

# MAT1375, Classwork4, Fall2025

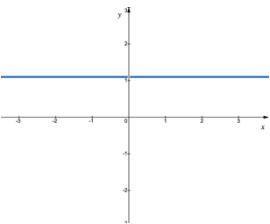
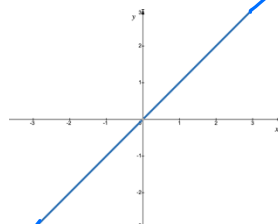
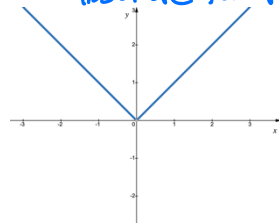
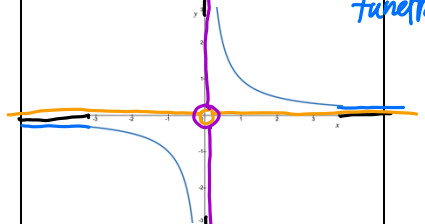
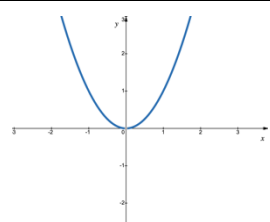
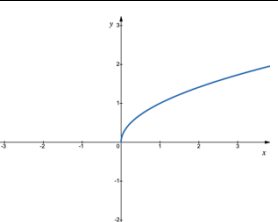
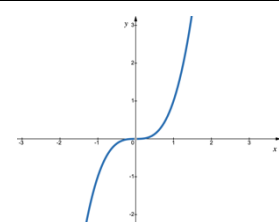
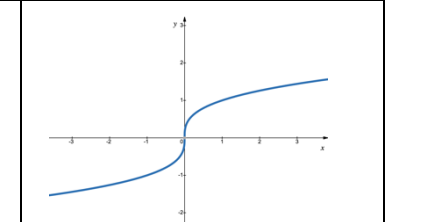
## Ch4. Basic Functions and Transformations

### 1. Even Functions and Odd Functions.

Even function: A function  $f$  is called even if  $f(-a) = f(a)$  for all  $a$ .

Odd function: A function  $f$  is called odd if  $f(-a) = -f(a)$  for all  $a$ .

### 2. Complete the table of basic functions:

$f(x) = \text{constant}$  Domain: $\mathbb{R}$ , $(-\infty, \infty)$ Range: $\{c\}$ Odd or Even: Even $f(-a) = c = f(a)$	$f(x) = x$  Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Odd or Even: Odd $f(-a) = -a = -f(a)$	$f(x) =  x $ absolute function  Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ Odd or Even: Even $f(-a) =  -a  = a = f(a)$	$f(x) = \frac{1}{x}$ hyperbolic function  Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$ Odd or Even: Odd $f(-a) = -\frac{1}{a} = -f(a)$
$f(x) = x^2$  Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ Odd or Even: Even $f(-a) = (-a)^2 = a^2 = f(a)$	$f(x) = \sqrt{x}$  Domain: $[0, \infty)$ Range: $[0, \infty)$ Odd or Even: Neither $f(-a) = \sqrt{-a}$ (not defined for a > 0)	$f(x) = x^3$  Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Odd or Even: Odd $f(-a) = (-a)^3 = -a^3 = -f(a)$	$f(x) = \sqrt[3]{x}$  Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Odd or Even: Odd $f(-a) = \sqrt[3]{-a} = -\sqrt[3]{a} = -f(a)$

### 3. Determine which of the following functions are even, odd, or neither.

$a > 0$

(a)  $f(x) = 2x^2 + x^4$  (b)  $f(x) = x + 3x^3$  (c)  $f(x) = x^3 + 4x^2$

$$\begin{aligned}
 (a) \quad f(a) &= 2(a)^2 + (a)^4 \\
 &= 2a^2 + a^4 \\
 f(-a) &= 2(-a)^2 + (-a)^4 \\
 &= 2a^2 + a^4
 \end{aligned}$$

