MAT1375, Classwork23, Fall2025

Ch21. Trigonometric Identities

1. Addition and Subtraction of angles formulas:

Let α, β be two angles. We have sin(θ) is good and cos(θ) is bad.

$$(1)\ \sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$(2)\ \sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$(3)\ \cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$(4)\ \cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

2. Half- and double-angle formulas: $\sin(\alpha + \alpha)$

$$(5)\ \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)\ \text{(From (1) and let } \beta = \alpha)$$

$$(6)\ \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)\ \text{(From (3) and let } \beta = \alpha)$$

$\cos(\alpha + \alpha)$

$$(7)\ \sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}}$$

$\frac{\alpha}{2}$ is in I, II $\quad +\sqrt{\frac{1-\cos(\alpha)}{2}}$

$\frac{\alpha}{2}$ is in III IV $\quad -\sqrt{\frac{1-\cos(\alpha)}{2}}$

$$(8)\ \cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$$

$\frac{\alpha}{2}$ is I IV $\quad +\sqrt{\frac{1+\cos(\alpha)}{2}}$

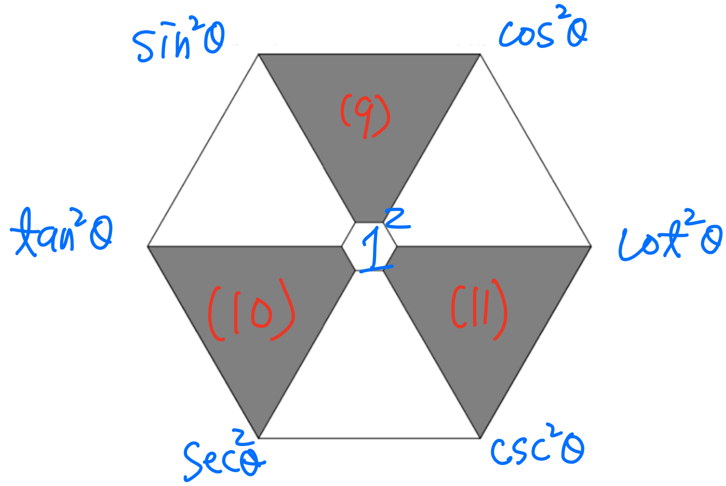
$\frac{\alpha}{2}$ is II III $\quad -\sqrt{\frac{1+\cos(\alpha)}{2}}$

3. Pythagorean Identities:

$$(9)\quad \sin^2(\theta) + \cos^2(\theta) = 1^2$$

$$(10)\quad 1^2 + \tan^2(\theta) = \sec^2(\theta)$$

$$(11)\quad 1^2 + \cot^2(\theta) = \csc^2(\theta)$$



4. Find the exact value of the trigonometric functions:

a) $\sin\left(\frac{11\pi}{12}\right)$ b) $\cos\left(\frac{7\pi}{8}\right)$ c) $\sin(15°)$ d) $\cos(75°)$

II

(a) $\sin\left(\frac{3}{12}\pi + \frac{8}{12}\pi\right) = \sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right)$

$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(+\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

(b) $\cos\left(\frac{1}{2}\cdot\frac{7}{4}\pi\right) = \pm\sqrt{\frac{1+\cos\left(\frac{7\pi}{4}\right)}{2}} = \pm\sqrt{\frac{\left(1+\frac{\sqrt{2}}{2}\right)\times 2}{(2)\times 2}} = \pm\sqrt{\frac{2+\sqrt{2}}{4}}$

$\rightarrow$ II $\rightarrow$ $\cos\left(\frac{7\pi}{8}\right) = -\sqrt{\frac{2+\sqrt{2}}{4}}$

(c) $\sin(15°) = \sin(45°-30°) = \sin(45°)\cos(30°) - \cos(45°)\sin(30°) = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

(d) $\cos(75°) = \cos(45°+30°) = \cos(45°)\cos(30°) - \sin(45°)\sin(30°)$
$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

5. Simplify the given function using the addition and subtraction formulas.

a) $\sin\left(\frac{\pi}{2}+x\right)$ b) $\sin\left(\frac{\pi}{2}-x\right)$ c) $\cos\left(\frac{\pi}{2}+x\right)$ d) $\cos\left(\frac{\pi}{2}-x\right)$

e) $\sin(-x) = \sin(0-x) = \underbrace{\sin(0)}_{=0}\cos(x) - \underbrace{\cos(0)}_{1}\sin(x) = -\sin(x)$ odd

f) $\cos(-x) = \cos(0-x) = \underbrace{\cos(0)}_{=1}\cos(x) + \underbrace{\sin(0)}_{=0}\sin(x) = \cos(x)$ even

a) $\sin\left(\frac{\pi}{2}+x\right) = \sin\left(\frac{\pi}{2}\right)\cos(x) + \cos\left(\frac{\pi}{2}\right)\sin(x) = 1\cdot\cos(x) + 0\cdot\sin(x) = \cos(x)$

b) $\sin\left(\frac{\pi}{2}-x\right) = \sin\left(\frac{\pi}{2}\right)\cos(x) - \cos\left(\frac{\pi}{2}\right)\sin(x) = 1\cdot\cos(x) - 0\cdot\sin(x) = \cos(x)$

c) $\cos\left(\frac{\pi}{2}+x\right) = \cos\left(\frac{\pi}{2}\right)\cos(x) - \sin\left(\frac{\pi}{2}\right)\sin(x) = 0\cdot\cos(x) - 1\cdot\sin(x) = -\sin(x)$

d) $\cos\left(\frac{\pi}{2}-x\right) = \cos\left(\frac{\pi}{2}\right)\cos(x) + \sin\left(\frac{\pi}{2}\right)\sin(x) = 0\cdot\cos(x) + 1\cdot\sin(x) = \sin(x)$