

Section 3.1

1. List all the steps used by Algorithm 1 to find the maximum of the list 1, 8, 12, 9, 11, 2, 14, 5, 10, 4.

ALGORITHM 1 Finding the Maximum Element in a Finite Sequence.

```
procedure max( $a_1, a_2, \dots, a_n$ : integers)
  max :=  $a_1$ 
  for  $i := 2$  to  $n$ 
    if  $max < a_i$  then  $max := a_i$ 
  return max{max is the largest element}
```

Initialization: $n=10$, $max=1$ \leftarrow if T, $max=a_i$

i	$max < a_i$ (T or F)	max
1		
2	$1 < 8$ (T)	8
3	$8 < 12$ (T)	12
4	$12 < 9$ (F)	12
5	$12 < 11$ (F)	12
6	$12 < 2$ (F)	12
7	$12 < 14$ (T)	14
8	$14 < 5$ (F)	14
9	$14 < 10$ (F)	14
10	$14 < 4$ (F)	14

return $max = 14$

3. procedure $AddUp(a_1, \dots, a_n$: integers)

```
sum :=  $a_1$ 
for  $i := 2$  to  $n$ 
  sum := sum +  $a_i$ 
return sum
```

5. procedure *duplicates*(a_1, a_2, \dots, a_n : integers in nondecreasing order)

$k := 0$ {this counts the duplicates}, $n =$ the length of list.

$j := 2$

while $j \leq n$

if $a_j = a_{j-1}$ **then**

$k := k + 1$

$c_k := a_j$

while $j \leq n$ and $a_j = c_k$

$j := j + 1$

$j := j + 1$

{ c_1, c_2, \dots, c_k is the desired list}

} if the number is repeated, save it into c_k
} if it is repeated more than twice, it will
 not be saved twice in c_k .

7. procedure *last even location*(a_1, a_2, \dots, a_n : integers)

$k := 0$, $n =$ the length of list. $\{a_i\}$

for $i := 1$ **to** n

if a_i is even **then** $k := i$

return k { $k = 0$ if there are no evens}

9. procedure *palindrome check*($a_1 a_2 \dots a_n$: string)

$answer := \text{true}$, $n =$ the length of $\{a_i\}$

for $i := 1$ **to** $[n/2]$

if $a_i \neq a_{n+1-i}$ **then** $answer := \text{false}$ and STOP

return $answer$

11. procedure *interchange*(x, y : real numbers)

$z := x$ {temp. save x in z }

$x := y$ {Set x being y }

$y := z$ {Set y being z which is x }

The minimum number of assignments needed is three.

13. List all the steps used to search for 9 in the sequence 1, 3, 4, 5, 6, 8, 9, 11 using

a) a linear search.

Sol: a) Initialization: $n=8$, $x=9$

i	$x = a_i$ (T or F)	$i = i + 1$
1	$q = 1$ (F)	2
2	$q = 3$ (F)	3
3	$q = 4$ (F)	4
4	$q = 5$ (F)	5
5	$q = 6$ (F)	6
6	$q = 8$ (F)	7
7	$q = 9$ (T)	Stop

return location = $i = 7$

b) a binary search.

b) Initialization: $n=8$, $x=9$, $i=1$, $j=8$

$i < j$ (T or F)	m	$x > a_m$	$i = m + 1$	$j = m$
$i=1, j=8$	$1 < 8$ (T)	$\left\lfloor \frac{1+8}{2} \right\rfloor = 4$	$9 > 5$ (T)	$i=5$
$i=5, j=8$	$5 < 8$ (T)	$\left\lfloor \frac{5+8}{2} \right\rfloor = 6$	$9 > 8$ (T)	$i=7$
$i=7, j=8$	$7 < 8$ (T)	$\left\lfloor \frac{7+8}{2} \right\rfloor = 7$	$9 > 9$ (F)	$i=7$
$i=7, j=7$	$7 < 7$ (F)	stop		

