MAT2440, Classwork15, Spring2025

ID:______________________________ Name:______________________________

1. The Second Method: A Proof by **Contradiction**

(a) To prove a statement $\boldsymbol{p}$ is **true**, we first find a contradition q such that $\neg p \to q$ is true. Since $\boldsymbol{q}$ **is false** and $\neg \boldsymbol{p} \to \boldsymbol{q}$ **is true**, it concludes that $\neg p$ is true which implies p is true.

(b) To prove a statement $\boldsymbol{p} \to \boldsymbol{q}$ is **true**, we first ***assume*** p and $\neg q$ are true. Then using $\neg q$ shows $\neg p$ is true. Because p and $\neg p$ are both true, we have a contradition. It implies the ***assumption*** "$\neg q$ is true" **is wrong** which means q is true.

2. Give a contradiction proof of the theorem "If n^2 is an odd integer, then n is odd."

$P(n)$ $Q(n)$

Assume n^2 is odd and n is even $(\neg Q(n))$

Then $n = 2k$ which implies $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$ and it is even

Here we get a $\boxed{\text{contradition}}$ since n^2 cannot both even and odd.

Therefore, n is odd.

3. Rational and Irrational numbers:

The real number r is rational if there exist integers a and b with $b \neq 0$ such that

$$r = \frac{a}{b}.$$

A real number that is not rational is called irrational..

4. Prove that a product of a non-zero rational number and an irrational number is irrational.

Assume "the product of a rational number and an irrational is rational"

$$\frac{a}{b} \cdot i = \frac{c}{d} \quad (a, b, c, d \text{ are non-zero integers})$$

i = non-zero irrational

Then $i = \frac{c}{d} \cdot \frac{b}{a} = \frac{cb}{da} \Rightarrow i$ is a rational number

Here is a contradition that i is both rational and irrational which implies the assumption is wrong, and a product of a non-zero rational number and an irrational one is irrational.

5. The Third Method: A Proof by **Contraposition**

Proofs by ________________ make use of the fact that the conditional statement $p \rightarrow q$ is **equivalent** to its contrapositive _____ $\rightarrow$ _____. This means that $p \rightarrow q$ can be proved by showing $\neg q \rightarrow \neg p$ is _________.

6. Give a proof by Contraposition of the theorem ''If n^2 is an odd integer, then n is odd.''

7. Mistakes in Proofs: An Example

What is wrong with this famous supposed ''proof'' that $1 = 2$?

Proof: We use these steps, where a and b are two equal positive integers.

Step	**Reason**
(1). $a = b$	Given
(2). $a^2 = ab$	Multiply both sides of (1) by a
(3). $a^2 - b^2 = ab - b^2$	Subtract b^2 from both sides of (2)
(4). $(a - b)(a + b) = b(a - b)$	Factor both sides of (3)
(5). $a + b = b$	Divide both sides of (4) by $a - b$
(6). $2b = b$	Replace a by b in (5) since $a = b$
(7). $2 = 1$	Divide both sides of (6) by b