

MAT1375, Classwork24, Fall2025

Ch22. Vectors in the Plane

1. Definition of a geometric vector:

A geometric vector \vec{PQ} is a directed line segment with a direction and a magnitude.

The magnitude of \vec{PQ} is its length, denoted by $|\vec{PQ}|$ or $|\overrightarrow{PQ}|$

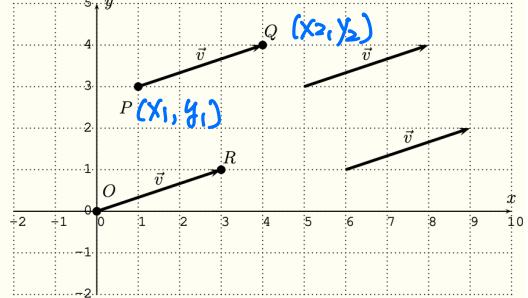
2. How to find and present a vector:

Given a vector $\vec{v} = \vec{PQ}$. We call P the initial point and Q the terminal point.

We find $\vec{v} = \vec{PQ}$ by $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$\vec{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} \text{ or } \langle x_2 - x_1, y_2 - y_1 \rangle,$$

where $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$.



The magnitude of \vec{v} is $\|\vec{v}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

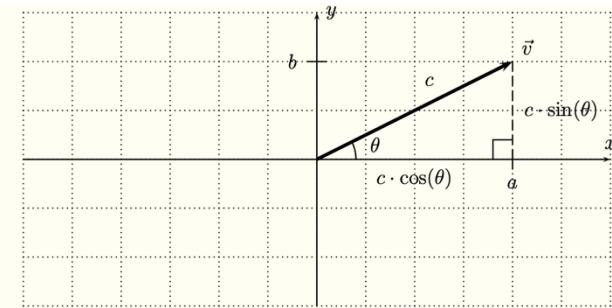
Any vectors with the same _____ and _____ are equivalent.

3. Direction angle:

Let $\vec{v} = \langle a, b \rangle = \overrightarrow{OR}$ be a vector with original point _____ as the initial point of \vec{v} and $R(,)$ as the terminal point of \vec{v} .

The _____ of \vec{v} is the angle θ determined by \overrightarrow{OR} :

$c = \sqrt{a^2 + b^2}$ is the _____ of \vec{v} and we have $\sin(\theta) = \frac{b}{c}$, $\cos(\theta) = \frac{a}{c}$, and $\tan(\theta) = \frac{b}{a}$.



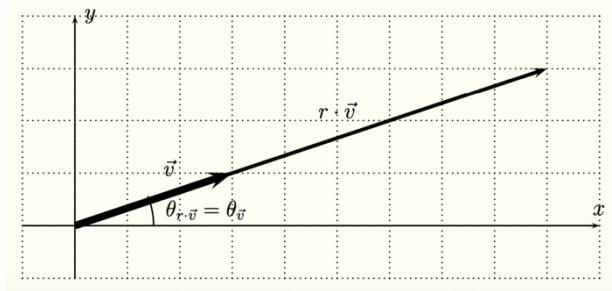
4. The vector \vec{v} can be presented by its length _____ and direction angle _____:

$$\vec{v} = \langle a, b \rangle = \langle \|\vec{v}\| \cos(\theta), \|\vec{v}\| \sin(\theta) \rangle$$

5. Operations on vectors: Let $\vec{v} = \langle a, b \rangle$ and $\vec{w} = \langle c, d \rangle$

Scalar multiplication: $r\vec{v} = r \cdot \langle a, b \rangle = \langle \quad, \quad \rangle$

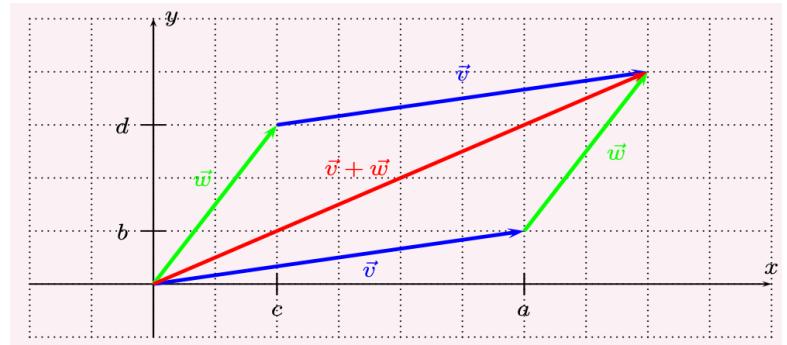
$r > 0$:



$r < 0$:

Unit vector of \vec{v} : $r\vec{v}$ where $r = \sqrt{a^2 + b^2}$ and we have $r\vec{v} = \langle \quad, \quad \rangle$

Vector addition: $\vec{v} + \vec{w} = \langle a, b \rangle + \langle c, d \rangle = \langle \quad, \quad \rangle$



6. Let $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = 4\mathbf{i} - 9\mathbf{j}$. Find (a) the directional angle of \vec{v} , (b) the unit vector of \vec{v} , (c) $\vec{v} + \vec{w}$, and (d) $2\vec{v} - 3\vec{w}$