Section 3.1

1. List all the steps used by Algorithm 1 to find the maximum of the list 1, 8, 12, 9, 11, 2, 14, 5, 10, 4.

ALGORITHM 1 Finding the Maximum Element in a Finite Sequence.

procedure *max*($a_1, a_2, \ldots, a_n$: integers)
$max := a_1$
for $i := 2$ **to** n
  if $max < a_i$ **then** $max := a_i$
return *max*{*max* is the largest element}

Initialization: $n=10$, $max=1$

← if T, $max = a_i$

i	$max < a_i$ (T or F)		max
2	1 < 8	(T)	8
3	8 < 12	(T)	12
4	12 < 9	(F)	12
5	12 < 11	(F)	12
6	12 < 2	(F)	12
7	12 < 14	(T)	14
8	14 < 5	(F)	14
9	14 < 10	(F)	14
10	14 < 4	(F)	14

return $max = 14$

3. procedure *AddUp*($a_1, \ldots, a_n$: integers)
$sum := a_1$
for $i := 2$ **to** n
  $sum := sum + a_i$
return *sum*

5. **procedure** *duplicates*($a_1, a_2, \ldots, a_n$: integers in nondecreasing order)

$k := 0$ {this counts the duplicates} , n= the length of list.
$j := 2$
while $j \leq n$
  if $a_j = a_{j-1}$ **then**
    $k := k + 1$
    $c_k := a_j$
    if the number is repeated, save it into c_k
    while $j \leq n$ and $a_j = c_k$
      $j := j + 1$
    if it is repeated more than twice, it will <u>not</u> be saved twice in c_k.
  $j := j + 1$
$\{c_1, c_2, \ldots, c_k$ is the desired list$\}$

7. **procedure** *last even location*($a_1,a_2, \ldots ,a_n$: integers)

$k := 0$, n= the length of list. $\{a_i\}$
for $i := 1$ **to** n
  if a_i is even **then** $k := i$
return k $\{k = 0$ if there are no evens$\}$

9. **procedure** *palindrome check*($a_1 a_2 \ldots a_n$: string)

answer := **true** , n = the length of $\{a_i\}$
for $i := 1$ **to** $\lfloor n/2 \rfloor$
  if $a_i \neq a_{n+1-i}$ **then** *answer* := **false** and STOP
return *answer*

11. **procedure** *interchange*(x, y: real numbers)

$z := x$ {temp. save x in z}
$x := y$ {set x being y}
$y := z$ {set y being z which is x}

The minimum number of assignments needed is three.

13. List all the steps used to search for 9 in the sequence 1, 3, 4, 5, 6, 8, 9, 11 using

a) a linear search. **b)** a binary search.

Sol: a) initialization: $n=8$, $x=9$

i	$x=a_i$ (T or F)		$i=i+1$
1	$9=1$	(F)	2
2	$9=3$	(F)	3
3	$9=4$	(F)	4
4	$9=5$	(F)	5
5	$9=6$	(F)	6
6	$9=8$	(F)	7
7	$9=9$	(T)	Stop

return location $= i = 7$

b) initialization: $n=8$, $x=9$, $i=1$, $j=8$

	$i<j$ (T or F)	m	$x>a_m$	(T) $i=m+1$	(F) $j=m$
$i=1, j=8$	$1<8$ (T)	$\lfloor\frac{1+8}{2}\rfloor=4$	$9>5$ (T)	$i=5$	* $(j=8)$
$i=5, j=8$	$5<8$ (T)	$\lfloor\frac{5+8}{2}\rfloor=6$	$9>8$ (T)	$i=7$	* $(j=8)$
$i=7, j=8$	$7<8$ (T)	$\lfloor\frac{7+8}{2}\rfloor=7$	$9>9$ (F)	* $(i=7)$	$j=7$
$(i=7, j=7)$	$7<7$ (F)	stop			

$9=a_7$ is True, location $= i = 7$

return location $= 7$

15. procedure *insert*($x, a_1, a_2, \dots, a_n$: integers)

{the list is in order: $a_1 \le a_2 \le \cdots \le a_n$}

$a_{n+1} := x+1$ ⟵ In case that x is larger than a_n

$i := 1$, $n=$ the length of $\{a_i\}$

while $x > a_i$

  $i := i+1$

for $j := 0$ **to** $n-i$

  $a_{n-j+1} := a_{n-j}$

} once the inserted position i is found, move $a_i, a_{i+1}, \cdots, a_n$ to $a_{i+1}, a_{i+2}, \cdots, a_n, a_{n+1}$

$a_i := x$

{x has been inserted into correct position}

16. Describe an algorithm for finding the smallest integer in a finite sequence of natural numbers.

procedure min($a_1, a_2, \cdots, a_n$: a list of n numbers)

$n :=$ the length of $\{a_i\}$

temp.min $:= a_1$

for $i := 2$ **to** n

  if *temp*min $>$ a_i **then** *temp*min $:= a_i$

return tempmin {*temp*min is the smallest element}

17. procedure *first largest*($a_1, \ldots, a_n$: integers)
$max := a_1$, n= the length of $\{a_n\}$
$location := 1$
for $i := 2$ **to** n
  if $max < a_i$ **then**
    $max := a_i$
    $location := i$
return *location*

If the next term $\leq$ max , the index i will not be changed.

