MAT2440, Classwork25, Spring2025

ID:____________________ Name:____________________

1. Summation of a sequence: series

   Given a sequence $a_1, a_2, \cdots, a_m, a_{m+1}, \cdots, a_n$. The summation from the m^{th} term to the n^{th} term of this sequence can be written by the summation notation:

$$a_m + a_{m+1} + \cdots + a_n = \sum_{i=m}^{n} a_i. \quad (m \le n)$$

2. The index of summation:

$$a_3 + a_4 + a_5 + a_6 + a_7 = \sum_{i=3}^{7} a_i = \sum_{j=3}^{7} a_j = \sum_{n=2}^{6} a_{n+1}$$

   where i, j, and n are the index of summation.

3. What is the value of $\sum_{i=1}^{4} (i+1)^2 - i^2$?

$$\sum_{i=1}^{4}[(i+1)^2 - i^2] = \underbrace{[4-1]}_{i=1} + \underbrace{[9-4]}_{i=2} + \underbrace{[16-9]}_{i=3} + \underbrace{[25-16]}_{i=4}$$

$$= 24$$

4. What is the value of $\sum_{j=3}^{6} 2$?

$$\sum_{j=3}^{6} 2 = \underset{j=3}{2} + \underset{j=4}{2} + \underset{j=5}{2} + \underset{j=6}{2} = 8$$

5. What is the value of $\sum_{n \in S} \frac{1}{n}$, where $S = \{1, 3, 4, 8\}$?

$$\sum_{n \in S} \frac{1}{n} = \frac{1}{1} + \frac{1}{3} + \frac{1}{4} + \frac{1}{8} = \frac{41}{24}.$$

6. Let $a, d,$ and r be real number and $r \neq 0$, then

$$\frac{a\cdot(r^{n+1}-1)}{r-1}$$

Arithmetic series

\# of terms

$$\sum_{i=1}^{n} a + (i-1)d = n\cdot\left(\frac{a + a + (n-1)d}{2}\right)$$

first term $= a$ (as $i=1$)

last term $= a+(n-1)d$ (as $i=n$)

Geometric Series

$$\sum_{j=0}^{n} ar^j = \begin{cases} \frac{a(1-r^{n+1})}{1-r}, & \text{if } r \neq 1; \\ (n+1)a, & \text{if } r = 1. \end{cases}$$

7. Some useful summation formulae:

first term, last term

$$\sum_{k=1}^{n} k = n\cdot\frac{1+n}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)\cdot(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n\cdot(1+n)}{2}\right)^2$$

$$\sum_{k=1}^{\infty} x^k = \frac{1}{1-x}$$

if $|x| < 1$.

8. The Double Summations:

Double summations arise in many contexts (as in the analysis of **nested loops** in Computer programs). For example,

$$\sum_{i=1}^{4}\sum_{j=1}^{3} ij = \sum_{i=1}^{4}\left[i\cdot 1 + i\cdot 2 + i\cdot 3\right] = \sum_{i=1}^{4} 6i$$

$$= 6\cdot 1 + 6\cdot 2 + 6\cdot 3 + 6\cdot 4 = 60.$$

- You can first expand inner summation, then expand the outer summation.

inner summation

9. Find $\sum_{i=1}^{4}\boxed{\sum_{j=1}^{3}(i-j)} = \sum_{i=1}^{4}\boxed{(i-1)+(i-2)+(i-3)}$ fix "i" and $j=1,2,3$

$$= \sum_{i=1}^{4}(i-1+i-2+i-3)$$

$$= \sum_{i=1}^{4}(3i-6) = \underbrace{3\cdot 1-6}_{i=1} + \underbrace{3\cdot 2-6}_{i=2} + \underbrace{3\cdot 3-6}_{i=3} + \underbrace{3\cdot 4-6}_{i=4}$$

$$= -3+0+3+6 = 6.$$