Mat 1375 HW24

Exercise 24.1

Find the first seven terms of the sequence.

a) $a_n = 3n$ b) $a_n = 5n + 3$ c) $a_n = n^2 + 2$

a) $a_n = 3n \Rightarrow \{\underset{a_1}{3\cdot 1}, \underset{a_2}{3\cdot 2}, \underset{a_3}{3\cdot 3}, \underset{a_4}{3\cdot 4}, \underset{a_5}{3\cdot 5}, \underset{a_6}{3\cdot 6}, \underset{a_7}{3\cdot 7}\}$

$= \{3, 6, 9, 12, 15, 18, 21\}$

b) $a_n = 5n + 3 \Rightarrow \{\underset{a_1}{5\cdot 1+3}, \underset{a_2}{5\cdot 2+3}, \underset{a_3}{5\cdot 3+3}, \underset{a_4}{5\cdot 4+3}, \underset{a_5}{5\cdot 5+3}, \underset{a_6}{5\cdot 6+3}, \underset{a_7}{5\cdot 7+3}\}$

$= \{8, 13, 18, 23, 28, 33, 38\}$

c) $a_n = n^2 + 2 \Rightarrow \{\underset{a_1}{(1)^2+2}, \underset{a_2}{(2)^2+2}, \underset{a_3}{(3)^2+2}, \underset{a_4}{(4)^2+2}, \underset{a_5}{(5)^2+2}, \underset{a_6}{(6)^2+2}, \underset{a_7}{(7)^2+2}\}$

$= \{3, 6, 11, 18, 27, 38, 51\}$

Exercise 24.3

Find the value of the series.

a) $\sum_{n=1}^{4} a_n$, where $a_n = 5n$ b) $\sum_{k=1}^{5} a_k$, where $a_k = k$

c) $\sum_{i=1}^{4} a_i$, where $a_n = n^2$ d) $\sum_{n=1}^{6} (n-4)$

a) $\sum_{n=1}^{4} a_n = \sum_{n=1}^{4} 5n = \underset{n=1}{5\cdot 1} + \underset{n=2}{5\cdot 2} + \underset{n=3}{5\cdot 3} + \underset{n=4}{5\cdot 4} = 5 + 10 + 15 + 20 = 50$

b) $\sum_{k=1}^{5} a_k = \sum_{k=1}^{5} k = \underset{k=1}{1} + \underset{k=2}{2} + \underset{k=3}{3} + \underset{k=4}{4} + \underset{k=5}{5} = 15$

c) $\sum_{i=1}^{4} a_i = \sum_{i=1}^{4} i^2 = (1)^2 + (2)^2 + (3)^2 + (4)^2 = 1 + 4 + 9 + 16 = 30$

d) $\sum_{n=1}^{6} (n-4) = (1-4) + (2-4) + (3-4) + (4-4) + (5-4) + (6-4)$

$= -3$

Exercise 24.4

Is the sequence below part of an arithmetic sequence? If it is part of an arithmetic sequence, find the formula for the nth term a_n in the form $a_n = a_1 + (n-1) \cdot d$.

a) $5, 8, 11, 14, 17, \ldots$ b) $-10, -7, -4, -1, 2, \ldots$
c) $-1, 1, -1, 1, -1, 1, \ldots$ d) $18, 164, 310, 474, \ldots$

a) $5, 8, 11, 14, 17, \cdots$

$+3 \quad +3 \quad +3 \quad +3 \longrightarrow$ common difference $d=3$ $\Rightarrow$ Yes, it is an arithmetic one

form of a_n : $a_n = 5 + (n-1) \cdot 3$.

b) $-10, -7, -4, -1, 2$

$+3 \quad +3 \quad +3 \quad +3 \longrightarrow$ common difference $d=3$ $\Rightarrow$ Yes, it is an arithmetic one

form of a_n: $a_n = -10 + (n-1) \cdot 3$

c) $-1, 1, -1, 1, -1, 1$

$+2 \quad -2 \quad +2 \quad -2 \quad +2 \longrightarrow$ there is no common difference

$\Rightarrow$ Not an arithmetic one.

d) $18, 164, 310, 474$

$+146 \quad +146 \quad +164 \longrightarrow$ there is no common difference

$(164-18) \quad (310-164) \quad (474-310=164)$

$\Rightarrow$ Not an arithmetic one

Exercise 24.5

Determine the general nth term a_n of an arithmetic sequence $\{a_n\}$ with the data given below.

