock returns affects the wealth distribution. Two exercise are conducted.

First, I compare stationary distributions in single-β models with different beliefs distribution that were calibrated to match the same aggregate wealth-to-GDP ratio. This exercise shows the additional wealth inequality generated by belief heterogeneity, relative to a standard Bewley-Aiyagari type model.

Second, I compare stationary distributions in the multi-β model with different beliefs distribution while keeping the distribution of discount factors the same. This exercise shows the wealth inequality reduction from heterogeneous beliefs to homogeneous beliefs, in a model calibrated to match US inequality statistics.

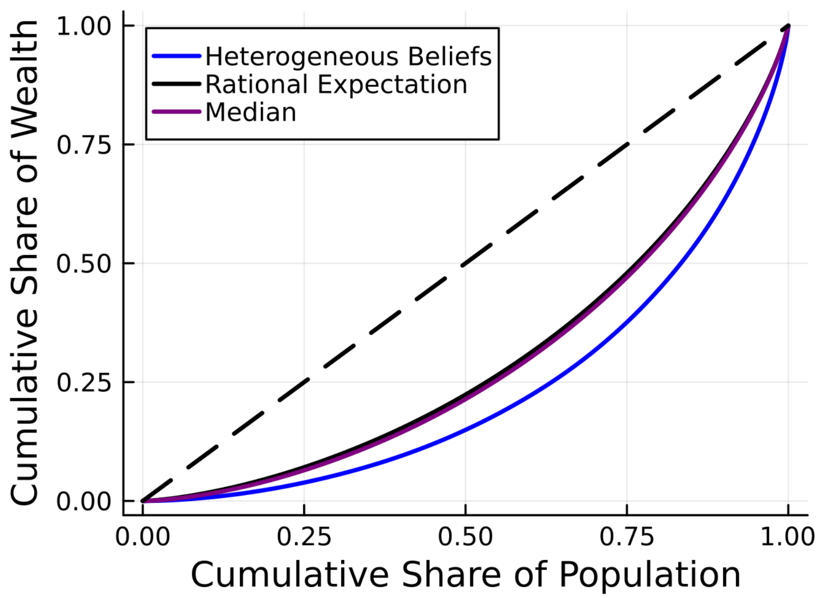
4.1 Wealth Distribution under Different Beliefs: Single-β Model

The dispersion in subjective beliefs about stock returns significantly affects the wealth distribution. Figure 6 shows the Lorenz curve of the wealth distribution under different beliefs. The Lorenz curve plots the cumulative share of population against the cumulative share of wealth. The dashed line represents perfect equality, when wealth is equally distributed to the households. The baseline model generates a Lorenz curve that is further away from the 45-degree line than the model with rational expectation and the model with median belief. The additional Gini coefficient of wealth, measured by the area between the blue line and the purple line, is around 0.12 in the baseline model compared to models with homogeneous beliefs.

To understand the reason behind the result, I decompose the growth rate of financial wealth into saving rate and portfolio return[14]. Beliefs about stock returns affect both the saving rate and the portfolio return. Figure 7 shows the growth rate of financial wealth, saving rate and portfolio return under different beliefs. The growth rate of financial wealth is higher for optimistic households and lower for pessimistic households. The saving rate is lower for optimistic households due to the EIS above one and the higher perceived return of the risky asset. The expected portfolio return is also

[14] In the appendix, I extend the formula to cover the labour income.

