

MAT2540, Quiz2, Spring2026

ID: _____

Name: Sol

- This quiz consists of 2 questions for a total of 10 points.
- You have 15 minutes to complete the quiz.
- Show all work and justify your answers.
- Wishing you success.

1. Find $f(1), f(2), f(3), f(4)$, and $f(5)$ if $f(n)$ is defined recursively by $f(0) = 3$, and for $n = 1, 2, 3, \dots$

$$f(n+1) = f(n)^2 - 2f(n) - 2.$$

$$n=0, f(1) = f(0+1) = f(0)^2 - 2f(0) - 2 = 3^2 - 2 \cdot 3 - 2 = 1$$

$$n=1, f(2) = f(1+1) = f(1)^2 - 2f(1) - 2 = 1^2 - 2 \cdot 1 - 2 = -3$$

$$n=2, f(3) = f(2+1) = f(2)^2 - 2f(2) - 2 = (-3)^2 - 2 \cdot (-3) - 2 = 13$$

$$n=3, f(4) = f(3+1) = f(3)^2 - 2f(3) - 2 = (13)^2 - 2 \cdot (13) - 2 = 169 - 26 - 2 = 141$$

$$n=4, f(5) = f(4+1) = f(4)^2 - 2f(4) - 2 = (141)^2 - 2 \cdot (141) - 2$$

2. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \dots$ if $a_n = n(n+1)$.

Sol 1.

$$a_n = n(n+1)$$

$$a_{n+1} = (n+1)(n+2)$$

$$a_{n+2} = (n+2)(n+3)$$

⋮

$$a_{n+1} = (n+1)(n+2) = \frac{n+2}{n} a_n$$

$$(n \neq 0)$$

Sol 2

$$\begin{array}{cccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \text{''} & \text{''} & \text{''} & \text{''} & \text{''} \\ 1 \cdot (1+1) & 2 \cdot (2+1) & 3 \cdot (3+1) & 4 \cdot (4+1) & 5 \cdot (5+1) \\ \text{''} & \text{''} & \text{''} & \text{''} & \text{''} \\ 2 & 6 & 12 & 20 & 30 \\ \text{''} & \text{''} & \text{''} & \text{''} & \text{''} \\ +4 & +6 & +8 & +10 & \\ \text{''} & \text{''} & \text{''} & \text{''} & \\ 2 \cdot 2 & 2 \cdot 3 & 2 \cdot 4 & 2 \cdot 5 & \end{array}$$

$$\cdots \cdots a_{n-1} a_n$$

$$\Rightarrow a_n = a_{n-1} + 2n$$

$$\text{or } a_{n+1} = a_n + 2(n+1)$$