MAT1375, Classwork26, Fall2025

Ch24. Sequences and Series & Ch25. Geometric Series

1. Definition of a sequence: $1, 2, 13, 2, 5, 100, 50$

A <u>sequence</u> is an enumerated list of numbers and it can be denoted by

$$a_1, a_2, a_3, a_4, a_5, a_6, \cdots, \text{ or } \underline{\{a_n\}}, \text{ or } \underline{\{a_n\}_{n\geq 1}}$$

2. A sequence with a **given pattern**: Find the first 6 terms of each sequence.

a) $a_n = 4n + 3$ b) $a_k = k^2$ c) $a_m = \frac{m}{m+1}$ d) $a_n = (-1)^n$

$n=1$ $a_1 = 4\cdot 1 + 3 = 7$ $+4$
$n=2$ $a_2 = 4\cdot 2 + 3 = 11$ $+4$
$n=3$ $a_3 = 4\cdot 3 + 3 = 15$ $+4$
$n=4$ $a_4 = 4\cdot 4 + 3 = 19$ $+4$
$n=5$ $a_5 = 4\cdot 5 + 3 = 23$

b) $k=1$, $a_1 = (1)^2 = 1$
$k=2$ $a_2 = (2)^2 = 4$
$k=3$ $a_3 = (3)^2 = 9$
$k=4$ $a_4 = (4)^2 = 16$
$k=5$ $a_5 = (5)^2 = 25$
$k=6$ $a_6 = (6)^2 = 36$

d) $n=1$, $a_1 = (-1)^1 = -1$
$n=2$ $a_2 = (-1)^2 = 1$
$n=3$ $a_3 = (-1)^3 = -1$
$n=4$ $a_4 = (-1)^4 = 1$
$n=5$ $a_5 = (-1)^5 = -1$
$n=6$ $a_6 = (-1)^6 = 1$

3. A sequence **without** a **given pattern**:

a) Find the 70th terms of the sequence: $22, 19, 16, 13, \cdots$ (-3 -3 -3)

$a_1 = 22$,
$19 = a_2 = 22 - 3\times 1$
$16 = a_3 = 19 - 3 = 22 - 3 - 3 = 22 - 3\times 2$ $\cdots$
$13 = a_4 = 16 - 3 = 22 - 3 - 3 - 3 = 22 - 3\times 3$
$a_{70} = 22 - 3\times \underline{69} = -185$

b) Find the 95th terms of the sequence: $-17, -12, -7, -2, \cdots$ ($+5$ $+5$ $+5$) $d=5$

$a_1 = -17$, $a_k = a_1 + (k-1)\cdot 5$
$k=95$, $a_{95} = a_1 + (95-1)\cdot 5 = -17 + 94\cdot 5 = -17 + 470 = 453$

c) Find the 10th terms of the sequence: $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \cdots$ ($\times(-\frac{1}{2})$ $\times(-\frac{1}{2})$ $\times(-\frac{1}{2})$)

common ratio $r = -\frac{1}{2}$

$a_1 = \frac{1}{2}$
$a_{10} = a_1 \cdot r^9 = \frac{1}{2}\left(-\frac{1}{2}\right)^9 = \frac{1}{2}\cdot\left(-\frac{1}{2^9}\right) = -\frac{1}{2^{10}}$

4. The **Arithmetic** Sequence: (for example, 3(a), (b))

A sequence $\{a_k\}$ is called <u>Arithmetic</u> sequence if any two consecutive terms have a common <u>difference</u> d.

The arithmetic sequence $\{a_k\}$ is determined by d and <u>a_1</u> (which is <u>the first term</u>):

$$\boldsymbol{a_k = a_{k-1}} + \underline{d} \text{ for } n \geq 2 \text{ or } \boldsymbol{a_k = a_1} + \underline{(k-1)\cdot d}.$$

5. The **Geometric** Sequence: : (for example, 3(c))

$\frac{a_2}{a_1} = r$, $\frac{a_3}{a_2} = r$, $\frac{a_4}{a_3} = r$, $\cdots$

A sequence $\{a_k\}$ is called <u>Geometric</u> sequence if any two consecutive terms have a common <u>ratio</u> r.

The geometric sequence $\{a_k\}$ is determined by r and <u>a_1</u> (which is <u>the first term</u>):

$$\boldsymbol{a_k = a_{k-1}} \cdot \underline{r} \text{ for } \cancel{n}\,k \geq 2 \text{ or } \boldsymbol{a_k = a_1} \cdot \underline{r^{k-1}}.$$

6. The **Series**:

Let $\{a_k\}$ be a sequence. The Series is the **sum** of all the term of a_k for $k \geq 1$:

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + \cdots$$

7. The Arithmetic **Series**: Let $\{a_k\}$ be an arithmetic sequence. Then the sum of the arithmetic sequence of **the first n term** is given by

the first term

the last term

sigma (sum)

$$\sum_{k=1}^{n} a_k = \frac{n}{2}(a_1 + a_n)$$

8. The Geometric **Series** : Let $\{a_k\}$ be a geometric sequence with the **common ratio** r that $-1<r<1$.

Then the sum of the geometric sequence of **the first n term** is given by

$$\sum_{k=1}^{n} a_k = a_1 \cdot \frac{1-r^n}{1-r}$$

Furthermore, the **infinite** geometric series is defined when $-1 < r < 1$ and given by

$$\sum_{k=1}^{\infty} a_k = a_1 \cdot \frac{1}{1-r}$$

→ fixed number (can't be ∞)

9. What is the infinite geometric series of $\{a_k\}$ if its common ratio $r \geq 1$ or $r \leq -1$?

$a_1 = 1$, $r = 2$ $\quad a_{100} = a_1 \cdot r^{99} = 1 \cdot 2^{99}$

$a_2 = a_1 \cdot r = 2$

10. Find the sum of the first 70 terms of the arithmetic sequence: $\overset{-3}{22}, \overset{-3}{19}, \overset{-3}{16}, 13, \cdots$.

$a_1 + a_2 + a_3 + \cdots + a_{70} = \sum_{n=1}^{70} a_n = \frac{70}{2}(a_1 + a_{70})$

$a_1 = 22$, $a_{70} = a_1 + (70-1)\cdot(-3)$ $\quad = \frac{70}{2} \cdot (22 - 185)$

$d = -3$ $\quad = -185$ $\quad = 35 \cdot (-163) = -5705$

11. Find the exact sum of infinite geometric sequence: $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \cdots$.

$$\sum_{n=1}^{\infty} a_n = a_1 \cdot \frac{1}{1-r} = \frac{1}{2} \cdot \frac{1}{1-(-\frac{1}{2})} = \frac{1}{2} \cdot \frac{1}{\frac{3}{2}} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} = \frac{1}{3}$$

$$\left(a_1 = \frac{1}{2},\ a_2 = -\frac{1}{4} \Rightarrow r = \frac{a_2}{a_1} = \frac{-\frac{1}{4}}{\frac{1}{2}} = -\frac{1}{4} \div \frac{1}{2} = -\frac{1}{4} \times \frac{2}{1} = -\frac{1}{2}\right)$$