

MAT1375, Classwork11, Fall2025

Ch10. Rational Functions II

6. The order of growth for functions: **Power functions.**

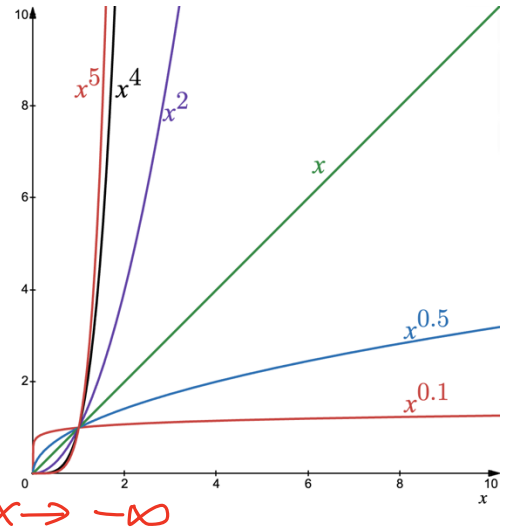
$$x^p, p > 0$$

For each $x > 1$, we have

$$\dots < x^{0.1} < x^{0.5} < x < x^2 < x^4 < x^5 < \dots$$

7. The definition of a **Horizontal Asymptote**:

The line $y = b$ is a horizontal asymptote of the graph of a function f if $f(x)$ approaches b as x increases or decreases without bound.



8. What is the difference of Vertical Asymptote and Horizontal Asymptote?

Vertical asymptote occurs at $x=c$ when $x \rightarrow c, f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$ and vertical asymptote is the line $x=c$. On the other hand, horizontal asymptote occurs at $x \rightarrow \infty$ and $f(x)$ approaches to a number c . and horizontal asymptote is the line $y=c$.

9. How to locate Horizontal Asymptotes: Let $f(x) = \frac{p(x)}{q(x)}$ be a rational function given by

$$f(x) = \frac{p_n x^n + p_{n-1} x^{n-1} + \dots + p_1 x + p_0}{q_m x^m + q_{m-1} x^{m-1} + \dots + q_1 x + q_0}, p_n \neq 0, q_m \neq 0.$$

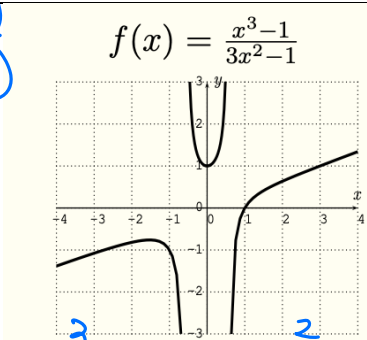
where the degree of the numerator is n and the degree of the denominator is m .

(1) If $n > m$, the graph of f has no horizontal asymptote.

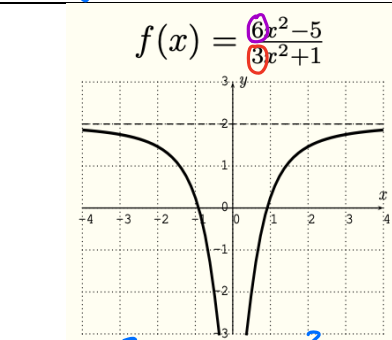
(2) If $n = m$, the line $y = \frac{p_n}{q_m}$ (which is the ratio of two leading coefficients) is the horizontal asymptote of the graph of f .

(3) If $n < m$, the x-axis (which is $y=0$) is the horizontal asymptote of the graph of f .

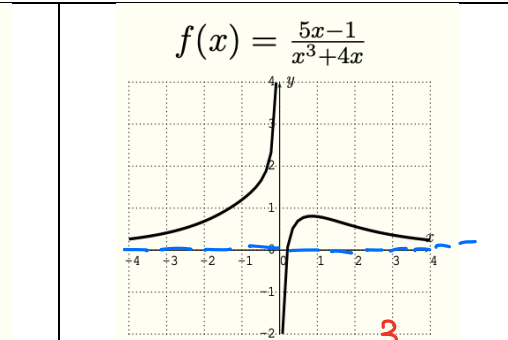
$$f(x) = \frac{p(x)}{q(x)}$$



$\deg(p(x)) > \deg(q(x))$
no horizontal asymp



$\deg(p(x)) = \deg(q(x))$
 $y = \frac{6}{3} = 2$



$\deg(p(x)) < \deg(q(x))$
 $y=0$ (x-axis)

Ch11. More on Rational Functions

1. The graph of $f(x) = \frac{p(x)}{g(x)}$ is displayed below, where $\deg(p(x))=1$ and $\deg(g(x))=3$.

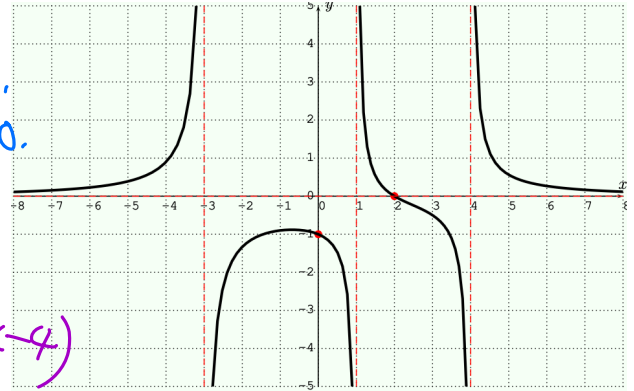
Find the intercepts, asymptotes, and a formula for $f(x)$.

\checkmark X-intercept $(2, 0)$
 \checkmark Y-intercept $(0, -1)$
 \checkmark V.A. $x = -3, x = 1, x = 4$
 \checkmark H.A. $y = 0$

$f(2) = \frac{p(2)}{g(2)} = 0$
 \Downarrow
 $p(2) = 0$
 $(g(2) \neq 0)$
 2 is a root of $p(x)$

$f(0) = \frac{p(0)}{g(0)} = -1$
 \Downarrow
 $p(0) = -1$

$g(x) = (x+3)(x-1)(x-4)$



$$f(x) = \frac{6 \cdot (x-2)}{(x+3)(x-1)(x-4)} \Rightarrow f(0) = \frac{6(0-2)}{(0+3)(0-1)(0-4)} = \frac{-2}{12} \cdot 6 = -1$$

For 2. And 3., let $f(x) = \frac{p(x)}{g(x)}$ be a rational function and $\deg(p(x)) > \deg(g(x))$.

2. Rational Function and Long Division:

If $p(x)$ divided by $g(x)$ can be represented with a quotient $q(x)$ and a remainder $r(x)$ where $\deg(r(x)) < \deg(g(x))$, one can rewrite $f(x)$ as

$$f(x) = \frac{p(x)}{g(x)} = \text{_____} + \frac{r(x)}{g(x)}.$$

3. Asymptotic Behavior with Slant Asymptote:

Since $\deg(r(x)) < \deg(g(x))$, for large $|x|$ (which is $x \rightarrow \pm \text{_____}$), we have

$$\frac{r(x)}{g(x)} \text{ approaches } \text{_____} \text{ so that } f(x) \text{ approaches } q(x).$$

If $q(x)$ is a linear function (which is a polynomial of degree _____), then q is called the _____ asymptote of f .

4. Find the **slant asymptote** of the rational function $f(x) = \frac{2x^3 - 13x^2 + 35x - 26}{x^2 - 4x + 6}$.