MAT 1375, Classwork17, Fall2024

ID:________________________ Name:________________________

1. Review: Even function and Odd function.

If $f(x)$ is an even function, then $f(-x) =$ $f(x)$. for example, $y = x^2$

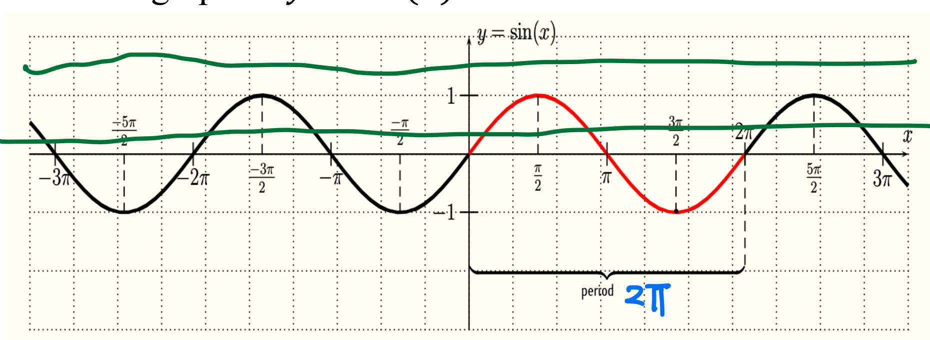
If $f(x)$ is an odd function, then $f(-x) =$ $-f(x)$. $y = x^3$

2. Definition of a **Periodic Function**:

A function f is periodic if there is a positive number p called a period such that

$$f(x+p) = f(x) \quad \text{for all } x.$$

3. The graph of $y = \sin(x)$:



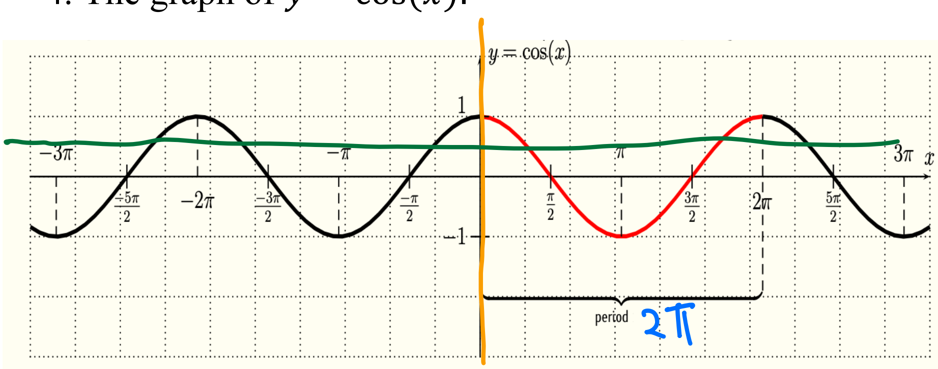
Characteristics:
Period: 2π
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$

Property: odd function with origin symmetry where $\sin(-x) = -\sin(x)$.
One-to-one function? NO

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

I II III IV

4. The graph of $y = \cos(x)$:



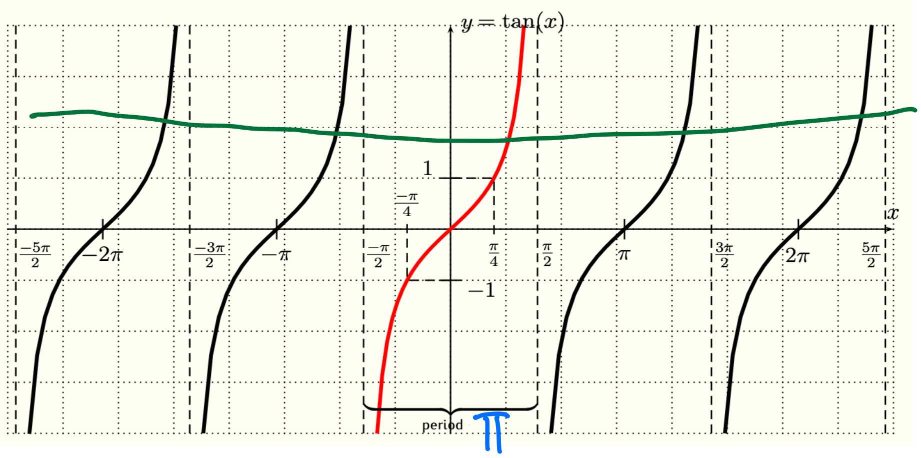
Characteristics:
Period: 2π
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$

Property: even function with y-axis symmetry where $\cos(-x) = \cos(x)$.
One-to-one function? NO

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1

I II III IV

5. The graph of $y = \tan(x)$:



Characteristics:

Period: π

Domain: All real numbers except odd multiples of $\frac{\pi}{2}$

Range: $(-\infty, \infty)$

Vertical Asymptotes: $x = -\frac{5\pi}{2}$, $x = -\frac{3\pi}{2}$, $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{5\pi}{2}$, ...

