Mat 1275 HW 14

14.4 Exercises

1. Solve $\sqrt{w} = 8$.

How to make "$\sqrt{\ }$" disapear? Take square on both sides.

$(\sqrt{w})^2 = (8)^2 \Rightarrow w = 64$.

Check: Left hand side $= \sqrt{64} = 8$
Right hand side $= 8$ ✓

2. Solve $\sqrt{w} = -8$.

$(\sqrt{w})^2 = (-8)^2 \Rightarrow w = 64$. However, Left hand side $= \sqrt{64} = 8$
Right hand side $= -8$

Left hand side $\neq$ Right hand side $\Rightarrow$ No solution.

3. Solve $\sqrt{w-2} = 8$.

$(\sqrt{w-2})^2 = (8)^2 \Rightarrow w-2 = 64 \Rightarrow w = 66$
$\quad\quad +2 \quad +2$

Check: Left hand side $= \sqrt{66-2} = \sqrt{64} = 8$
Right hand side $= 8$
$\Rightarrow$ Left hand side = Right hand side

4. Solve $2\sqrt{w-2} + w = 8$.

① Isolate the "$\sqrt{\ }$" term

$2\sqrt{w-8} + w = 8$
$\quad\quad -w \quad -w$

$\Rightarrow (2\sqrt{w-2})^2 = (8-w)^2$

	8	$-w$
8	64	$-8w$
$-w$	$-8w$	$+w^2$

② square it on both sides $\Rightarrow 2^2 \cdot (w-2) = 64 - 16w + w^2$

$\Rightarrow 4(w-2) = 64 - 16w + w^2$

$\Rightarrow 4w - 8 = 64 - 16w + w^2$
$\quad -4w \ +8 \quad +8 \ -4w$

$\Rightarrow 0 = 72 - 20w + w^2 \Rightarrow w^2 - 20w + 72 = 0$

$112 = 4 \cdot 28$
$\quad\quad 1-4$
$\sqrt{112} = 4\sqrt{7}$

by formula, $w = \dfrac{20 \pm \sqrt{(-20)^2 - 4 \cdot 1 \cdot 72}}{2} = \dfrac{20 \pm \sqrt{112}}{2} = \dfrac{20 \pm 4\sqrt{7}}{2}$
$= 10 \pm 2\sqrt{7}$

③ Check the answers:

When $w = 10 + 2\sqrt{7}$.

(LHS) Left hand side $= 2 \cdot \sqrt{10 + 2\sqrt{7} - 2} + 10 + 2\sqrt{7}$

$8 + 2\sqrt{7} = 1 + 2\sqrt{7} + 7$
$= 1 + 2\sqrt{7} + (\sqrt{7})^2 = (1 + \sqrt{7})^2$

$= 2\cdot\sqrt{(1+\sqrt{7})^2} + 10 + 2\sqrt{7} = 2\cdot(1+\sqrt{7}) + 10 + 2\sqrt{7}$

$= 12 + 4\sqrt{7}$

(RHS) Right hand side $= 8$ $\Rightarrow$ LHS $\neq$ RHS $\Rightarrow$ NOT A solution

When $W = 10 - 2\sqrt{7}$

$8 - 2\sqrt{7} = 7 - 2\sqrt{7} + 1 = (\sqrt{7})^2 - 2\sqrt{7} + 1 = (\sqrt{7}-1)^2$

$LHS = 2\cdot\sqrt{10 - 2\sqrt{7} - 2} + 10 - 2\sqrt{7}$

$= 2\cdot\sqrt{(\sqrt{7}-1)^2} + 10 - 2\sqrt{7}$

$= 2\cdot(\sqrt{7}-1) + 10 - 2\sqrt{7} = 2\sqrt{7} - 2 + 10 - 2\sqrt{7} = 8$

$RHS = 8 \Rightarrow LHS = RHS$ ✓

④ $\boxed{W = 10 - 2\sqrt{7}}$

5. Solve $\sqrt{w-2} = 4 - \sqrt{w+2}$. (challenge)

① Take square on both sides: $(\sqrt{w-2})^2 = (4 - \sqrt{w+2})^2$

$\Rightarrow w - 2 = 16 - 8\sqrt{w+2} + w + 2$

$-w \quad +16-2 \quad +16 \qquad -w \quad -2$

	4	$-\sqrt{w+2}$
4	16	$-4\sqrt{w+2}$
$-\sqrt{w+2}$	$-4\sqrt{w+2}$	$(\sqrt{w+2})^2 = w+2$

