Quiz10, MAT1375 Professor Chiu

ID:_____   Name: _Sol_____

- This quiz consists of 3 sets of questions for a total of 10 points.
- You have 15 minutes to complete the quiz.
- Scientific calculators are allowed.
- Wishing you success.

True or False. Circle your answers (either T (true) or F (false)) on this sheet.

1. (T)/ F) The product rule for logarithms states that $\log_b(MN) = \log_b M + \log_b N$.

2. (T)/ F) -10 is a solution of $\log_5(x + 35) = 2$.

3. (T /(F)) -3 is a solution of $\log_5 9 = 2\log_5 x$.

4. ((T)/ F) Since the exponential function b^x is one to one, then $b^x = b^y \Leftrightarrow x = y$.

5. ((T)/ F) $\log_b(x) = \frac{\log_e(x)}{\log_e(b)} = \frac{\ln(x)}{\ln(b)}$

Show all your work and justify your answer:

6. Solve the exponential equation $7^{2x+1} = 3^{x+2}$.

power rule

① Take "ln" on the both sides : $\ln(7^{2x+1}) = \ln(3^{x+2})$

$(2x+1)\cdot \ln(7) = (x+2)\ln(3)$

② Isolate x: $2x\cdot \ln(7) + \ln(7) = x\cdot \ln(3) + 2\ln(3)$

$\Rightarrow 2x\ln(7) - x\ln(3) = 2\ln(3) - \ln(7)$

$\Rightarrow x(2\ln(7) - \ln(3)) = 2\ln(3) - \ln(7)$

$\Rightarrow x = \dfrac{2\ln(3) - \ln(7)}{2\ln(7) - \ln(3)}$

7. In 2021, the population of a city is 80,000 people, and is growing at a rate of 5% per year.

(a) What will the population be in 2025?

(b) In what year will the population be triple?

(a) $r = 0.05$. Let $P(t)$ be the population. We have

$P(t) = 80000 \cdot e^{0.05t}$ when $t=0$ in 2021

$\Rightarrow t = 2025 - 2021 = 4$ in 2025, the population is $P(4) = 80000\, e^{0.05\cdot 4}$

$= 80000 \cdot e^{0.2}$

(b) The triple of $80000 = 240000$

$\Rightarrow P(t) = 80000\, e^{0.05t} = 240000$

$\Rightarrow e^{0.05t} = 3$

Take "ln" $\ln(e^{0.05t}) = \ln(3)$

$0.05t \cdot \ln(e)^{=1} = \ln(3)$

$\Rightarrow 0.05t = \ln(3) \Rightarrow t = 20\cdot \ln(3)$