Figure 6. Lorenz Curve of Wealth Inequality

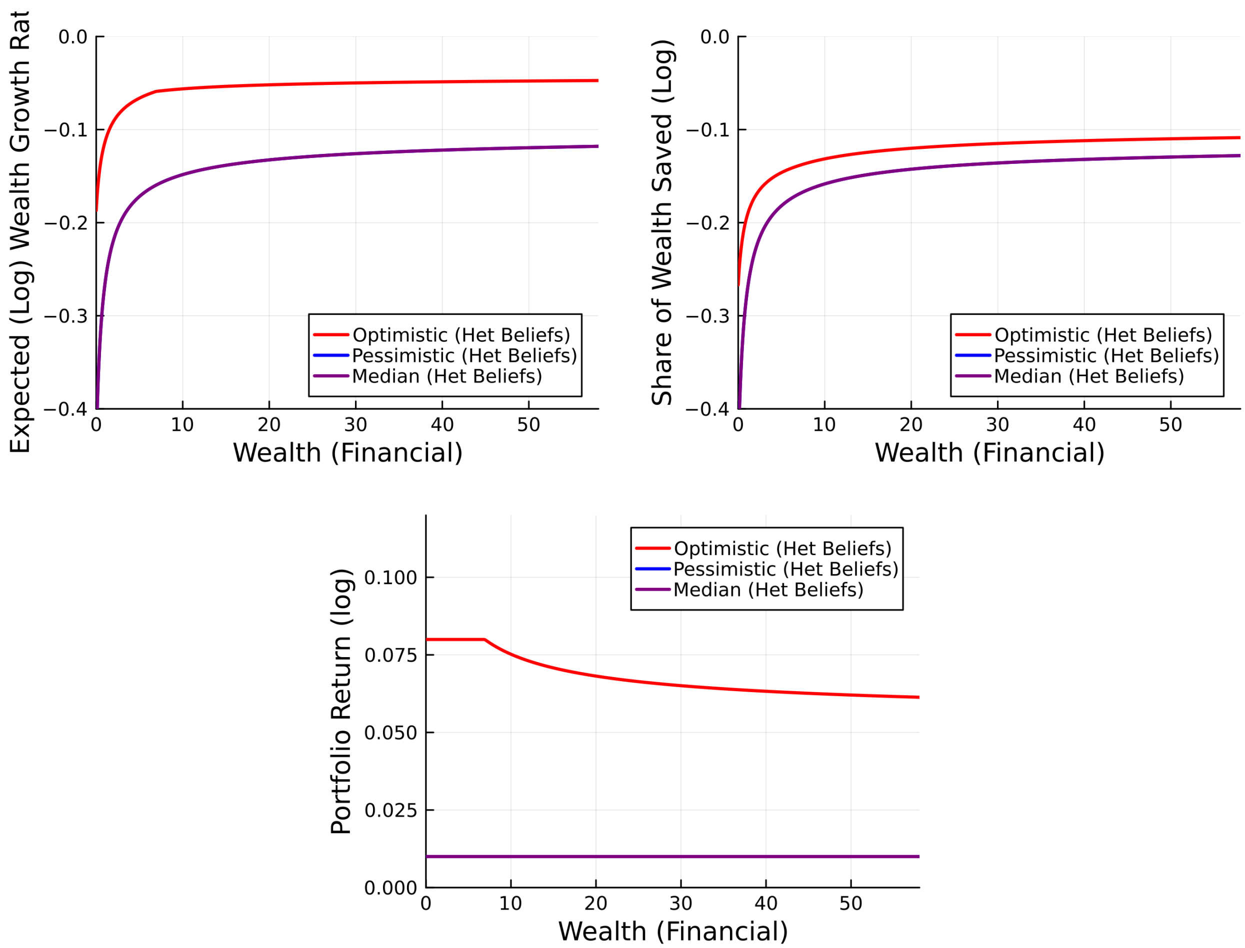


Note: Lorenz Curve plots the cumulative share of population against the the cumulative share of wealth. Heterogeneous Beliefs refers to the model with calibrated beliefs. Rational Expectation refers to the model with rational expectation on stock returns. Median refers to the model with median beliefs on stock returns. Each model's discount factor is calibrated to match $Asset/GDP = 10$ Dashed 45-degree line represents perfect equality.

higher for optimistic households due to their higher risky share of the portfolio.

$$\underbrace{\log(\frac{W_{it+1}}{W_{it}})}_{\text{Growth Rate of Financial Wealth}} = \underbrace{\log(\frac{X_{it}}{W_{it}})}_{\text{Share of Financial Wealth Saved}} + \underbrace{\log(R^f + \omega_{it}(R^m - R^f))}_{\text{Portfolio Return}} \tag{13}$$

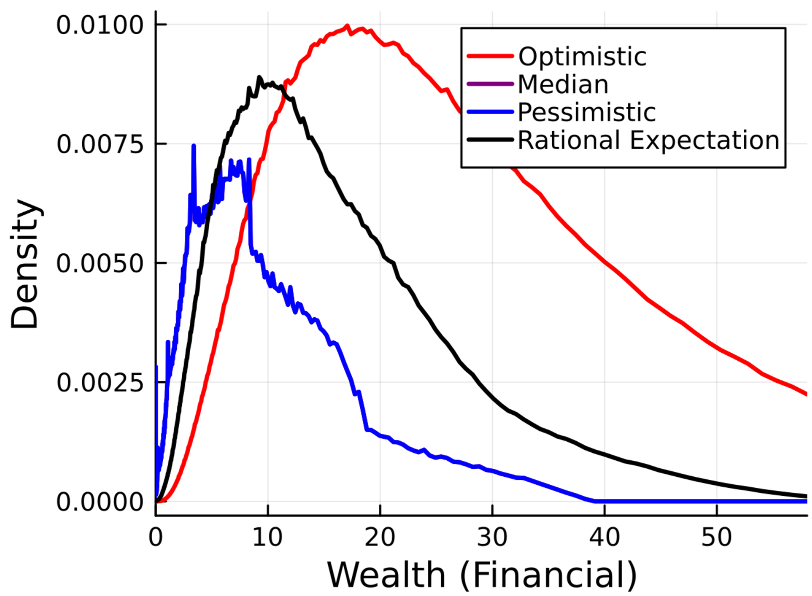
Figure 7. Wealth Growth Rate under Different Beliefs



Note: The figure shows the growth rate of financial wealth, saving rate and portfolio return under different beliefs for a median income household in the model. The discount factor is the same for all three types of beliefs, that is calibrated to match Asset/Income = 10.

Difference in wealth growth rate lead to a disparity in the wealth position between the households with different beliefs. Figure 8 depicts the stationary wealth distribution under different beliefs. The optimistic households accumulate more wealth and have a thicker right tail than the pessimistic households. They accumulate more wealth because of the higher wealth growth rate, which is driven by the higher risky share of the portfolio and the higher saving rate. The thicker right tail is due to the higher exposure to the risky asset. Households who experienced multiple high returns would end up on the right-tail of the wealth distribution.

