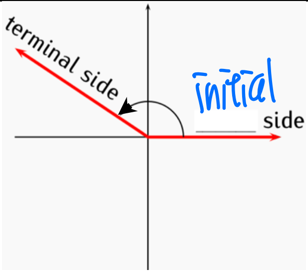
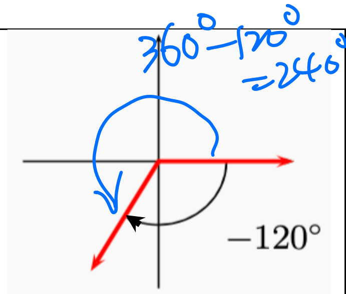
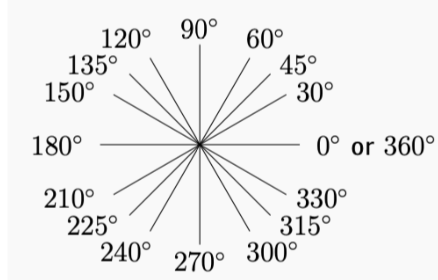
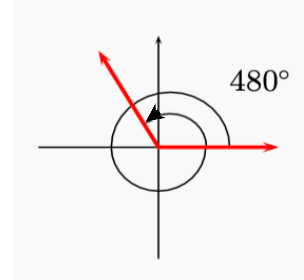
# MAT1375, Classwork18, Fall2025

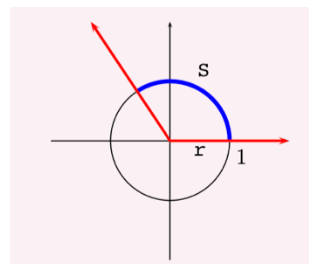
*Ch17. Trigonometric Functions reviewed*

1. Angle in standard position:

An angle in the plane is in standard position if its vertex is at the origin and the initial side is at the positive $x$-axis.

| | | | |
|---|---|---|---|
|  |  |  |  |
| Counterclockwise direction: Angle $>$ 0. | Clockwise direction: Angle $<$ 0. | A full rotation measure as $360^\circ$ | An angle can be more than 360° |

2. The **Central angle** is an angle whose vertex is at the center of the circle.



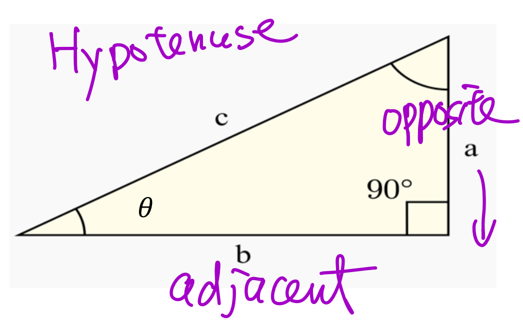

The **radian** measure of the central angle of a circle is **ratio** of the length of the intercept arc ***s*** with the circle radius ***r***:

$$\text{Angle in radian} = \frac{s}{r}$$

When $r = 1$ and half circumference of this circle is $\pi$, then we have the central angle to be $180$ in degree or $\frac{\pi}{r} = \pi$ in radian. $180^\circ \to \pi$ $\quad 360^\circ \to 2\pi$ $\quad 90^\circ \to \frac{\pi}{2}$

3. Right Triangle Definitions of **Trigonometric Functions** and **Reciprocal Identities**:

| | |
|---|---|
| sine<br>$\sin(\theta) = \dfrac{\text{length of side opposite angle } \theta}{\text{length of Hypotenuse}} = \dfrac{a}{c}$ | $\csc(\theta) = \dfrac{1}{\sin\theta} = \dfrac{c}{a}$ |
| cosine<br>$\cos(\theta) = \dfrac{\text{length of side adjacent to angle } \theta}{\text{length of Hypotenuse}} = \dfrac{b}{c}$ | $\sec(\theta) = \dfrac{1}{\cos\theta} = \dfrac{c}{b}$ |
| tangent<br>$\tan(\theta) = \dfrac{\text{length of side opposite angle } \theta}{\text{length of side adjacent to angle } \theta} = \dfrac{a}{b}$ | $\cot(\theta) = \dfrac{1}{\tan\theta} = \dfrac{b}{a}$ |




4. Quotient Identities:

$$\tan(\theta) = \frac{\sin\theta}{\cos\theta} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

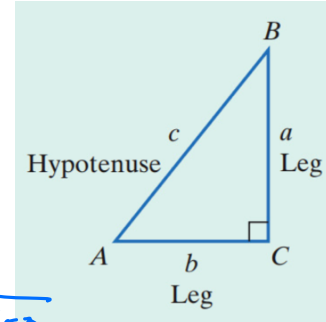
5. Reciprocal Identities:

$$\csc(\theta) = \frac{1}{\sin\theta} \qquad \sec(\theta) = \frac{1}{\cos\theta} \qquad \cot(\theta) = \frac{1}{\tan\theta}$$

6. The Pythagorean Theorem:

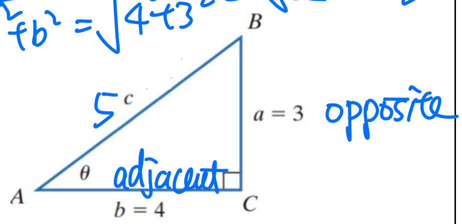
The sum of the square of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

$a^2$ + $b^2$ = $c^2$.