18. Describe an algorithm that locates the last occurrence of the smallest element in a finite list of integers, where the integers in the list are not necessarily distinct.

procedure last smallest ($a_1, a_2, \ldots, a_n$: integers)
n= the length of $\{a_i\}$, $min = a_1$
location := 1
for $i := 2$ to n
  if $min \geq a_i$ then
    $min = a_i$
    location $= i$
return location

36. Use the bubble sort to sort 6, 2, 3, 1, 5, 4, showing the lists obtained at each step.

Initialization: $n=6$, $i = 1$ to 5 ($n-1$) and $j=1$ to $6-i$

Round 1, $i=1$

$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	
6	2	2	2	2	2
2	6	3	3	3	3
3	3	6	1	1	1
1	1	1	6	5	5
5	5	5	5	6	4
4	4	4	4	4	$\boxed{6}$

Round 2
$i=2$

$j=1$	$j=2$	$j=3$	$j=4$	
2	2	2	2	2
3	3	1	1	1
1	1	3	3	3
5	5	5	5	4
4	4	4	4	$\boxed{5}$
$\boxed{6}$	6	6	6	$\boxed{6}$

Round 3
$i=3$

$j=1$	$j=2$	$j=3$	
2	1	1	1
1	2	2	2
3	3	3	3
4	4	4	$\boxed{4}$
$\boxed{5}$	5	5	$\boxed{5}$
$\boxed{6}$	6	6	$\boxed{6}$

Round 4
$i=4$

$j=1$	$j=2$	
1	1	1
2	2	2
3	3	$\boxed{3}$
$\boxed{4}$	4	$\boxed{4}$
$\boxed{5}$	5	$\boxed{5}$
$\boxed{6}$	6	$\boxed{6}$

Round 5
$i=5$

$j=1$	
1	1
2	2
$\boxed{3}$	3
$\boxed{4}$	4
$\boxed{5}$	5
$\boxed{6}$	6

Done.

37. Use the bubble sort to sort 3, 1, 5, 7, 4, showing the lists obtained at each step.

Initialization: $n=5$, $i=1$ to 4 ($n-1$) and $j=1$ to $5-i$ ($n-i$)

Round 1
$i=1$

$j=1$	$j=2$	$j=3$	$j=4$	
3	1	1	1	1
1	3	3	3	3
5	5	5	5	5
7	7	7	7	4
4	4	4	4	$\boxed{7}$

Round 2
$i=2$

$j=1$	$j=2$	$j=3$	
1	1	1	1
3	3	3	3
5	5	5	4
4	4	4	$\boxed{5}$
$\boxed{7}$	7	7	$\boxed{7}$

Round 3
$i=3$

$j=1$	$j=2$	$j=3$	
1	1	1	1
3	3	3	3
4	4	4	$\boxed{4}$
$\boxed{5}$	5	5	$\boxed{5}$
$\boxed{7}$	7	7	$\boxed{7}$

Round 4
$i=4$

$j=1$	
1	1
3	3
$\boxed{4}$	4
$\boxed{5}$	5
$\boxed{7}$	7

Done.

39. **procedure** *better bubblesort*($a_1, \ldots, a_n$: integers)

$i := 1$; *done* := **false**

while $i < n$ and *done* = **false**

  done := **true**

  for $j := 1$ **to** $n - i$

    if $a_j > a_{j+1}$ **then**

      interchange a_j and a_{j+1}

      done := **false**

  $i := i + 1$

$\{a_1, \ldots, a_n$ is in increasing order$\}$

40. Use the insertion sort to sort the list in Exercise 36, showing the lists obtained at each step.

Input $\{6, 2, 3, 1, 5, 4\}$

Initialization: $n = 6$, $i = 2$ to 6, $j = 1$

① Round 1. $i=2$, insert $a_2 = 2$ to the right place

$a_i > a_j$ and $i > j$ (T or F) $j = j+1$ (if "T")

$i=2, j=1$ $\boxed{2 > 6}$ $\wedge$ $2 > 1$ (F) STOP

$m = a_2$ ($m = 2$. temp storge)

