MAT2440, Classwork7, Spring2025

ID:_____ Name:_____

1. Use identities to prove "$\neg (p \rightarrow q) \equiv p \wedge \neg q$"

~~$\neg(p \rightarrow q) \equiv p \wedge (\neg q)$~~

$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$ first one in Group II

$\equiv \neg(\neg p) \wedge \neg q$ De Morgan's law

$\equiv p \wedge (\neg q)$ Double Negation

2. Use identities to prove "$(p \wedge q) \rightarrow (p \vee q)$" is a tautology.

$(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ De Morgan's

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$ associative and commutive

$\equiv T \vee T$ negation

$\equiv T$ domination

3. Group III of the logically equivalences: Identities related to biconditional statements.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ ← "definition" of "$p \leftrightarrow q$"

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

(3) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Prove "$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$"

Proof: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$

Group II, the 1st one →

$\equiv ((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \vee ((\neg p \wedge p) \vee (q \wedge p))$

Negation law $\equiv ((\neg p \wedge \neg q) \vee F) \vee (F \vee (q \wedge p))$

Identity law $\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

prove $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

proof: Group III (3)

$\neg(p \leftrightarrow q) \equiv \neg((p \wedge q) \vee (\neg p \wedge \neg q))$

De Morgan's $\equiv \neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q)$

De Morgan: $\equiv (p \rightarrow \neg q) \wedge (p \vee q)$

$\neg(p \rightarrow \neg q) \equiv p \wedge q$ and negation in Group II

$\equiv (p \rightarrow \neg q) \wedge (p \vee \neg(\neg q))$

$\equiv (p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$ (Group II: $\neg p \rightarrow q \equiv p \wedge \neg q$)

$\equiv p \leftrightarrow \neg q$ (Group III first one)

4. Predicate logic and Propositional function:

The predicate logic allows variables in propositions and enables us to reason and explore relationships between objects. A propositional function is a statement with variables and has been used on predicate logic. Once the values have been assigned to the variables, the propositional function becomes a proposition, and has truth value

5. Let $P(x)$ denote the statement "$x > 3$". What are the truth values of $P(2)$ and $P(4)$.

P(x) denotes " x is greater than 3"
x: variable (subject); is greater than 3: predicate (property)

P(2) is " 2 > 3" false
P(4) is " 4 > 3" True

6. Let $Q(x, y)$ denote the statement "$x = y + 3$". What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$.

Q(1,2) → x=1, y=2 means " 1 = 2 + 3" which is false.

Q(3,0) → x=3, y=0 means " 3 = 0 + 3" which is true.

7. Given a computer programing "If $x > 0$, then $x := x + 1$". Using the terminology of propositional function to explain it.

this is P(x)

If P(x) is true, then x is increased by 1.
If P(x) is false, then x is not changed.