Figure 8. Wealth Distribution under Different Beliefs



Note: The figure shows the stationary wealth distribution under different beliefs.

A variance decomposition exercise is conducted to understand the role of belief heterogeneity in wealth inequality. The variance of log wealth in my model with belief heterogeneity is around 1.33 while the variance of log wealth in the model with homogeneous beliefs is around 0.78. The additional variance of log wealth from belief heterogeneity can stem from two sources: the dispersion of log wealth conditional on beliefs and the dispersion of log wealth across beliefs.

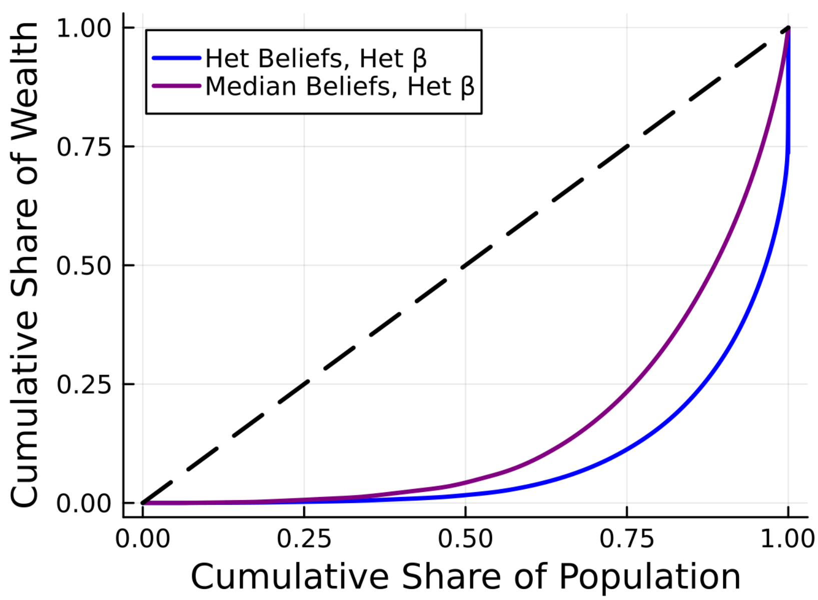
$$Var[\log(w)] = \underbrace{\sum_{\Pi_i} Var[\log(w)|\Pi_i]P(\Pi_i)}_{\text{Within Variance}} + \underbrace{\sum_{\Pi} (E[\log(w)|\Pi_i] - E[\log(w)])^2 P(\Pi_i)}_{\text{Differences across Beliefs}} \tag{14}$$

The within variance is around 0.93 and the differences across beliefs is around 0.40. The differences across beliefs is originated from the different growth rate of financial wealth under different beliefs as discussed earlier. The increase in the within variance is due to the lower discount factor β calibrated for the heterogeneous beliefs calibration to match the aggregate asset to income ratio. The lower discount factor pushes the pessimistic and median households to save less and consume more, leading to a higher percentage of hand-to-mouth households. This is verified by comparing the MPC of the households under different calibrations. The MPCs of the pessimistic and median households are 5 percent higher in the heterogeneous beliefs calibration than in the homogeneous belief calibration.

4.2 Wealth Distribution under Different Beliefs: Het-β Model

The findings from the single-β model are robust to the multi-β model. Heterogeneous beliefs still explain a sizable portion of the wealth inequality when heterogeneity in discount factors is considered. Figure 9 shows the Lorenz curve of the wealth distribution under different beliefs distribution in the multi-β model. The Gini coefficient of wealth inequality decreases from 0.81 to 0.67 when the belief heterogeneity is removed. The reduction mostly comes from the middle and the top of the wealth distribution. In the calibrated model, the top 10% households own around 85 percent of total wealth while the top 10% households own only 46 percent of total wealth in the counterfactual model with homogeneous beliefs. The bottom distribution of wealth is unaffected by the belief heterogeneity, as the bottom 50% of the households own less than 5 percent of the total wealth in both models.

