Math 1431
Final Exam Review

1. Find the following limits (if they exist):

a. $\lim\limits_{x->0} \frac{\sin 7x}{9x}$

b. $\lim\limits_{x->0} \frac{5x}{\tan(2x)} =$

c. $\lim\limits_{x->2} \frac{x^3-8}{x^2-4} =$

d. $\lim\limits_{x->1} \frac{x^2-1}{x+1} =$

e. $\lim\limits_{h\to 0} \frac{\sin(h)}{h}$

f. $\lim\limits_{h\to 0} \frac{1-\cos(h)}{h}$

g. $\lim\limits_{x\to 1} \frac{1}{x-1}$

h. $\lim\limits_{x\to 0} \frac{1-\cos(2x)}{x^2}$

i. $\lim\limits_{n\to\infty} \left(\frac{n}{n+1}\right)^{2n}$

j. $\lim\limits_{x->0} \frac{e^{x^2}-1}{3x^2}$

k. $\lim\limits_{x->0} \frac{\arctan(2x)}{4x}$

l. $\lim\limits_{x->0} \frac{3x}{\sin(4x)} =$

m. $\lim\limits_{x->0} 2x\cot(3x)$=

n. $\lim\limits_{x->-1} \frac{x+1}{x^2+3x+2} =$

o. $\lim\limits_{x->-1} \frac{x^2+3x+2}{x^2+1} =$

p. $\lim\limits_{x\to 1} \frac{x^2+2x}{x+1}$

q. $\lim\limits_{x\to -1} \frac{x^2-x-2}{x^2-1}$

r. $\lim\limits_{x\to 2} \left(3x^2-2x+1\right)$

s. $\lim\limits_{n\to\infty} \frac{\ln(n+4)}{\ln(2n+1)}$

t. $\lim\limits_{x->\infty} \left(\cos\left(\frac{1}{x}\right)\right)^x$

u. $\lim\limits_{x->0} \left[\frac{1}{\ln(x+1)} - \frac{1}{x}\right]$

2. Use the definition of the derivative to compute the derivative of $f(x) = \sqrt{x+1}$.

3. Find values of A and/or B so that the function is continuous:

a. $f(x) = \begin{cases} x^2 & x<1 \\ Ax-3 & x \geq 1 \end{cases}$

b. $f(x) = \begin{cases} Ax-B & x \leq 1 \\ 3x & 1<x\leq 2 \\ Bx^2-A & x>2 \end{cases}$

