Mat 1275 HW15

15.4 Exercises

1. ① Is $(2, -1)$ a solution to the equation $x^3 - y^3 + y = 3$? ② Is the point $(2, -1)$ on the graph of $x^3 - y^3 + y = 3$?

① Check if $(2,-1)$ is a solution of $x^3-y^3+y=3$

⇒ replace x by "2" and y by "-1" and see if the equation holds.

Left hand side $= (2)^3 - (-1)^3 + (-1) = 8 - (-1) - 1 = 8 + 1 - 1 = 8$

Right hand side $= 3$

⇒ Left hand side $\neq$ Right hand side

⇒ $(2,-1)$ is NOT a solution of $x^3-y^3+y=3$.

② since, from ①, $(2,-1)$ is NOT a solution of $x^3-y^3+y=3$, then point $(2,-1)$ is NOT on the graph of $x^3-y^3+y=3$.

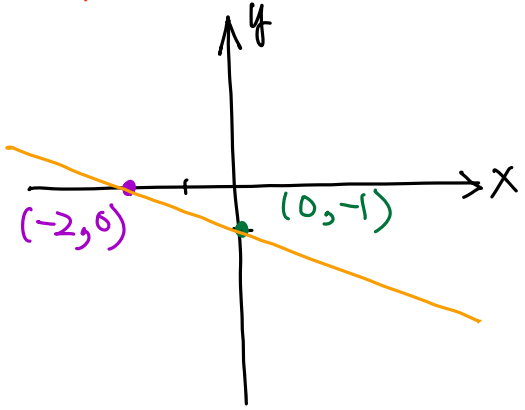
2. Identify the slope and y-intercept of $y = -\frac{1}{2}x - 1$, and graph the line.

General form of a line: $y = (\text{slope}) \cdot x + (y\text{-intercept})$

for $y = -\frac{1}{2}x - 1$, slope $= -\frac{1}{2}$, y-intercept is $(0,-1)$ (where $x=0$)

Graph:

x	y	(x,y)
0	−1	(0,−1)
−2	0	(−2,0)

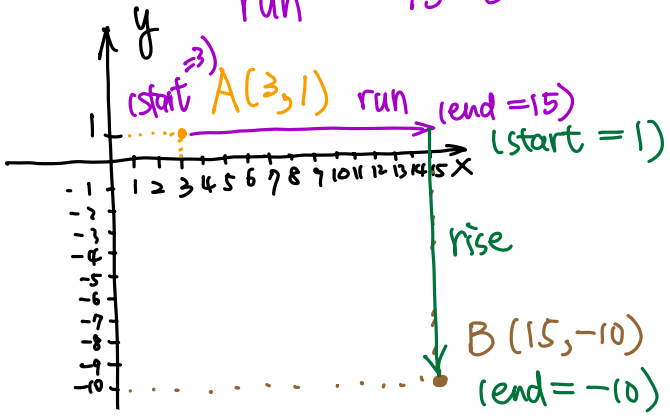


3. Write the equation of the line passing through $(3, 1)$ and $(15, -10)$.

A B

General form of a line: $y = (\text{slope}) \cdot x + \text{constant}$

① $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-10-1}{15-3} = \frac{-11}{12}$



② $y = -\frac{11}{12}x + ?$, find "?"

Since A(3,1) and B(15,-10) is on the graph of this line, then we can find "?" by A or B:

$1 = -\frac{11}{12} \cdot 3 + ? \Rightarrow 1 = -\frac{11}{4} + ?$

$+\frac{11}{4}$ $+\frac{11}{4}$

$\Rightarrow 1 + \frac{11}{4} = ? \Rightarrow \frac{15}{4} = ?$

By ①, ②, the equation of this line is $\boxed{y = -\frac{11}{12}x + \frac{15}{4}}$

4. Find two solution of the equation $y = 2$ as an equation with two variables, and use them to represent all solutions on a coordinate plane.

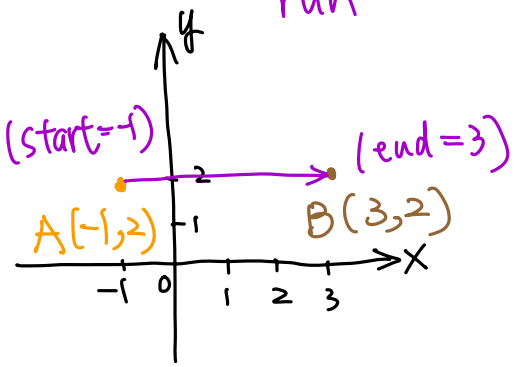
For two solutions of equation $y=2$, it means as long as $y=2$, x can be any number: $\boxed{(-1, 2)}$, $\boxed{(3, 2)}$, $(100, 2)$, $(-500, 2)$, ...

For general form of a line who passes $A(-1,2)$, $B(3,2)$,

we have $y = (\text{slope}) \cdot x + ?$

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{0}{3-(-1)} = \frac{0}{4} = 0 \Rightarrow y = 0 \cdot x + ?$

$\Rightarrow y = ?$



but No rise! (rise = 0)

since this line passes $A(-1,2)$ and $B(3,2)$, then $\boxed{y = 2}$ which represent all solution on coordinate plane.

