Mat 1372 HW4

3.3 Four games, one winner. Below are four versions of the same game. Your archnemesis gets to pick the version of the game, and then you get to choose how many times to flip a coin: 10 times or 100 times. Identify how many coin flips you should choose for each version of the game. It costs \$1 to play each game. Explain your reasoning.

(a) If the proportion of heads is larger than 0.60, you win \$1.
(b) If the proportion of heads is larger than 0.40, you win \$1.
(c) If the proportion of heads is between 0.40 and 0.60, you win \$1.
(d) If the proportion of heads is smaller than 0.30, you win \$1.

(a) 10 times. Since the desired outcome ($> 60\%$) is larger than the expected proportion of head (50%), we would want few trails. But fewer trails might lead to lower outcome ($< 50\%$) as well.

(b) 100 times, as long as you flip a coin for a larger number, the result of proportion of heads will close to 50% which is larger than 40%

(c) 100 times, just like (b), you will get the result which closes to 50% since what you want is between 40% and 60%

(d) 10 times. Although you might get a result which is larger than 50%, but you might also get a result which is smaller 50%, even 30%.

3.5 Coin flips. If you flip a fair coin 10 times, what is the probability of

(a) getting all tails?
(b) getting all heads?
(c) getting at least one tails?

Since it is a fair coin, then
$P(\text{tail}) = \frac{1}{2}$ and $P(\text{head}) = \frac{1}{2}$

(a) Since each flipping is independent with another flipping, then

$$P(\text{all tails 10 times}) = \underbrace{P(\text{tail}) \times P(\text{tail}) \times \cdots \times P(\text{tail})}_{\text{10 times}}$$

$$= \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$$

(b) Like (a), similarly, $P(\text{all heads 10 times}) = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

(c) To find P (at least 1 tails), it is easier to think about it by applying complement of "at least 1 tails" which is "no tails at all" or "all heads" Then

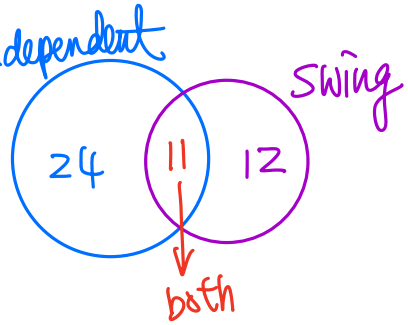
$P(\text{at least 1 tails}) = 1 - P(\text{all head}) = 1 - \frac{1}{1024} = \frac{1023}{1024}$

3.7 Swing voters. A Pew Research survey asked 2,373 randomly sampled registered voters their political affiliation (Republican, Democrat, or Independent) and whether or not they identify as swing voters. 35% of respondents identified as Independent, 23% identified as swing voters, and 11% identified as both.[21]

(a) Are being Independent and being a swing voter disjoint, i.e. mutually exclusive?
(b) Draw a Venn diagram summarizing the variables and their associated probabilities.
(c) What percent of voters are Independent but not swing voters?
(d) What percent of voters are Independent or swing voters?
(e) What percent of voters are neither Independent nor swing voters?
(f) Is the event that someone is a swing voter independent of the event that someone is a political Independent?

(a) NO, since 11% identified as both and it means they can be both independent and swing.

(b)



(c) 24% (=35% − 11%)

(d) $P(\text{Independent or swing}) = P(\text{Independent}) + P(\text{swing}) - P(\text{both})$
$= 35\% + 23\% - 11\% = 47\%$

(e) $P(\text{neither Independent nor swing}) = 1 - P(\text{Independent or swing})$
$= 1 - 47\% = 53\%$

(f) $P(\text{Independent}) \times P(\text{swing}) = 35\% \cdot 23\% \doteq 8\%$ which does not equal $P(\text{Independent AND swing}) = P(\text{both}) = 11\%$, so the events are

dependent.

3.8 Poverty and language. The American Community Survey is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services. The 2010 American Community Survey estimates that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English (foreign language) at home, and 4.2% fall into both categories.[22]

(a) Are living below the poverty line and speaking a foreign language at home disjoint?

(b) Draw a Venn diagram summarizing the variables and their associated probabilities.

(c) What percent of Americans live below the poverty line and only speak English at home?

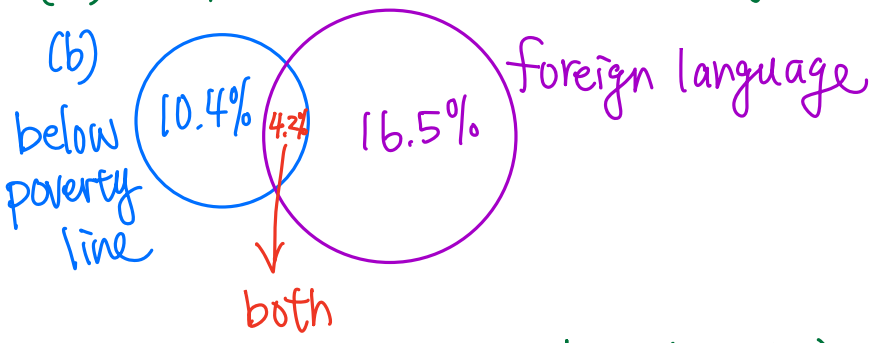
(d) What percent of Americans live below the poverty line or speak a foreign language at home?

(e) What percent of Americans live above the poverty line and only speak English at home?

(f) Is the event that someone lives below the poverty line independent of the event that the person speaks a foreign language at home?

(a) NO, 4.2% fall into both categories and it means it is NOT disjoint.

(b)



(c) P(below poverty and speak English)
$= P(\text{below poverty}) - P(\text{both}) = 14.6\% - 4.2\% = 10.4\%$

(d) P(below poverty or speak foreign language)
$= P(\text{below poverty}) + P(\text{speak foreign language}) - P(\text{both})$
$= 14.6\% + 20.7\% - 4.2\% = 31.1\%$

(e) P(Above poverty and speak English)
$= 1 - P(\text{below poverty or speak foreign language}) = 1 - 31.1\% = 68.9\%$

(f) Using the multiplication rule:
$P(\text{below PL}) \times P(\text{speak FL}) = 14.6\% \cdot 20.7\% = 3\%$ which does not equal $P(\text{below PL AND speak FL}) = 4.2\%$, therefore the events are dependent.

3.10 Guessing on an exam. In a multiple choice exam, there are 5 questions and 4 choices for each question (a, b, c, d). Nancy has not studied for the exam at all and decides to randomly guess the answers. What is the probability that:

(a) the first question she gets right is the 5^{th} question?

(b) she gets all of the questions right?

(c) she gets at least one question right?

(a) Since there are four options and one of them is correct.
then $P(\text{correct}) = \frac{1}{4}$ and $p(\text{incorrect}) = \frac{3}{4}$.

Thus $P(\text{Incorrect, Incorrect, Incorrect, Incorrect, correct})$

$= \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{3^4}{4^5} \approx 0.0791.$

(b) $P(\text{correct, correct, correct, correct, correct})$

$= \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$

(c) $P(\text{at least one question right}) = 1 - P(\text{All incorrect})$

$= 1 - \left(\frac{3}{4}\right)^5 = 1 - 0.2373 = 0.7627$