

Quiz7, MAT1375 Professor Chiu

ID: _____

Name: Sol

- This quiz consists of 2 sets of questions for a total of 10 points.
- You have 15 minutes to complete the quiz.
- Wishing you success.

True or False. Circle your answers (either T (true) or F (false)) on this sheet.

1. (T / F) Let $f(x) = \frac{(x-2)(x+3)}{x-3}$. Then $x = 3$ is a vertical asymptote of $f(x)$.

2. (T / F) Let $f(x) = \frac{3x-2}{x-3}$. Then $y = 3$ is a horizontal asymptote of $f(x)$.

3. (T / F) Let $f(x) = \frac{x^2-2}{x-3}$. Then there is no horizontal asymptote of $f(x)$.

4. (T / F) Let $f(x) = \frac{(x-2)(x-3)}{x-3}$. Then $x = 3$ is a vertical asymptote of $f(x)$.

5. (T / F) Let $f(x) = x - 3 + \frac{9}{x+3}$. Then $y = x - 3$ is a slant asymptote of $f(x)$.

$\rightarrow x=3$ is a removable discontinuity.

Show all your work and justify your answer:

6. The graph of $f(x) = \frac{p(x)}{q(x)}$ is displayed below, where $\deg(p(x)) = 1$ and $\deg(q(x)) = 3$. All intercepts and asymptotes are at integer values. Find all intercepts, asymptotes, and a formula for $f(x)$.

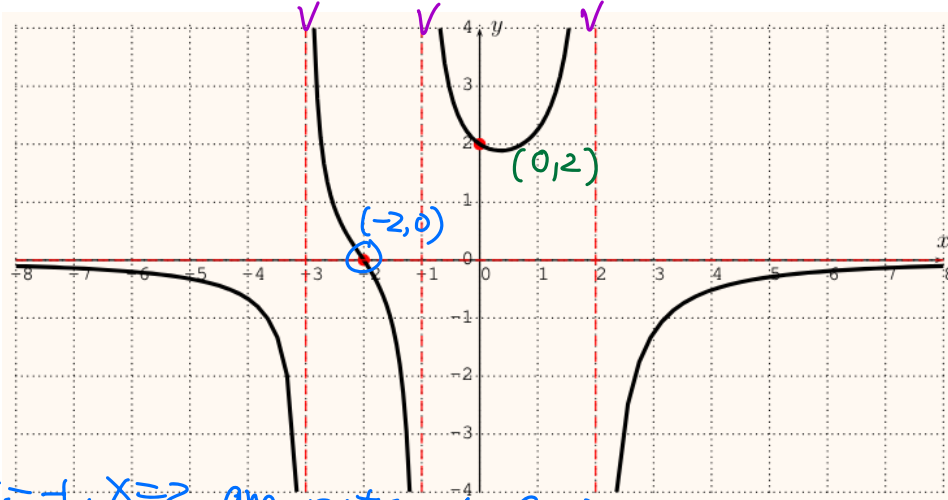
① X-intercept, $(-2, 0)$

② y-intercept, $(0, 2)$

③ H.A. $y=0$

④ V.A. $x=-3, x=-1, x=2$

⑤ $f(x) = \frac{p(x)}{q(x)}$



From ④, we know $x=-3, x=-1, x=2$ are roots of $q(x)$

$$\Rightarrow q(x) = (x+3)(x+1)(x-2)$$

From ①, $f(-2) = \frac{p(-2)}{q(-2)} = 0 \Rightarrow p(-2) = 0 \Rightarrow x = -2$ is a root of $p(x) \Rightarrow p(x) = (x+2)$

From ②, $f(0) = 2$, then let $f(x) = \frac{A(x+2)}{(x+3)(x+1)(x-2)}$ and when $x=0$,

$$\text{we have } 2 = f(0) = \frac{A(0+2)}{(0+3)(0+1)(0-2)} = \frac{2A}{(3)(1)(-2)} = \frac{2A}{-6}$$

$$\Rightarrow \frac{2A}{-6} = 2 \Rightarrow A = -6, \text{ Therefore, } f(x) = \frac{-6(x+2)}{(x+3)(x+1)(x-2)}$$