

MAT1372, Quiz6, Fall2025

ID: _____

Name: Sol.

- This quiz consists of 2 question for a total of 10 points.
- You have 15 minutes to complete the quiz.
- Show all work and justify your answers.
- Wishing you success.

1. Consider a random variable X that takes on either the value 1 or 0 with respective probabilities p and $1 - p$. That is, $P(X = 1) = p$, and $P(X = 0) = 1 - p$. Find $E[X]$.

$$\begin{aligned} E(X) &= 1 \cdot P(X=1) + 0 \cdot P(X=0) \\ &= 1 \cdot p + 0 \cdot (1-p) = p. \end{aligned}$$

2. A building contractor has sent in bids for three jobs. If the contractor obtains these jobs, they will yield respective profits of 20, 25, and 40 (in units of \$1000). On the other hand, for each job the contractor does not win, he will incur a loss (due to time and money already spent in making the bid) of 2 (in units of \$1000). If the probabilities that the contractor will get these jobs are, respectively, 0.3, 0.6, and 0.2, what is the expected total profit? (for example, let the first job be X_1 , we have $P(X_1: \text{win}) = 0.3$ and $P(X_1: \text{loss}) = 1 - 0.3 = 0.7$)

Let X_1, X_2, X_3 be the profits of these three jobs, respectively.

✓ if win, get 20k

$$P(X_1 = 20) = 0.3$$

✓ if lose, loss 2k

$$\begin{aligned} P(X_1 = -2) &= 1 - 0.3 \\ &= 0.7 \end{aligned}$$

$$\begin{aligned} E(X_1) &= 20 \cdot P(X_1 = 20) + (-2) \cdot P(X_1 = -2) \\ &= 20 \cdot 0.3 + (-2) \cdot 0.7 \\ &= 6 - 1.4 = 4.6 \end{aligned}$$

$$P(X_2 = 25) = 0.6$$

$$\begin{aligned} P(X_2 = -2) &= 1 - 0.6 \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} E(X_2) &= 25 \cdot P(X_2 = 25) + (-2) \cdot P(X_2 = -2) \\ &= 25 \cdot 0.6 + (-2) \cdot 0.4 \\ &= 15 - 0.8 = 14.2 \end{aligned}$$

$$P(X_3 = 40) = 0.2$$

$$\begin{aligned} P(X_3 = -2) &= 1 - 0.2 \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} E(X_3) &= 40 \cdot P(X_3 = 40) + (-2) \cdot P(X_3 = -2) \\ &= 40 \cdot 0.2 + (-2) \cdot 0.8 \\ &= 8 - 1.6 = 6.4 \end{aligned}$$

Let X be the total profit, then $X = X_1 + X_2 + X_3$. and

$$E(X) = E(X_1) + E(X_2) + E(X_3) = 4.6 + 14.2 + 6.4 = 25.2 \text{ k}$$