5. Write an equation for a line perpendicular to $y = 2x - 1$ which passes through $(-2, 1)$.

line 1: $y = 2x - 1$

Find a line, denote line 2, which is perpendicular to line 1

The general form of line 2: $y = (\text{slope of line 2}) \cdot x + (\text{a constant})$

Since line 2 is perpendicular to line 1, we have slope of line 1 = "2"

$$(\text{slope of line 1}) \cdot (\text{slope of line 2}) = -1$$

$$\Rightarrow \underbrace{2 \cdot (\text{slope of line 2})}_{2} = \frac{-1}{2} \Rightarrow \text{slope of line 2} = -\frac{1}{2}$$

The equation of line 2 is $y = -\frac{1}{2}x + ?$ which passes $(-2, 1)$

To find "?", we have $1 = -\frac{1}{2} \cdot (-2) + ?$

$$\Rightarrow 1 = 1 + ? \Rightarrow 0 = ?$$

$\Rightarrow$ the equation of line 2 is $\boxed{y = -\frac{1}{2}x}$

6. Are the following lines parallel: line 1 $2x - 4y = 7$ and line 2 $3x - 5y = 8$? Explain.

line 1: $2x - 4y = 7 \Rightarrow$

$-2x \quad -2x$

$$-4y = -2x + 7 \Rightarrow y = \frac{-2x+7}{-4} \Rightarrow y = \frac{-2x}{-4} + \frac{7}{-4}$$

$$\Rightarrow y = \frac{-2}{-4}x - \frac{7}{4} \Rightarrow y = \underbrace{\frac{1}{2}}_{m}x - \frac{7}{4}$$

slope of line 1 $= \frac{1}{2}$

line 2: $3x - 5y = 8$

$-3x \quad -3x$

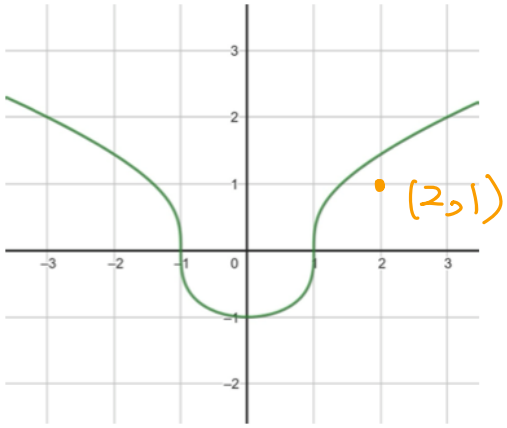
$$\Rightarrow -5y = -3x + 8 \Rightarrow y = \frac{-3x+8}{-5}$$

$$\Rightarrow y = \frac{-3}{-5}x + \frac{8}{-5} \Rightarrow y = \frac{3}{5}x - \frac{8}{5}$$

slope of line 2 $= \frac{3}{5}$

Since slope of line 1 $\neq$ slope of line 2, then they are NOT parallel.

7. Is $(2,1)$ a solution to the equation whose graph is given below?



NO, since $(2,1)$ is NOT on the graph of this equation, then $(2,1)$ is NOT a solution of this equation.

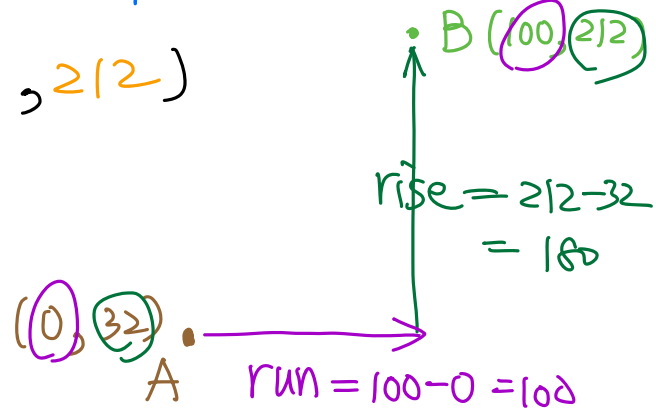
8. Find an equation representing the relationship between Celsius and Fahrenheit temperature scales noting the freezing point of water is $0°C$ and $32°F$ and boiling point of water is $100°C$ and $212°F$. If it is $76°F$ outside, what is the temperature in Celsius (use your equation)?

C F

We can see Celsius and Fahrenheit as a pair:

$(0, 32)$ and $(100, 212)$

point A point B



$$F = \frac{\text{rise}}{\text{run}} \cdot C + ?$$

$$F = \frac{180}{100} \cdot C + ?$$

Using point A $(0, 32)$, we have $32 = \frac{180}{100} \cdot 0 + ? \Rightarrow 32 = ?$

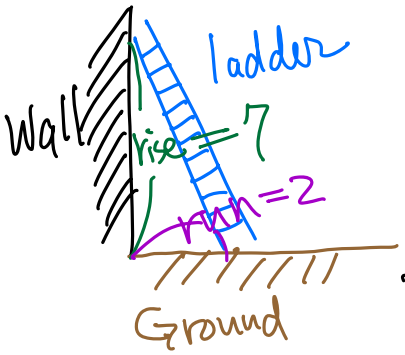
Then $F = \frac{180}{100} C + 32.$

If $F = 76\ (°F) \Rightarrow 76 = \frac{180}{100} C + 32$

$-32 \qquad -32$

$$\Rightarrow \frac{\cancel{100}^{\,22}}{\cancel{180}_{\,9}} 44 = \frac{100}{180} \frac{180}{100} \cdot C \Rightarrow \frac{220}{9} = C \Rightarrow C = 24.44\ldots$$

9. A ladder is leaning against a wall so that it meets the wall 7 feet off the ground and the base of the ladder is 2 feet from the wall. If you orient yourself so that the slope of the ladder is positive, a safe slope is 3.87. Is your ladder safe to climb? Explain.



$\Rightarrow$ slope of the ladder $= \frac{\text{rise}}{\text{run}} = \frac{7}{2} = 3.5$

since $3.5 \neq$ safe slope 3.87, then it is NOT safe to climb.