MAT 1375, Classwork23, Fall2024

ID:______________________ Name:______________________

1. Definition of a sequence:

A ___Sequence___ is an enumerated list of numbers and it can be denoted by

$$a_1, a_2, a_3, a_4, a_5, a_6, \cdots \text{ or } \{\ a_n\ \} \text{ or } \{\ a_n\ \}_{n\geq 1}.$$

2. A sequence with a **given pattern**: Find the first 6 terms of each sequence.

$a_1=1,\ a_2=4,\ a_3=9,\ a_4=16,\ a_5=25,\ a_6=36$

a) $a_n = 4n+3$ b) $a_k = k^2$ c) $a_m = \frac{m}{m+1}$ d) $a_n = (-1)^n$

$a_1=7,\ a_2=11,\ a_3=15,\ a_4=19,\ a_5=23,\ a_6=27$

3. A sequence **without** a **given pattern**:

$22-3$ $\quad 19-3 = 22-3-3 = 22-2\cdot 3$

a) Find the 70th terms of the sequence: $22, 19, 16, 13, \cdots$

$16-3 = 19-3-3 = 22-3-3-3 = 22\cdot 3\cdot 3$

b) Find the 95th terms of the sequence: $-17, -12, -7, -2, \cdots$

$\underbrace{-17,-12}_{+5},\ \underbrace{-12,-7}_{+5},\ \underbrace{-7,-2}_{+5}$ Arithmetic $a_1=-17,\ d=5$

c) Find the 10th terms of the sequence: $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \cdots$

$a_1=22,\quad a_2=19=22-1\cdot 3,\quad a_3=16=22-2\cdot 3 \quad \cdots \quad a_{70}=22-69\cdot 3$
$=22-207$
$=-185$

$a_{95}=a_1+(95-1)\cdot d = -17+(95-1)\cdot 5 = -17+470$
$=45$

$a_1=\frac{1}{2},\quad r=\frac{a_2}{a_1}=\frac{-\frac{1}{4}}{\frac{1}{2}}=-\frac{1}{4}\times\frac{2}{1}=-\frac{1}{2}$

$a_{10}=a_1\cdot r^{10-1}=\frac{1}{2}\cdot\left(-\frac{1}{2}\right)^9=-\frac{1}{1024}$

4. The **Arithmetic** Sequence:

A sequence $\{a_k\}$ is called ___arithmetic___ sequence if any two consecutive terms have a common ___difference___ d. The arithmetic sequence $\{a_k\}$ is determined by d and ___a_1___: the first term

$$\boldsymbol{a_k = a_{k-1} + \underline{\ d\ }} \text{ for } n \geq 2 \text{ or } \boldsymbol{a_k = a_1 + \underline{(k-1)\cdot d}}.$$

5. The **Geometric** Sequence:

A sequence $\{a_k\}$ is called ___geometric___ sequence if any two consecutive terms have a common ___ratio___ r. The ~~arithmetic~~ geometric sequence $\{a_k\}$ is determined by ~~d~~ r and ___a_1___ :

$$\boldsymbol{a_k = a_{k-1} \cdot \underline{\ r\ }} \text{ for } n \geq 2 \text{ or } \boldsymbol{a_k = a_1 \cdot \underline{r^{k-1}}}.$$

6. The **Series**:

Let $\{a_k\}$ be a sequence. The ~~arithmetic~~ Series is the **sum** of all the term of a_k for $k \geq 1$:

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + \cdots$$

7. The Arithmetic **Series**: Let $\{a_k\}$ be an arithmetic sequence. Then the sum of the arithmetic sequence of the first n term is given by

$$\sum_{k=1}^{n} a_k = \frac{n}{2}(a_1 + a_n)$$

the first term + the last term

8. The Geometric **Series** : Let $\{a_k\}$ be a geometric sequence with the **common ratio** r that $-1 < r < 1$.

Then the sum of the geometric sequence of the first n term is given by

$$\sum_{k=1}^{n} a_k = a_1 \cdot \frac{1 - r^n}{1 - r}$$

Furthermore, the **infinite** geometric series is defined when $-1 < r < 1$ and given by

$$\sum_{k=1}^{\infty} a_k = a_1 \cdot \frac{1}{1 - r}$$

9. What is the infinite geometric series of $\{a_k\}$ if its common ratio $r \geq 1$ or $r \leq -1$?

10. Find the sum of the first 70 terms of the arithmetic sequence: $22, 19, 16, 13, \cdots$.

$$\begin{array}{r} 163 \\ 35 \\ \hline 815 \\ 489 \end{array}$$

$a_1 = 22,\ a_2 = 19,\ a_3 = 16,\ d = a_2 - a_1 = 19 - 22 = -3$

$$a_1 + \cdots + a_{70} = \sum_{k=1}^{70} a_k = \frac{70}{2}(22 + a_{70}) = \frac{70}{2}(22 - 185) = \frac{\cancel{70}^{35}}{2}(-163)$$

$$= -5705$$

$a_{70} = a_1 + (70 - 1) \cdot d = 22 + 69 \cdot (-3) = -185$

11. Find the exact sum of infinite geometric sequence: $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \cdots$.

$a_1 = \frac{1}{2},\ a_2 = -\frac{1}{4},\ a_3 = \frac{1}{8},\ \ldots \quad \frac{a_2}{a_1} = \frac{-\frac{1}{4}}{\frac{1}{2}} = -\frac{1}{4} \cdot 2 = -\frac{1}{2}$

$$\sum_{k=1}^{\infty} a_k = a_1 \cdot \frac{1}{1 - r} = \frac{1}{2} \cdot \frac{1}{1 - (-\frac{1}{2})} = \frac{1}{2} \cdot \frac{1}{\frac{3}{2}} = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$