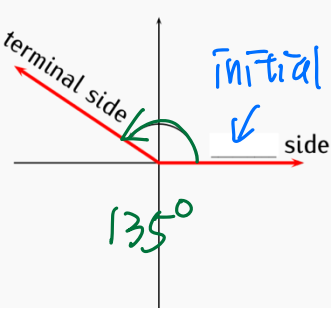
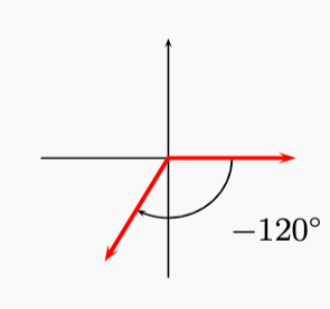
MAT 1275, Classwork20, Fall2024

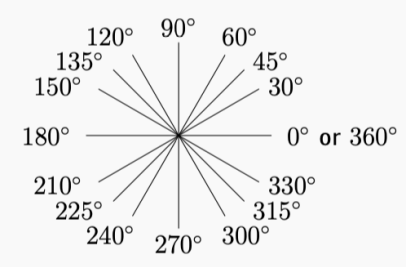
ID:_____ Name:_____

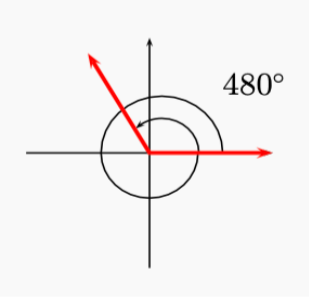
1. Angle in standard position:

An angle in the plane is in _standard position_ if its vertex is at the origin and the initial side is at the positive x-axis.









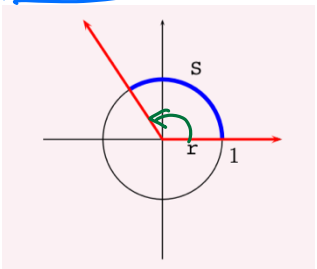
Counterclockwise direction: Angle _>_ 0.

Clockwise direction: Angle _<_ 0.

A full rotation measure as _360°_

An angle can be _more than_ 360°

2. The **Central angle** is an angle whose vertex is at the _center_ of the circle.

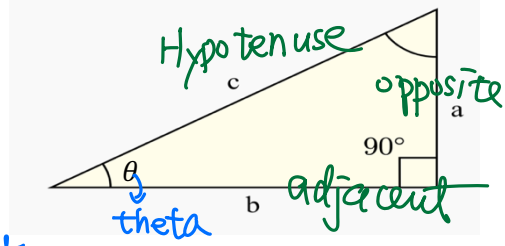


The **radian** measure of the central angle of a circle is **ratio** of the _length_ of the intercept arc ***s*** with the circle radius ***r***:

$$\text{Angle in radian} = \frac{s}{r}.$$

When $r = 1$ and half circumference of this circle is π, then we have the central angle to be 180° in degree or π in radian.

3. Right Triangle Definitions of **Trigonometric Functions** and **Reciprocal Identities**: SOHCAHTOA



sine O/H	$\sin(\theta) = \frac{\text{length of side } \underline{\text{opposite}} \text{ angle } \theta}{\text{length of } \underline{\text{Hypotenuse}}} = \frac{a}{c}$	cosecant $\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{c}{a}$
cosine A/H	$\cos(\theta) = \frac{\text{length of side } \underline{\text{adjacent}} \text{ to angle } \theta}{\text{length of } \underline{\text{Hypotenuse}}} = \frac{b}{c}$	$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{c}{b}$ secant
tangent O/A	$\tan(\theta) = \frac{\text{length of side } \underline{\text{opposite}} \text{ angle } \theta}{\text{length of side } \underline{\text{adjacent}} \text{ to angle } \theta} = \frac{a}{b}$	$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{b}{a}$ cotangent

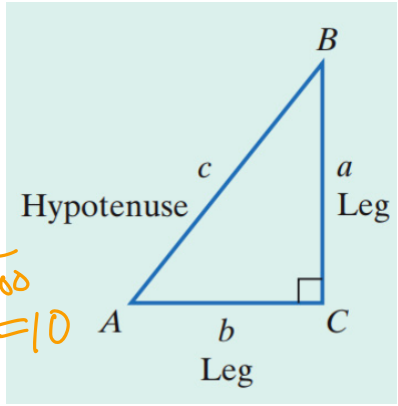
4. Quotient Identities:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

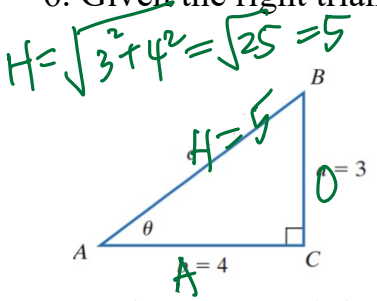
5. The Pythagorean Theorem:

The sum of the square of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

$a^2 + b^2 = c^2$.