c. $f(x) = \begin{cases} 3x^2-1 & x<4 \\ A & x=4 \\ Bx-1 & x>4 \end{cases}$

4. Determine if it is possible to find A so that f(x) is continuous and if it is, find A.

a. $$f(x)=\begin{cases} Ax & x>3 \\ x^2-3 & x\le 3 \end{cases}$$

b. $$f(x)=\begin{cases} x^2-1 & x<3 \\ 8 & x=3 \\ Ax-1 & x>3 \end{cases}$$

c. $$f(x)=\begin{cases} x^2+2 & x<-1 \\ 1 & x=-1 \\ Ax-2 & x>-1 \end{cases}$$

d. $$f(x)=\begin{cases} 2x-1, & x\le 2 \\ Ax^2-3x & x>2 \end{cases}$$

5. Find the derivative of the following:

a. $f(x)=\dfrac{3}{\sqrt{x^3-2x}}$

b. $y=\cos^3(2x)$

c. $f(x)=x\tan x$

d. $f(x)=-3x\cos(2x)$

e. $f(x)=\dfrac{2x^2}{x^2+3x}$

f. $f(x)=\sin(x^2+2x)$

g. $f(x)=\dfrac{x}{x^2+2}$

h. $y=-\cos(x^2-3x+5)$

i. $f(x)=(\sin(2x)-\cos(3x))^3$

j. $f(x)=(4\sin x+\cos(5x))^5$

k. $f(x)=\sin(2x)$

l. $f(x)=\cos(3x)$

m. $f(x)=\tan(4x)$

n. $f(x) = \cot(x)$

o. $f(x) = \sec(x)$

p. $f(x) = \csc(2x)$

q. $f(x) = -2x^3 + 4x^2 - 7$

r. $f(x) = x\sin(x)$

s. $f(x) = \sqrt{1+x}$

t. $f(x) = 1/(x+1)$

u. $f(x) = (\tan(2x) + x)^4$

v. $y = (3x-1)^{\sin(x)}$

w. $y = (x+1)^{\ln(x)}$

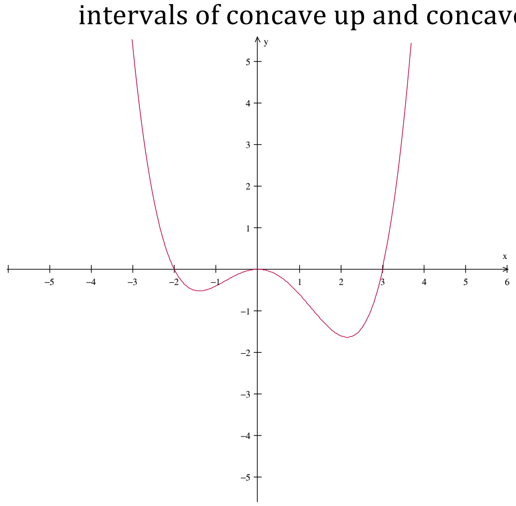
x. $y = (x^2+2)^{(\frac{1}{\ln x})}$

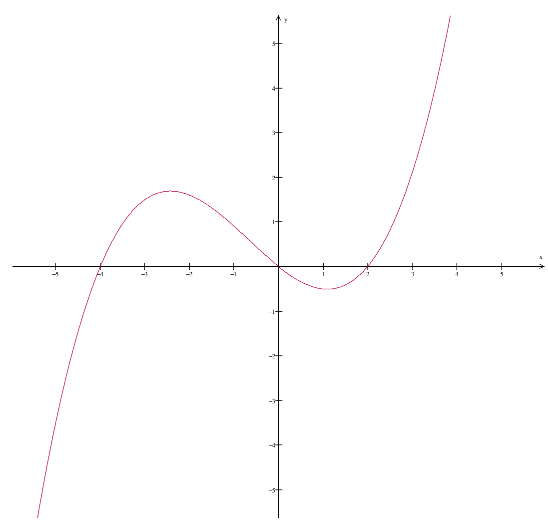
6. Use the intermediate value theorem to show that the function $f(x) = 2x^5 + 3x + 1$ has a root on the interval $[-1,2]$.

7. Notice that $(x,y) = (1,2)$ is a solution to the equation $xy^3 + y = 10x$. Compute dy/dx at $(x,y) = (1,2)$.

8. Give an equation for the tangent line to the graph of the function $f(x) = 2x^2 - 3x + 1$ at the point where $x = -1$.

9. Each of the graphs below are graphs of f'. Determine where f is increasing, decreasing, intervals of concave up and concave down:



