

$$\binom{5}{2} = \frac{5!}{2!3!} = 10$$

Hypergeometric Distribution (QR Code5)

1. A committee of 4 people is to be selected from a group of 5 men and 7 women. If the selection is made randomly, what is the probability the committee will consist of 2 men and 2 women?

$$P(2m, 2w \mid 4 \text{ ppl out of } 12) = \frac{\binom{5}{2} \binom{7}{2}}{\binom{12}{4}} = 0.3333$$

2. Hypergeometric Random Variable.

How to describe a hypergeometric random variable in word?

It describes the number of successes in a fixed number of trials without replacement.

Example: The classical application of the hypergeometric distribution is sampling w/o replacement.

How to describe a hypergeometric random variable in math?

It describes the probability of k successes in n draws w/o replacement.

Example: Find the probability of the outcome of drawing k green marbles out of r total green marbles, and draw $n-k$ red marbles out of b red marbles, in n rounds.

3. Definition of the Hypergeometric Distribution.

Suppose X is a R.V. with a Hypergeometric distri, denoted by

$$X \sim H(r, b, n)$$

The probability of exactly k observation be selected from r observation (the size of 1st group) given n (the size of the chosen sample) be picked from $r+b$.

$$P(\text{exactly } k \text{ from } r \mid \text{picked } n \text{ from } r+b) = P(X = k) = \frac{\binom{r}{k} \binom{b}{n-k}}{\binom{r+b}{n}}$$

with its mean, variance, and standard deviation of the number of observed successes

$$\mu = \frac{nr}{r+b}, \quad \sigma^2 = n \frac{r}{r+b} \left(1 - \frac{r}{r+b}\right) \frac{r+b-n}{r+b-1}, \quad \sigma = \sqrt{n \frac{r}{r+b} \left(1 - \frac{r}{r+b}\right) \frac{r+b-n}{r+b-1}}$$

4. A school site committee is to be chosen randomly from 6 men and 5 women. If the committee consists of 4 members chosen randomly, what is the probability that 2 of them are women? How many women do you expect to be on the committee?

5. Let $X \sim H(r, b, 1)$. (a) Find $P(X = 0)$ and $P(X = 1)$. (b) Do we have $P(X = 2)$? Or $P(X = n)$ for $n > 2$?

4.5 Poisson Distribution

1. Poisson Distribution.

How to describe a Poisson distribution in word?

Example:

How to describe a Poisson distribution in math?

Example:

2. If a call center receives 20 calls between 8 a.m. and 12 p.m. What is the average calling this center gets in 15 minutes?

3. Definition of the Poisson Distribution.

$$P(\quad) = \quad$$

The mean and standard deviation of this distribution are

$$\mu = \quad, \quad \sigma = \quad.$$

4. Tom receives about 6 telephone calls between 8 a.m. and 10 a.m. What is the probability that Tom receives more than one call in the next 15 minutes?