✓a) $d = 4$, and $a_8 = 57$ ✓b) $d = -3$, and $a_{99} = -70$

a) $a_n = a_8 + (n-8)\cdot d$
$= 57 + (n-8)\cdot 4$
$= 57 + 4n - 32$
$= 4n + 25$

b) $a_n = a_{99} + (n-99)\cdot d.$
$= -70 + (n-99)\cdot(-3)$
$= -70 - 3n + 297$
$= -3n + 227$

Exercise 24.7

Determine the sum of the arithmetic sequence.

✓a) Find the sum $a_1 + \cdots + a_{48}$ for the arithmetic sequence $a_n = 4n + 7$.

✓b) Find the sum $\sum_{n=1}^{21} a_n$ for the arithmetic sequence $a_n = 2 - 5n$.

a) $a_1 + \cdots + a_{48} = \sum_{n=1}^{48}(4n+7) = \frac{48}{2}(a_1 + a_{48}) = \frac{48}{2}(11+199) = \frac{\cancel{48}^{24}}{\cancel{2}}(210)$
$= 5040$
$a_1 = 4\cdot 1 + 7 = 11$
$a_{48} = 4\cdot 48 + 7 = 199$

b) $\sum_{n=1}^{21} a_n = \sum_{n=1}^{21}(2-5n) = \frac{21}{2}(a_1 + a_{21}) = \frac{21}{2}(-3 - 103)$
-53
$= \frac{21}{\cancel{2}}\cdot(-\cancel{106}) = -1113$
$a_1 = 2 - 5\cdot 1 = -3$
$a_{21} = 2 - 5\cdot 21 = -103$

✓e) Find the sum of the first 100 terms of the arithmetic sequence:

$$2, 4, 6, 8, 10, 12, \ldots$$

$+2$ $+2$ $+2$ $+2$ $+2$ $\longrightarrow d = 2.$ $a_n = 2 + (n-1)\cdot 2$

$\sum_{n=1}^{100} a_n = \frac{100}{2}(a_1 + a_{100}) = 50\cdot(2+200) = 10100$

$(a_{100} = 2 + (100-1)\cdot 2 = 200)$

f) Find the sum of the first 83 terms of the arithmetic sequence:

$$25, 21, 17, 13, 9, 5, \ldots$$

$-4 \quad -4 \quad -4 \quad -4 \quad -4 \longrightarrow d=-4$

$a_n = 25+(n-1)\cdot(-4)$

$$\sum_{n=1}^{83} a_n = \frac{83}{2}(25+a_{83}) = \frac{83}{2}(25-323)$$

-159

$$\boxed{\begin{aligned} a_{83} &= 25+(82)\cdot(-4) \\ &= 25-328 \\ &= -323 \end{aligned}}$$

$$= \frac{83}{\cancel{2}}(-\cancel{318}) = -13197$$

$$\begin{array}{r} -159 \\ 83 \\ \hline 477 \\ 1272 \end{array}$$

g) Find the sum of the first 75 terms of the arithmetic sequence:

$$2012, 2002, 1992, 1982, \ldots$$

$-10 \quad -10 \quad -10 \longrightarrow d=-10$

$a_n = 2012+(n-1)\cdot(-10)$

$$\sum_{n=1}^{75} a_n = \frac{75}{2}(2012+a_{75}) = \frac{75}{2}(2012+1272)$$

$$\boxed{\begin{aligned} a_{75} &= 2012+(75-1)\cdot(-10) \\ &= 2012-740 \\ &= 1272 \end{aligned}}$$

1642

$$= \frac{75}{\cancel{2}}(\cancel{3284}) = 123150$$

$$\begin{array}{r} 1642 \\ 75 \\ \hline 8210 \\ 11494 \end{array}$$

i) Find the sum of the first 99 terms of the arithmetic sequence:

$$-8, -8.2, -8.4, -8.6, -8.8, -9, -9.2, \ldots$$

$-0{,}2 \quad -0{,}2 \quad -0{,}2 \quad -0{,}2 \quad -0{,}2 \quad -0{,}2 \longrightarrow d = -0.2$

$a_n = -8 + (n-1)\cdot(-0{,}2)$

$$\sum_{n=1}^{99} a_n = \frac{99}{2}(a_1 + a_{99}) = \frac{99}{2}(-8 - 27.6) = \frac{99}{\cancel{2}}(\cancel{-35{,}6})^{-17{,}8}$$

$$\boxed{\begin{aligned} a_{99} &= -8 + 98\cdot(-0{,}2) \\ &= -27{,}6 \end{aligned}}$$

$= -1778{,}2$