MAT2440, Classwork9, Spring2025

ID:______________________________ Name:______________________________

1. Quantifier with **Restricted Domain**:

Given the following statements of quantifier with restricted domain:

(a) $\forall x < 0(x^2 > 0)$ (b) $\forall y \neq 0(y^3 \neq 0)$ (c) $\exists z > 0(z^2 = 2)$

What do the statements mean where the domain in each case consists of the real numbers?

(a) $\forall x<0(x^2>0)$ means "for every real number x with $x<0$, $x^2>0$"
or "for every negative real number x, $x^2>0$"
and it is true. $\boxed{\forall x(x<0 \rightarrow x^2>0)}$ conditional proposition

(b) $\forall y\neq 0(y^3\neq 0)$ means "for every nonzero y, $y^3\neq 0$"
and it is true. $\boxed{\forall y(y\neq 0 \rightarrow y^3\neq 0)}$

(c) $\exists z>0(z^2=2)$ means "there exists a positive real number z such that $z^2=2$"
and it is true, (which is $z=\sqrt{2}$) $\boxed{\exists z(z>0 \wedge z^2=2)}$

2. Observation:

Restriction of a universal quantification is the same as universal quantification of a conditional statement example 1(a)&(b)

Restriction of an existential quantification is the same as existential quantification of a conjunction "$\wedge$" example 1(c)

3. Quantifier over **Finite Domain**:

When the domain of a quantifier is finite (that is, when all its elements can be listed), quantifier statement can be expressed using propositional logic:

Let the elements of the domain be $x_1, x_2, x_3, \cdots, x_n$, then

$\forall x P(x)$ is the same as $P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \cdots \wedge P(x_n)$

$\exists x P(x)$ is the same as $P(x_1) \vee P(x_2) \vee P(x_3) \vee \cdots \vee P(x_n)$

4. Given $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4. What is the truth value of $\forall x P(x)$?

Finite domain = 1, 2, 3, 4

$\forall x\, P(x)$ means "$P(1) \wedge P(2) \wedge P(3) \wedge P(4)$" and it is false.

$1^2<10 \quad 2^2<10 \quad 3^2<10 \quad 4^2<10$

$T \wedge T \wedge T \wedge F$

5. Given $P(x)$ be the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4. What is the truth value of $\exists x P(x)$?

Finite domain = 1, 2, 3, 4

$\exists x\, P(x)$ means "$P(1) \vee P(2) \vee P(3) \vee P(4)$" and it is true.

$1^2<10 \quad 2^2<10 \quad 3^2<10 \quad 4^2<10$

$T \vee T \vee T \vee F$

6. Given a proposition: "Every $\underbrace{\text{student in your class}}_{x}$ has taken a course in calculus."

(a) Using the **universal quantification** to express this proposition. $\forall x P(x)$

(b) Write down the negation of this proposition.

(c) Using the quantification to express the negation of this proposition.

(a) x : student in your class, $P(x)$: x has taken a course in calculus

$\Rightarrow \forall x\, P(x)$

7. Given a proposition: "There is at least a student in your class who takes calculus."

(a) Using the **existential quantification** to express this proposition.

(b) Write down the negation of this proposition.

(c) Using the quantification to express the negation of this proposition.