MAT1375, Classwork13, Fall2025

Ch13. Exponential and Logarithmic Functions I

1. Definition of the **Exponential Function**:

A function f is called <u>an exponential function</u> with <u>base</u> b for **any real number** $\boldsymbol{x}$ if

$$f(x) = c \cdot b^x,$$

for some <u>real</u> number c and <u>positive</u> real number b which is called the <u>base</u>.

2. Please circle the given function if it is an **exponential function**:

(1) $f(x) = 2^x$. (2) $g(x) = 3^{x+1}$. (3) $h(x) = e^x$. π^x (4) $k(x) = \left(\frac{1}{5}\right)^x$. base$=\frac{1}{5}>0$ ~~(5)~~ $l(x) = x^2$. polynomial

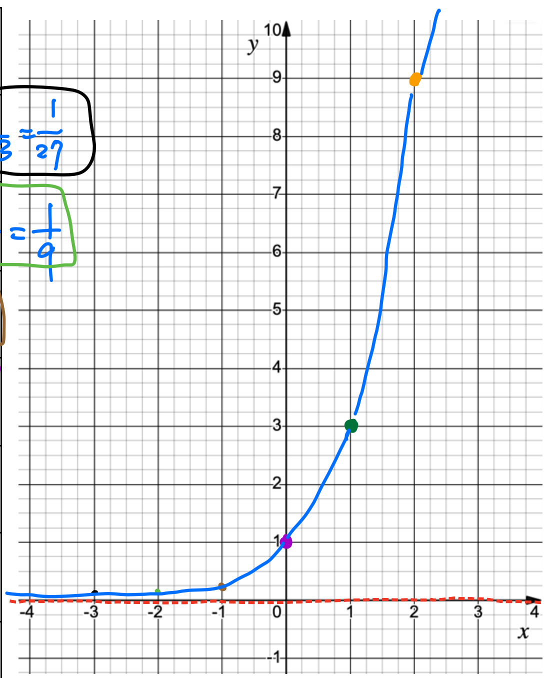
~~(6)~~ $m(x) = (-1)^x$. base $=-1<0$ ~~(7)~~ $n(x) = x^x$.

Euler's number $e = 2.71828182\cdots$
irrational number which is still a real number

3. Graph the given functions:

(a) $f(x) = 3^x$.

x	$f(x)$
-3	$3^{-3}=\frac{1}{3^3}=\frac{1}{27}$
-2	$3^{-2}=\frac{1}{3^2}=\frac{1}{9}$
-1	$3^{-1}=\frac{1}{3}$
0	$3^0=1$
1	$3^1=3$
2	$3^2=9$
3	$3^3=27$

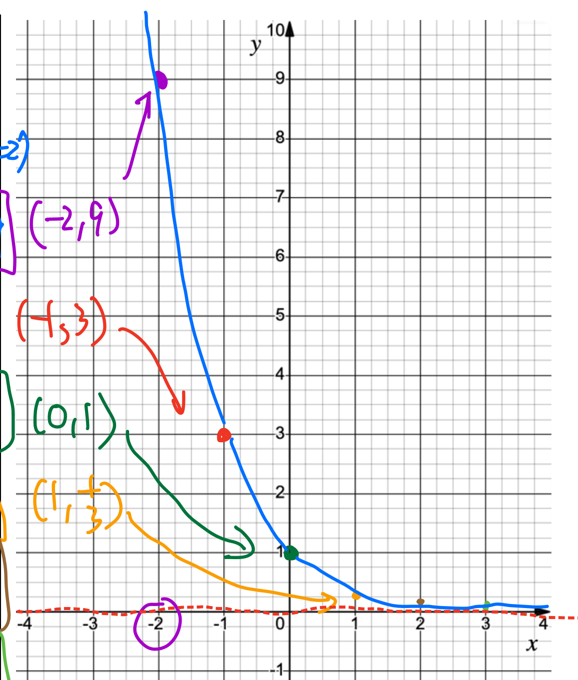


Domain: $(-\infty,\infty)$; Range: $(0,\infty)$

Asymptote: H.A. $y=0$.

(b) $g(x) = \left(\frac{1}{3}\right)^x = (3^{-1})^x = 3^{-x}$

x	$g(x)$
-3	$3^{-(-3)}=3^3=27$
-2	$3^{-(-2)}=3^2=9$
-1	$3^{-(-1)}=3$
0	$3^{-(0)}=1$
1	$3^{-(1)}=\frac{1}{3}$
2	$\left(\frac{1}{3}\right)^2=\frac{1}{9}$
3	$\left(\frac{1}{3}\right)^3=\frac{1}{27}$



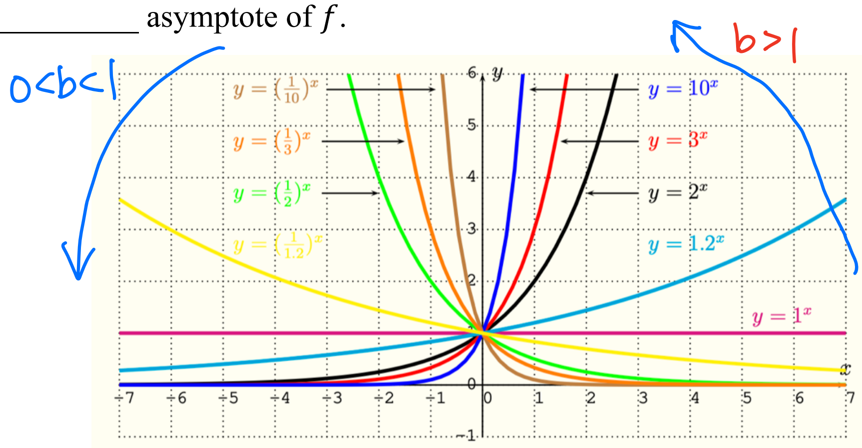
Domain: $(-\infty,\infty)$; Range: $(0,\infty)$.

Asymptote: H.A. $y=0$.

4. Characteristics of Exponential Function of $f(x) = b^x$.

(a) The domain of f:____________________; The Range of f: ______________________.

(b) There is _______ x-intercept. In fact, f approaches, but never touches _______________ which is a _________________ asymptote of f.



(c) Its y-intercept is _________.

(d) For $b > 1$, $f(x) \to$______ as $x \to \infty$, $f(x) \to$______ as $x \to -\infty$.

(e) For $0 < b < 1$, $f(x) \to$______ as $x \to \infty$, $f(x) \to$______as $x \to -\infty$.

(f) f is one-to-one and has an ___________ function.

6. What is the 4-steps strategy to find the inverse of a given function? Can it be used to find the inverse function of $f(x) = b^x$?

7. Definition of **Logarithmic Function**:

For $x > 0$ and $b > 0, b \neq$___, the logarithmic of x with base b is defined by the equivalence

$$x = b^y \quad \Leftrightarrow \quad y = \log_b(x).$$

This computes the inverse of the exponential function $y = b^x$ with base b. (We exchange _____ and _____ to get $x = b^y$ and solve for _____).

8. Rewrite the equation as a logarithmic equation.

a) $3^4 = x$. b) $e^x = 17$. c) $2^{7a} = 53$. d) $b^3 = 8$.