$k = 0$ to 0 $(i - j - 1 = 2-1-1 = \underline{\underline{0}})$

when $k = 0$ $a_2 = a_1$ (move "6" to a_2)

$a_1 = m$ $(a_1 = 2)$

* End of Round 1: 2, 6, 3, 1, 5, 4

② Round 2. $i=3$ insert $a_3 = 3$ to the right place

$a_i > a_j$ and $i > j$ (T or F) $j = j+1$

$i=3, j=1$ $3 > 2$ $\wedge$ $3 > 1$ (T) $j=2$

$i=3, j=2$ $\boxed{3 > 6}$ $\wedge$ $3 > 2$ (F) STOP

$m = a_3$ $(m = 3)$

$k = 0$ to 0 $(i - j - 1 = 3-2-1 = 0)$

when $k=0$, $a_3=a_2$ (move "6" to a_3)

$a_2=m$ ($a_2=3$)

* End of Round 2: 2, 3, 6, 1, 5, 4

③ Round 3: $i=4$ Insert $a_4=1$ to the right place.

	$a_i > a_j$ and $i > j$ (T or F)	$j=j+1$
$i=4, j=1$	$\boxed{1>2}$ ∧ $4>1$ (F)	STOP.

$m=a_4$ ($m=1$)

$k=0$ to 2 ($i-j-1=4-1-1=2$)

when $k=0$. $a_4=a_3$ (move "6" to a_4)

when $k=1$ $a_3=a_2$ (move "3" to a_3)

when $k=2$ $a_2=a_1$ (move "2" to a_2)

$a_1=m$ ($a_1=1$)

* End of Round 3: 1, 2, 3, 6, 5, 4

④ Round 4: $i=5$ insert $a_5=5$ to the right place.

	$a_i > a_j$ and $i > j$ (T or F)	$j=j+1$
$i=5, j=1$	$5>1$ ∧ $5>1$ (T)	$j=2$
$j=2$	$5>2$ ∧ $5>2$ (T)	$j=3$
$j=3$	$5>3$ ∧ $5>3$ (T)	$j=4$
$j=4$	$\boxed{5>6}$ ∧ $5>4$ (F)	STOP.

$m=a_5$ ($m=5$)

$k=0$ to 0 ($i-j-1=5-4-1=0$)

when $k=0$, $a_5=a_4$ (move "6" to a_5)

$a_4=m$ ($a_4=5$)

* End of Round 4: 1, 2, 3, 5, 6, 4

⑤ Round 5 $i=6$, insert $a_6=4$ to the right place.

	$a_i > a_j$ and $i > j$ (T or F)	$j=j+1$
$i=6, j=1$	$4>1$ and $6>1$ (T)	$j=2$
$j=2$	$4>2$ and $6>2$ (T)	$j=3$
$j=3$	$4>3$ and $6>3$ (T)	$j=4$
$j=4$	$\boxed{4>5}$ and $6>4$ (F)	STOP

$m=a_6$ ($m=4$)

$k = 0$ to 1 $(i-j-1 = 6-4-1 = 1)$

when $k=0$, $a_6 = a_5$ (move "6" to a_6)

when $k=1$, $a_5 = a_4$ (move "5" to a_5)

$a_4 = m$ $(a_4 = 4)$

End of Round 5: 1, 2, 3, 4, 5, 6.

56. Use the cashier's algorithm to make change using quarters, dimes, nickels, and pennies for

a) 87 cents. **b)** 49 cents.

c) 99 cents. **d)** 33 cents.

Sol: a) 87 cents. Initialization: $n = 87$, $r = 4$, $i = 1$ to 4.

