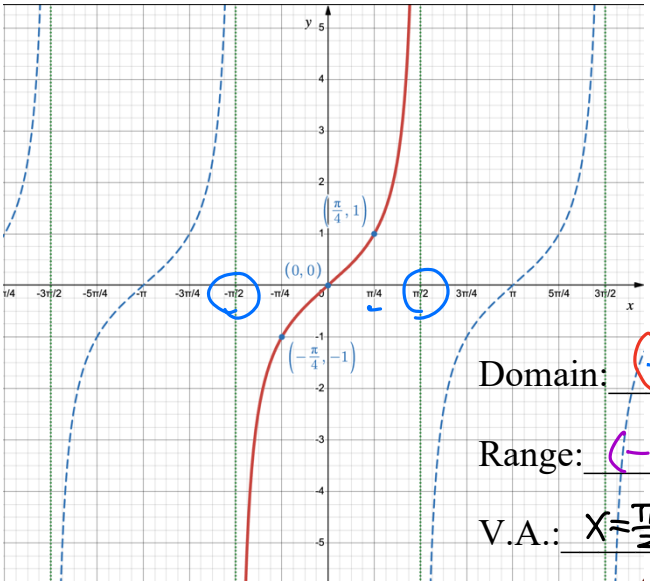
MAT 1375, Classwork19, Fall2024

ID:_____ Name:_____

1. The graph of $y = \tan^{-1}(x)$:

arctan(x)

$y = \tan(x)$ (the restricted tangent)

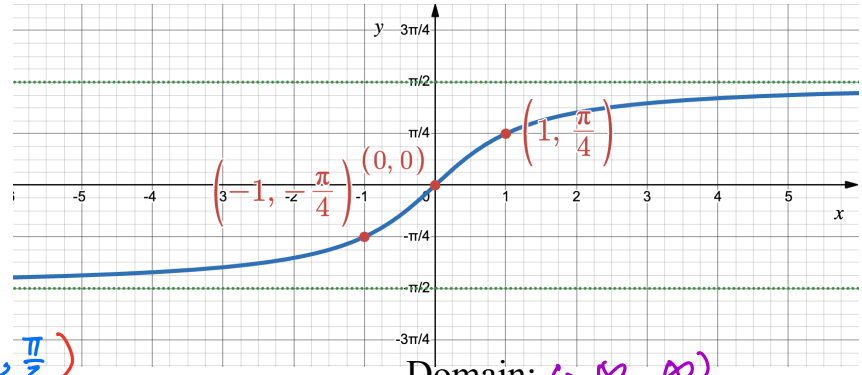


Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Range: $(-\infty, \infty)$.

V.A.: $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$.

$y = \tan^{-1}(x)$



Domain: $(-\infty, \infty)$.

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

H.A.: $y = -\frac{\pi}{2}$, $y = \frac{\pi}{2}$

2. How to find the value of $\tan^{-1}(x)$:

Let $\theta = \tan^{-1}(x)$. It implies $x =$ tan(θ) where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Then find the θ from the following table:

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan(\theta)$	$\frac{\sin(\theta)}{\cos(\theta)}$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	$\frac{0}{1}=0$	$\frac{1}{2}$	$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=1$	$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$	undefined.

undefined

3. Find the value of the given inverse tangent functions.

a) $\tan^{-1}(\sqrt{3})$ d) $\tan^{-1}(-1)$

a) Let $\theta = \tan^{-1}(\sqrt{3})$

$\tan(\theta) = \sqrt{3}$

$\theta = \frac{\pi}{3}$

b)

$\theta = -\frac{\pi}{4}$

4. Composition of Functions Involving with Inverse Trigonometry Functions.

a) $\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)$ b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$ c) $\cos(\cos^{-1}(0.6))$ d) $\cos(\cos^{-1}(1.5))$

e) $\sin^{-1}\left(\sin\left(\frac{3\pi}{2}\right)\right)$ f) $\sin^{-1}(\sin(\pi))$ g) $\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$

a) $\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

$\sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

c) $\cos(\cos^{-1}(0.6)) = \cos(\theta) = 0.6$

Let $\theta = \cos^{-1}(0.6) \Rightarrow \cos(\theta) = 0.6$

d) $\cos(\cos^{-1}(1.5)) =$ undefined

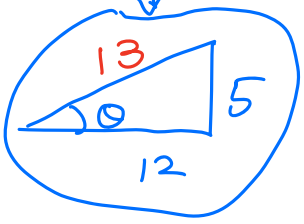
Let $\theta = \cos^{-1}(1.5) \Rightarrow \cos(\theta) = 1.5 \Rightarrow \theta$ is undefined

e) $\sin^{-1}\left(\sin\left(\frac{3\pi}{2}\right)\right) = \sin^{-1}(-1) = -\frac{\pi}{2}$

f) $\sin^{-1}(\sin(\pi)) = \sin^{-1}(0) = 0$

g) $\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \cos(\theta)$

Let $\theta = \tan^{-1}\left(\frac{5}{12}\right) \Rightarrow \tan(\theta) = \frac{5}{12}$ and $0 \le \theta < \frac{\pi}{2}$ ← why?



Since $0 \le \theta < \frac{\pi}{2}$, $\sin(\theta) > 0$, $\cos(\theta) > 0$

Then $\sin(\theta) = \frac{5}{13}$, $\cos(\theta) = \frac{12}{13}$

$\Rightarrow \cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \cos(\theta) = \frac{12}{13}$