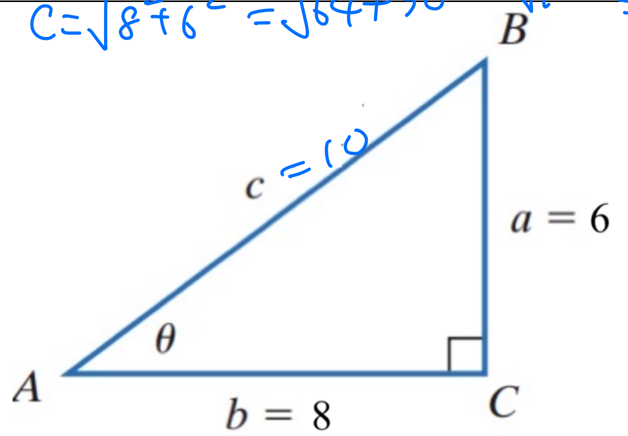

7. Given the right triangles. Find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$.

$C=\sqrt{a^2+b^2}=\sqrt{4^2+3^2}=\sqrt{25}=5$




$\sin(\theta) = \frac{3}{5}$, $\cos(\theta) = \frac{4}{5}$, $\tan(\theta) = \frac{3}{4}$

$C=\sqrt{8^2+6^2}=\sqrt{64+36}=\sqrt{100}=10$




$\sin(\theta) = \frac{6}{10}=\frac{3}{5}$

$\cos(\theta) = \frac{8}{10}=\frac{4}{5}$

$\tan(\theta) = \frac{6}{8}=\frac{3}{4}$

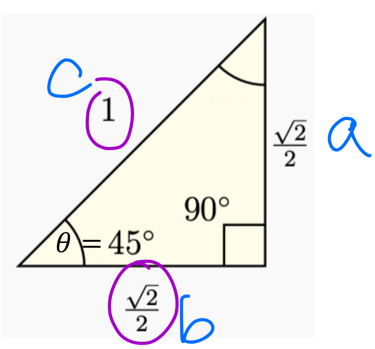
8. The values of trigonometric functions only depend on the size of angle, not the size of the triangle.

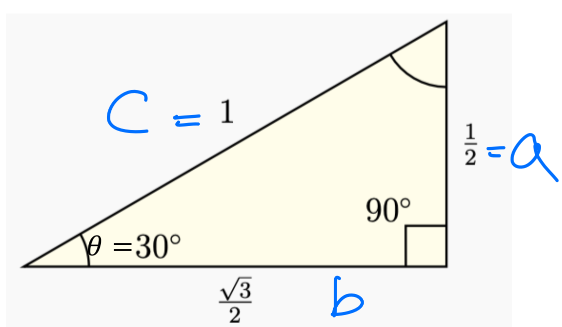
9. Pythagorean Triple: (leg, leg, hypotenuse) (a, b, c)

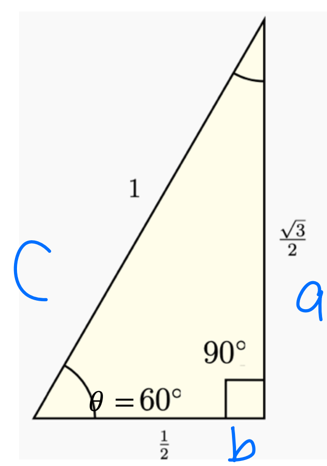
( 3, 4, 5), ( 5, 12, 13), ( 7, 24, 25), ( 8, 15, 17)

(9, 12, 15) (6, 8, 10)

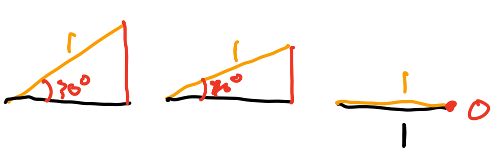
10. The values of trigonometric functions with special angles:




| $\theta$ | 0 (or 0°) | $\frac{\pi}{6}$ (or 30°) | $\frac{\pi}{4}$ (or 45°) | $\frac{\pi}{3}$ (or 60°) | $\frac{\pi}{2}$ (or 90°) |
|---|---|---|---|---|---|
| $\sin(\theta)$ | $\frac{0}{1}=0$ | $\sin(\frac{\pi}{6})=\frac{\frac{1}{2}}{1}=\frac{1}{2}$ | $\sin(\frac{\pi}{4})=\frac{\frac{\sqrt{2}}{2}}{1}=\frac{\sqrt{2}}{2}$ | $\sin(\frac{\pi}{3})=\frac{\frac{\sqrt{3}}{2}}{1}=\frac{\sqrt{3}}{2}$ | $\sin(\frac{\pi}{2})=\frac{oppo}{H}=\frac{1}{1}=1$ |
| $\cos(\theta)$ | $\frac{1}{1}=1$ | $\cos(\frac{\pi}{6})=\frac{\frac{\sqrt{3}}{2}}{1}=\frac{\sqrt{3}}{2}$ | $\cos(\frac{\pi}{4})=\frac{\frac{\sqrt{2}}{2}}{1}=\frac{\sqrt{2}}{2}$ | $\cos(\frac{\pi}{3})=\frac{\frac{1}{2}}{1}=\frac{1}{2}$ | $\cos(\frac{\pi}{2})=\frac{A}{H}=\frac{0}{1}=0$ |
| $\tan(\theta)$ | $\frac{0}{1}=0$ | $\tan(\frac{\pi}{6})=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}$ | $\tan(\frac{\pi}{4})=\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=1$ | $\tan(\frac{\pi}{3})=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$ | $\tan(\frac{\pi}{2})=\frac{oppo}{A}=\frac{1}{0}$ ⇒ undefined. |




$\frac{1\cdot\sqrt{3}}{\sqrt{3}\sqrt{3}}=\frac{\sqrt{3}}{3}$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\frac{\sqrt{3}}{2}\cdot\frac{2}{1}=\sqrt{3}$