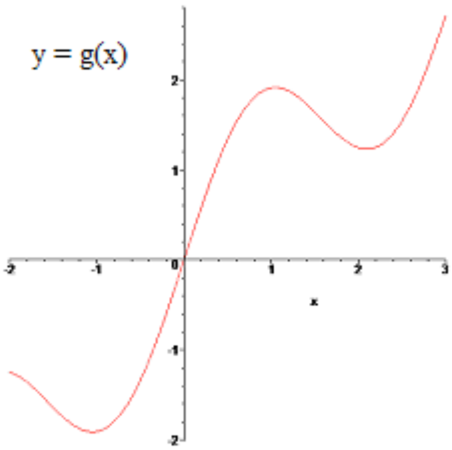


10. The value *x=a* is a critical number for *f(x)*, Classify *a* as a local maximum, local minimum or neither.

a. $f'(x) = x^3 - 12x^2, x = 0$

b. $f''(x) = x^2 - 2x + 1, x = 1$

11. Use the plot of the function on the interval [-2,3] to give a geometric depiction of the mean value theorem.



12. Give the differential of $f(x) = x^2 - 3x$ at $x = 1$ with respect to the increment 1/10.

13. Use differentials to approximate $\sqrt{65}$.

14. Evaluate:

a. $\dfrac{d}{dx}\displaystyle\int_{3x}^{x^3} \frac{1}{t}\,dt$

b. $\dfrac{d}{dx}\displaystyle\int_0^x \sin\left(t^2\right)dt$

c. $\dfrac{d}{dx}\displaystyle\int_0^{2-3x} \sin\left(t^2\right)dt$

15. The function f(x) given below is continuous, find a formula for f(x):

a. $-2x^4 - 3x^2 - 6 = \int_2^x \frac{f(t)}{t+2}\,dt$

b. $2x^3 - 3x^2 + x - 1 = \int_x^{-1} f(t)dt$

16. Integrate:

a. $\int \frac{\csc^2 x}{\sqrt{\cot x}}\,dx$

b. $\int \frac{x+2}{x^3}\,dx$

c. $\int \left(3x^3 - 2x^2 + 5\right)dx$

d. $\int_1^4 \sqrt{x}dx$

e. $\int_{-8}^0 \frac{1}{\sqrt{1-x}}\,dx$

f. $\int \sin^3 3x \cos 3x\,dx$

g. $\int_2^7 x\sqrt{x^2 + 2}dx$

h. $\int \left(x^2 - 2\right)\cos\left(x^3 - 6x\right)dx$

i. $\int \frac{2x}{\sqrt{9 - x^2}}\,dx$

j. $\int_0^1 \frac{2x}{\left(x^2 + 3\right)^4}\,dx$

k. $\int \sin(2x)dx$

l. $\int \cos(3x)dx$

m. $\int \sec^2(2x)dx$

n. $\int \csc^2(3x)dx$

o. $\int \sec(2x)\tan(2x)dx$

p. $\int \sqrt{x+1}dx$

q. $\int x(x^2+1)^4\,dx$

r. $\int (\cosh(3x)+\sinh(2x))dx$

s. $\int 4^{3x}\,dx$

t. $\int \frac{\log_2(x^3)}{x}dx$

u. $\int (2^{7x}-\sinh(5x))dx$

v. $\int \frac{\sin(3x)}{16+\cos^2(3x)}dx$

w. $\int \frac{6x}{4+x^4}dx$

x. $\int \tan(3x)dx$

y. $\int \frac{\arctan(3x)}{1+9x^2}dx$

17. Use the definition of derivative to find the derivative of

a. $f(x)=\sqrt{x+1}$

b. $f(x)=\dfrac{2}{x-3}$

18. Write the equation of the tangent line to

a. $y^2-xy+6=0$ at the point $(5,2)$.

b. $2x^2-5xy+y^2=4$ at the point $(3,1)$.

19. As a balloon in the shape of a sphere is being blown up, the volume is increasing at a rate of 4 in^3/sec. At what rate is the radius increasing when r = 1 in.?

20. Sand is falling off a conveyor onto a conical pile at a rate of 15 cubic feet per minute. The diameter of the base of the cone is twice the altitude. At what rate is the height of the pile changing when it is 10 feet high?

21. You have a rectangular piece of cardboard 6 inches wide and 16 inches long that you wish to fold into a box. It occurs to you that you can cut an equal square from each corner of the cardboard, make a crease along each side and fold the sides up to make the box. How much should you cut from the corners to form the box with maximum volume?

22. A man is walking away from a light pole at a rate of 5 feet per second. If the light pole is 20 feet tall and the man is 6 feet tall, how fast is his shadow growing when the man is 30 feet from the light pole?

23. A rectangle is drawn in the first quadrant so that its base is on the x axis and its left side is on the y axis. What is the maximum area of this rectangle if its upper right vertex lies on the line segment connecting the points $(4,0)$ and $(0,8)$?

24. Suppose f is a differentiable function on the interval $[a,b]$.

    a. Explain how to find the absolute maximum and absolute minimum values of f on the interval $[a,b]$.

    b. Use a graph to demonstrate that a function can have its absolute maximum value occur at exactly 3 places.

25. State the mean value theorem.

26. List the domain, critical numbers, intervals of increase, intervals of decrease, inflection points, intervals of concave up, and intervals of concave down for the function given. Then graph the function and carefully label any local maximums, local minimums or points of inflection.

    a. $f(x) = (x+1)^2(x-2)$

    b. $f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{9}{4}$

27. Let $f(x) = x^3 - 3x$ be defined on [-1, 1]. Find *c* on (-1, 1) that satisfies the conclusion of the Mean Value Theorem.

28. Estimate $\cos(28^\circ)$ using differentials.

29. Compute the Riemann sum for the function $f(x) = x^2 + 1$ on the interval [-1,2] associated with the partition P={-1,0,1,2}, given that the heights of the rectangles are created by using the midpoint of each subinterval.