## MAT2440, Classwork37, Spring2025

ID:

1. Find all integers that are congruent to 3 modulo 5.

Sol: 
$$Q = 3 \pmod{5} \Rightarrow Q = 5 \cdot N + 3$$
  
 $N = 0, \ Q = 5 \cdot 0 + 3 = 3$   
 $N = 1, \ Q = 5 \cdot 1 + 3 = 8$   
 $N = -2, \ Q = 5 \cdot 2 + 3 = 13$   
 $N = 2, \ Q = 5 \cdot 2 + 3 = 13$   
 $Q = \frac{1}{2} + \frac{$ 

2. Find the integer a such that  $a \equiv 3 \pmod{12}$  and  $11 \le a \le 22$ .

Sol 
$$Q \equiv 3 \pmod{12} \Rightarrow Q = 12 \cdot N + 3$$
  
if  $11 \le Q \le 22$ , then  $11 \le 12 \cdot N + 3 \le 22$   
 $\Rightarrow \theta \le 12N \le 19$   
 $\Rightarrow \frac{\theta}{12} \le N \le \frac{19}{12} \Rightarrow 0 < N \le 1$   
 $N = 1, Q = 12 \cdot 1 + 3 = 15$   
 $(N = 0, Q = 12 \cdot 0 + 3 = 3)$   $Q = 15$ .

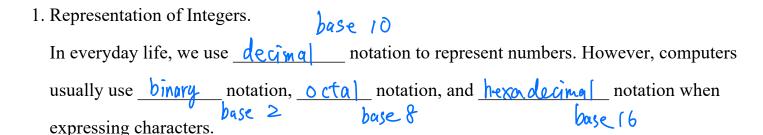
3. Find all integers between -50 and 50 that are congruent to 6 modulo 11.

Sol: if 
$$\alpha = 6 \pmod{11}$$
 and  $-5 \times \alpha \leq 50$ , we have

 $\alpha = 11 \cdot n + 6 = 50$ 
 $\Rightarrow -56 \leq 11 \cdot n + 6 \leq 50$ 
 $\Rightarrow -56 \leq 11 \cdot n \leq 44$ 
 $\Rightarrow -56 \leq 11 \cdot$ 

$$N = -4$$
,  $Q = 11 \cdot (-4) + 6 = -38$   
 $N = -3$   $Q = 11 \cdot (-3) + 6 = -2$   
 $N = -2$ ,  $Q = 11 \cdot (-2) + 6 = -16$   
 $N = -1$ ,  $Q = 11 \cdot (-1) + 6 = -5$ 

$$N=0$$
,  $\alpha=11.0$   $+6$   $=6$ )  $+11$   
 $N=1$ ,  $\alpha=11.1$   $+6$   $=17$ )  $+11$   
 $N=2$ ,  $\alpha=11.2$   $+6$   $=26$ )  $+11$   
 $N=3$ ,  $\alpha=11.3$   $+6$   $=39$ )  $+11$   
 $N=4$ ,  $\alpha=11.4$   $+6$   $=50$ 



2. Decimal notation (base 10):

Acceptable digits: 0, 1, 2, 3, 4, 5, 6, 1, 8, 9.

For example,  $2845 = (2845)_{10} = 2000 + 800 + 40 + 5$   $= 2 \times 10^{3} + 8 \times 10^{2} + 4 \times 10^{1} + 5 \times 10^{2}$ 

What is the decimal expansion of the number with binary expansion  $(10101)_2$ ?

$$(10101)_{2} = 1 \times 2^{4} + 0 \times 2^{3} + 1 \times 2^{2} + 0 \times 2^{1} + 1 \times 2^{0}$$

$$= 1 \times 16 + 0 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$$

$$= 2 (21)_{10}$$

4. Octal notation (base 8): Acceptable digits: 0, 1, 2, 3, 4, 5, 6, 1.

What is the decimal expansion of the number with octal expansion  $(7016)_8$ ?

$$(7016)_{8} = \frac{7 \times 8^{3} + 0 \times 8^{2} + 1 \times 8 + 6 \times 8^{6}}{2 \times 5!^{2} + 0 \times 6! + 1 \times 8 + 6 \times 1}$$

$$= \frac{7 \times 5!^{2} + 0 \times 6! + 1 \times 8 + 6 \times 1}{2 \times 5!^{2} + 0 \times 6! + 1 \times 8 + 6 \times 1}$$

$$= \frac{3598}{12 \times 5!^{2} + 0 \times 6! + 1 \times 8 + 6 \times 1}$$

5. Hexadecimal notation (base 16):

Acceptable digits:

What is the decimal expansion of the number with hexadecimal expansion  $(2AE0B)_{16}$ ?

$$(2AE0B)_{16} = \frac{2 \times [6^{4} + A \times 16^{3} + E \times 16^{2} + D \times 16^{4} + B \times 16^{6}]}{= 2 \times 16^{4} + 10 \times 16^{3} + 14 \times 16^{2} + D \times 16^{4} + 11 \times 1}$$
$$= \frac{2 \times 65536 + 10 \times 4696 + 14 \times 256 + D + 11}{2}$$

The above examples show the meaning of integer representation of different bases as well as how to convert them to decinal.