MAT2540, Classwork5, Spring2026

6.3 Permutations and Combinations

1. How many three letter "words" can be made from the letters a, b, and c with **no letters repeating**? A "word" is just an ordered group of letters. It doesn't have to be a real word in a dictionary.

abc, acb, bac, bca, cab, cba

six ways.

1st 2nd 3rd

$3 \times 2 \times 1 = 6$

2. Definition of Permutation and r-Permutation.

A permutation of a set of distinct objects is an ordered arrangement of these objects. Let A be a finite set with n elements. For $1 \le r \le n$, an r-permutation of A is an ordered selection of r distinct elements from A.

3. Example.

Let $S = \{1, 2, 3\}$. A 2-permutation of S: The **ordered** arrangement: 1,2; 1,3; 2,3; 2,1; 3,1; 3,2

A permutation of S: The **ordered** arrangement: 1,2,3; 1,3,2; 2,1,3; 2,3,1; 3,1,2; 3,2,1;

$n=3, r=2.$ $P(2,3) = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 6$ $P(3,3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 6$

4. The notation ~~$P(r, n)$~~. $P(n,r)$

Let n be a positive integer and r an integer with $1 \le r \le n$. The number of r-permutations of a set with n elements is denoted by ~~$P(r, n)$~~: $P(n,r)$

$P(n,r)$ ~~$P(r, n)$~~ $= n \times (n-1) \times (n-2) \times \cdots \times (n-r+2) \times (n-r+1)$ or $\frac{n!}{(n-r)!}$

where factorial $n! = n \times (n-1) \times (n-2) \times \cdots \times (2) \times (1)$.

$0! = 1$

5. Example. $P(5,3)$ ~~$P(3,5)$~~ $= \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$

$P(7,0)$ ~~$P(0,7)$~~ $= \frac{7!}{(7-0)!} = \frac{7!}{7!} = 1$

We can extend the definition of ~~$P(r, n)$~~ $P(n,r)$ when $0 \le r \le n$ if we use ~~$P(r, n)$~~ $P(n,r)$ $= \frac{n!}{(n-r)!}$.

6. How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

1st 2nd 3rd

$100 \cdot 99 \cdot 98 = 970200$ or

$P(100,3) = \frac{100!}{(100-3)!} = \frac{100!}{97!}$

$= \frac{100 \times 99 \times 98 \times 97 \times 96 \times \cdots \times 1}{97 \times 96 \times \cdots \times 1}$

$= 100 \times 99 \times 98$

7. How many permutations of the letters $\boxed{\text{ABC}}$DEFGH contain the string ABC ?

ABC cannot be separated, we see ABC as one block.
With the other five letters, we totally have 6 blocks.

$P(6,6) = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6! = 720$

8. How many different committees of three students can be formed from a group of four students?

student A, B, C, D, ⇒ ABC (no D)   ABD (no C)   ACD (no B)   BCD (no A)

9. Definition of r-Combination.

Let n be a positive integer and r an integer with $0 \le r \le n$. An r-combination of elements of a finite set with n elements is an unordered selection of r elements from the set. Thus, an r-combination is simply a subset of the n-element set with r elements.

10. The notation $\cancel{C(r,n)}$. $C(n,r)$, C^n_r, $\binom{n}{r}$

Let n be a positive integer and r an integer with $0 \le r \le n$. The number of r-combinations of a set with n elements is called a binomial coefficient and is denoted by $C(r,n)$ or $\binom{n}{r}$:

$$C(n,r)\ \cancel{C(r,n)} = \frac{n!}{r!\,(n-r)!} \text{ or } \frac{P(n,r)}{r!}$$

11. Let $S = \{1,2,3\}$. A 2- combination of S:

$\{1,2\}$; $\{1,3\}$; $\{2,3\}$ $C(3,2) = \frac{3!}{1!\,2!} = 3$

12. Binomial Coefficient, Pascal's Triangle, and Pascal's Identity.

n	$\binom{n}{r} = \frac{n!}{r!(n-r)!}$ Binomial Coefficient	Pascal's Triangle
0	$\binom{0}{0}$ = 1	1
1	$\binom{1}{0}$ = 1, $\binom{1}{1}$ = 1	1 1
2	$\binom{2}{0}$ = 1, $\binom{2}{1}$ = 2, $\binom{2}{2}$ = 1	1 2 1
3	$\binom{3}{0}$ = 1, $\binom{3}{1}$ = 3, $\binom{3}{2}$ = 3, $\binom{3}{3}$ = 1	1 3 3 1
4	$\binom{4}{0}$ = 1, $\binom{4}{1}$ = 4, $\binom{4}{2}$ = 6, $\binom{4}{3}$ = 4, $\binom{4}{4}$ = 1	1 4 6 4 1
5	$\binom{5}{0}$ = 1, $\binom{5}{1}$ = 5, $\binom{5}{2}$ = 10, $\binom{5}{3}$ = 10, $\binom{5}{4}$ = 5, $\binom{5}{5}$ = 1	1 5 10 10 5 1

Pascal's Identity: $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$ For example, $\binom{5}{2} = \binom{4}{1} + \binom{4}{2}$

13. Observation: $\binom{5}{3} = \binom{5}{2}$, $\binom{4}{0} = \binom{4}{4}$, and $\binom{5}{4} = \binom{5}{1}$

Let n and r be nonnegative integers with $r \le n$. Then $\binom{n}{r} = \binom{n}{n-r}$.

14. How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

15. Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?