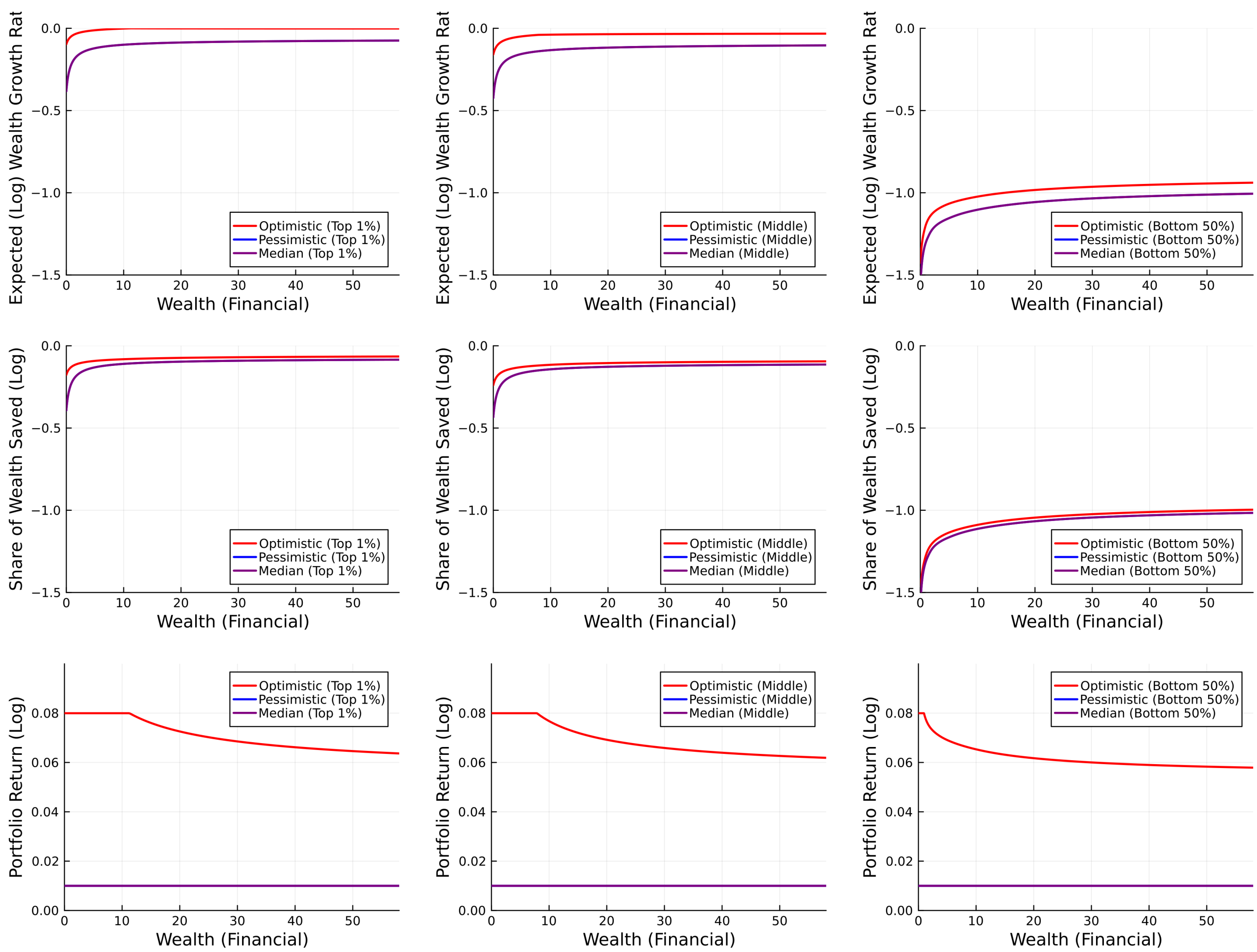
Figure 9. Lorenz Curve of Wealth Inequality: Heterogeneous β



Note: Lorenz Curve plots the cumulative share of population against the the cumulative share of wealth. "Het Beliefs, Het β"'s discount factor is calibrated to match $Asset/GDP = 10$. "Median Beliefs, Het β" uses the same distribution of discount factors while beliefs are set to median belief. Dashed 45-degree line represents perfect equality.

Figure 10 shows the growth rate of financial wealth, saving rate and portfolio return under different beliefs in the multi-β model. The effect of beliefs to saving is similar across households to different discount factors. Optimistic households tend to save more and allocate more to the risky asset. As wealth is concentrated in the top 50% of the households who have a high exposure to risky asset, changing the beliefs of the top 50% of the households would have a significant impact on the wealth distribution. Beliefs still affect the bottom 50% of the households, but since they save very little out of their income, the overall effect to inequality is small.

Figure 10. Wealth Growth Rate under Different Beliefs: Heterogeneous β



Note: The figure shows the growth rate of financial wealth, saving rate and portfolio return under different beliefs for top 1% (Left), middle (Mid) and bottom 50% (Right) income households in the model with heterogeneous discount factors. The discount factor is calibrated to match Asset/Income $= 10$ and wealth shares of 2%, 68%, 30% for bottom 50%, middle and top 1% households.

The analysis on the heterogeneous discount factors model reinforces the importance of belief heterogeneity in explaining wealth inequality. The effect of belief heterogeneity is similar to the single-β model, except that the effect is more pronounced in the middle to top wealth distribution.

5 Conclusion

In this paper, I argue that the dispersion in subjective beliefs about stock returns could have a significant impact on wealth inequality. Comparing optimistic and pessimistic households in the US, I find that optimistic households are more likely to participate in the stock market and invest more in stocks. Using these empirical findings, I calibrate a heterogeneous-agent model that matches data on income inequality, beliefs distribution and portfolio choice. The model with belief heterogeneity generates an additional 0.07 in the Gini coefficient of wealth and increases the share of wealth owned by the top 15% of the households by 9%, compared to the counterfactual models where all households share the same belief. This suggests that the dispersion in subjective beliefs about stock returns could be an important factor in explaining the size of wealth inequality in the US.

