Mat 1375 HW 17

Exercise 17.1

Convert from radian to degree.

✓a) $\frac{\pi}{4}$ ✓b) $\frac{2\pi}{3}$ ✓c) $\frac{5\pi}{6}$ ✓d) $\frac{7\pi}{4}$

Key : π in radian $\longleftrightarrow 180^\circ$

a) $\frac{\pi}{4} \cdot \frac{180^\circ}{\pi} = 45^\circ$ b) $\frac{2\pi}{3} \frac{180^\circ}{\pi} = 120^\circ$

c) $\frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} = 150^\circ$ d) $\frac{7\pi}{4} \cdot \frac{180^\circ}{\pi} = 315^\circ$

Exercise 17.2

Convert from degree to radian.

Key : π in radian $\longleftrightarrow 180^\circ$

✓a) 120° ✓b) 60° ✓c) 300° ✓d) 135°

a) $120^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{3}$ b) $60^\circ \frac{\pi}{180^\circ} = \frac{\pi}{3}$ c) $300^\circ \frac{\pi}{180^\circ} = \frac{5\pi}{3}$

d) $135^\circ \cdot \frac{\pi}{180^\circ} = \frac{3\pi}{4}$

Exercise 17.3

Find $\sin(x)$, $\cos(x)$, and $\tan(x)$ for the following angles.

✓a) $x = 150^\circ$ ✓b) $x = 45^\circ$ ✓c) $x = 210^\circ$ ✓d) $x = 60^\circ$

a) $150^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{6}$. $\sin(150^\circ) = \frac{1}{2}$, $\cos(150^\circ) = -\frac{\sqrt{3}}{2}$, $\tan(150^\circ) = -\frac{\sqrt{3}}{3}$

b) $\sin(45^\circ) = \frac{\sqrt{2}}{2}$, $\cos(45^\circ) = \frac{\sqrt{2}}{2}$, $\tan(45^\circ) = 1$

c) $210^\circ \cdot \frac{\pi}{180^\circ} = \frac{7\pi}{6}$, $\sin(210^\circ) = -\frac{1}{2}$, $\cos(210^\circ) = -\frac{\sqrt{3}}{2}$, $\tan(210^\circ) = \frac{\sqrt{3}}{3}$

d) $\sin(60^\circ) = \frac{\sqrt{3}}{2}$, $\cos(60^\circ) = \frac{1}{2}$, $\tan(60^\circ) = \sqrt{3}$

✓e) $x = 30^\circ$ ✓f) $x = 300^\circ$ ✓g) $x = 90^\circ$ ✓h) $x = 315^\circ$

e) $\sin(30^\circ) = \frac{1}{2}$, $\cos(30^\circ) = \frac{\sqrt{3}}{2}$, $\tan(30^\circ) = \frac{\sqrt{3}}{3}$

f) $300^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{3}$, $\sin(300^\circ) = -\frac{\sqrt{3}}{2}$, $\cos(300^\circ) = \frac{1}{2}$, $\tan(300^\circ) = -\sqrt{3}$

g) $\sin(90°) = 1$, $\cos(90°) = 0$, $\tan(90°) =$ undefined

h) $315° \cdot \frac{\pi}{180°} = \frac{7\pi}{4}$, $\sin(315°) = -\frac{\sqrt{2}}{2}$, $\cos(315°) = \frac{\sqrt{2}}{2}$, $\tan(315°) = -1$

q) $x = \frac{5\pi}{3}$ r) $x = \frac{\pi}{6}$ s) $x = \frac{4\pi}{3}$ t) $x = \frac{5\pi}{6}$

q) $\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$, $\tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$

r) $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$

s) $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $\cos\left(\frac{4}{3}\pi\right) = -\frac{1}{2}$, $\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$

Exercise 17.4

Find the trigonometric function values by using the addition and subtraction formulas.

a) $\sin(75°)$ b) $\cos(15°)$ c) $\tan(105°)$ d) $\sin(195°)$

a) $\sin(75°) = \sin(30° + 45°)$

$= \sin(30°)\cos(45°) + \cos(30°)\sin(45°)$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

b) $\cos(15°) = \cos(60° - 45°)$

$= \cos(60°)\cos(45°) + \sin(60°)\cdot\sin(45°)$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

c) $\tan(105°) = \frac{\sin(105°)}{\cos(105°)} = \frac{\sin(45° + 60°)}{\cos(45° + 60°)}$

$= \frac{\sin(45°)\cos(60°) + \cos(45°)\sin(60°)}{\cos(45°)\cos(60°) - \sin(45°)\sin(60°)}$

$$= \frac{\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}} = \frac{\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}} = \frac{\frac{\sqrt{2}+\sqrt{6}}{4}}{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}} = \frac{(\sqrt{2}+\sqrt{6})(\sqrt{2}+\sqrt{6})}{(\sqrt{2}-\sqrt{6})(\sqrt{2}+\sqrt{6})} = \frac{2+6+2\sqrt{12}}{2-6} = \frac{8+4\sqrt{3}}{-4}$$

$$= -2-\sqrt{3}.$$

Exercise 17.5

Find the exact trigonometric function values by using the half-angle formulas.

a) $\cos(22.5°)$ b) $\sin(15°)$ c) $\cos(15°)$ d) $\tan(15°)$

a) $$\cos(22.5°) = \sqrt{\frac{1+\cos(2\cdot 22.5°)}{2}} = \sqrt{\frac{1+\cos(45°)}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

b) $$\sin(15°) = \sqrt{\frac{1-\cos(2\cdot 15°)}{2}} = \sqrt{\frac{1-\cos(30°)}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$$

d) $$\tan(15°) = \frac{\sin(15°)}{\cos(15°)} = \frac{\sqrt{\frac{1-\cos(30°)}{2}}}{\sqrt{\frac{1+\cos(30°)}{2}}} = \sqrt{\frac{1-\cos(30°)}{1+\cos(30°)}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}} = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$$