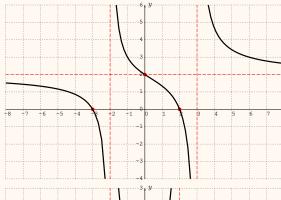
11.3 Exercises

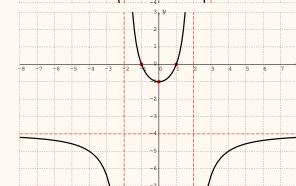
Exercise 11.1

Below are the graphs of rational functions whose numerators and denominators are polynomials of degree 2. All intercepts and asymptotes are at integer values, indicated in red. Find all intercepts and asymptotes, and find a formula for each function.





b)

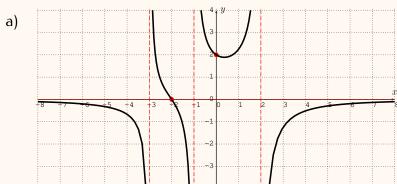


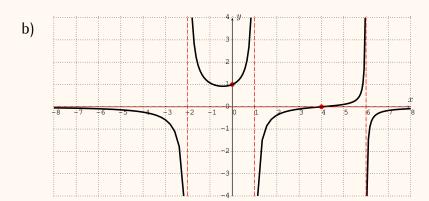
c)

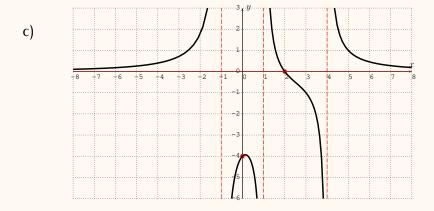


Exercise 11.2

Below are the graphs of rational functions whose numerators are polynomials of degree 1 and whose denominators are polynomials of degree 3. All intercepts and asymptotes are at integer values indicated in red. Find all intercepts and asymptotes, and find a formula for each function.







Exercise 11.3

Find the domain of each rational function below. Identify the removable discontinuities and find their x- and y-coordinates.

$$\mathbf{Va)} \ f(x) = \frac{(x-3)(x-4)}{(x+5)(x-4)}$$

$$\begin{array}{ll} \text{Va)} \ f(x) = \frac{(x-3)(x-4)}{(x+5)(x-4)} & \text{Vb)} \ f(x) = \frac{3(x+2)(x-5)}{(x+3)(x-5)} \\ \text{C)} \ f(x) = \frac{7(x-2)}{(x+3)(x-2)(x-6)} & \text{Vd)} \ f(x) = \frac{x^2+6x+8}{x^2+x-12} \\ \text{e)} \ f(x) = \frac{x^2-9}{x^2-x-6} & \text{f)} \ f(x) = \frac{x^2-4x+3}{x^3+x^2-2x} \end{array}$$

$$V$$
d) $f(x) = \frac{x^2 + 6x + 8}{x^2 + x - 12}$

e)
$$f(x) = \frac{x^2 - 9}{x^2 - x - 6}$$

f)
$$f(x) = \frac{x^2 - 4x + 3}{x^3 + x^2 - 2x}$$

Find the slant asymptote of the rational function.

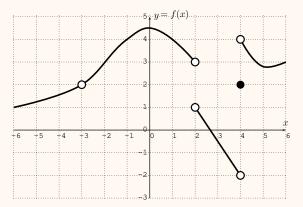
Va)
$$f(x) = \frac{2x^3 + 9x^2 - 20x - 21}{2x^2 - 3x - 4}$$
 b) $f(x) = \frac{2x^3 - 13x^2 + 35x - 26}{x^2 - 4x + 6}$ c) $f(x) = \frac{12x^3 + 10x^2 - 4x - 9}{3x^2 + x - 2}$ d) $f(x) = \frac{-3x^3 - 4x^2 + 20x - 16}{x^2 + 2x - 5}$

b)
$$f(x) = \frac{2x^3 - 13x^2 + 35x - 26}{x^2 - 4x + 6}$$

c)
$$f(x) = \frac{12x^3 + 10x^2 - 4x - 9}{3x^2 + x - 2}$$

d)
$$f(x) = \frac{-3x^3 - 4x^2 + 20x - 16}{x^2 + 2x - 5}$$

The graph of the function y = f(x) is shown below.



Find the limits of f(x) as x approaches the values indicated below.

- $\begin{array}{lll} \text{a)} \ x \rightarrow 2^+ & \text{b)} \ x \rightarrow 2^- & \text{c)} \ x \rightarrow 2 \\ \text{d)} \ x \rightarrow -3^+ & \text{e)} \ x \rightarrow -3^- & \text{f)} \ x \rightarrow -3 \\ \text{g)} \ x \rightarrow -1^+ & \text{h)} \ x \rightarrow -1^- & \text{i)} \ x \rightarrow -1 \\ \text{j)} \ x \rightarrow 4^+ & \text{k)} \ x \rightarrow 4^- & \text{l)} \ x \rightarrow 4 \end{array}$

Choose inputs that approach the given value from the indicated side (right or left). (Note that there is not just one unique answer for this part of the problem!)

Then, use the graphing calculator to compute the corresponding output values and guess what the limit might be.

a)
$$\lim_{x \to 3^{-}} \frac{x-3}{|x-3|}$$

b)
$$\lim_{x \to 1^+} \frac{x^3 - 1}{\sqrt{x} - 1}$$

c)
$$\lim_{x \to 2^{-}} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$$

d)
$$\lim_{x \to -5^+} \frac{x^3 + 5x^2}{|x+5|}$$

a)
$$\lim_{x \to 3^{-}} \frac{x-3}{|x-3|}$$
 b) $\lim_{x \to 1^{+}} \frac{x^3-1}{\sqrt{x}-1}$ c) $\lim_{x \to 2^{-}} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$ d) $\lim_{x \to -5^{+}} \frac{x^3+5x^2}{|x+5|}$ e) $\lim_{x \to -5^{-}} \frac{x^3+5x^2}{|x+5|}$ f) $\lim_{x \to 4^{-}} \frac{x-1}{x-4}$

f)
$$\lim_{x \to 4^-} \frac{x-1}{x-4}$$