MAT1375, Classwork7, Fall2025

leading coefficient

degree

$2x^3+5x^2+1 \Rightarrow$ 3 2

$3x^4+10x^6+5x \Rightarrow$ 6 10

Ch7. Dividing Polynomials

1. Definition of **Polynomial function of degree *n*** in one variable:

A polynomial function of degree n in one variable is a function f of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x^1 + a_0,$$

for some constants $a_0, a_1, \cdots, a_n$, where $a_n \neq 0$ and n is a non-negative integer. The numbers $a_0, a_1, \cdots, a_n$ are called coefficients. The **number a_n**, the coefficient of the variable to the highest power, is called the leading coefficient and ***n*** is the degree of the polynomial.

2. Rational Function.

A rational function is a fraction of two polynomials $\frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are both polynomials, and $g(x) \neq 0$ is not the zero function. The domain of this rational function is

$D_{\frac{f}{g}} = \{x \in D_f \cap D_g,\ g(x) \neq 0\}$

3. Divide the following fraction via long division: $\frac{x^3+5x^2+4x+2}{x+3}$

$$
\begin{array}{rl}
 & x^2 + 2x - 2 \\
x+3 & \overline{)\, x^3+5x^2+4x+2} \\
x^2(x+3) \Rightarrow & -(x^3+3x^2) \\
 & 2x^2+4x \\
2x(x+3) \Rightarrow & -(2x^2+6x) \\
 & -2x+2 \\
-2(x+3) \Rightarrow & -(-2x-6) \\
 & 8
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & 1 & +2 & -2 & \\
1+3 & 1 & +5 & +4 & +2 \\
 & -(1 & +3) & & \\
 & & 2 & +4 & \\
 & & -(2 & +6) & \\
 & & & -2 & +2 \\
 & & & -(-2 & -6) \\
 & & & & 8
\end{array}
$$

$$\frac{x^3+5x^2+4x+2}{x+3} = x^2+2x-2 + \frac{8}{x+3}$$

or $x^3+5x^2+4x+2 = (x^2+2x-2)\cdot(x+3) + 8$

4. Dividend, Divisor, Quotient, and Remainder.

When dividing $\frac{f(x)}{g(x)}$, $f(x)$ is called the dividend and $g(x)$ is called the divisor. As a results of dividing $f(x)$ by $g(x)$ via long division with quotient $q(x)$ and remainder $r(x)$, we can write

$$\frac{f(x)}{g(x)} = \boxed{Q(x) + \frac{r(x)}{g(x)}}$$

If we multiply this equation by $g(x)$, we obtain the following alternative version:

$f(x) = Q(x)\cdot g(x) + r(x)$.

If $g(x)$ is a **factor** of $f(x)$, then we have

$f(x) = Q(x)\cdot g(x) \iff r(x) = 0$.

$x^2+2x-3 = (x-1)(x+3)$

5. Given $f(x) = x^5 - 3x^3 + 5x^2 - 12$, and $g(x) = x + 3$. (a) Divide the fraction via long division: $\frac{f(x)}{g(x)}$

(b) Find $f(-3)$. (c) Is $g(x)$ a factor of $f(x)$? Why or why not?

$$
\begin{array}{r|l}
 & +x^4 - 3x^3 + 6x^2 - 13x + 39 \\
x+3 & x^5 + 0x^4 - 3x^3 + 5x^2 + 0x - 12 \\
 & -)\ x^5 + 3x^4 \\
 & \quad -3x^4 - 3x^3 \\
 & -)\ -3x^4 - 9x^3 \\
 & \qquad 6x^3 + 5x^2 \\
 & -)\ 6x^3 + 18x^2 \\
 & \qquad -13x^2 + 0x \\
 & -)\ -13x^2 - 39x \\
 & \qquad 39x - 12 \\
 & -)\ 39x + 117 \\
 & \qquad -129
\end{array}
$$

6. Remainder Theorem, and Factor Theorem.

Assume $g(x) = x - c$, and the long division of $f(x)$ by $g(x)$ has remainder r, that is,

____

(1) The remainder when dividing $f(x)$ by $(x - c)$ is ________ since ______________________________.

(2) $f(c) = 0 \Leftrightarrow$ ______________________________. Here c is a ______________ of $f(x)$.

7. Given $f(x) = x^5 - 3x^3 + 5x^2 - 12$, and $g(x) = x + 2$. (a) Divide the fraction via long division: $\frac{f(x)}{g(x)}$

(b) Find $f(-2)$. (c) Is $g(x)$ a factor of $f(x)$? Why or why not?