Ch5. Operations on Functions

$(f-g)(x)$

1. Complete the definition of **the Algebra of Functions**:

Let $f(x)$ and $g(x)$ be two functions with the domain D_f and D_g ,respectively. We have sum, difference, product, and quotient of functions:

intersection

The Algebra of functions	Notation	Definition	Domain
Sum	$(f+g)(x) :=$	$f(x) + g(x)$	$D_{f+g} = D_f \cap D_g$
Difference	$(f-g)(x) :=$	$f(x) - g(x)$	$D_{f-g} = D_f \cap D_g$
Product	$(f \cdot g)(x) :=$	$f(x) \cdot g(x)$	$D_{f \cdot g} = D_f \cap D_g$
Quotient	$\left(\frac{f}{g}\right)(x) :=$	$\frac{f(x)}{g(x)}$, provided $g(x) \neq 0$	$D_{\frac{f}{g}} = D_f \cap D_g$ but $g(x) \neq 0$

Here, $D_f \cap D_g = \{ x \mid$ _____ $\}$

2. Let $f(x) = x^2 + 5x + 6$ and $g(x) = x + 2$. Find the following functions and state their domains.
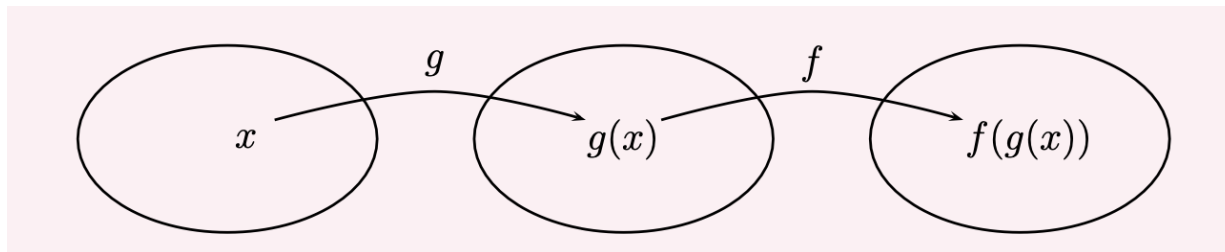
$(f + g)(x) =$

$(f - g)(x) =$

$(f \cdot g)(x) =$

$\left(\frac{f}{g}\right)(x) =$

3. Complete the definition of **the Composition of Functions**:

Let $f(x)$ and $g(x)$ be two functions. The composition of the function f with g is denoted by

_____ and is defined by the equation

_____ := _____ .

The domain of the composition of the function $f \circ g$ is the set of all x such that x is the _____ of $g(x)$ and

_____ is the domain of $f(x)$.

The notation of the domain of the composition of the function $f \circ g$ is

_____ $= \{ \, x \,|$ _____ $\}$

4. Are $f(g(x))$ and $g(f(x))$ the same functions?

5. Find $(f \circ g)(x)$ for the following functions and state their domains.

a) $f(x) = x^2 + 2$ and $g(x) = x - 3$

b) $f(x) = \frac{2}{x-3}$ and $g(x) = x^2 + 2x$