$q = a_7$ is True, location = $i = 7$

return location = 7

15. procedure $insert(x, a_1, a_2, \dots, a_n: \text{integers})$

{the list is in order: $a_1 \leq a_2 \leq \dots \leq a_n$ }

$a_{n+1} := x + 1$ ← In case that x is larger than a_n

$i := 1$, $n = \text{the length of } \{a_i\}$

while $x > a_i$

$i := i + 1$

for $j := 0$ to $n - i$ } once the inserted position i is found,
 $a_{n-j+1} := a_{n-j}$ } move a_i, a_{i+1}, \dots, a_n to $a_{i+1}, a_{i+2}, \dots, a_n, a_{n+1}$

$a_i := x$

{ x has been inserted into correct position}

16. Describe an algorithm for finding the smallest integer in a finite sequence of natural numbers.

procedure $\min(a_1, a_2, \dots, a_n: \text{a list of } n \text{ numbers})$

$n := \text{the length of } \{a_i\}$

$\text{temp_min} := a_1$

for $i := 2$ to n

if $\text{temp_min} > a_i$ then $\text{temp_min} := a_i$

return temp_min : { temp_min is the smallest element }

17. procedure *first largest*(a_1, \dots, a_n : integers)

$\max := a_1$, $n = \text{the length of } \{a_i\}$

location := 1

for $i := 2$ to n

if $\max < a_i$ then } If the next term $\leq \max$, the index i
} will not be changed.
 $\max := a_i$
 $\text{iocation} := i$

return *location*

18. Describe an algorithm that locates the last occurrence of the smallest element in a finite list of integers, where the integers in the list are not necessarily distinct.

procedure *last smallest* (a_1, a_2, \dots, a_n : integers)

$n = \text{the length of } \{a_i\}$, $\min = a_1$

location := 1

for $i := 2$ to n

if $\min \geq a_i$ then

$\min = a_i$

location = i

return *location*

36. Use the bubble sort to sort 6, 2, 3, 1, 5, 4, showing the lists obtained at each step.

Initialization: $n=6$, $i=1$ to $5^{\downarrow n-1}$ and $j=1$ to $6-i$

Round 1 $j=1$ $j=2$ $j=3$ $j=4$ $j=5$

$i=1$	6	2	3	2	2	2
	2	6	3	3	3	3
	3	3	6	1	1	1
	1	1	1	6	5	5
	5	5	5	5	6	4
	4	4	4	4	4	6

Round 2	$j=1$	$j=2$	$j=3$	$j=4$		Round 3	$j=1$	$j=2$	$j=3$	
$i=2$	x(2 3 1 5 4 6	2 3 1 5 4 6	2 1 3 5 4 6	2 1 3 5 4 6	2 1 3 5 4 6	$i=3$	2 3 4 5 6	1 3 4 5 6	2 3 4 5 6	1 2 3 4 5 6
	x(2 3 1 5 4 6	2 3 1 5 4 6	2 1 3 5 4 6	2 1 3 5 4 6	2 1 3 5 4 6		x(2 3 4 5 6	1 3 4 5 6	2 3 4 5 6	1 2 3 4 5 6
	x(1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6				x(1 2 3 4 5 6	1 2 3 4 5 6		

Round 4	$j=1$	$j=2$	
$i=4$	x(1 2 3 4 5 6	1 2 3 4 5 6	
	x(1 2 3 4 5 6	1 2 3 4 5 6	

Round 5	$j=1$	$j=2$	
$i=5$	x(1 2 3 4 5 6	1 2 3 4 5 6	
	x(1 2 3 4 5 6	1 2 3 4 5 6	

Done.

