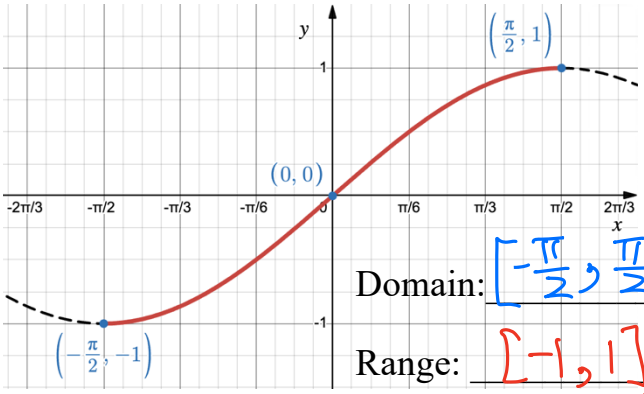
MAT 1375, Classwork18, Fall2024

ID:_____ Name:_____

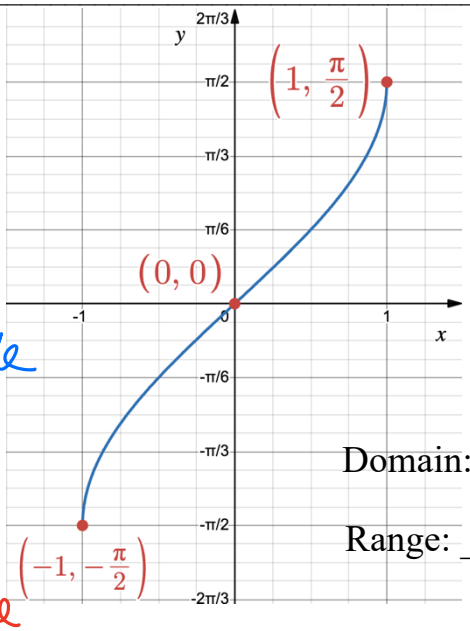
1. The graph of $y = \sin^{-1}(x)$:

$y = \sin(x)$ (the restricted sine)



Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ← angle

Range: $[-1, 1]$. ← value



$y = \sin^{-1}(x)$ or arcsin(x)

Domain: $[-1, 1]$. ← value

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. ← angle

2. How to find the value of $\sin^{-1}(x)$:

Let $\theta = \sin^{-1}(x)$. It implies $x =$ $\sin(\theta)$ where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Then find the θ from the following table:

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

3. Find the value of the given inverse sine functions.

a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ b) $\sin^{-1}\left(-\frac{1}{2}\right)$ c) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ d) $\sin^{-1}(-4.3)$

a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

Let $\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\sin(\theta) = \frac{\sqrt{2}}{2}$

(What "θ" will make $\sin(\theta) = \frac{\sqrt{2}}{2}$?)

$\theta = \frac{\pi}{4}$.

b) $\sin^{-1}\left(-\frac{1}{2}\right)$

Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$

$\sin(\theta) = -\frac{1}{2}$

$\theta = -\frac{\pi}{6}$

c) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Let $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\sin(\theta) = -\frac{\sqrt{2}}{2}$

$\theta = -\frac{\pi}{4}$

d) $\sin^{-1}(-4.3)$

Let $\theta = \sin^{-1}(-4.3)$

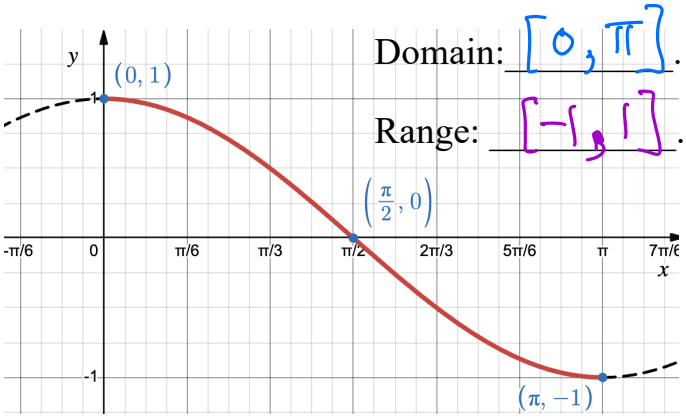
$\sin(\theta) = -4.3$

θ is undefined.

4. The graph of $y = \cos^{-1}(x)$:

cosine

$y = \cos(x)$ (the restricted sine)

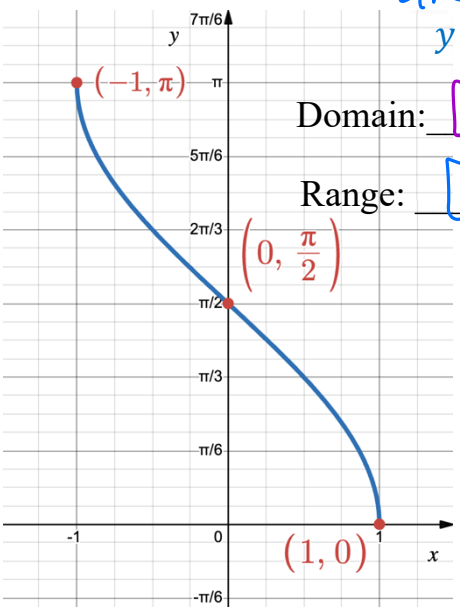


Domain: $[0, \pi]$.

Range: $[-1, 1]$.

arcos (x)

$y = \cos^{-1}(x)$



Domain: $[-1, 1]$.

Range: $[0, \pi]$.

5. How to find the value of $\cos^{-1}(x)$:

Let $\theta = \cos^{-1}(x)$. It implies $x =$ $\cos(\theta)$ where $0 \le \theta \le \pi$. Then find the angle θ from the following table:

I II

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

6. Find the value of the given inverse cosine functions.

a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ b) $\cos^{-1}\left(-\frac{1}{2}\right)$ c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ d) $\cos^{-1}(-4.3)$

a) Let $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \cos(\theta) = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$

$(0 \le \theta \le \pi)$

b) Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow \cos(\theta) = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

c) Let $\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\Rightarrow \cos(\theta) = -\frac{\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}$

d) Let $\theta = \cos^{-1}(-4.3)$

$\cos(\theta) = -4.3$

θ is undefined.