MAT1375, Classwork24, Fall2025

Ch22. Vectors in the Plane

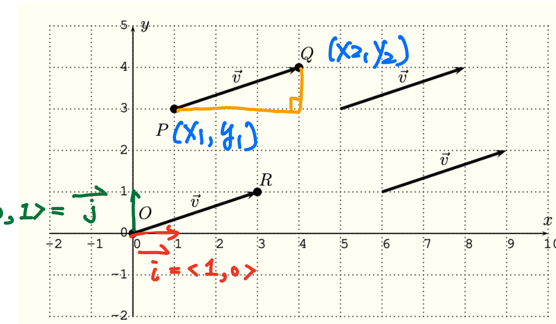
1. Definition of a geometric vector:

A geometric vector $\overrightarrow{PQ}$ is a directed line segment with a direction and a magnitude.

The magnitude of $\overrightarrow{PQ}$ is its length, denoted by $|\overrightarrow{PQ}|$ or $|\overrightarrow{PQ}|$.

2. How to find and present a vector:

Given a vector $\vec{v} = \overrightarrow{PQ}$. We call P the initial point and Q the terminal point.



We find $\vec{v} = \overrightarrow{PQ}$ by $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$\vec{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} \text{ or } \langle x_2 - x_1, y_2 - y_1 \rangle$$

where $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$.

The magnitude of $\vec{v}$ is $\|\vec{v}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Any vectors with the same direction and magnitude are equivalent.

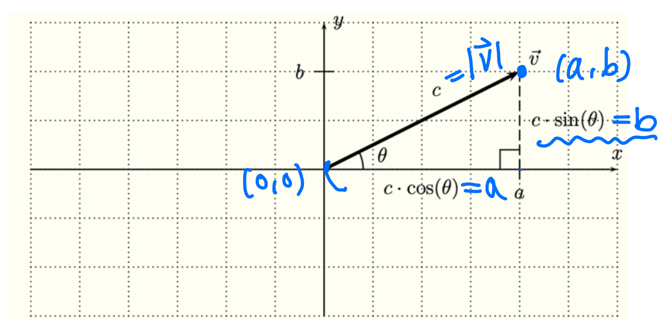
3. Direction angle:

$\overrightarrow{OR}$ $R(a, b) - (0, 0) = \langle a, b \rangle$

Let $\vec{v} = \langle a, b \rangle = \overrightarrow{OR}$ be a vector with original point $(0, 0)$ as the initial point of $\vec{v}$ and $R(a, b)$ as the terminal point of $\vec{v}$.

The direction angle of $\vec{v}$ is the angle θ determined by $\overrightarrow{OR}$:

$c = |\vec{v}|$ is the length of $\vec{v}$ and we have $\sin(\theta) = \frac{b}{c}$, $\cos(\theta) = \frac{a}{c}$, and $\tan(\theta) = \frac{b}{a}$.

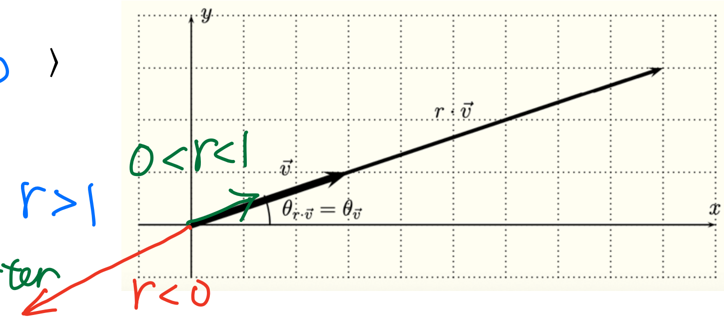


4. The vector $\vec{v}$ can be presented by its length c and direction angle θ:

$$\vec{v} = \langle a, b \rangle = \langle c\cos(\theta), c\sin(\theta) \rangle$$

5. Operations on vectors: Let $\vec{v} = \langle a, b\rangle$ and $\vec{w} = \langle c, d\rangle$

Scalar multiplication: $r\vec{v} = r \cdot \langle a, b\rangle = \langle ra, rb \rangle$



$r > 0$: Keep the same direction but with a longer length as $r>1$

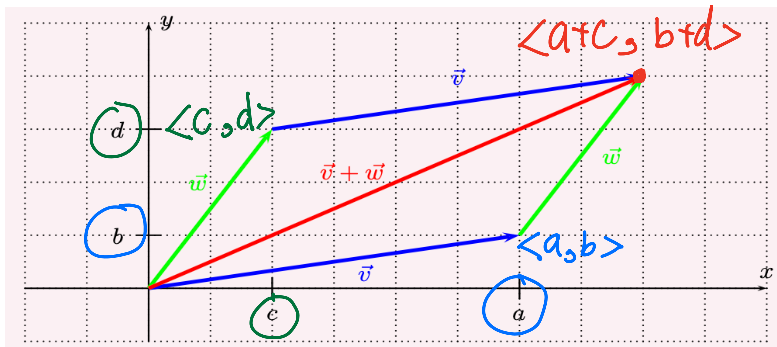
as $0<r<1$, the length is shorter

$r < 0$: this vector is having an opposite direction

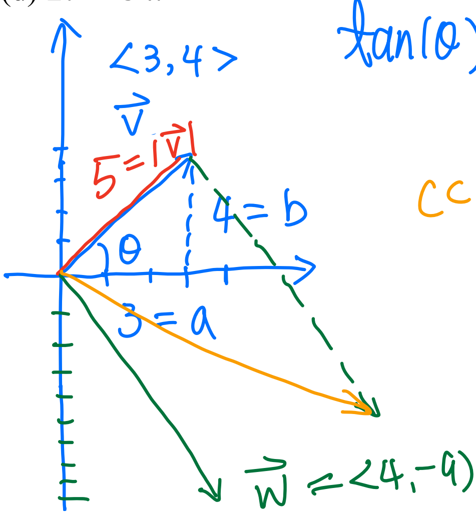
$\|\vec{v}\|$

Unit vector of $\vec{v}$: $r\vec{v}$ where $r = \frac{1}{|\vec{v}|}$ and we have $r\vec{v} = \frac{\vec{v}}{|\vec{v}|}$

Vector addition: $\vec{v} + \vec{w} = \langle a, b\rangle + \langle c, d\rangle = \langle a+c, b+d \rangle$



6. Let $\vec{v} = \langle 3, 4\rangle$ and $\vec{w} = 4\mathbf{i} - 9\mathbf{j}$ $= \langle 4,-9\rangle$. Find (a) the directional angle of $\vec{v}$, (b) the unit vector of $\vec{v}$, (c) $\vec{v} + \vec{w}$, and (d) $2\vec{v} - 3\vec{w}$



(a) $\tan(\theta) = \frac{b}{a} = \frac{4}{3}$

(b) $\frac{\vec{v}}{|\vec{v}|} = \frac{\langle 3,4\rangle}{5} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

(c) $\vec{v} + \vec{w}$

$= \langle 3,4\rangle + \langle 4,-9\rangle$

$= \langle 3+4, 4-9\rangle = \langle 7, -5\rangle$

(d) $2\vec{v} - 3\vec{w}$

$2\langle 3,4\rangle - 3\langle 4,-9\rangle$

$= \langle 2\cdot 3, 2\cdot 4\rangle + \langle (-3)4, (-3)(-9)\rangle$

$= \langle 6, 8\rangle + \langle -12, 27\rangle$

$= \langle 6-12, 8+27\rangle = \langle -6, 35\rangle$