$n \geq c_i$ $\quad d_i = d_i + 1$; $n = n - c_i$

$i=1$, $d_1 = 0$, $87 \geq 25$ (Yes) $d_1 = 1$; $n = 87 - 25 = 62$

$d_1 = 1$, $62 \geq 25$ (Yes) $d_1 = 2$; $n = 62 - 25 = 37$

$d_1 = 2$, $37 \geq 25$ (Yes) $d_1 = 3$, $n = 37 - 25 = 12$

$\boxed{d_1 = 3}$, $12 \geq 25$ (NO) and STOP

$i=2$, $d_2 = 0$ $12 \geq 10$ (Yes) $d_2 = 1$, $n = 12 - 10 = 2$

$\boxed{d_2 = 1}$ $2 \geq 10$ (NO) STOP

$i=3$, $\boxed{d_3 = 0}$ $2 \geq 5$ (NO) STOP

$i=4$, $d_4 = 0$ $2 \geq 1$ (Y) $d_4 = 1$, $n = 2 - 1 = 1$

$d_4 = 1$ $1 \geq 1$ (Y), $d_4 = 2$, $n = 1 - 1 = 0$

$\boxed{d_4 = 2}$ $0 \geq 1$ (N) STOP

87 cents = 3 quarter(s) + 1 dime(s) + 0 nickel(s) + 2 pennies

b) 49 cents. Initialization: $n = 49$, $r = 4$. $i = 1$ to 4

$n \geq c_i$ $\quad d_i = d_i + 1$; $n = n - c_i$

$i=1$, $d_1 = 0$. $49 \geq 25$ (Yes) $d_1 = 1$; $n = 49 - 25 = 24$

$\boxed{d_1 = 1}$, $24 \geq 25$ (NO) STOP

$i=2$ $d_2 = 0$ $24 \geq 10$ (Yes) $d_2 = 1$; $n = 24 - 10 = 14$

$d_2 = 1$ $14 \geq 10$ (Yes) $d_2 = 2$; $n = 14 - 10 = 4$

$\boxed{d_2 = 2}$ $4 \geq 10$ (NO) STOP

$i=3.\ \boxed{d_3=0}$ $\quad 4 \geq 5$ (NO) STOP

$i=4.\ d_4=0$ $\quad 4 \geq 1$ (Yes); $d_4=1$; $n=4-1=3$

$d_4=1$ $\quad 3 \geq 1$ (Yes); $d_4=2$, $n=3-1=2$

$d_4=2$ $\quad 2 \geq 1$ (Yes); $d_4=3$, $n=2-1=1$

$d_4=3$ $\quad 1 \geq 1$ (Yes); $d_4=4$ $\quad n=1-1=0$

$\boxed{d_4=4}$ $\quad 0 \geq 1$ (NO) STOP

49 cents = 1 quarter(s) + 2 dime(s) + 0 nickel(s) + 4 pennies

c) 99 cents. Initialization: $n=99$, $r=4$, $i=1$ to 4.

$n \geq c_i$ $\qquad d_i = d_i + 1$, $n = n - c_i$

$i=1,\ d_1=0$ $\quad 99 \geq 25$ (Yes) $d_1=1$, $n=99-25=74$

$d_1=1$ $\quad 74 \geq 25$ (Yes) $d_1=2$, $n=74-25=49$

$d_1=2$ $\quad 49 \geq 25$ (Yes) $d_1=3$, $n=49-25=24$

$\boxed{d_1=3}$ $\quad 24 \geq 25$ (NO) STOP

$i=2\ \ d_2=0$ $\quad 24 \geq 10$ (Yes) $d_2=1$, $n=24-10=14$

$d_2=1$ $\quad 14 \geq 10$ (Yes) $d_2=2$, $n=14-10=4$

$\boxed{d_2=2}$ $\quad 4 \geq 10$ (NO) STOP

$i=3,\ \boxed{d_3=0}$ $\quad 4 \geq 5$ (NO) STOP

$i=4.\ d_4=0$ $\quad 4 \geq 1$ Yes. $d_4=1$, $n=4-1=3$

$d_4=1$, $\quad 3 \geq 1$ Yes $d_4=2$ $\quad n=3-1=2$

$d_4=2$, $\quad 2 \geq 1$ Yes $d_4=3$ $\quad n=2-1=1$

$d_4=3$, $\quad 1 \geq 1$ Yes $d_4=4$ $\quad n=1-1=0$

$\boxed{d_4=4}$ $\quad 0 \geq 1$ NO STOP

99 cents = 3 quarter(s) + 2 dime(s) + 0 nickel(s) + 4 pennies

d) 33 cents. Initialization: $n=33$, $r=4$, $i=1$ to 4

$n \geq c_i$ $\qquad d_i = d_i + 1$; $n = n - c_i$

$i=1,\ d_1=0$ $\quad 33 \geq 25$ (Yes), $d_1=1$, $n=33-25=8$

$\boxed{d_1=1}$ $\quad 8 \geq 25$ (NO) STOP

$i=2.\ \boxed{d_2=0}$ $\quad 8 \geq 10$ (NO) STOP

$i=3$, $d_3=0$ $\quad 8 \geq 5$ (Yes) $d_3=1$, $n=8-5=3$

$\boxed{d_3=1}$ $\quad 3 \geq 5$ (NO) STOP

$i=4$, $d_4=0$ $\quad 3 \geq 1$ (Yes) $d_4=1$, $n=3-1=2$

$d_4=1$ $\quad 2 \geq 1$ (Yes) $d_4=2$ $\quad n=2-1=1$

$d_4=2$ $\quad 1 \geq 1$ (Yes) $d_4=3$ $\quad n=1-1=0$

$\boxed{d_4=3}$ $\quad 0 \geq 1$ (NO) STOP

33 cents = 1 quarter(s) + 0 dime(s) + 1 nickel(s) + 3 pennies