MAT 1375, Classwork21, Fall2024

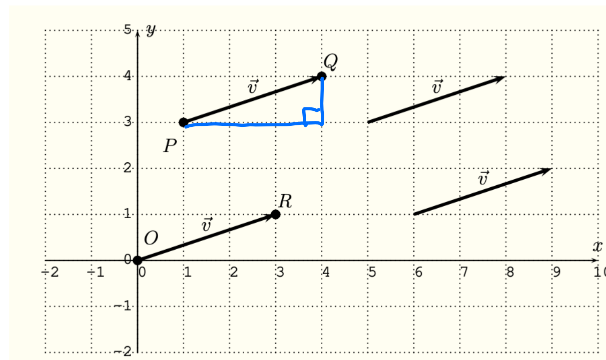
ID:______________________ Name:______________________

1. Definition of a geometric vector:

A geometric vector $\overrightarrow{PQ}$ is a directed line segment with a direction and a magnitude.

The magnitude of $\overrightarrow{PQ}$ is its length, denoted by $\|\overrightarrow{PQ}\|$.

2. How to find and present a vector:



Given a vector $\vec{v} = \overrightarrow{PQ}$. We call P the initial point and Q the terminal point.

We find $\vec{v} = \overrightarrow{PQ}$ by $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$\vec{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$ or $\langle x_2 - x_1, y_2 - y_1 \rangle$,

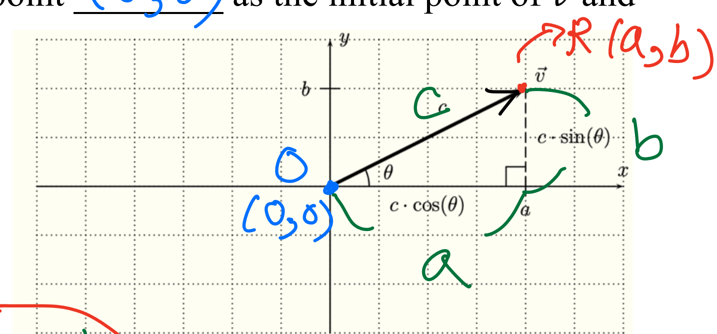
where $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$.

The magnitude of $\vec{v}$ is $\|\vec{v}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Any vectors with the same direction and magnitude are equivalent.

3. Direction angle: $\langle a - 0, b - 0 \rangle$

Let $\vec{v} = \langle a, b \rangle = \overrightarrow{OR}$ be a vector with original point $(0, 0)$ as the initial point of $\vec{v}$ and $R(a, b)$ as the terminal point of $\vec{v}$.



The direction angle of $\vec{v}$ is the angle θ determined by $\overrightarrow{OR}$:

$c = \|\vec{v}\|$ is the length of $\vec{v}$ and

we have $\sin(\theta) = \frac{b}{c}$, $\cos(\theta) = \frac{a}{c}$, and $\tan(\theta) = \frac{b}{a}$.

4. The vector $\vec{v}$ can be presented by its length c and direction angle θ:

$$\vec{v} = \langle a, b \rangle = \langle c \cdot \cos(\theta), c \sin(\theta) \rangle$$

$$\left(\cos(\theta) = \frac{a}{c}, \quad \sin(\theta) = \frac{b}{c} \right)$$

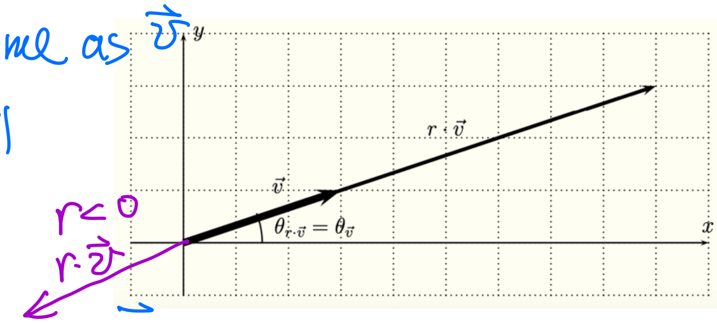
5. Operations on vectors: Let $\vec{v} = \langle a, b \rangle$ and $\vec{w} = \langle c, d \rangle$

