MAT 1275, Classwork17, Fall2024
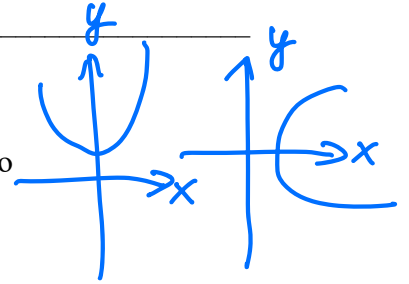
ID:_____   Name:_____

1. Equations of a parabola and a circle:

   A quadratic equation with two variables x and y is an equation that is equivalent to

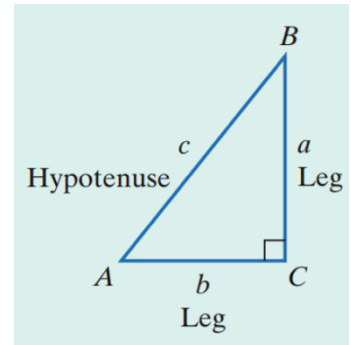 $$Ax^2 + By^2 + Cx + Dy + Exy + F = 0.$$

   1) For $A = E = 0$ or $B = E = 0$, we get a __parabola__.

   2) For $A = B$ and $E = 0$, we get a __circle__.

2. The Pythagorean Theorem:

   The sum of the square of the lengths of the legs of a right triangle equals the square

   of the length of the hypotenuse: $a^2 + b^2 = c^2$.

3. The Distance from point $A(x_1, y_1)$ to point $B(x_2, y_2)$:

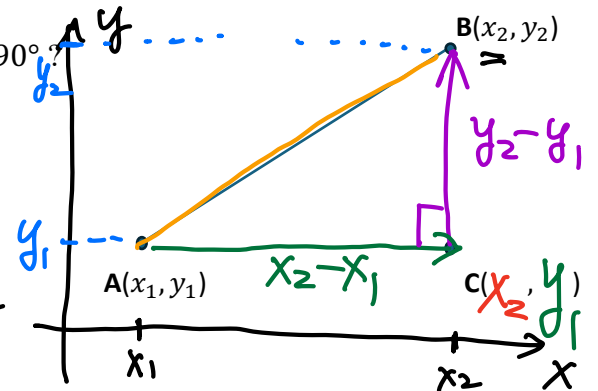   What is a point C to make $\triangle ABC$ a right triangle where angle $C = 90°$? $C(x_2, y_1)$

   What is the length of $\overline{AC}$? $x_2 - x_1$

   What is the length of $\overline{BC}$? $y_2 - y_1$

   What is the length of $\overline{AB}$ (the distance from A to B)?

   $$\overline{AB}^2 = (\overline{AC})^2 + (\overline{BC})^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

   $$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

4. Circle: All points on a circle's circumference are having the same __distance__ to a fixed-point C.

   The fixed-point C is called __center__.

   The distance from a point on the circle's circumference to the center C is called __radius__.

5. Find the coordinates from a graph of a circle:

   $C(\,0\,,\,0\,)$; $A(\,1\,,\,0\,)$; $B(\,0\,,\,1\,)$; $D(\,-1,\,0\,)$; $E(\,0\,,\,-1\,)$.

   The radius of this circle = __1__.
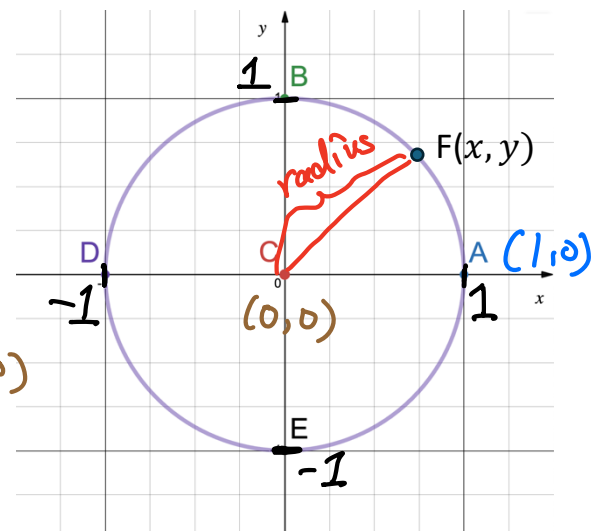
   Find the distance from A to C:

   $(\overline{AC})^2 = (1 - 0)^2 + (0 - 0)^2, \Rightarrow \overline{AC} = $ __1__.

