MAT1372, Classwork9, Fall2025

3.3 Sampling from a small population & Counting Techniques(QR Code3)

1. Sampling with/without replacement

Sampling with replacement: selected subjects are put back into the population before another subject are sampled.

Sampling without replacement: Selected subjects will not be in the "pool" for selection

2. Professors sometimes select a student at random to answer a question and each student has an equal chance of being selected and there are 10 people in your class. If the professor asks 3 questions and assume that one **will not** be picked twice in a given lecture, then

(a) what is the probability that you will not be selected?

$P(Q_1 = \text{Not picked}, Q_2 = \text{not picked}, Q_3 = \text{not picked}) = \frac{9}{10}\cdot\frac{8}{9}\cdot\frac{7}{8} = \frac{7}{10}$

$= P(Q_1 = \text{not picked}) \cdot P(Q_2 = \text{not picked given } Q_1 = \text{not picked}) \cdot P(Q_3 = \text{not picked} \mid Q_1, Q_2 = \text{not picked})$

(b) what is the probability that you will be selected for the first question? $P(Q_1 = \text{picked}) = \frac{1}{10}$

(c) what is the probability that you will be selected for the second question?

$P(Q_1 = \text{not picked and } Q_2 = \text{picked}) = \frac{9}{10}\cdot\frac{1}{9} = \frac{1}{10}$

not independent

fair chance to get picked

(d) what is the probability that you will be selected for the third question?

$P(Q_1 = \text{not picked}, Q_2 = \text{not picked}, Q_3 = \text{picked}) = \frac{9}{10}\cdot\frac{8}{9}\cdot\frac{1}{8} = \frac{1}{10}$

(e) what is the probability that you will be selected for one question?

(b)+(c)+(d) $= \frac{3}{10}$ or $1-(a) = 1-\frac{7}{10} = \frac{3}{10} = 30\%$

3. In 2., If the professor asks 3 questions and assume that one **could be** picked more than twice in a given lecture, then

(a) what is the probability that you will not be selected?

$P(Q_1 = \text{not picked}, Q_2 = \text{not picked}, Q_3 = \text{not picked}) = \frac{9}{10}\cdot\frac{9}{10}\cdot\frac{9}{10} = \frac{729}{1000}$

(b) what is the probability that you will be selected for the first question? $P(Q_1 = \text{picked}) = \frac{1}{10}$.

(c) what is the probability that you will be selected for the second question?

$P(Q_1 = \text{not picked}, Q_2 = \text{picked}) = \frac{9}{10}\cdot\frac{1}{10} = \frac{9}{100}$

(d) what is the probability that you will be selected for the third question?

$P(Q_1 = \text{not picked}, Q_2 = \text{not picked}, Q_3 = \text{picked}) = \frac{9}{10}\cdot\frac{9}{10}\cdot\frac{1}{10} = \frac{81}{1000}$

(e) what is the probability that you will be selected for at least one question?

(b)+(c)+(d) or $1-(a) = \frac{271}{1000} = 27.1\%$

4. In 2., assume there are **100 students** and the professor asks 3 questions.

(a) If one **will not** be picked twice then what is the probability that you will be selected for one question?

$1-P(Q_1=\text{not picked}, Q_2=\text{not picked}, Q_3=\text{not picked})=1-\frac{99}{100}\cdot\frac{98}{99}\cdot\frac{97}{98}=1-\frac{97}{100}=\frac{3}{100}=3\%$

(b) If one **could** be picked twice then what is the probability that you will be selected for at least one question?

$1-P(Q_1=\text{not picked}, Q_2=\text{not picked}, Q_3=\text{not picked})=1-\frac{99}{100}\cdot\frac{99}{100}\cdot\frac{99}{100}=\frac{2.9701}{100}=2.97\%$

5. What can you observe the difference of the results once the sample space getting larger?

In 2.3 30% (without replacement) → 3%

27.1% (with replacement) → 2.97%

10 → 100 students = sample size

6. How many three letter "words" can be made from the letters a, b, and c with **no letters repeating**? A "word" is just an ordered group of letters. It doesn't have to be a real word in a dictionary.

abc bac cab
acb bca cba

6 combination

$\underset{3\text{ options}}{\downarrow}\ \underset{2\text{ options}}{\downarrow}\ \underset{1\text{ option}}{\downarrow} = 6$ combination

7. Factorial $n!$: $n! = n\cdot(n-1)\cdot(n-2)\cdot(n-3)\cdot\cdots\cdot 2\cdot 1$

8. How many three letter "words" can be made from the letters a, b,…, z with **no letters repeating**?

$\underset{26\text{ options}}{\downarrow}\ \underset{25\text{ options}}{\downarrow}\ \underset{24\text{ options}}{\downarrow} = 26\cdot 25\cdot 24$

${}_{26}P_3=\frac{26!}{(26-3)!}=\frac{26!}{23!}=\frac{26\times25\times24\times23\times\cdots\times2\times1}{23\times22\times21\times\cdots\times2\times1}=26\times25\times24$

9. Permutation Formula ${}_nP_r$: ${}_nP_r=\frac{n!}{(n-r)!}$ (picked r objects from n objects when order matters)

10. How many ways you choose three letters from the letters a, b,…, z with **no letters repeating**?

$\underset{26\text{ options}}{\downarrow}\ \underset{25\text{ options}}{\downarrow}\ \underset{24\text{ options}}{\downarrow} = 26\cdot25\cdot24$

$=\frac{26\cdot25\cdot24}{6}=\frac{26\cdot25\cdot24}{3!}$

${}_{26}C_3=\frac{26!}{3!\,(26-3)!}$

11. Combination Formula ${}_nC_r$: ${}_nC_r=\frac{n!}{r!\,(n-r)!}$ (picked r objects from n objects when order doesn't matter)

12. How many three letter "words" can be made from the letters a, b, and b?

13. How many six letter "words" can be made from the letters a, b, b, c, c, and c?