Scalar multiplication: $r\vec{v} = r \cdot \langle a, b \rangle = \langle\ ra\ ,\ rb\ \rangle$

$r > 0$: the direction is the same as $\vec{v}$

the magnitude is $r \cdot \|\vec{v}\|$

$r < 0$: the direction is the opposite to $\vec{v}$



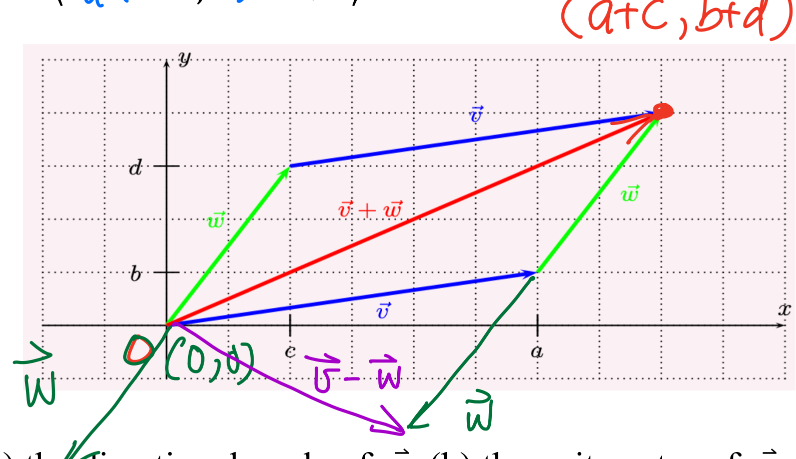
Unit vector of $\vec{v}$: $r\vec{v}$ where $r = \dfrac{1}{\|\vec{v}\|}$ and we have $\dfrac{\vec{v}}{\|\vec{v}\|}$.

Vector addition: $\vec{v} + \vec{w} = \langle a, b \rangle + \langle c, d \rangle = \langle\ a+c\ ,\ b+d\ \rangle$

$\langle a+c-0,\ b+d-0 \rangle$

$= \langle a+c,\ b+d \rangle$

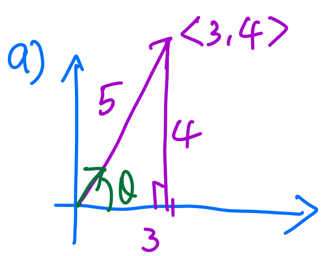
$\vec{i}$ $\vec{j}$



6. Let $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = 4\mathbf{i} - 9\mathbf{j}$. Find (a) the directional angle of $\vec{v}$, (b) the unit vector of $\vec{v}$, (c) $\vec{v} + \vec{w}$, (d) $2\vec{v} - 3\vec{w}$

$\vec{w} = \langle 4, -9 \rangle$

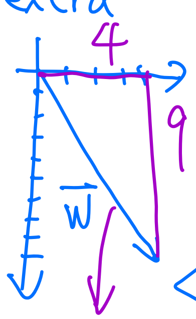
a)



$\tan(\theta) = \frac{4}{3}$

b) unit vector of $\vec{v} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{\langle 3,4 \rangle}{5} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

extra



unit vector of $\vec{w} = \langle \frac{4}{\sqrt{97}}, \frac{-9}{\sqrt{97}} \rangle$

$\sqrt{4^2+9^2} = \sqrt{16+81} = \sqrt{97}$

(c) $\vec{v} + \vec{w} = \langle 3,4 \rangle + \langle 4,-9 \rangle = \langle 3+4, 4-9 \rangle = \langle 7, -5 \rangle$

(d) $2\vec{v} - 3\vec{w} = 2\langle 3,4 \rangle - 3\langle 4,-9 \rangle = \langle 6, 8 \rangle + \langle -12, 27 \rangle$

$= \langle 6-12, 8+27 \rangle = \langle -6, 35 \rangle$