How should policy maker respond to the finding? My shows that a significant portion of wealth inequality could be attributed to the difference in subjective beliefs but not solely driven by the difference in risk preferences. The difference in subjective beliefs could be originated from information frictions or behavorial biases. Further reseach is needed to understand the source of the difference in subjective beliefs to answer important normative policy questions.

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A First Order Conditions

For a household i with financial wealth W_{it}, income Y_{it}, beliefs type Π_{it} and non-beliefs permanent type Ξ_i, the maximization problem is given by:

$$V_t(W_{it}, Y_{it}, \Pi_{it}, \Xi_i) = \max_{C_{it},\omega_{it}} \left((1-\beta)C_{it}^{1-1/\psi} + \beta \Big(CE_{it}[V_{it+1}] \Big)^{1-1/\psi} \right)^{1/(1-1/\psi)} \tag{15}$$

where $CE_{it}[.]$ denotes household i certainty equivalent operator at time t and future value is denoted as V_{it+1}, with $V_{it+1} = V_{t+1}(W_{it+1}, Y_{it+1}, \Pi_{it+1}, \Xi_i)$. The certainty equivalent of future value with a risk aversion parameter θ is given by

$$CE_{it}(V_{it+1}) = \left(E \left[\Big(V_{t+1}(W_{it+1}, Y_{it+1}, \Pi_{it+1}, \Xi_i) \Big)^{1-\theta} | \Pi_{it}, Y_{it} \right] \right)^{\frac{1}{1-\theta}}$$

subject to the budget constraint

$$W_{it+1} = (W_{it} + Y_{it} - C_{it}) \left(R_{t,t+1}\omega_{it} + R^f(1-\omega_{it}) \right) \tag{16}$$

and constraints on borrowing, short-selling and leveraging

$$W_{it+1} \geq 0 \quad \text{and} \quad 0 \leq \omega_{it} \leq 1$$

The first order conditions for the maximization problem are given by

$$\beta (CE_{it}[V_{it+1}])^{\theta - 1/\psi} E_t \left[\Big(V_{it+1} \Big)^{-\theta} \frac{\partial V_{it+1}}{\partial W_{it+1}} \Big(R_{t,t+1}\omega_{it} + R^f(1-\omega_{it}) \Big) \right] = (1-\beta) C_{it}^{-1/\psi} \tag{17}$$

$$E_t \left[\Big(V_{it+1} \Big)^{-\theta} \frac{\partial V_{it+1}}{\partial W_{it+1}} (R_{t,t+1} - R^f) \right] = 0 \tag{18}$$

The first equation is the standard Euler equation and the second equation is the optimality condition for the portfolio choice.

B Additional Data Description

The *Michigan Survey of Consumers* is used for a majority of the analysis. The survey is conducted by the Institute for Social Research under University of Michigan. Each month, 600 households are surveyed through telephone interviews to collect information on consumer sentiment and households characteristics. The survey started in 1946 and has been conducted continuously since then.

The survey aims at capturing the US population, as the summary statistics closely track the other national statistics.

B.1 Summary Statistics

C Additional Empirical Results

This section presents additional results. subsection C.1 shows the regression results with risk-adjusted returns as the regressors.

C.1 Regression with Risk-adjusted Returns

The regression results are robust to regressing on subjective risk-adjusted returns. Under the log-normal returns assumption, the subjective risk-adjusted return is a monotone transformation of the subjective probability of positive stock returns. However, there are a number of survey respondents who report 100 percent or 0 percent probability of positive stock returns, which corresponds to risk-adjusted returns of infinity or negative infinity. Both of them are dropped from the regression analysis. All the regressions produce same signs and similar quantitative results as the baseline regressions.

Table 8. Stock Market Participation on Beliefs

Dependent Variable:	Participation in Stock Market			
Model:	(1)	(2)	(3)	(4)
Sub. Risk-adjusted Return	0.118***	0.006***	0.119***	0.006**
	(0.002)	(0.002)	(0.002)	(0.003)
Individual FE		Yes		Yes
Time FE			Yes	Yes
Observations	114,815	114,815	114,815	114,815
Adjusted R^2	0.04543	0.75264	0.04934	0.75325

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Note: Participation in Stock Market is a binary indicator of whether the individual has stock market investment. The subjective risk-adjusted return is inferred from the subjective probability of positive stock return under log-normal distribution assumptoin.

The standard deviation of risk-adjusted return is around 0.84. Thus, one standard deviation increase in optimism corresponds to around 10 percentage point increase in stock market participation and around 7 percent increase in stock investment conditional on the same income.

