MAT1375, Classwork8, Fall2025

Ch8. Graphing Polynomials

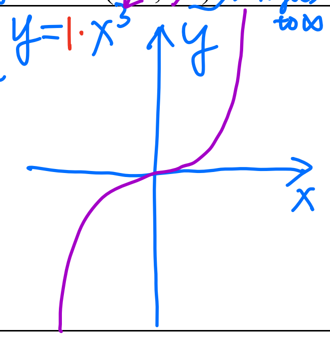
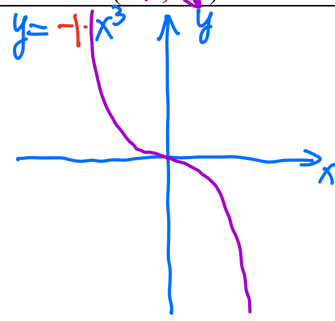
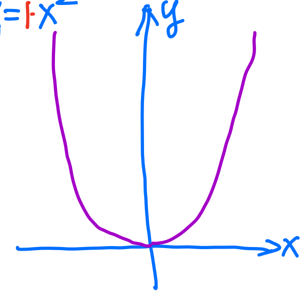
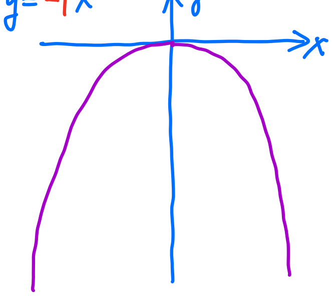
1. The **End Behavior of the polynomials** and the **Leading Coefficient Test**:

As x goes to ∞ or $-\infty$, the graph of polynomial function

$2x^4+2x+1$

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0, \qquad (a_n \neq 0)$$

either **rises** or **falls** eventually. Here, we can conclude this into the following table

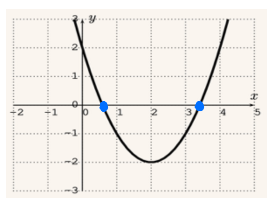
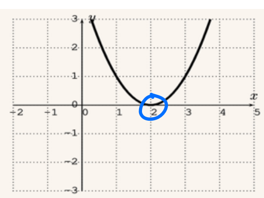
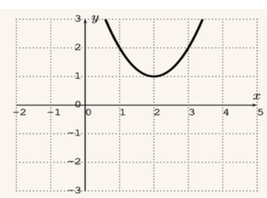
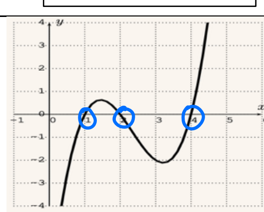
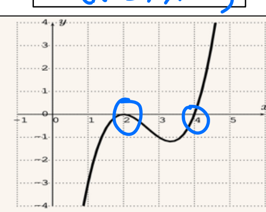
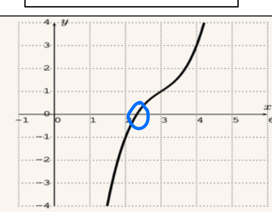
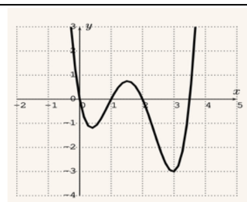
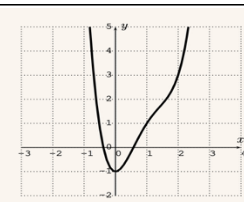
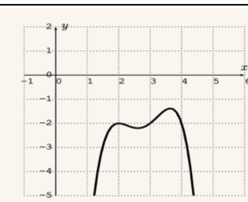
degree is odd n is an **odd** number		n is an **even** number	
leading coefficient is positive $a_n > 0$	$a_n < 0$	$a_n > 0$	$a_n < 0$
x goes to $-\infty$ (↙ , ↗) x goes to ∞	(↖ , ↘)	(↖ , ↗)	(↙ , ↘)
Example $y=1\cdot x^3$ 	$y=-1\cdot x^3$ 	$y=1\cdot x^2$ 	$y=-1x^2$ 

2. Roots of a Function and x-intercepts.

A <u>root</u>, or <u>zero</u>, or <u>solution</u> of a polynomial $f(x)$ is **a number c** so that $f(c)$ = <u>0</u>.

