Math 1372 HW8

72. Consider the following experiment. You are one of 100 people enlisted to take part in a study to determine the percent of nurses in America with an R.N. (registered nurse) degree. You ask nurses if they have an R.N. degree. The nurses answer "yes" or "no." You then calculate the percentage of nurses with an R.N. degree. You give that percentage to your supervisor.

a. What part of the experiment will yield discrete data?
b. What part of the experiment will yield continuous data?

Sol: (a). The "yes-no" question is a discrete data
(b) The percentage of the "Yes/no" will be the continuous data

73. When age is rounded to the nearest year, do the data stay continuous, or do they become discrete? Why?

Sol: Discrete. Because the age will become an integer when it is rounded to the nearest year.

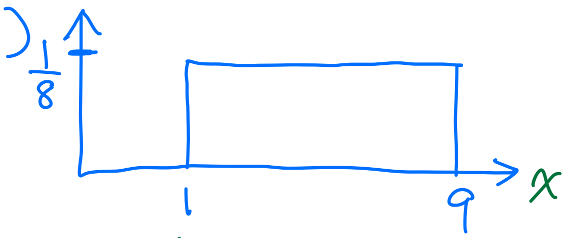
For example, $12\frac{7}{12}$ (12 years and 7 months) becomes 13 years old.

75. A random number generator picks a number from one to nine in a uniform manner.

a. $X \sim$ ________
b. Graph the probability distribution.
c. $f(x) =$ ________
d. $\mu =$ ________
e. $\sigma =$ ________
f. $P(3.5 < x < 7.25) =$ ________
g. $P(x > 5.67)$
h. $P(x > 5 \mid x > 3) =$ ________
i. Find the 90th percentile.

(a) $X \sim U(1,9)$

(b)



(c) $f(x) = \frac{1}{9-1} \cdot x \ , \ 1 \le x \le 9$

$\Rightarrow f(x) = \frac{x}{8} \ , \ 1 \le x \le 9$

(d) $\mu = \frac{9+1}{2} = \frac{10}{2} = 5$

(e) $\sigma = \sqrt{\frac{(9-1)^2}{12}} = \sqrt{\frac{64}{12}} = 2.309$

(f) $P(3.5 < X < 7.25) = (7.25 - 3.5) \cdot \frac{1}{8} = \frac{3.75}{8} = 0.46875$

(g) $P(X > 5.67) = (9 - 5.67)\frac{1}{8} = \frac{3.33}{8} = 0.41625$

(h) $P(X>5 \mid X>3) = \frac{P(X>5 \ \& \ X>3)}{P(X>3)} = \frac{P(X>5)}{P(X>3)} = \frac{(9-5)\frac{1}{8}}{(9-3)\frac{1}{8}} = \frac{4}{6} = 0.667$

(i) $P(X<k) = 0.9 \Rightarrow \frac{k-1}{8} = 0.9 \Rightarrow k-1 = 7.2 \Rightarrow k = 8.8$

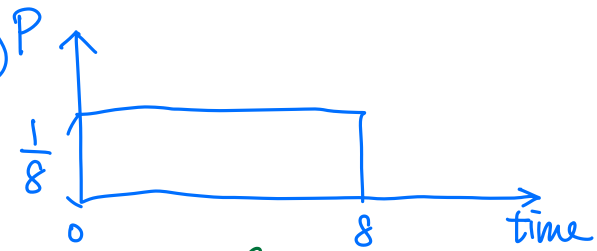
77. A subway train arrives every eight minutes during rush hour. We are interested in the length of time a commuter must wait for a train to arrive. The time follows a uniform distribution.

a. Define the random variable. X = _______
b. X ~ _______
c. Graph the probability distribution.
d. $f(x)$ = _______
e. μ = _______
f. σ = _______
g. Find the probability that the commuter waits less than one minute.
h. Find the probability that the commuter waits between three and four minutes.
i. Sixty percent of commuters wait more than how long for the train? State this in a probability question, similarly to parts g and h, draw the picture, and find the probability.

Sol (a) X = the length of time a commuter must wait for a train to arrive

(b) Train arrives every 8 minutes which means the waiting time is from 0 to 8 mins. Thus, $X \sim U(0,8)$

(c)



(d) $f(x) = \frac{1}{8-0} = \frac{1}{8}, \ 0 \le x \le 8$

(e) $\mu = \frac{0+8}{2} = 4$

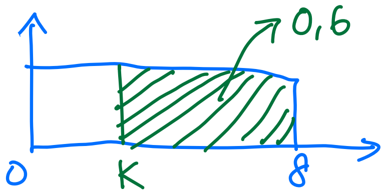
(f) $\sigma = \sqrt{\frac{(8-0)^2}{12}} = \sqrt{\frac{64}{12}} = 2.309$

(g) $P(X<1) = P(0<X<1) = \frac{1}{8} \cdot (1-0) = \frac{1}{8} = 0.125$

(h) $P(3<X<4) = (4-3) \cdot \frac{1}{8} = \frac{1}{8} = 0.125$

(i) Find the k minutes, $k \in [0,8]$, such that

$P(X>k) = 0.6 \Rightarrow (8-k)\frac{1}{8} = 0.6$



$\Rightarrow 8-k = 4.8$

$\Rightarrow k = 8-4.8 = 3.2$ (mins).