Table 9. Investment Share on Beliefs

Dependent Variable:	Log Investment to Income			
Model:	(1)	(2)	(3)	(4)
Sub. Risk-adjusted Return	0.095^{***}	0.035^{***}	0.084^{***}	0.029^{***}
	(0.007)	(0.007)	(0.007)	(0.009)
Individual FE		Yes		Yes
Time FE			Yes	Yes
Observations	64,250	64,250	64,250	64,250
Adjusted R^2	0.00273	0.83450	0.01559	0.83568

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 10. Beliefs Persistence

Dependent Variable:	Sub. Risk-adjusted Return	
Model:	(1)	(2)
Sub. Risk-adjusted Return 6m lag	0.418^{***}	0.407^{***}
	(0.005)	(0.006)
Time FE		Yes
Observations	36,553	36,553
Adjusted R^2	0.17742	0.19394

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

C.2 Regression with Macro Expectations

I analyze how beliefs about stock returns correlate with investment decisions. In this section, I present the regression results using the macro expectations as the regressors. The *Michigan Survey of Consumers* also asks respondents about their expectation on business conditions and employment conditions in the next year. However, they are coarsely measured by the survey questions. The survey asks respondents whether they expect business conditions to be better, the same, or worse in the next year. Similarly, they were asked whether they expect unemployment rate to be higher, the same, or lower in the next year.

I construct two binary variables, "Expect Better Economy" and "Expect More Employment", for each question, with a value of one representing a positive expectation and zero otherwise. Both set of regressions produce similar signs but the magnitudes are smaller. This may be caused by the coarseness of the survey questions, as the households cannot express their beliefs beyond the three categories. However, it may also suggest that the expectation on the macro conditions are less important than the expectation on stock returns in determining the investment decisions.

Expectation on Business Conditions

Table 11. Stock Market Participation on Beliefs

Dependent Variable:	Participation in Stock Market			
Model:	(1)	(2)	(3)	(4)
Expect Better Economy	0.054***	0.003	0.051***	0.002
	(0.003)	(0.003)	(0.004)	(0.003)
Individual FE		Yes		Yes
Time FE			Yes	Yes
Observations	163,159	163,159	163,159	163,159
Adjusted R^2	0.00261	0.73539	0.02646	0.73622

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 12. Investment Share on Beliefs

Dependent Variable:	Log Investment to Income			
Model:	(1)	(2)	(3)	(4)
Expect Better Economy	0.123***	0.009	0.133***	0.017
	(0.011)	(0.010)	(0.012)	(0.014)
Individual FE		Yes		Yes
Time FE			Yes	Yes
Observations	82,872	82,872	82,872	82,872
Adjusted R^2	0.00148	0.81999	0.01831	0.82127

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Expectation on Employment Conditions

Table 13. Stock Market Participation on Beliefs

Dependent Variable:	Participation in Stock Market			
Model:	(1)	(2)	(3)	(4)
Expect More Empolyment	0.045***	0.005	0.028***	0.005
	(0.003)	(0.003)	(0.004)	(0.004)
Individual FE		Yes		Yes
Time FE			Yes	Yes
Observations	163,159	163,159	163,159	163,159
Adjusted R^2	0.00144	0.73540	0.02473	0.73623

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 14. Investment Share on Beliefs

Dependent Variable:	Log Investment to Income			
Model:	(1)	(2)	(3)	(4)
Expect More Empolyment	0.173***	0.017	0.131***	0.019
	(0.012)	(0.012)	(0.015)	(0.016)
Individual FE		Yes		Yes
Time FE			Yes	Yes
Observations	82,872	82,872	82,872	82,872
Adjusted R^2	0.00234	0.82000	0.01791	0.82127

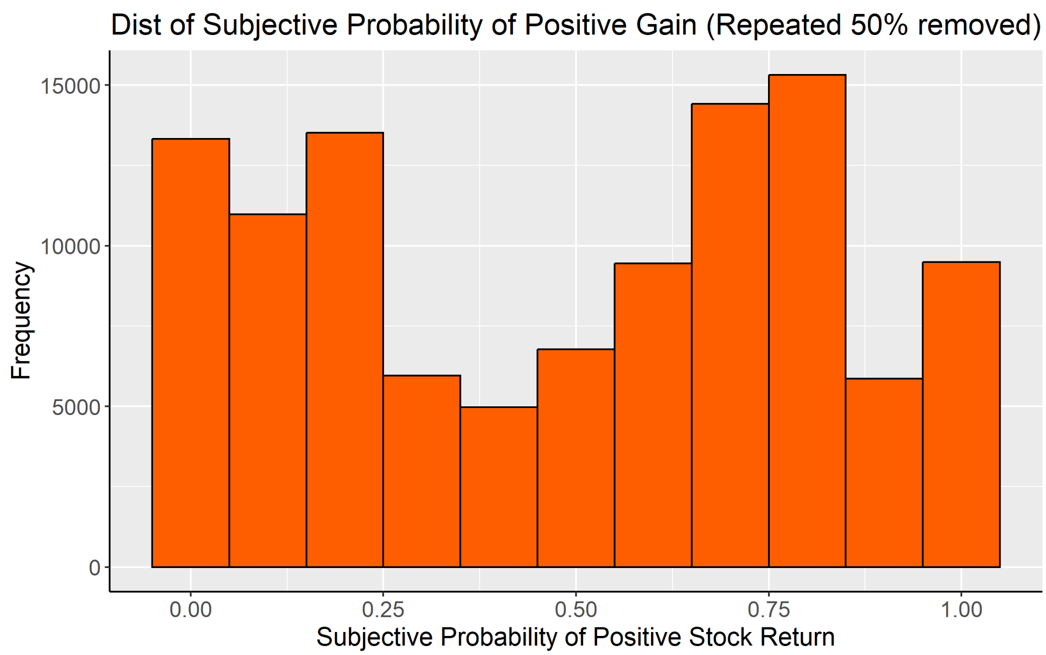
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

C.3 Regression by Groups

C.4 Remove Noisy Respondents

In this section, I consider the regression results after removing respondents who reported the same answer of 50 percent in both survey waves. They are likely to be noisy respondents as uninformed people might answer a common response of 50% to any probability question. In the data, they constitutes only around 3% of the samples. All the empirical findings still hold true after removing those responses. Figure 11 shows the distribution of subjective beliefs about stock returns after removing the noisy respondents.

