MAT1375, Classwork19, Fall2025

Ch18. Graphing Trigonometric Functions

1. Review: Even function and Odd function.

   If $f(x)$ is an even function, then $f(-a) =$ f(a).

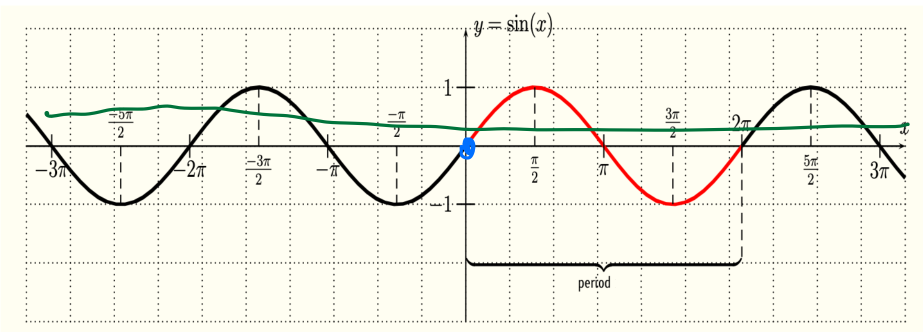
   If $f(x)$ is an odd function, then $f(-a) =$ − f(a).

2. Definition of a **Periodic Function**:

   A function f is periodic if there is a positive number p called a period such that

$$f(x+p) = f(x) \quad \text{for all } x.$$

3. The graph of $y = \sin(x)$:



Characteristics:

Period: 2π

Angle
Domain: $(-\infty, \infty)$

value
Range: $[-1, 1]$

One-to-one function? NO

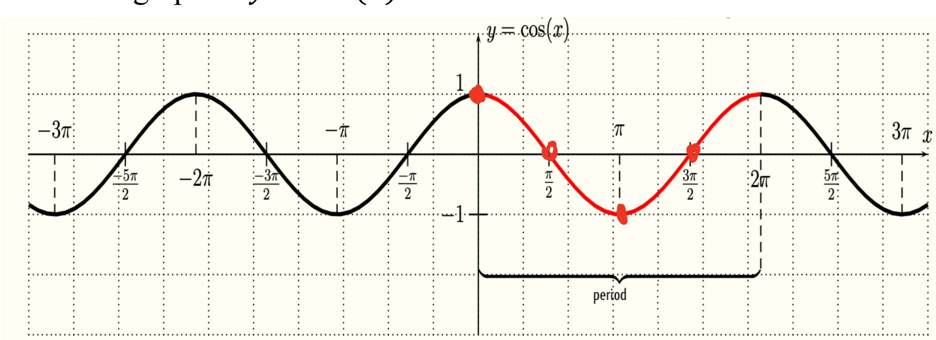
Inverse function? NO

Property: odd function with origin symmetry where $\sin(-x) =$ $-\sin(x)$.

x	0	$\frac{\pi}{6}$ $\frac{\pi}{4}$ $\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$ $\frac{3\pi}{4}$ $\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$ $\frac{5\pi}{4}$ $\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$ $\frac{7\pi}{4}$ $\frac{11\pi}{6}$	2π
$\sin(x)$	0	$\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{1}{2}$	0	$-\frac{1}{2}$ $-\frac{\sqrt{2}}{2}$ $-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$ $-\frac{\sqrt{2}}{2}$ $-\frac{1}{2}$	0

I II III IV

4. The graph of $y = \cos(x)$:



Characteristics:

Period: 2π

angle
Domain: $(-\infty, \infty)$

value
Range: $[-1, 1]$

One-to-one function? NO

Inverse function? NO

$x=0$

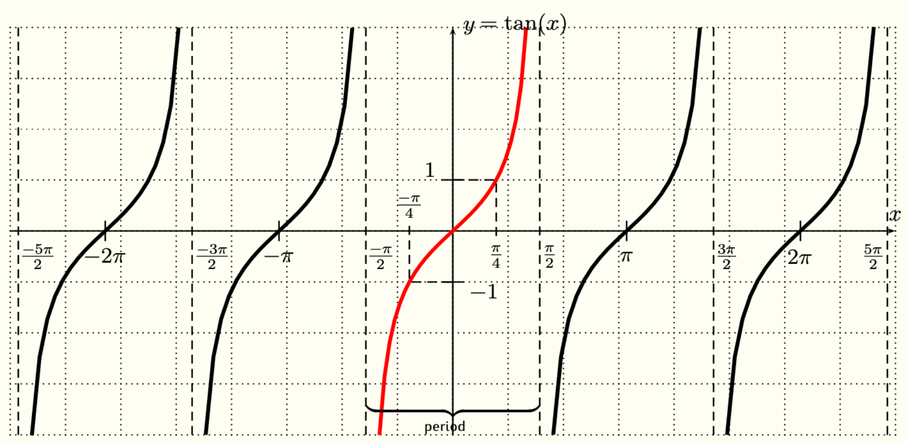
Property: even function with y-axis symmetry where $\cos(-x) =$ $\cos(x)$.