6. Given the right triangles. Find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$.

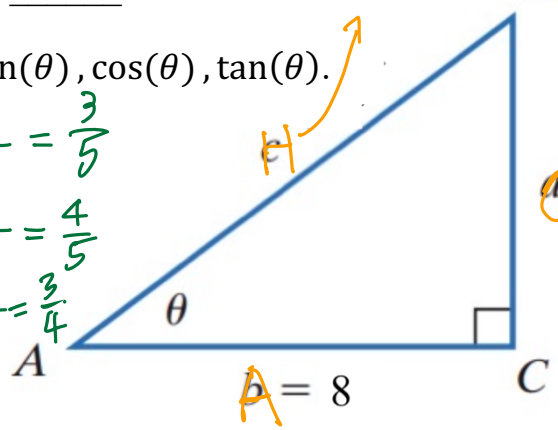


$H = \sqrt{3^2+4^2} = \sqrt{25} = 5$

$\sin(\theta) = \frac{O}{H} = \frac{3}{5}$

$\cos(\theta) = \frac{A}{H} = \frac{4}{5}$

$\tan(\theta) = \frac{O}{A} = \frac{3}{4}$



$H = \sqrt{6^2+8^2} = \sqrt{100} = 10$

$\sin(\theta) = \frac{6}{10} = \frac{3}{5}$

$\cos(\theta) = \frac{8}{10} = \frac{4}{5}$

$\tan(\theta) = \frac{6}{8} = \frac{3}{4}$

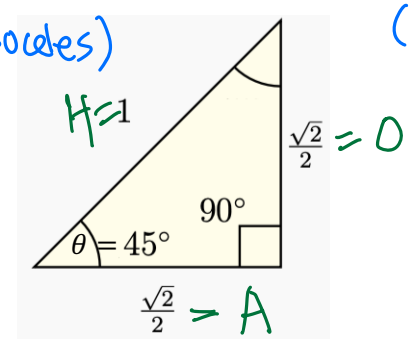
7. Pythagorean Triple:

(3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17)

8. The values of trigonometric functions only depend on the size of angle, not the size of the triangle.

9. The values of trigonometric functions with special angles:

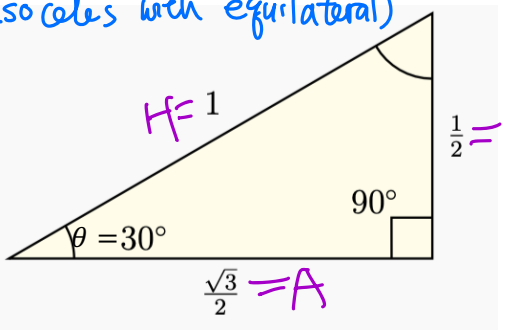
(isosceles)



$\frac{\sqrt{2}}{2} = O$

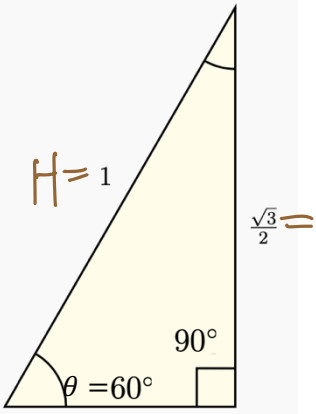
$\frac{\sqrt{2}}{2} = A$

(Isosceles with equilateral)



$\frac{1}{2} = O$

$\frac{\sqrt{3}}{2} = A$



$\frac{\sqrt{3}}{2} = O$

$\frac{1}{2} = A$

θ	0 (or 0°)	$\frac{\pi}{6}$ (or 30°)	$\frac{\pi}{4}$ (or 45°)	$\frac{\pi}{3}$ (or 60°)	$\frac{\pi}{2}$ (or 90°)
$\sin(\theta)$	0	$\frac{O}{H} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$	$\frac{O}{H} = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$	$\frac{O}{H} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{A}{H} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$	$\frac{A}{H} = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$	$\frac{A}{H} = \frac{1}{2}$	0
$\tan(\theta)$	$\frac{O}{A} = 0$	$\frac{1}{2} \div \frac{\sqrt{3}}{2}$	$\frac{O}{A} = 1$	$\frac{O}{H} = \sqrt{3}$	$\frac{1}{0}$ = undefined

$= \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3}$