

MAT1375, Classwork23, Fall2025

Ch21. Trigonometric Identities

1. Addition and Subtraction of angles formulas:

Let α, β be two angles. We have $\sin(\alpha)$ is good and $\cos(\alpha)$ is bad.

$$(1) \sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$(2) \sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$(3) \cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$(4) \cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

2. Half- and double-angle formulas:

$$\sin(\alpha + \alpha)$$

$$(5) \sin(2\alpha) = 2 \sin(\alpha)\cos(\alpha) \quad (\text{From (1) and let } \beta = \alpha)$$

$$(6) \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (\text{From (3) and let } \beta = \alpha)$$

$$\downarrow \\ \cos(\alpha + \alpha)$$

$$(7) \sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1-\cos(\alpha)}{2}}$$

$\frac{\alpha}{2}$ is in I, II

$$+ \sqrt{\frac{1-\cos(\alpha)}{2}}$$

$\frac{\alpha}{2}$ is in III, IV

$$- \sqrt{\frac{1-\cos(\alpha)}{2}}$$

$$(8) \cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1+\cos(\alpha)}{2}}$$

$\frac{\alpha}{2}$ is in I, IV

$$+ \sqrt{\frac{1+\cos(\alpha)}{2}}$$

$\frac{\alpha}{2}$ is in II, III

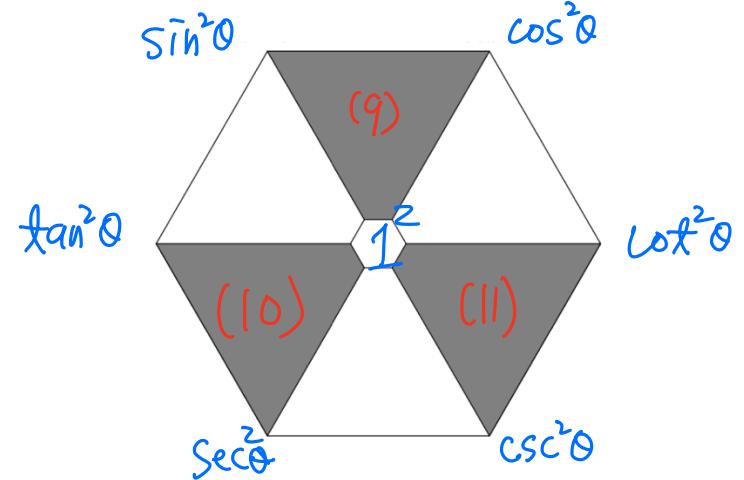
$$- \sqrt{\frac{1+\cos(\alpha)}{2}}$$

3. Pythagorean Identities:

$$(9) \sin^2(\theta) + \cos^2(\theta) = 1^2$$

$$(10) 1^2 + \tan^2(\theta) = \sec^2(\theta)$$

$$(11) 1^2 + \cot^2(\theta) = \csc^2(\theta)$$



4. Find the exact value of the trigonometric functions:

a) $\sin\left(\frac{11\pi}{12}\right)$ b) $\cos\left(\frac{7\pi}{8}\right)$ c) $\sin(15^\circ)$ d) $\cos(75^\circ)$

5. Simplify the given function using the addition and subtraction formulas.

a) $\sin\left(\frac{\pi}{2} + x\right)$ b) $\sin\left(\frac{\pi}{2} - x\right)$ c) $\cos\left(\frac{\pi}{2} + x\right)$ d) $\cos\left(\frac{\pi}{2} - x\right)$