$x =$ odd multiples of $\frac{\pi}{2}$.

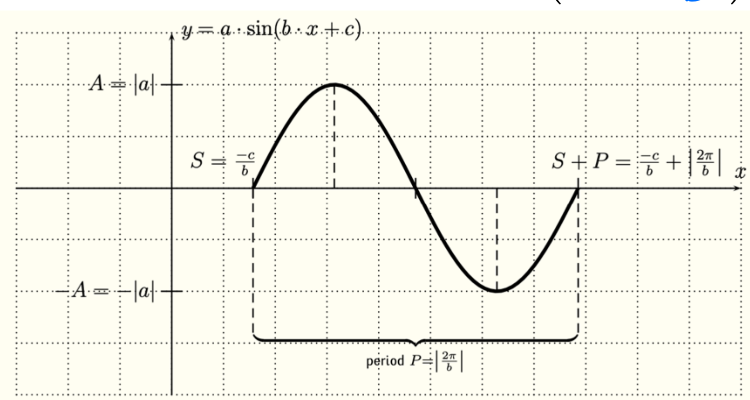
One-to-one function? NO

Property: Odd function with origin symmetry where $\tan(-x) = -\tan(x)$.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan(x)$ $\frac{\sin(x)}{\cos(x)}$	undefined $(\to -\infty)$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	$\frac{0}{1} = 0$	$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$ $\frac{\sqrt{3}}{3}$	$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$	$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$	$\frac{1}{0}$ undefined $(\to \infty)$

6. Amplitude, period, and phase shift:

Let $f(x) = a \cdot \sin(b \cdot x + c) = a \cdot \sin\left(b \cdot \left(x + \frac{c}{b}\right)\right)$ or $f(x) = a \cdot \cos(b \cdot x + c) = a \cdot \cos\left(b \cdot \left(x + \frac{c}{b}\right)\right)$.

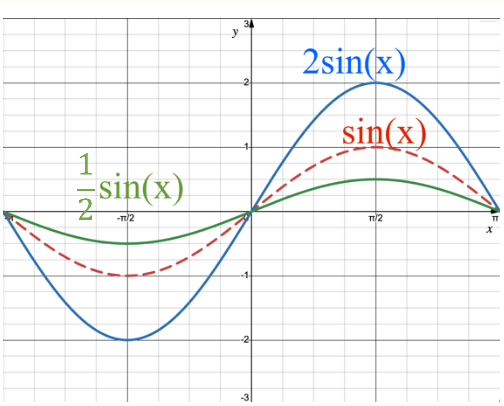


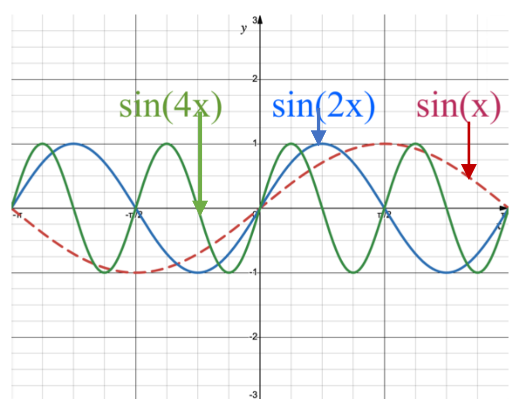
We define

1) the amplitude $A = |a|$;

2) the period $P = \left|\frac{2\pi}{b}\right|$;

3) the phase shift $S = -\frac{c}{b}$.





For $f(x) = 2\sin(x)$, its A is 2.

For $f(x) = \frac{1}{2}\sin(x)$, its A is $\frac{1}{2}$.

For $f(x) = \sin(2x)$, its P is $\frac{2\pi}{2} = \pi$

For $f(x) = \sin(4x)$, its P is $\frac{2\pi}{4} = \frac{\pi}{2}$