37. Use the bubble sort to sort 3, 1, 5, 7, 4, showing the lists obtained at each step.

Initialization: $n=5$, $i=1$ to 4 \leftarrow^{n-1} and $j=1$ to $5-i$ \leftarrow^{n-i}

Round 1	$j=1$	$j=2$	$j=3$	$j=4$
$i=1$	3 1 5 7 4	1 3 5 7 4	1 3 5 7 4	1 3 5 7 4
	x(3 1 5 7 4	1 3 5 7 4	1 3 5 7 4	1 3 5 7 4
	x(1 3 5 7 4	1 3 5 7 4	1 3 5 7 4	1 3 5 7 4

Round 2	$j=1$	$j=2$	$j=3$	
$i=2$	1 3 5 7 4	1 3 5 7 4	1 3 5 7 4	1 3 5 7 4
	x(1 3 5 7 4	1 3 5 7 4	1 3 5 7 4	1 3 5 7 4
	x(1 3 5 7 4	1 3 5 7 4	1 3 5 7 4	1 3 5 7 4

Round 3	$j=1$	$j=2$	$j=3$	
$i=3$	1 3 4 5 7	1 3 4 5 7	1 3 4 5 7	1 3 4 5 7
	x(1 3 4 5 7	1 3 4 5 7	1 3 4 5 7	1 3 4 5 7
	x(1 3 4 5 7	1 3 4 5 7	1 3 4 5 7	1 3 4 5 7

Round 4	$j=1$	$j=2$	
$i=4$	1 3 4 5 7	1 3 4 5 7	
	x(1 3 4 5 7	1 3 4 5 7	

Done.

39. procedure better bubblesort(a_1, \dots, a_n : integers)

$i := 1$; $done := \text{false}$

while $i < n$ and $done = \text{false}$

$done := \text{true}$

for $j := 1$ **to** $n - i$

if $a_j > a_{j+1}$ **then**

interchange a_j and a_{j+1}

$done := \text{false}$

$i := i + 1$

{ a_1, \dots, a_n is in increasing order}

40. Use the insertion sort to sort the list in Exercise 36, showing the lists obtained at each step.

Input {6, 2, 3, 1, 5, 4}

Initialization: $n = 6$, $\bar{i} = 2$ to 6, $\bar{j} = 1$

① Round 1. $\bar{i} = 2$, insert $a_2 = 2$ to the right place

$a_i > a_j$ and $i > j$ (T or F) $\bar{j} = \bar{j} + 1$ (if "T")

$\bar{i} = 2, \bar{j} = 1$ $2 > 6$ \wedge $2 > 1$ (F) STOP

$m = a_2$ ($m = 2$. temp storage)

$K = 0$ to 0 $\leftarrow (i - j - 1 = 2 - 1 - 1 = 0)$

When $K = 0$ $a_2 = a_1$ (move "6" to a_2)

$a_1 = m$ ($a_1 = 2$)

* End of Round 1 : 2, 6, 3, 1, 5, 4

② Round 2. $\bar{i} = 3$ insert $a_3 = 3$ to the right place

$a_i > a_j$ and $i > j$ (T or F) $\bar{j} = \bar{j} + 1$

$\bar{i} = 3, \bar{j} = 1$ $3 > 2$ \wedge $3 > 1$ (T) $\bar{j} = 2$

$\bar{i} = 3, \bar{j} = 2$ $3 > 6$ \wedge $3 > 2$ (F) STOP

$m = a_3$ ($m = 3$)

$K = 0$ to 0 ($i - j - 1 = 3 - 2 - 1 = 0$)

when $k=0$, $a_3=a_2$ (move "6" to a_3)

$a_2=m$ ($a_2=3$)

* End of Round 2: $2, 3, 6, 1, 5, 4$

③ Round 3: $i=4$ Insert $a_4=1$ to the right place.

$\bar{i}=4, \bar{j}=1$ $a_i > a_j$ and $i > j$ (T or F) $\bar{j}=\bar{j}+1$
 $\boxed{1 > 2} \wedge 4 > 1$ (F) STOP.

