MAT1375, Classwork5, Fall2025

Ch5. Operations on Functions

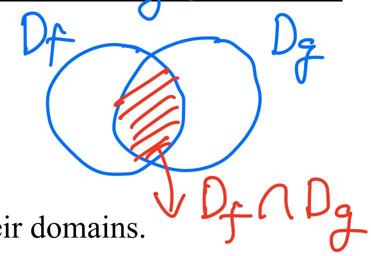
1. Complete the definition of **the Algebra of Functions**: $(f-g)(x)$

Let $f(x)$ and $g(x)$ be two functions with the domain D_f and D_g ,respectively. We have sum, difference, product, and quotient of functions:

intersection

The Algebra of functions	Notation	Definition	Domain
Sum	$(f+g)(x) :=$	$f(x)+g(x)$	$D_{f+g} = D_f \cap D_g$
Difference	$\boxed{(f-g)(x)} :=$	$f(x)-g(x)$	$D_{f-g} = D_f \cap D_g$
Product	$(f \cdot g)(x) :=$	$f(x)\cdot g(x)$	$D_{f \cdot g} = D_f \cap D_g$
Quotient	$\left(\frac{f}{g}\right)(x) :=$	$\frac{f(x)}{g(x)}$, provided $g(x) \neq 0$	$D_{\frac{f}{g}} = D_f \cap D_g$ but $g(x) \neq 0$

Here, $D_f \cap D_g = \{\, x \mid$ x is from D_f and x is from D_g $\}$



2. Let $f(x) = x^2 + 5x + 6$ and $g(x) = x + 2$. Find the following functions and state their domains.

$(f+g)(x) =$ $f(x)+g(x) = x^2+5x+6 + x+2$
$= x^2+6x+8$

$D_f = \mathbb{R}$ $(-\infty,\infty)$, $D_g = \mathbb{R}$ $(-\infty,\infty)$

$D_{f+g} = D_f \cap D_g = \mathbb{R}$ $(-\infty,\infty)$

$(f-g)(x) =$ $f(x)-g(x) = x^2+5x+6-(x+2)$
$= x^2+5x+6-x-2$
$= x^2+4x+4$

$D_f = \mathbb{R}$ $D_g = \mathbb{R}$

$D_{f-g} = D_f \cap D_g = \mathbb{R}$

$(f \cdot g)(x) =$ $f(x)\cdot g(x) = (x^2+5x+6)\cdot(x+2) = x^3+7x^2+16x+12$

$$\begin{array}{c|ccc} & x^2 & +5x & +6 \\ \hline x & x^3 & +5x^2 & +6x \\ +2 & 2x^2 & +10x & +12 \end{array}$$

$D_f = \mathbb{R}$. $D_g = \mathbb{R}$

$D_{fg} = D_f \cap D_g = \mathbb{R}$

$\left(\frac{f}{g}\right)(x) =$

$\frac{f(x)}{g(x)} = \frac{x^2+5x+6}{x+2} = \frac{(x+3)(x+2)}{(x+2)}$

$= (x+3)$ assuming $x+2 \neq 0$
$\Rightarrow x \neq -2$

$D_f = \mathbb{R}$, $D_g = \mathbb{R}$

$D_f \cap D_g = \mathbb{R}$ and $g(x) \neq 0$

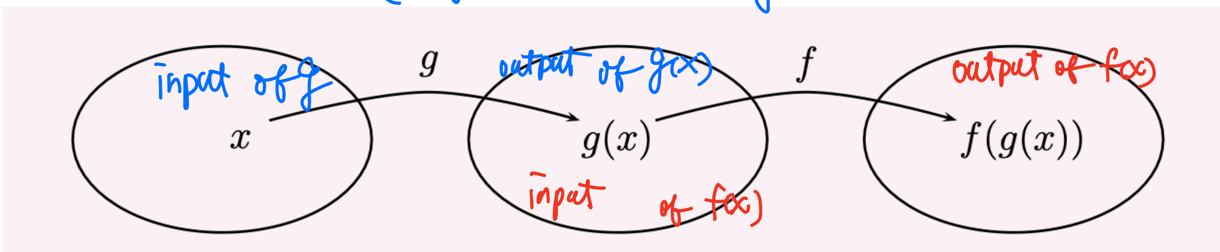
$\Rightarrow D_{\frac{f}{g}} = \{\, x \mid x \in \mathbb{R},\ x \neq -2 \}$

$x \in (-\infty,-2) \cup (-2,\infty)$

3. Complete the definition of **the Composition of Functions**:

Let $f(x)$ and $g(x)$ be two functions. The composition of the function f with g is denoted by $(f \circ g)(x)$ and is defined by the equation

$$(f \circ g)(x) := f(g(x)).$$



The domain of the composition of the function $f \circ g$ is the set of all x such that x is the input of $g(x)$ and output of g(x) is the domain of $f(x)$.

The notation of the domain of the composition of the function $f \circ g$ is

$D_{f \circ g} = \{ x \mid$ x is from D_g and output of g(x) is from $D_f \}$

NO 4. Are $f(g(x))$ and $g(f(x))$ the same functions?

$f(x)$: 30% off $\Rightarrow f(x) = 0.7 \cdot x$

$g(x)$: 500 Gift Card $\Rightarrow g(x) = x - 500$

\$3000 laptop

pay

$f(g(3000)) = f(2500) = 2500 \cdot 0.7 = 1750$

$g(f(3000)) = g(3000 \cdot 0.7) = g(2100) = 2100 - 500 = 1600$

5. Find $(f \circ g)(x)$ for the following functions and state their domains.

a) $f(x) = x^2 + 2$ and $g(x) = x - 3$

$(f \circ g)(x) = f(g(x)) = f(x-3) = (x-3)^2 + 2$

$= x^2 - 6x + 9 + 2 = x^2 - 6x + 11$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

$D_{f \circ g} = (-\infty, \infty)$ or $\mathbb{R}$ or all real number

b) $f(x) = \frac{2}{x-3}$ and $g(x) = x^2 + 2x$

$(f \circ g)(x) = f(g(x)) = f(x^2 + 2x) = \frac{2}{(x^2+2x) - 3}$

$= \frac{2}{x^2 + 2x - 3}$ denominator $\neq 0$

$x^2 + 2x - 3 = (x+3)(x-1) = 0$

$\Rightarrow x + 3 = 0$ or $x - 1 = 0$

$\Rightarrow x = -3$ or $x = 1$

$D_{f \circ g} = \{x \mid x$ is from $\mathbb{R}$ but $x \neq -3$ and $x \neq 1\}$

$= \{x \mid x \in \mathbb{R}$ but $x \neq -3$ and $x \neq 1\}$

$x \in (-\infty, -3) \cup (-3, 1) \cup (1, \infty)$