Each **real** root/zero/solution of the polynomial $f(x)$ appears as an <u>x-intercepts</u> of the graph of $f(x)$.

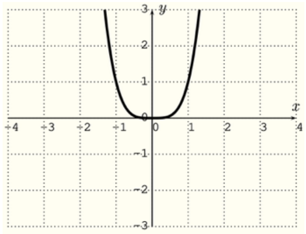
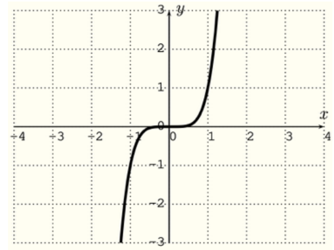
(Here '**real**' means not a complex number)

Degree	Number of the roots	Graph of the polynomial of that degree		
2	At most 2 (real) roots	 2 (real) roots	 1 (real) roots (x=2, x=2)	 No (real) roots
3	At most 3 (real) roots	 3 (real) roots (x=1, x=2, x=4)	 2 (real) roots (x=2, x=2, x=4)	 1 (real) root (x=2.3)
4	At most 4 (real) roots	 4 (real) roots (x=0, x=1, x=2, x=3.5)	 2 (real) roots	 0 (real) root

3. **Multiplicity** of the root and x-intercepts.

Let $f(x) = (x - r)^n$ where r is the __________ of f and this root repeats _____ times. We call r a root with

_____________________ n.

Even Multiplicity (n is even)	Odd Multiplicity (n is odd)
 $y = x^n$, for n even The graph ____________________ the x-axis and ___________________________ at the root r.	 $y = x^n$, for n odd The graph ____________ the x-axis at the root r.
The graph tends to **flatten out** near the roots with **multiplicity** greater than _________	

4. **Turning Points** of Polynomial Functions:

Let $f(x)$ be a polynomial function of **degree n**, then the graph of f has at most _______ turning points.

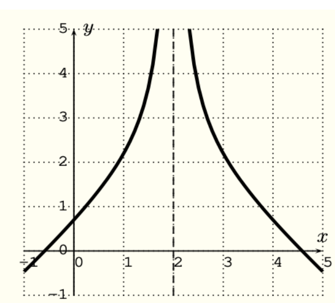
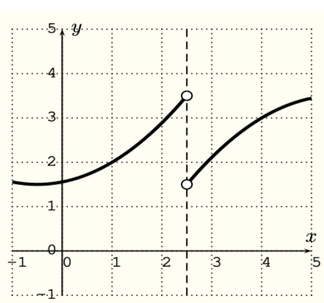
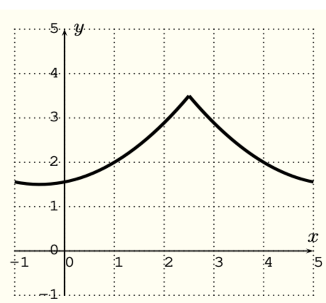
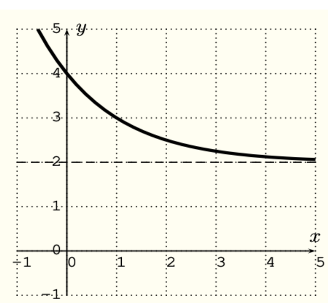
5. The essential part for drawing **a complete graph of f**:

- End Behavior by _________________ test (how the function behaves when ____ approaches _______)
- All roots (which are ___- intercepts) with the Multiplicities
- All y-intercepts (the values by computing _______)
- All asymptotes (for rational functions in next chapter)
- Turning points with Extrema (that is all _________ and ___________)

6. The domain of a polynomial f is ___________________, and it is continuous for all real numbers and there

are no ______________, no ________________ or ______________ asymptotes, and no ________________

The following graphs **cannot** be graphs of polynomials:

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