Figure 11. Distribution of Subjective Beliefs about Stock Returns



Note: Data from *Michigan Survey of Consumers*. Subjective Probability of Positive Stock Return is measured by respondents prediction of the probability that a diversified stock mutual fund would have a positive return in the next year. This histogram excludes respondents who reported the same answer of 50 percent in both survey waves.

Table 15 reports the same estimate for the persistence of subjective beliefs about stock returns. Since the noisy observations is exactly 50 percent, their effect on the persistence of subjective beliefs on a regression with time-fixed effect is negligible. Optimism today still predicts optimism in six months after removing the noisy respondents.

Table 15. Beliefs Persistence

Dependent Variable:	Sub. Prob of Positive Return	
Model:	(1)	(2)
Sub. Prob of Positive Return 6m lag	0.432***	0.415***
	(0.004)	(0.005)
Time FE		Yes
Observations	42,113	42,113
Adjusted R^2	0.19212	0.21502

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Note: The subjective probability of positive stock return is the individual's subjective probability of positive stock market return. Sub. Prob of Positive Return 6m lag is the belief reported by the same household six mouths before. Respondents who reported the same answer of 50 percent in both survey waves are removed.

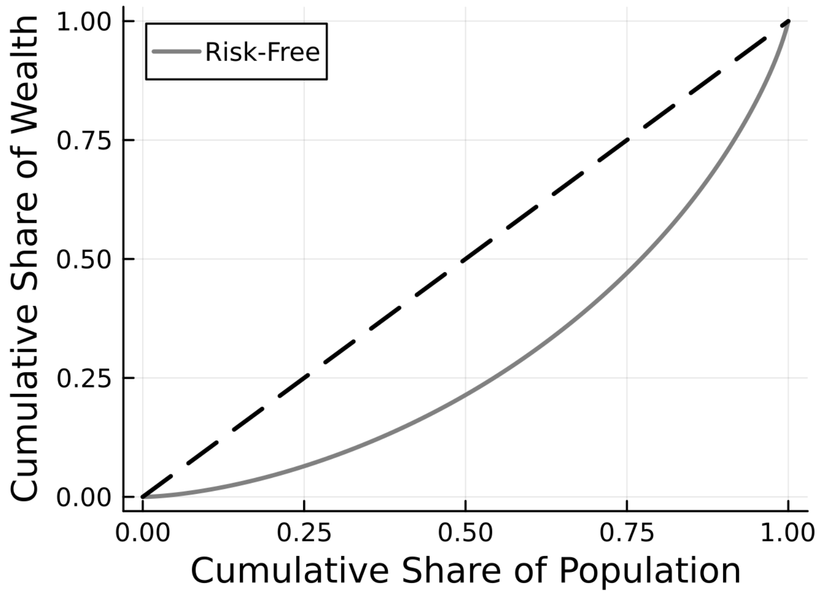
D Additional Results

In this section, I present some additional simulation results. subsection D.1 shows the wealth distribution in the model without the risky asset, similar to the setup of Bewley (1977). This is a useful exercise to understand the effect of calibrated labour income process on the wealth distribution.

D.1 Without Risky Asset

In the model with only a risk-free asset, I calibrate the discount factor β to match the asset-to-GDP ratio of 10. Figure 12 depicts the Lorenz curve of the wealth distribution in this model. The Gini coefficient of wealth inequality is 0.41, in line with the literature that models with only precautionary savings motive when matched to aggregate wealth data generate wealth inequality similar to the income inequality.

Figure 12. Lorenz Curve of Wealth Distribution: Standard Incomplete Market



This figure shows the Lorenz curve of the wealth distribution in the calibrated model with only the risk-free asset. The dashed 45-degree line represents perfect equality. The Gini coefficient of wealth inequality is 0.41. β is calibrated to match the $Asset/GDP = 10$

E Solution Algorithm

E.1 Endogenous Grid Method for the Portfolio Problem

The policy function is solved by the endogenous grid method by Carroll (2006). I modified the steps to incorporate the portfolio problem. I first describe the method for solving a one-period backward induction problem. The policy function in the stationary equilibrium could be obtained by iterating the one-period backward induction problem.

1. **Initialization**: We are given a grid of future value V_{it+1} and $\partial V_{it+1}/\partial W_{it+1}$ defined on grid points of W_{it+1}, Y_{it+1}, Π_{it+1} and Ξ_i. The grid is linearly interpolated to obtain the value of V_{it+1} and $\partial V_{it+1}/\partial W_{it+1}$ along the continuous state space W_{it+1}, the interpolated function is denoted as $\tilde{V}_{t+1}(W|Y_{it+1}, \Pi_{it+1}, \Xi_i)$ and $\tilde{V}'_{t+1}(W|Y_{it+1}, \Pi_{it+1}, \Xi_i)$.

