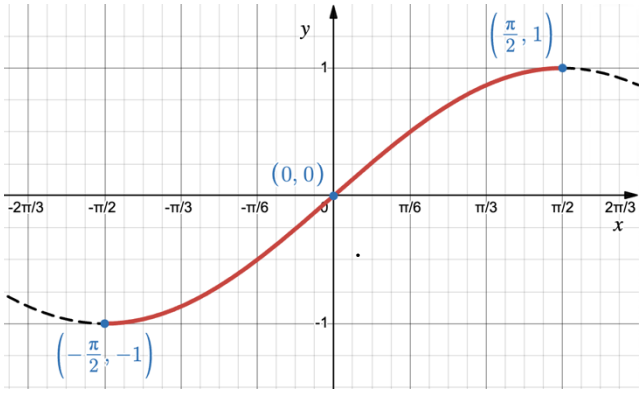
MAT1375, Classwork20, Fall2025

Ch19. Inverse Trigonometric Functions I

1. The graph of $y = \sin^{-1}(x)$:

$y = \sin(x)$ (the restricted sine) Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (angle)

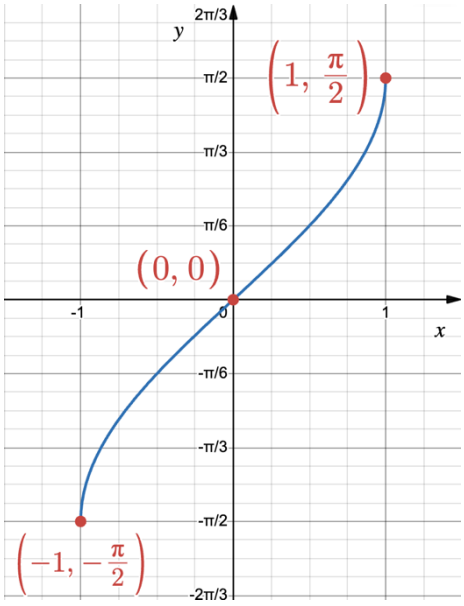
Range: $[-1, 1]$ (value).



$y = \sin^{-1}(x)$

Domain: $[-1, 1]$ (value).

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (angle).



2. How to find the value of $\sin^{-1}(x)$:

Let $\theta = \sin^{-1}(x)$. It implies $x =$ $\sin(\theta)$ where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Then find the θ from the following table:

θ (or $\sin^{-1}(x)$)	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$ (or x)	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

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3. Find the value of the given inverse sine functions.

a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ b) $\sin^{-1}\left(-\frac{1}{2}\right)$ c) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ d) $\sin^{-1}(-4.3)$

a) $\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\sin(\theta) = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

b) $= -\frac{\pi}{6}$

c) $= -\frac{\pi}{4}$

d) $x = \sin^{-1}(-4.3)$

$\sin(x) = -4.3$

since $-1 \le \sin(x) \le 1$

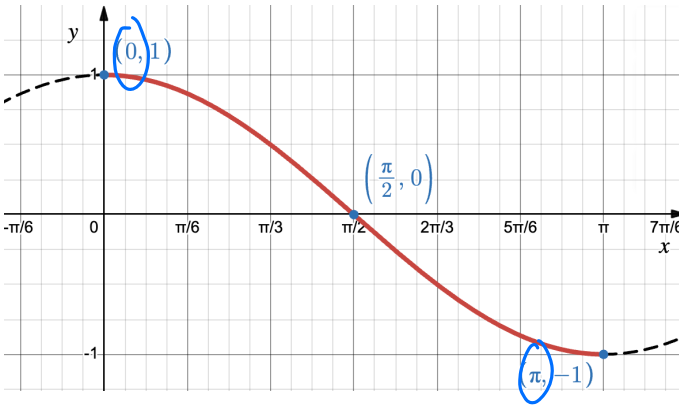
if $\sin(x) = -4.3$

x is undefined.

4. The graph of $y = \cos^{-1}(x)$:

$y = \cos(x)$ (the restricted cosine) Domain: angle $[0, \pi]$.

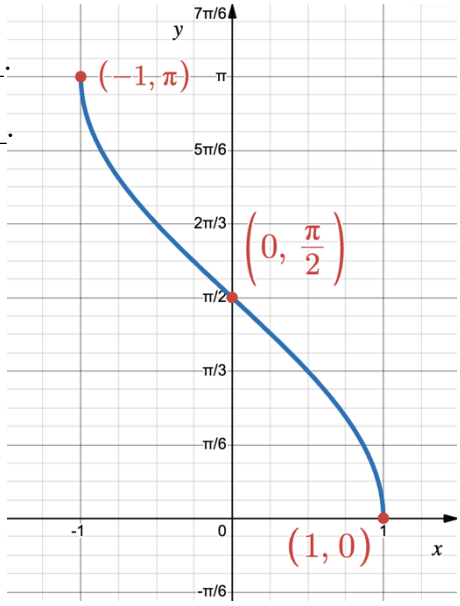
Range: value $[-1, 1]$.



$y = \cos^{-1}(x)$

Domain: value $[-1, 1]$.

Range: angle $[0, \pi]$.



5. How to find the value of $\cos^{-1}(x)$:

Let $\theta = \cos^{-1}(x)$. It implies $x =$ $\cos(\theta)$ where $0 \le \theta \le \pi$. Then find the angle θ from the following table:

θ(or $\cos^{-1}(x)$)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos(\theta)$(or x)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

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6. Find the value of the given inverse cosine functions.

a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\pi}{6}$

b) $\cos^{-1}\left(-\frac{1}{2}\right)$ $= \frac{2\pi}{3}$

c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $= \frac{3\pi}{4}$

d) $\cos^{-1}(-4.3)$

$X = \cos^{-1}(-4.3)$

$\cos(X) = -4.3$

$-1 \le \cos(X) \le 1$

$\Rightarrow$ If $\cos(X) = -4.3$, then X is undefined.