MAT2440, Classwork24, Spring2025

ID:______________________ Name:______________________

1. The special Sequence with explicit formula: **Arithmetic Sequences**

An arithmetic sequence $\{a_n\}$ is a sequence of the form $\boldsymbol{a_n = a_1 + (n-1)d}$:

$$a_1 = a_1,\ a_2 = a_1 + d,\ a_3 = a_1 + 2d,\ \cdots,\ a_k = a_1 + (k-1)d,\ \cdots,$$

where the initial (first) term a_1 and the common difference d are real numbers.

2. List the first five terms $a_1, a_2, \cdots, a_5$ of the arithmetic sequence $\{a_n\}$ and find the common difference d of the sequence. (a) $a_n = 3 + (n-1)(-4)$. (b) $a_n = -1 + 4n$.

(a)
$a_1 = 3 + (1-1)\cdot(-4) = 3$) -4
$a_2 = 3 + (2-1)\cdot(-4) = -1$) -4
$a_3 = 3 + (3-1)\cdot(-4) = -5$) -4
$a_4 = 3 + (4-1)\cdot(-4) = -9$) -4
$a_5 = 3 + (5-1)(-4) = -13$

$d = -4$

(b)
$a_1 = -1 + 4\cdot 1 = 3$) $+4$
$a_2 = -1 + 4\cdot 2 = 7$) $+4$
$a_3 = -1 + 4\cdot 3 = 11$) $+4$
$a_4 = -1 + 4\cdot 4 = 15$) $+4$
$a_5 = -1 + 4\cdot 5 = 19$

$d = 4$

3. . The special Sequence with explicit formula: **Geometric Sequences**

An geometric sequence $\{a_n\}$ is a sequence of the form $a_n = a_1 r^{n-1}$:

$$a_1 = a_1,\ a_2 = a_1 r,\ a_3 = a_1 r^2,\ \cdots,\ a_k = a_1 r^{k-1},\ \cdots,$$

where the initial term a_1 and the common ratio. r are real numbers.

4. List the first five terms $a_1, a_2, \cdots, a_5$ of the geometric sequence $\{a_n\}$ and find the common ratio r of the sequence. (a) $a_n = (-1)^n$. (b) $a_n = \left(-\frac{1}{2}\right)^{n-1}$.

(a)
$a_1 = (-1)^1 = -1$
$a_2 = (-1)^2 = 1$
$a_3 = (-1)^3 = -1$
$a_4 = (-1)^4 = 1$
$a_5 = (-1)^5 = -1$

$r = \frac{a_2}{a_1} = \frac{1}{-1} = -1$

(b)
$a_1 = \left(-\frac{1}{2}\right)^{1-1} = \left(-\frac{1}{2}\right)^0 = 1$
$a_2 = \left(-\frac{1}{2}\right)^{2-1} = -\frac{1}{2}$
$a_3 = \left(-\frac{1}{2}\right)^{3-1} = \frac{1}{4}$
$a_4 = \left(-\frac{1}{2}\right)^{4-1} = -\frac{1}{8}$
$a_5 = \left(-\frac{1}{2}\right)^{5-1} = \frac{1}{16}$

$r = \frac{a_2}{a_1} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$

5. Define a Sequence by **Recursive Relations**:

Another popular method to define a sequence is to provide one or more ___________ terms together with a ___________ rule for determining subsequent terms from those that precede them.

6. Let $\{a_n\}$ be a sequence that satisfies the **initial term** $a_0 = 2$ and the **recurrence relation**

$$a_n = a_{n-1} + 3 \text{ for } n = 1, 2, 3, \cdots.$$

What are a_1, a_2, and a_3?

7. Let $\{a_n\}$ be a sequence that satisfies the **initial term** $a_0 = 3$ and the **recurrence relation**

$$a_n = \frac{1}{3}a_{n-1} \text{ for } n = 1, 2, 3, \cdots.$$

What are a_1, a_2, and a_3?

8. (*Fibonacci sequence*) Let $\{f_n\}$ be a sequence that satisfies the **initial term** $f_0 = 1$, $f_1 = 1$, and **recurrence relation**

$$f_n = f_{n-1} + f_{n-2} \text{ for } n = 2, 3, 4, \cdots.$$

What are the first five terms?

Explicit formula (also called a closed formula) of *Fibonacci sequence*:

$$f_n =$$