MAT1375, Classwork12, Fall2025

Ch12. Solving inequalities

1. The Strategy for Solving Inequalities (Application of **Number Line Test**):

Step1. Replace ‘>’ (‘≥’) or ‘<’ (‘≤’) by ‘=’ and solve the equation.

Step2. Mark the solutions on the number line and check sign (positive / negative) in each subinterval.

Step3. Check the end point(s) of the subintervals to see if they are included in the solution set.

2. Given $x^3 + 15x \ge 7x^2 + 9$. Solve for x.

① $x^3 - 7x^2 + 15x - 9 \ge 0$.

Replace "≥" with "=", $f(x) = x^3 - 7x^2 + 15x - 9 = 0$

② Find the root(s) of $f(x) = x^3 - 7x^2 + 15x \boxed{-9}$

Possible roots: $\underline{1}, 3, 9, -1, -3, -9$ (the factors of "-9")

$f(1) = 1^3 - 7\cdot1^2 + 15\cdot1 - 9 = 1 - 7 + 15 - 9 = 0 \Rightarrow x = 1$ is a root of $f(x)$

$\Rightarrow (x-1)$ is a factor of $f(x)$

$$x^2 - 6x + 9$$

$$x-1 \,\big|\, x^3 - 7x^2 + 15x - 9$$

$x^2(x-1) \rightarrow x^3 - x^2$

$-6x^2 + 15x$

$-6x(x-1) \rightarrow -6x^2 + 6x$

$9x - 9$

$9(x-1) \rightarrow 9x - 9$

0

$f(x) = (x^2 - 6x + 9)(x-1)$

$x \quad -3$

$x \quad -3$

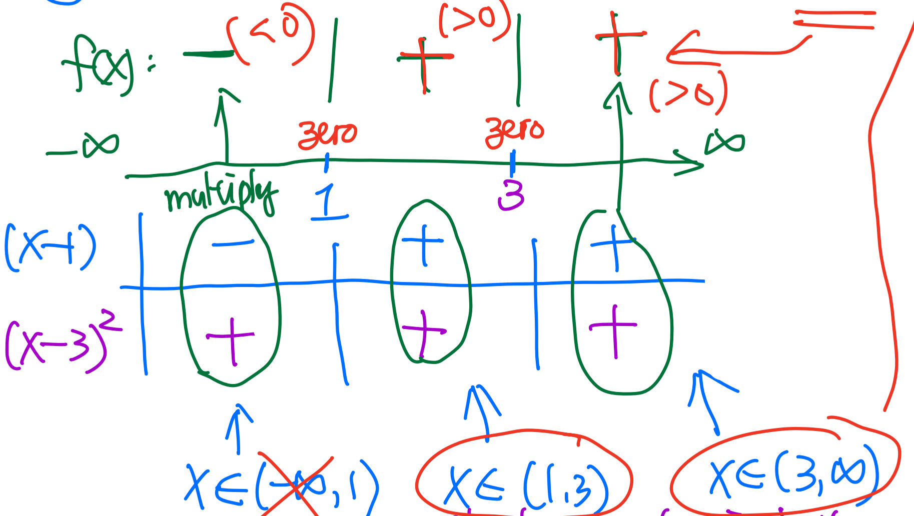
$= (x-3)(x-3)(x-1)$

$f(x) = (x-3)(x-3)(x-1) = 0$

$x-3=0$ or $x-3=0$ or $x-1=0$

$x=3$ or $x=3$ or $x=1$.

③ Check $f(x) = x^3 - 7x^2 + 15x - 9 \ge 0$



check x=2 (2 is between 1 and 3)

$x \in (1,3) \cup (3,\infty)$

④ Check end points

$x=1$, $f(1) = 0$ included or ~~excluded~~

$x=3$, $f(3) = 0$

$x \in [1,3] \cup [3,\infty)$

$\boxed{\Rightarrow x \in [1,\infty)}$

3. Solve for x: $|2x-3| \geq 7$.

① $|2x-3| \geq 7$. Replace "$\geq$" with "$=$", $|2x-3| = 7$

② $2x-3=7$ or $2x-3=-7$

$x=5$ or $x=-2$ → $x\in(-\infty,-2)\cup(5,\infty)$

③ Check $|2x-3| \geq 7$

$x\in(-\infty,-2)$ | ~~$x\in(-2,5)$~~ | $x\in(5,\infty)$

Try $x=-3$ | -2 | Try $x=0$ | 5 | Try $x=6$

$|2\cdot(-3)-3|=9\geq 7$ | $|2(0)-3|=3<7$ | $|2(6)-3|=9\geq 7$

True | False | True

④ Check end points

$x=-2$ $|2\cdot(-2)-3|=7$ included

$x=5$ $|2\cdot(5)-3|=7$ included

$$\boxed{x\in(-\infty,-2]\cup[5,\infty)}$$

4. Solve for x: $\frac{x^2-5x+6}{x^2-5x} \geq 0$.