Since  $f(-a) = 2a^2 + a^4 = f(a)$   
 $f$  is an even function

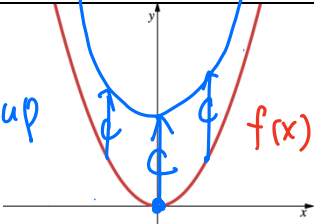
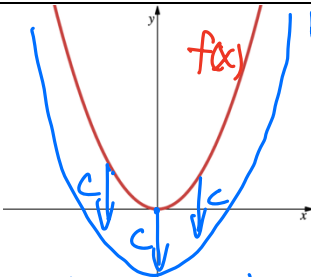
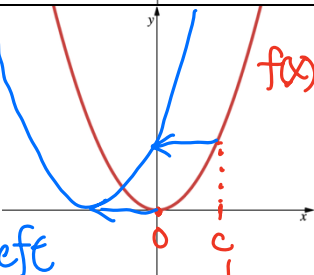
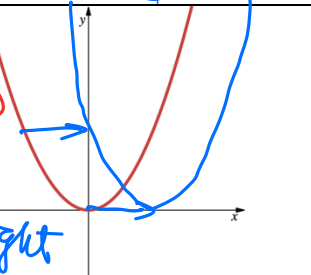
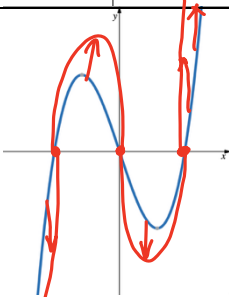
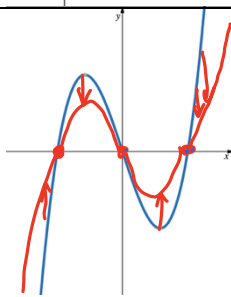
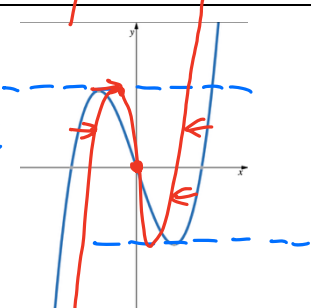
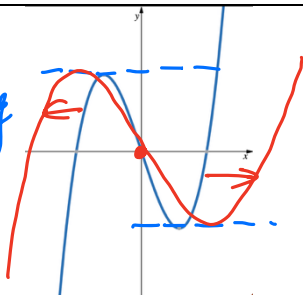
$$\begin{aligned}
 (b) \quad f(a) &= a + 3a^3 \\
 f(-a) &= (-a) + 3(-a)^3 \\
 &= -a - 3a^3 \\
 &= -1 \cdot (a + 3a^3) \\
 &= -f(a)
 \end{aligned}$$

$f(x)$  is odd

$$\begin{aligned}
 (c) \quad f(a) &= a^3 + 4a^2 \\
 f(-a) &= (-a)^3 + 4(-a)^2 \\
 &= -a^3 + 4a^2 \\
 -f(a) &= -(a^3 + 4a^2) \\
 &= -a^3 - 4a^2
 \end{aligned}$$

$f(x)$  is neither even nor odd.

4. Complete the table

Transformation of $f(x)$	Given $f(x)$ and then draw the transformation	
<u>Vertical shifts</u> $y = f(x) + c$ Add/subtract a constant on output	$c > 0$ : shift up 	$c < 0$ : shift down 
<u>Horizontal shifts</u> $y = f(x + c)$ Add/subtract a constant on input	$c > 0$ : shift to the left 	$c < 0$ : shift to the right 
<u>Vertical stretching/shrinking</u> $y = cf(x)$ Multiply a constant on output(s)	$c > 1$ : stretching 	$0 < c < 1$ : shrinking 
<u>Horizontal stretching/shrinking</u> $y = f(cx)$ Multiply a constant on input.	$c > 1$ : shrinking 	$0 < c < 1$ : stretching 

<u>Reflection about the <math>x</math>-axis</u> $y = -f(x)$ Multiply "-1" on output(s)	<u>Reflection about the <math>y</math>-axis</u> $y = f(-x)$ Multiply "-1" on the input(s)