② Isolate "$\sqrt{\ }$" term

$\Rightarrow \frac{-20}{-4} = \frac{-8\sqrt{w+2}}{-4}$

$\Rightarrow 5 = 2\sqrt{w+2}$

③ Take square on the both sides $(5)^2 = (2\sqrt{w+2})^2$

$\Rightarrow 25 = 4\cdot(w+2) \Rightarrow 25 = 4w + 8$

$-8 \qquad -8$

$\Rightarrow 17 = 4w \Rightarrow w = \frac{17}{4}$

$\frac{}{4} \quad \frac{}{4}$

④ Check "$w = \frac{17}{4}$"

$LHS = \sqrt{\frac{17}{4} - 2} = \sqrt{\frac{17}{4} - \frac{8}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$RHS = 4 - \sqrt{\frac{17}{4} + 2} = 4 - \sqrt{\frac{17}{4} + \frac{8}{4}} = 4 - \sqrt{\frac{25}{4}} = 4 - \frac{5}{2}$

$= \frac{8}{2} - \frac{5}{2} = \frac{3}{2}$

$\Rightarrow$ LHS = RHS $\Rightarrow$ $W = \frac{12}{4}$ is an answer.

6. If a boat is 22 meters long and has a displacement of 22 cubic meters then the largest that area the sails S can be to qualify for a race satisfies

$$\frac{22 + 1.25\sqrt{S} - 9.8\sqrt[3]{22}}{0.686} = 24.$$

What is the largest that the area of the sails can be in cubic meters?

① Isolate $\sqrt{S}$: $\frac{22 + 1.25\sqrt{S} - 9.8\sqrt[3]{22}}{0.686} \times 0.686 = 24 \times 0.686$

$\Rightarrow 22 + 1.25\sqrt{S} - 9.8\sqrt[3]{22} = 16.464$

$-22 \quad +9.8\sqrt[3]{22} \qquad -22 + 9.8\sqrt[3]{22}$

$\Rightarrow 1.25\sqrt{S} = 9.8\sqrt[3]{22} - 5.536$

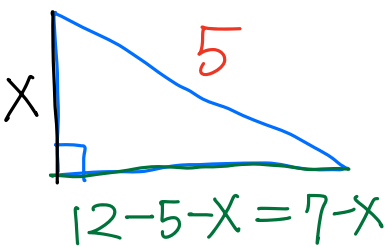
$\Rightarrow \sqrt{S} = \frac{9.8\sqrt[3]{22} - 5.536}{1.25}$

② square it on the both sides:

$$\left(\sqrt{S}\right)^2 = \left(\frac{9.8\sqrt[3]{22} - 5.536}{1.25}\right)^2$$

$$\Rightarrow S = \left(\frac{9.8\sqrt[3]{22} - 5.536}{1.25}\right)^2 = 307.623128$$

7. If a right triangle has hypotenuse 5 feet and the perimeter is 12 feet, what are the lengths of the legs of the triangle? Be sure to draw a picture, label it, and form an appropriate equation whose solution leads to an answer.



$\Rightarrow X^2 + (7-X)^2 = (5)^2$

$\Rightarrow X^2 + 49 - 14X + X^2 = 25$

$-25 \qquad -25$

$\Rightarrow 2X^2 - 14X + 24 = 0$

$\Rightarrow 2 \cdot (X^2 - 7X + 12) = 0$

$X \quad -3$

$X \quad -4$

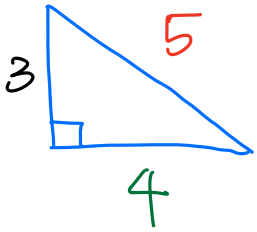
$\Rightarrow (X-3)(X-4) = 0$

$X-3=0$, $X-4=0$

$+3 \quad +3 \qquad +4 \quad +4$

$X = 3$ or $X = 4$

either



or