$m=a_4$ ($m=1$)

$k=0$ to ∞ ($i-\bar{j}-1 = 4-1-1 = 2$)

when $k=0$, $a_4=a_3$ (move "6" to a_4)

when $k=1$, $a_3=a_2$ (move "3" to a_3)

when $k=2$, $a_2=a_1$ (move "2" to a_2)

$a_1=m$ ($a_1=1$)

* End of Round 3: $1, 2, 3, 6, 5, 4$

④ Round 4: $i=5$ Insert $a_5=5$ to the right place.

$a_i > a_j$ and $i > j$ (T or F) $\bar{j}=\bar{j}+1$

$\bar{i}=5, \bar{j}=1$ $5 > 1 \wedge 5 > 1$ (T) $\bar{j}=2$

$\bar{j}=2$ $5 > 2 \wedge 5 > 2$ (T) $\bar{j}=3$

$\bar{j}=3$ $5 > 3 \wedge 5 > 3$ (T) $\bar{j}=4$

$\bar{j}=4$ $\boxed{5 > 6} \wedge 5 > 4$ (F) STOP.

$m=a_5$ ($m=5$)

$k=0$ to ∞ ($i-\bar{j}-1 = 5-4-1 = 0$)

when $k=0$, $a_5=a_4$ (move "6" to a_5)

$a_4=m$ ($a_4=5$)

* End of Round 4: $1, 2, 3, 5, 6, 4$

⑤ Round 5 $i=6$, Insert $a_6=4$ to the right place.

$a_i > a_j$ and $i > j$ (T or F) $\bar{j}=\bar{j}+1$

$\bar{i}=6, \bar{j}=1$ $4 > 1$ and $6 > 1$ (T) $\bar{j}=2$

$\bar{j}=2$ $4 > 2$ and $6 > 2$ (T) $\bar{j}=3$

$\bar{j}=3$ $4 > 3$ and $6 > 3$ (T) $\bar{j}=4$

$\bar{j}=4$ $\boxed{4 > 5}$ and $6 > 4$ (F) STOP

$m=a_6$ ($m=4$)

$$k=0 \text{ to } 1 \quad (i-j-1 = 6-4-1 = 1)$$

when $k=0$, $a_6=a_5$ (move "6" to a_5)

when $k=1$, $a_5=a_4$ (move "5" to a_5)

$$a_4=m \quad (a_4=4)$$

End of Round 5: 1, 2, 3, 4, 5, 6.

56. Use the cashier's algorithm to make change using quarters, dimes, nickels, and pennies for

- a) 87 cents.
c) 99 cents.

- b) 49 cents.
d) 33 cents.

Sol: a) 87 cents

$$\frac{87}{25} = 3 + \frac{12}{25}, \quad \frac{12}{10} = 1 + \frac{2}{10}, \quad \frac{2}{5} = 0 + \frac{2}{5}, \quad \frac{2}{1} = 2$$

cents = 3 quarters + 1 dime(s) + 0 nickel(s) + 2 pennies

b) 49 cents

$$\frac{49}{25} = 1 + \frac{24}{25}, \quad \frac{24}{10} = 2 + \frac{4}{10}, \quad \frac{4}{5} = 0 + \frac{4}{5}, \quad \frac{4}{1} = 4$$

cents = 1 quarter(s) + 2 dime(s) + 0 nickel(s) + 4 pennies

c) 99 cents

$$\frac{99}{25} = 3 + \frac{24}{25}, \quad \frac{24}{10} = 2 + \frac{4}{10}, \quad \frac{4}{5} = 0 + \frac{4}{5}, \quad \frac{4}{1} = 4$$

cents = 3 quarter(s) + 2 dime(s) + 0 nickel(s) + 4 pennies

d) 33 cents

$$\frac{33}{25} = 1 + \frac{8}{25}, \quad \frac{8}{10} = 0 + \frac{8}{10}, \quad \frac{8}{5} = 1 + \frac{3}{5}, \quad \frac{3}{1} = 3$$

cents = 1 quarter(s) + 0 dime(s) + 1 nickel(s) + 3 pennies