MAT 1375, Classwork20, Fall2024

ID:________________________ Name:________________________

1. Find all exact solution in radians.

$$2\sin^2(x) + \sqrt{3}\sin(x) = 0.$$

$$2\boxed{\sin(x)}\cdot\sin(x) + \sqrt{3}\,\boxed{\sin(x)} = 0$$

$$\sin(x)\left(2\cdot\sin(x) + \sqrt{3}\right) = 0 \Rightarrow \sin(x) = 0 \quad \text{or} \quad 2\sin(x) + \sqrt{3} = 0$$

$$\sin(x) = -\frac{\sqrt{3}}{2}$$

$x = 0, \pi, 2\pi, 3\pi, 4\pi, \cdots$

$-\pi, -2\pi, -3\pi, \cdots$

$x = n\cdot\pi$, n is all integers

$x = \frac{4\pi}{3}, \frac{4\pi}{3}+2\pi, \cdots$

$\frac{4\pi}{3}-2\pi, \cdots$

$\frac{5\pi}{3}, \frac{5\pi}{3}+2\pi, \frac{5\pi}{3}-2\pi \cdots$

$x = \frac{4\pi}{3} + n\cdot(2\pi)$ or $\frac{5\pi}{3} + n\cdot(2\pi)$, n is all integers

2. Find all exact solution in radians.

$$2\cos^2(x) - \cos(x) - 1 = 0.$$

$\cos(x) \quad -1$

$2\cos(x) \quad +1$

$$\Rightarrow (\cos(x) - 1)\cdot(2\cos(x) + 1) = 0$$

$\cos(x) - 1 = 0 \Rightarrow \cos(x) = 1$

$x = (2\pi)\cdot n$, where n is all integers

$2\cos(x) + 1 = 0 \Rightarrow \cos(x) = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{2\pi}{3}+2\pi, \frac{2\pi}{3}+4\pi, \cdots$

$\frac{4\pi}{3}, \frac{4\pi}{3}+2\pi, \cdots$

$x = \frac{2\pi}{3} + n\cdot(2\pi)$ or $\frac{4\pi}{3} + n\cdot(2\pi)$ where n is all integers

3. Find all exact solution in radians.

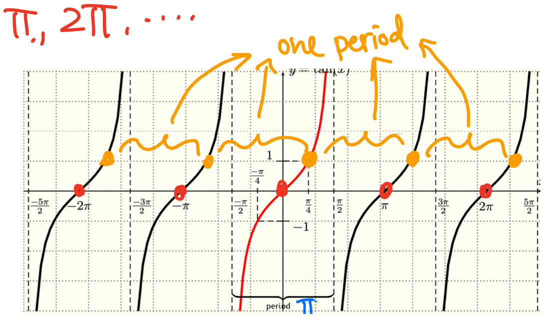
$$\tan^2(x) - \tan(x) = 0.$$

$$\tan(x)\left(\tan(x) - 1\right) = 0 \Rightarrow \tan(x) = 0 \quad \text{or} \quad \tan(x) = 1$$

$x = \cdots, \frac{\pi}{4}-2\pi, \frac{\pi}{4}-\pi, \frac{\pi}{4}, \frac{\pi}{4}+\pi, \cdots$

$x = \cdots, -2\pi, -\pi, 0, \pi, 2\pi, \cdots$

$\Rightarrow x = n\pi$ where n is all integers.



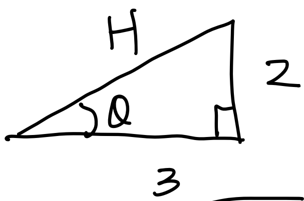
$\Rightarrow x = \frac{\pi}{4} + n\cdot\pi$ where n is all integers.

4. Given $\tan(\theta) = -\frac{2}{3}$ and $\cos(\theta) > 0$. Find $\sin(\theta)$, $\cos(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$.

$\cos(\theta) > 0 \Rightarrow \theta$ is in I or IV

since $\tan(\theta) = -\frac{2}{3} < 0 \Rightarrow \theta$ is in IV

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{-2}{3}$ $\quad \sin(\theta) = \frac{-2}{\sqrt{13}}$ $\quad \cos(\theta) = \frac{3}{\sqrt{13}}$



$\cot(\theta) = \frac{1}{\tan(\theta)} = -\frac{3}{2}$

$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{\sqrt{13}}{3}$, $\csc(\theta) = \frac{1}{\sin(\theta)} = -\frac{\sqrt{13}}{2}$

$H = \sqrt{2^2+3^2} = \sqrt{13}$

5. Given $\cos(\theta) = -\frac{1}{4}$ and $\sin(\theta) < 0$. Find $\sin(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$.

$\sin(\theta) < 0 \Rightarrow \theta$ is in III or IV $\Rightarrow$ III.

$\cos(\theta) = -\frac{1}{4} \Rightarrow \theta$ is in II or III

$\sin^2(\theta) + \cos^2(\theta) = 1 \Rightarrow \sin^2(\theta) + \left(-\frac{1}{4}\right)^2 = 1 \Rightarrow \sin^2(\theta) = 1 - \frac{1}{16} = \frac{15}{16}$

$\Rightarrow \sin(\theta) = \pm\frac{\sqrt{15}}{4}$ (since $\sin(\theta) < 0$) $\quad \sec(\theta) = \frac{1}{\cos(\theta)} = -4$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{-\frac{\sqrt{15}}{4}}{-\frac{1}{4}} = \sqrt{15}$ $\quad \csc(\theta) = \frac{1}{\sin(\theta)} = -\frac{4}{\sqrt{15}} = -\frac{4}{15}\sqrt{15}$.

$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$

6. Given $\sin(\alpha) = -\frac{4}{5}$ and α be in quadrant III. Find the exact values of the trigonometric functions of $\frac{\alpha}{2}$ and of 2α by using the half-angle and double-angle formulas.

$\sin(2\alpha) = \sin(\alpha)\cdot\underbrace{\cos(\alpha)} + \underbrace{\cos(\alpha)}_{\text{unknown}}\sin(\alpha)$

$\sin\left(\frac{\alpha}{2}\right) = \pm\cancel{-}\sqrt{\frac{1-\cos(\alpha)}{2}}$ (since $\frac{\alpha}{2}$ will be in I or II and $\sin\left(\frac{\alpha}{2}\right) > 0$ in I or II)

$\cos^2(\alpha) + \sin^2(\alpha) = 1 \Rightarrow \cos^2(\alpha) + \left(-\frac{4}{5}\right)^2 = 1 \Rightarrow \cos^2(\alpha) = 1 - \frac{16}{25} = \frac{9}{25}$

$\cos(\alpha) = \cancel{+}-\frac{3}{5}$ (since α is in III)

$\Rightarrow \boxed{\sin(2\alpha)} = \sin(\alpha)\cos(\alpha) + \cos(\alpha)\cdot\sin(\alpha) = \left(-\frac{4}{5}\right)\cdot\left(-\frac{3}{5}\right) + \left(-\frac{3}{5}\right)\cdot\left(-\frac{4}{5}\right)$

$= \frac{12}{25} + \frac{12}{25} = \boxed{\frac{24}{25}}$ and

$$\boxed{\sin\left(\frac{\alpha}{2}\right)} = +\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{8}{10}} = \frac{2}{\sqrt{5}} = \boxed{\frac{2}{5}\sqrt{5}}$$