

# MAT2540, Classwork5, Spring2026

## 6.3 Permutations and Combinations

1. How many three letter “words” can be made from the letters a, b, and c with **no letters repeating**? A “word” is just an ordered group of letters. It doesn’t have to be a real word in a dictionary.

2. Definition of Permutation and  $r$ -Permutation.

A \_\_\_\_\_ of a set of distinct objects is an \_\_\_\_\_ of these objects. Let  $A$  be a finite set with  $n$  elements. For \_\_\_\_\_, an \_\_\_\_\_ of  $A$  is an ordered selection of \_\_\_\_\_ from  $A$ .

3. Example.

Let  $S = \{1, 2, 3\}$ . A 2-permutation of  $S$ : The **ordered** arrangement: \_\_\_\_\_

A permutation of  $S$ : The **ordered** arrangement: \_\_\_\_\_

4. The notation  $P(r, n)$ .

Let  $n$  be a positive integer and  $r$  an integer with  $1 \leq r \leq n$ . The number of  $r$ -permutations of a set with  $n$  elements is denoted by  $P(r, n)$ :

$$P(r, n) = \text{_____} \text{ or } \text{_____}$$

where factorial  $n! = \text{_____}$ .

5. Example.  $P(3, 5) =$

$$P(0, 7) =$$

We can extend the definition of  $P(r, n)$  when \_\_\_\_\_ if we use  $P(r, n) = \frac{n!}{(n-r)!}$ .

6. How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

7. How many permutations of the letters ABCDEFGH contain the string ABC ?

8. How many different committees of three students can be formed from a group of four students?

9. Definition of  $r$ -Combination.

Let  $n$  be a positive integer and  $r$  an integer with  $0 \leq r \leq n$ . An \_\_\_\_\_ of elements of a finite set with  $n$  elements is an \_\_\_\_\_ selection of  $r$  elements from the set. Thus, an  $r$ -combination is simply a \_\_\_\_\_.

10. The notation  $C(r, n)$ .

Let  $n$  be a positive integer and  $r$  an integer with  $0 \leq r \leq n$ . The number of  $r$ -combinations of a set with  $n$  elements is called a binomial coefficient and is denoted by  $C(r, n)$  or  $\binom{n}{r}$ :

$$C(r, n) = \text{_____} \text{ or } \text{_____}$$

11. Let  $S = \{1, 2, 3\}$ . A 2-combination of  $S$ :

12. Binomial Coefficient, Pascal's Triangle, and Pascal's Identity.

| $n$ | Binomial Coefficient   | Pascal's Triangle |
|-----|--|-------------------|
| 0   | $\binom{0}{0} = \underline{\hspace{2cm}}$  | 1                 |
| 1   | $\binom{1}{0} = \underline{\hspace{2cm}}, \binom{1}{1} = \underline{\hspace{2cm}}$   | 1 1               |
| 2   | $\binom{2}{0} = \underline{\hspace{2cm}}, \binom{2}{1} = \underline{\hspace{2cm}}, \binom{2}{2} = \underline{\hspace{2cm}}$  | 1 2 1             |
| 3   | $\binom{3}{0} = \underline{\hspace{2cm}}, \binom{3}{1} = \underline{\hspace{2cm}}, \binom{3}{2} = \underline{\hspace{2cm}}, \binom{3}{3} = \underline{\hspace{2cm}}$   | 1 3 3 1           |
| 4   | $\binom{4}{0} = \underline{\hspace{2cm}}, \binom{4}{1} = \underline{\hspace{2cm}}, \binom{4}{2} = \underline{\hspace{2cm}}, \binom{4}{3} = \underline{\hspace{2cm}}, \binom{4}{4} = \underline{\hspace{2cm}}$  | 1 4 6 4 1         |
| 5   | $\binom{5}{0} = \underline{\hspace{2cm}}, \binom{5}{1} = \underline{\hspace{2cm}}, \binom{5}{2} = \underline{\hspace{2cm}}, \binom{5}{3} = \underline{\hspace{2cm}}, \binom{5}{4} = \underline{\hspace{2cm}}, \binom{5}{5} = \underline{\hspace{2cm}}$ | 1 5 10 10 5 1     |

Pascal's Identity: \_\_\_\_\_

For example, \_\_\_\_\_

13. Observation: \_\_\_\_\_

Let  $n$  and  $r$  be nonnegative integers with  $r \leq n$ . Then \_\_\_\_\_.

14. How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

15. Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?