MAT1375, Classwork9, Fall2025

Ch9. Roots of Polynomials

1. Factors and roots of polynomials.

degree "n" polynomial

Every n-degree polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x^1 + a_0$, $(a_n \neq 0)$ can be factored as

$$f(x) = a_n(x-c_1)\cdot(x-c_2)\cdot(x-c_3)\cdot\cdots\cdot(x-c_n)$$

Thus, the polynomial $f(x)$ of degree n has **at most** ___n___ roots (which are $c_1, c_2, \cdots, c_n$) and these roots may be either ___real___ or ___complex___. $(3+2i,\ \sqrt{2}+5i)$

2. The Repeat roots and its **Multiplicity.**

Let $f(x) = (x-r)^n$ where r is the ___root___ of f and this root repeats ___n___ times. We call r a root with ___multiplicity___ n.

3. The complex root and its Conjugate.

Let f be a polynomial with all **real coefficients**. The complex roots are always found as a **pair**, that is, if $c = a + bi$ is a complex root of f, then the complex ___conjugate___ $\bar{c} =$ ___$a-bi$___ is also a root of f. real part imagary part

$c = 3+2i$, $\bar{c} = 3-2i$

$c = 4-i$, $\bar{c} = 4+i$

4. The Relation between Roots and Coefficient a_0.

For a n-degree polynomial $f(x) = 1x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x^1 + a_0$, we can factorize it as

$$f(x) = 1\cdot(x-c_1)\cdot(x-c_2)\cdot(x-c_3)\cdot\cdots\cdot(x-c_n).$$

where $c_1, c_2, \cdots, c_n$ are the roots of $f(x)$. Then we have

(1) $f(c_1) =$ ___0___, $f(c_2) =$ ___0___, $f(c_3) =$ ___0___, … , $f(c_n) =$ ___0___.

(2) $a_0 = (-c_1)\times(-c_2)\times(-c_3)\times\cdots\times(-c_n)$.

(3) The ______ of a_0 might be the possible candidates for ________ of $f(x)$.

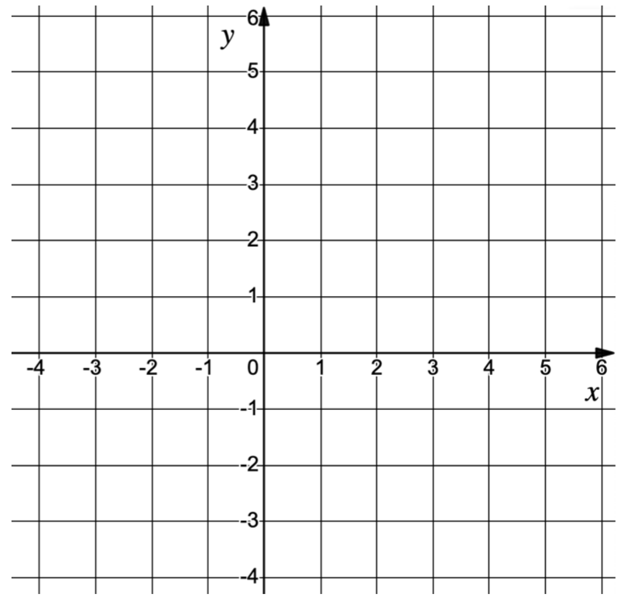
5. Let $f(x) = x^3 - x^2 + 2$. Find all the roots of $f(x)$.

6. The **Number Line Test**:

Step1. Solve the ________________ and find all the _____________.

Step2. Mark the roots on the number line and check ______________________________in each subinterval.

7. Let $f(x) = x^3 - 3x^2 + 4$. Find all the roots of $f(x)$. Sketch a complete graph and label all roots.



8. Let $f(x) = x^3 - x^2 - 9x + C$ where C is a real number. If $x = 3$ is a root of $f(x)$, find C so that $f(x)$ has this root as indicated. Then, for this choice of C, find all remaining roots of $f(x)$.

9. Find a polynomial $f(x)$ that fits the given data.

$f(x)$ has degree 4. $f(x)$ has roots $0, 2, -1, -4$, and $f(1) = 20$.