x	0	$\frac{\pi}{6}$ $\frac{\pi}{4}$ $\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$ $\frac{3\pi}{4}$ $\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$ $\frac{5\pi}{4}$ $\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$ $\frac{7\pi}{4}$ $\frac{11\pi}{6}$	2π
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{1}{2}$	0	$-\frac{1}{2}$ $-\frac{\sqrt{2}}{2}$ $-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$ $-\frac{\sqrt{2}}{2}$ $-\frac{1}{2}$	0	$\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$	1

I II III IV

5. The graph of $y = \tan(x)$: $= \dfrac{\sin(x)}{\cos(x)} = 0$

$\bigcup_{n=-\infty}^{\infty} \left(\frac{(2n-1)\pi}{2}, \frac{(2n+1)\pi}{2}\right)$



Characteristics:

Period: $\frac{3\pi}{2} - \frac{\pi}{2} = \left(\frac{3}{2} - \frac{1}{2}\right)\pi = \pi$

Domain: All real numbers except odd multiples of $\frac{\pi}{2}$

Range: $(-\infty, \infty)$

Vertical Asymptotes: $x = -\frac{\pi}{2}$, $x = -\frac{3\pi}{2}$, $x = -\frac{5\pi}{2}$, $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{5\pi}{2}$, $x = \frac{7\pi}{2}$ ....

One-to-one function? NO

Inverse function? NO

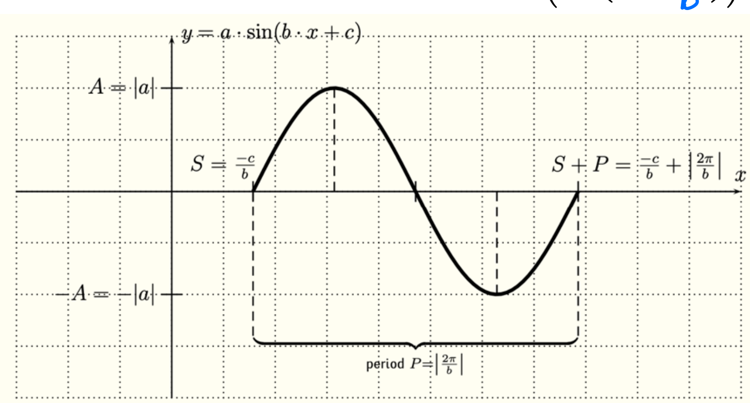
Property: odd function with origin symmetry where $\tan(-x) = \dfrac{\sin(-x)}{\cos(-x)} = \boxed{-\dfrac{\sin(x)}{\cos(x)}} = -\tan(x)$.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\frac{\sin(x)}{\cos(x)} =$ $\tan(x)$	undefined								undefined

$\cos(-\frac{\pi}{2}) = 0$ $\qquad$ $\cos(\frac{\pi}{2}) = 0$

6. Amplitude, period, and phase shift:

Let $f(x) = a \cdot \sin(b \cdot x + c) = a \cdot \sin\left(b \cdot \left(x + \frac{c}{b}\right)\right)$ or $f(x) = a \cdot \cos(b \cdot x + c) = a \cdot \cos\left(b \cdot \left(x + \frac{c}{b}\right)\right)$.

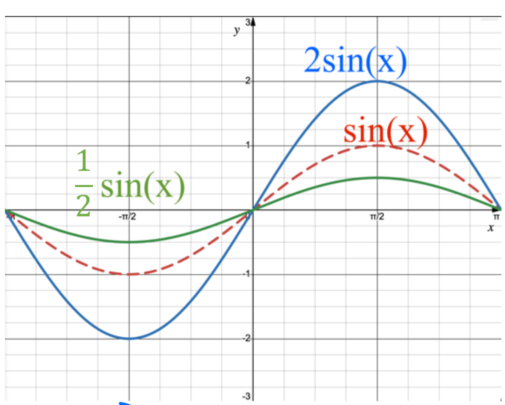


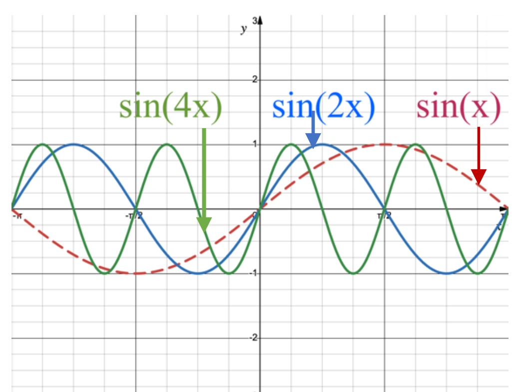
We define

1) the amplitude $A = |a|$; shrink/stretching vertically

2) the period $P = \left|\frac{2\pi}{b}\right|$; shrink/stretch horizontally

3) the phase shift $S = -\frac{c}{b}$. shifting horizontally





For $f(x) = 2\sin(x)$, its A is 2

For $f(x) = \frac{1}{2}\sin(x)$, its A is $\frac{1}{2}$

b=2

For $f(x) = \sin(2x)$, its P is $\frac{2\pi}{2} = \pi$.

For $f(x) = \sin(4x)$, its P is $\frac{2\pi}{4} = \frac{\pi}{2}$.

b=4