25.3 Exercises

Exercise 25.1

Which of these sequences is geometric, arithmetic, neither, or both. Write the sequence in the usual form $a_n = a_1 + (n-1)\cdot d$ if it is an arithmetic sequence, and $a_n = a_1 \cdot r^{n-1}$ if it is a geometric sequence.

a) $7, 14, 28, 56, \dots$

b) $3, -30, 300, -3000, \dots$

c) $81, 27, 9, 3, 1, \frac{1}{3}, \dots$

d) $-7, -5, -3, -1, 1, 3, 5, 7, \dots$

e) $-6, 2, -\frac{2}{3}, \frac{2}{9}, -\frac{2}{27}, \dots$

f) $-2, -2\cdot\frac{2}{3}, -2\cdot\left(\frac{2}{3}\right)^2, -2\cdot\left(\frac{2}{3}\right)^3, \dots$

g) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

h) $2, 2, 2, 2, 2, \dots$

i) $5, 1, 5, 1, 5, 1, 5, 1, \dots$

j) $-2, 2, -2, 2, -2, 2, \dots$

k) $0, 5, 10, 15, 20, \dots$

l) $5, \frac{5}{3}, \frac{5}{3^2}, \frac{5}{3^3}, \frac{5}{3^4}, \dots$

m) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

n) $\log(2), \log(4), \log(8), \log(16), \dots$

o) $a_n = -4^n$

p) $a_n = -4n$

q) $a_n = 2\cdot(-9)^n$

r) $a_n = \left(\frac{1}{3}\right)^n$

s) $a_n = -\left(\frac{5}{7}\right)^n$

t) $a_n = \left(-\frac{5}{7}\right)^n$

u) $a_n = \frac{2}{n}$

v) $a_n = 3n + 1$

Exercise 25.2

A geometric sequence, $a_n = a_1 \cdot r^{n-1}$, has the given properties. Find the term a_n of the sequence.

a) $a_1 = 3$, and $r = 5$, find a_4

b) $a_1 = 200$, and $r = -\frac{1}{2}$, find a_6

c) $a_1 = -7$, and $r = 2$, find a_n (for all n)

d) $r = 2$, and $a_4 = 48$, find a_1

e) $r = 100$, and $a_4 = 900,000$, find a_n (for all n)

f) $a_1 = 20$, $a_4 = 2500$, find a_n (for all n)

g) $a_1 = \frac{1}{8}$, and $a_6 = \frac{3^5}{8^6}$, find a_n (for all n)

h) $a_3 = 36$, and $a_6 = 972$, find a_n (for all n)

i) $a_8 = 4000$, $a_{10} = 40$, and r is negative, find a_n (for all n)

Exercise 25.3

Find the value of the finite geometric series using formula (25.2). Confirm the formula either by adding the the summands directly, or alternatively by using the calculator.

a) Find the sum $\sum_{j=1}^{4} a_j$ for the geometric sequence $a_j = 5 \cdot 4^{j-1}$.

b) Find the sum $\sum_{i=1}^{7} a_i$ for the geometric sequence $a_n = \left(\frac{1}{2}\right)^n$.

c) Find: $\sum_{m=1}^{5} \left(-\frac{1}{5}\right)^m$

d) Find: $\sum_{k=1}^{6} 2.7 \cdot 10^k$

e) Find the sum of the first 5 terms of the geometric sequence:

$$2, 6, 18, 54, \ldots$$

f) Find the sum of the first 6 terms of the geometric sequence:

$$-5, 15, -45, 135, \ldots$$

g) Find the sum of the first 8 terms of the geometric sequence:

$$-1, -7, -49, -343, \ldots$$

h) Find the sum of the first 10 terms of the geometric sequence:

$$600, -300, 150, -75, 37.5, \ldots$$

i) Find the sum of the first 40 terms of the geometric sequence:

$$5, 5, 5, 5, 5, \ldots$$

Exercise 25.4

Find the value of the infinite geometric series.

a) $\sum_{j=1}^{\infty} a_j$, for $a_j = 3 \cdot \left(\frac{2}{3}\right)^{j-1}$
b) $\sum_{j=1}^{\infty} 7 \cdot \left(-\frac{1}{5}\right)^{j}$
c) $\sum_{j=1}^{\infty} 6 \cdot \frac{1}{3^j}$
d) $\sum_{n=1}^{\infty} -2 \cdot (0.8)^n$
e) $\sum_{n=1}^{\infty} (0.99)^n$
f) $27 + 9 + 3 + 1 + \frac{1}{3} + \dots$
g) $-2 + 1 - \frac{1}{2} + \frac{1}{4} - \dots$
h) $-6 - 2 - \frac{2}{3} - \frac{2}{9} - \dots$
i) $100 + 40 + 16 + 6.4 + \dots$
j) $-54 + 18 - 6 + 2 - \dots$

Exercise 25.5

Rewrite the decimal using an infinite geometric sequence, and then use the formula for the infinite geometric series to rewrite the decimal as a fraction (see Example 25.11).

a) $0.44444\dots$
b) $0.77777\dots$
c) $5.55555\dots$
d) $0.2323232323\dots$
e) $39.393939\dots$
f) $0.248248248\dots$
g) $20.02002\dots$
h) $0.5040504\dots$