2. **Portfolio Choice on Endogenous Grid**: For a grid point of saving X_{it}^{endo} find the optimal portfolio choice $\omega_{it}(X_{it}^{endo})$

(a) **Solve FOC**: Solve the numerical counterpart of Equation 18, which is given by

$$E_t\left[\left(\tilde{V}_{t+1}(X_{it}^{endo}R_{t,t+1}^m(\omega_{it})|Y_{it+1},\Pi_{it+1},\Xi_i)\right)^{-\theta}\tilde{V}_{t+1}'(X_{it}^{endo}R_{t,t+1}^m(\omega_{it})|Y_{it+1},\Pi_{it+1},\Xi_i)(R_{t,t+1}-R^f)\right]=0$$

$$\text{where} \quad R_{t,t+1}^m(\omega_{it})=R_{t,t+1}\omega_{it}+R^f(1-\omega_{it})$$

(b) **Impose Portfolio Constraint**: If $\omega_{it}>1$, set $\omega_{it}=1$ and if $\omega_{it}<0$, set $\omega_{it}=0$.

Notice that this

3. **Future Value of Saving on Endogenous Grid**: Using the optimal portfolio choice ω_{it}, calculate the future value of saving on the grid points of X_{it}^{endo}. The left-hand side of Equation 17 is approximated by

$$\begin{aligned}\mathbb{W}_{it}(X_{it}^{endo}) =&\beta(CE_{it}[\tilde{V}_{t+1}(X_{it}^{endo}R_{t,t+1}^m(\omega_{it})|Y_{it+1},\Pi_{it+1},\Xi_i)])^{\theta-1/\psi}\\ &\times E_t\left[\left(\tilde{V}_{t+1}(X_{it}^{endo}R_{t,t+1}^m(\omega_{it})|Y_{it+1},\Pi_{it+1},\Xi_i)\right)^{-\theta}\tilde{V}_{t+1}'(X_{it}^{endo}R_{t,t+1}^m(\omega_{it})|Y_{it+1},\Pi_{it+1},\Xi_i)\right.\\ &\left.\times\left(R_{t,t+1}\omega_{it}+R^f(1-\omega_{it})\right)\right]\\ &\text{where} \quad R_{t,t+1}^m(\omega_{it})=R_{t,t+1}\omega_{it}+R^f(1-\omega_{it})\end{aligned}$$

4. **Consumption on Endogenous Grid**: The optimal consumption is given by Equation 17

$$C_{it}(X_{it}^{endo})=(\frac{1}{1-\beta}\mathbb{W}_{it}(X_{it}^{endo}))^{-\psi}$$

Notice that $C_{it}(X_{it}^{endo})$ and $\omega_{it}(X_{it}^{endo})$ are represented as a function of X_{it}^{endo} to highlight that the obtained consumption and portfolio choice are conditional on endogenous saving point X_{it}^{endo}.

5. **Policy Function on the Original Grid**: The policy function is obtained by linearly interpolating the optimal consumption and portfolio choice on the original grid points of W_{it}, Y_{it}, Π_{it} and Ξ_i.

   - **Financial Wealth on the Endogenous Grid**: The cash-on-hand on the endogenous grid is given by $X_{it}^{endo}+C_{it}(X_{it}^{endo})$
   - **Interpolation**: Find the policy function X_{it} on the original grid of cash-on-hand $W_{it}+Y_{it}$ by linear interpolating using the saving grid X_{it}^{endo} as the y-variable and the cash-on-hand

on the endogenous grid $X_{it}^{endo} + C_{it}(X_{it}^{endo})$ as the x-variable. Similarly, interpolate the portfolio choice ω_{it}

- **Impose Constraints**: If the policy function violates the borrowing, short-selling and leveraging constraints, set the policy function to the nearest boundary.

6. **Value Function on the Original Grid**: The value function on the original grid is obtained by

$$V_{it}(W_{it}, Y_{it}, \Pi_{it}, \Xi_i) = \left((1-\beta) C_{it}^{1-1/\psi} + \beta \Big(CE_{it}[\tilde{V}_{t+1}(X_{it} R_{t,t+1}^m(\omega_{it}) | Y_{it+1}, \Pi_{it+1}, \Xi_i)] \Big)^{1-1/\psi} \right)^{1/(1-1/\psi)}$$

and the partial derivative of the value function wrt financial wealth is given by

$$\frac{\partial}{\partial W_{it}} V_{it+1}(W_{it}, Y_{it}, \Pi_{it}, \Xi_i) = (1-\beta) \times V_{it}(W_{it}, Y_{it}, \Pi_{it}, \Xi_i)^{1/\psi} \times C_{it}^{-1/\psi}$$

Finally, for the policy function and the value function in the stationary equilibrium, iterate the above backward induction step until the policy function and the value function converge.

E.2 Solving the Stationary Distribution

E.3 Calibration

F Details on Discretization