Use the following information to answer the next three exercises. The Sky Train from the terminal to the rental–car and long–term parking center is supposed to arrive every eight minutes. The waiting times for the train are known to follow a uniform distribution.

79. What is the average waiting time (in minutes)?

a. zero
b. two
c. three
d. four

$X \sim U(0,8)$ be the waiting time

$$\mu_X = \frac{0+8}{2} = 4$$

81. The probability of waiting more than seven minutes given a person has waited more than four minutes is?

a. 0.125
b. 0.25
c. 0.5
d. 0.75

$$P(X>7 \mid X>4) = \frac{(8-7)\cdot\frac{1}{8}}{(8-4)\cdot\frac{1}{8}} = \frac{1}{4} = 0.25$$

83. Suppose that the value of a stock varies each day from \$16 to \$25 with a uniform distribution.

a. Find the probability that the value of the stock is more than \$19.
b. Find the probability that the value of the stock is between \$19 and \$22.
c. Find the upper quartile - 25% of all days the stock is above what value? Draw the graph.
d. Given that the stock is greater than \$18, find the probability that the stock is more than \$21.

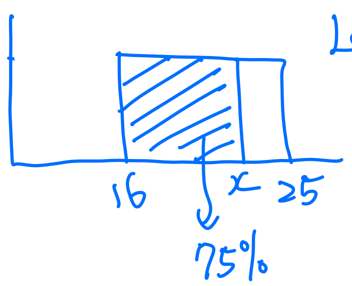
Assume X be the random variable of the value of stock and

$$X \sim U(16,25) \Rightarrow f(x) = \frac{1}{25-16} = \frac{1}{9}, \quad 16 \le x \le 25$$

(a) $P(X>19) = P(19<X<25) = \frac{1}{9}(25-19) = \frac{6}{9} = \frac{2}{3} = 0.67$

(b) $P(19<X<22) = \frac{1}{9}(22-19) = \frac{3}{9} = \frac{1}{3} = 0.33$

(c)



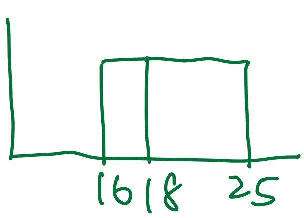
Let x be the upper quartile of the all day stock above. We have

$P(X<x) = 0.75$

$$\Rightarrow \frac{x-16}{25-16} = 0.75 \Rightarrow x = 15+9\cdot 0.75 = 21.75$$

$x = 21.75$

(d)



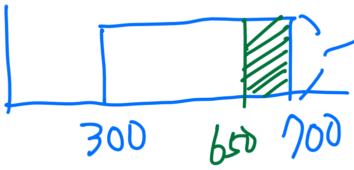
$$P(X>21 \mid X>18) = \frac{P(X>21)}{P(X>18)} = \frac{\frac{25-21}{9}}{\frac{25-18}{9}} = \frac{4}{7}$$

85. The number of miles driven by a truck driver falls between 300 and 700, and follows a uniform distribution.

a. Find the probability that the truck driver goes more than 650 miles in a day.
b. Find the probability that the truck drivers goes between 400 and 650 miles in a day.
c. At least how many miles does the truck driver travel on the furthest 10% of days?

Sol

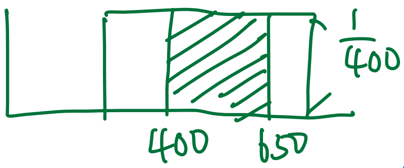
(a)



$$f(x) = \frac{1}{700-300} = \frac{1}{400}, \quad 300 \le x \le 700.$$

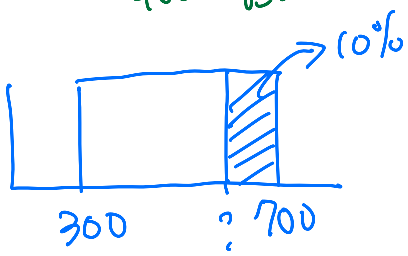
$$P(x>650) = (700-650) \times \frac{1}{400} = \frac{50}{400} = \frac{1}{8}$$

(b)



$$P(400<x<650) = \frac{650-400}{400} = \frac{250}{400} = \frac{5}{8}$$

(c)



$$P(x>?) = 10\%$$

$$\Rightarrow \frac{700-?}{400} = \frac{1}{10}$$

$$\Rightarrow 700-? = 40$$

$$\Rightarrow ? = 700-40 = 660. \text{ (miles)}$$