   Find the distance from F to C:

   $$\text{radius} = \overline{FC}^2 = (x - 0)^2 + (y - 0)^2$$
   $$\text{or } 1$$

   point $F (x, y)$ $\longrightarrow$ $x^2 + y^2 = 1$

   $C(0, 0)$

6. Find the coordinates from a graph of a circle:

C(2,-1); A(4,-1); B(2, 1); D(0,-1); E(2,-3).

The radius of this circle = ___2___.

Find the distance from A to C:

$(\overline{AC})^2 = (4-2)^2 + (-1-(1))^2, \Rightarrow \overline{AC} = $ ___2___.

$2^2 + 0 = 4$

Find the distance from F to C:

$$\overline{FC}^2 = (x-2)^2 + (y-(-1))^2$$

$$2^2 = (radius)^2 = (x-2)^2 + (y+1)^2 \Rightarrow 2^2 = (x-2)^2 + (y+1)^2$$
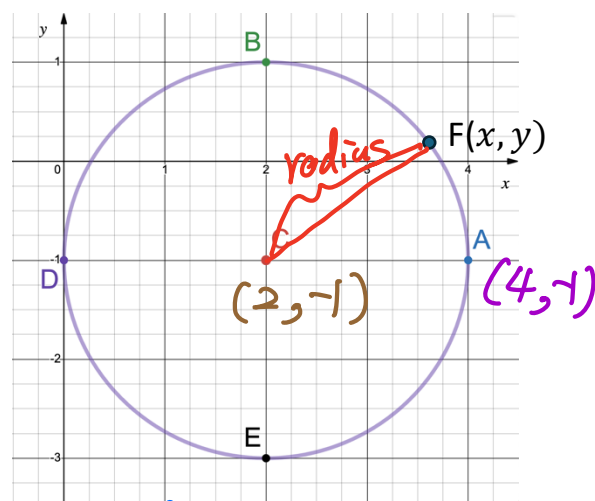
7. General equation of a circle:

If there is a circle with center point $C(a, b)$ and radius r, the equation of this circle is

$$(x - a)^2 + (y - b)^2 = (r)^2$$

8. Find a constant C that completes the square for the expression.

a) $x^2 - 2x + C$

$x^2 + 2ax + a^2$

$2a = -2 \Rightarrow a = -1$

$C = a^2 = (-1)^2 = 1$

$x^2 - 2x + 1 = (x-1)^2 \longrightarrow a$

b) $y^2 - 4y + C$

$y^2 + 2ay + a^2$

$2a = -4 \Rightarrow a = -2$

$C = a^2 = (-2)^2 = 4$

$y^2 - 4y + 4 = (y-2)^2$

c) $x^2 + 3x + C$

$x^2 + 2ax + a^2$

$2a = 3 \Rightarrow a = \frac{3}{2}$

$C = a^2 = (\frac{3}{2})^2 = \frac{9}{4}$

$x^2 + 3x + \frac{9}{4} = (x+\frac{3}{2})(x+\frac{3}{2})$

$\begin{array}{c} x \\ x \end{array}$ $\begin{array}{c} \frac{3}{2} \\ \frac{3}{2} \end{array}$ $= (x+\frac{3}{2})^2$

9. Graph $x^2 + y^2 - 4y - 2x + 1 = 0$.

Complete the square:

$x^2 - 2x + 1 \quad + \quad y^2 - 4y + 4 \quad +1 = 0 + 1 + 4$

$(x-1)^2 \quad + \quad (y-2)^2 + 1 = 5$

$\qquad\qquad -1 \quad -1$

$(x-1)^2 + (y-2)^2 = 4$

radius $= \sqrt{4} = 2$

center $= (1, 2)$

Center C (1 , 2)

Radius $r = $ 2