MAT 1375, Classwork22, Fall2024

ID:____________________ Name:____________________

1. The Imaginary Unit and the Complex Number: Real Number $\mathbb{R}$

We define the **Imaginary Unit** or **complex unit** to be

$$i = \underline{\sqrt{-1}} \quad (\text{since } i^2 = \underline{-1}).$$

A complex number is a number with the form

$$a + bi$$

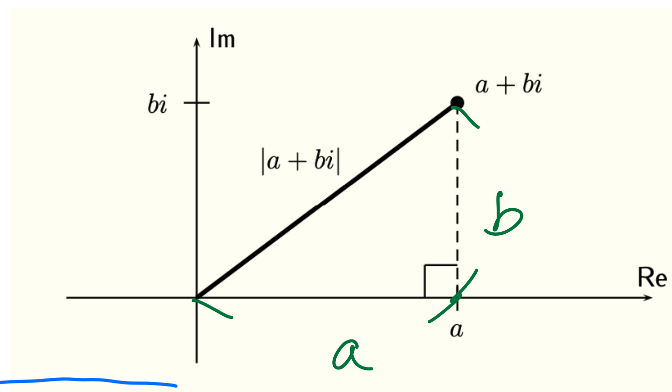
where a and b are any __real__ numbers, i is the __imaginary__ unit. The number a is called the __real__ part of $a + bi$, and b is called the __imaginary__ part of $a + bi$.

The set of all complex numbers is denoted by __$\mathbb{C}$__.

2. Complex Plane:

A complex number $z = a + bi$ can be represented as a point __(a, b)__ in a **Coordinate Plane** with the horizontal axis which is called __real__ axis and the vertical axis which is called __imaginary__ axis.

The absolute value or __the length__ of $z = a + bi$ is the __distance__ between z in the complex plane and the origin (0,0) and it denoted by



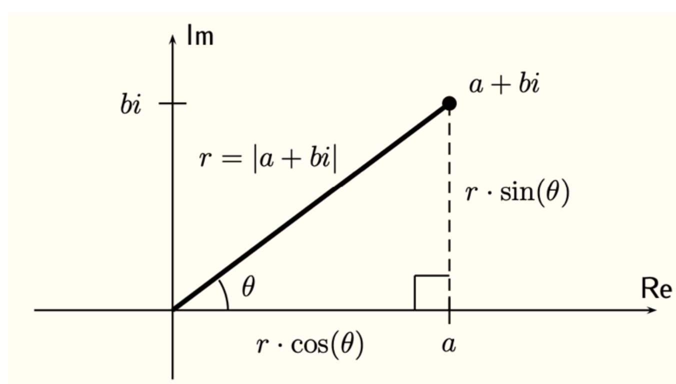
$$r = |z| = |a + bi| = \underline{\sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2+b^2}}.$$

3. Polar Form of a Complex number:

The complex number $z = a + bi$ is written in **Polar Form** as

$$z = a + bi = \underline{r\cos(\theta)} + i(\underline{r\sin(\theta)})$$

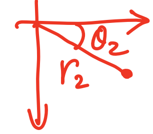
where $\tan(\theta) = \dfrac{b}{a}$ and $r = |z|$.



abs value, norm.

$\vec{v} = \langle a, b\rangle = \langle c\cdot\cos(\theta), c\sin(\theta)\rangle$

4. Product and Quotient in polar form:



Let $r_1(\cos(\theta_1) + i\sin(\theta_1))$ and $r_2(\cos(\theta_2) + i\sin(\theta_2))$ be two complex numbers in polar form. We have

θ : theta

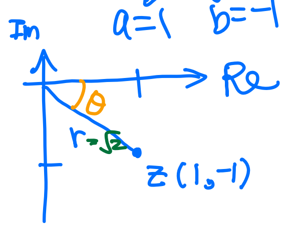
$$r_1(\cos(\theta_1) + i\sin(\theta_1)) \cdot r_2(\cos(\theta_2) + i\sin(\theta_2)) = \underline{r_1 r_2} \cdot (\underline{\cos(\theta_1+\theta_2)} + i\,\underline{\sin(\theta_1+\theta_2)}).$$

De Moivre's Theorem

$$\frac{r_1(\cos(\theta_1)+i\sin(\theta_1))}{r_2(\cos(\theta_2)+i\sin(\theta_2))} = \frac{r_1}{r_2} \cdot (\underline{\cos(\theta_1-\theta_2)} + i\,\underline{\sin(\theta_1-\theta_2)}).$$

5. Let $z = 1 - i$. Find the polar form of z.

$\sqrt{2}\,(\cos(\theta) + i\sin(\theta))$ where $\tan(\theta) = -1$



$a = 1 \quad b = -1. \quad z = a + bi$

$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$z = 1 - i = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$\left(\cos(\theta) = \frac{1}{\sqrt{2}},\ \sin(\theta) = -\frac{1}{\sqrt{2}} \Rightarrow \theta = -\frac{\pi}{4} \text{ or } \frac{7\pi}{4}\right)$

6. Let $z_1 = 2(\cos(210°) + i\sin(210°))$ and $z_2 = 4(\cos(90°) + i\sin(90°))$. Find

a) $z_1 \cdot z_2$ in $\boxed{\text{standard}}$ complex form. b) $\frac{z_1}{z_2}$ in standard complex form.

a) $z_1 \cdot z_2 = 2(\cos(210°) + i\sin(210°)) \cdot 4(\cos(90°) + i\sin(90°))$

$= 2 \cdot 4(\cos(210° + 90°) + i\sin(210° + 90°))$

$= 8(\cos(300°) + i\sin(300°))$

$= 8\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = 4 - 4\sqrt{3}\,i$

b) $\frac{z_1}{z_2} = \frac{2(\cos(210°) + i\sin(210°))}{4(\cos(90°) + i\sin(90°))} = \frac{2}{4}(\cos(210° - 90°) + i\sin(210° - 90°))$

$= \frac{1}{2}(\cos(120°) + i\sin(120°))$

$= \frac{1}{2}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= -\frac{1}{4} + i\frac{\sqrt{3}}{4}$