MAT2440, Classwork32, Spring2025

ID:______________________________ Name:______________________________

1. The order of growth for functions: **Exponential functions**.

$$b^x, b > 1$$

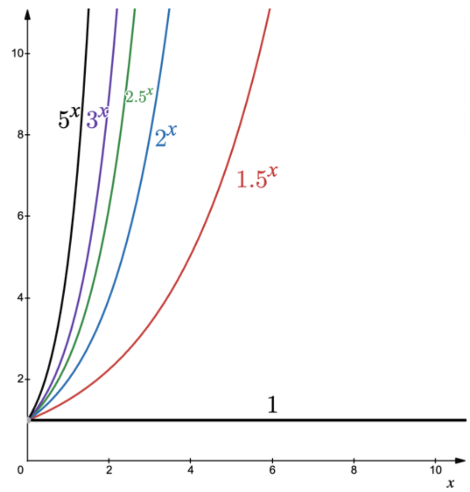
For each $x > 0$, we have

$$\cdots < 1.5^x \underline{<} 2^x \underline{<} 2.5^x \underline{<} 3^x \underline{<} 5^x < \cdots.$$

For each $x >$ certain positive number, we have

$$x^b < b^x$$

For example, $x^2 < 2^x$ for $x > 4$.



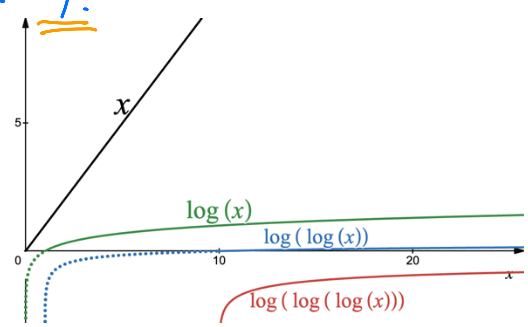
upper bound

2. Find the big-O for $f(x) = x^2 + 2^x$.

$g(x)$

$f(x) = x^2 + 2^x \le 2^x + 2^x = 2 \cdot (2^x)$ for $x > 4$

$\Rightarrow f(x)$ is $O(2^x)$ with $C = 2$, $k = 4$.

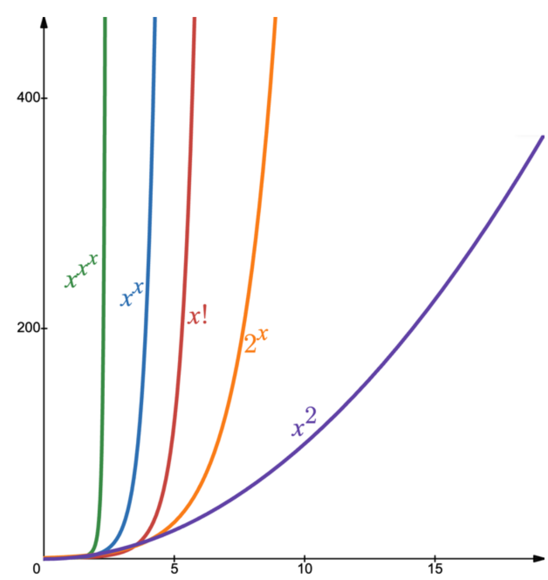


3. The order of growth for functions: **Logarithmic functions**.

$$\log(\log(\log(x))) \underline{<} \log(\log(x)) \underline{<} \log(x) \underline{\ll} x^p$$

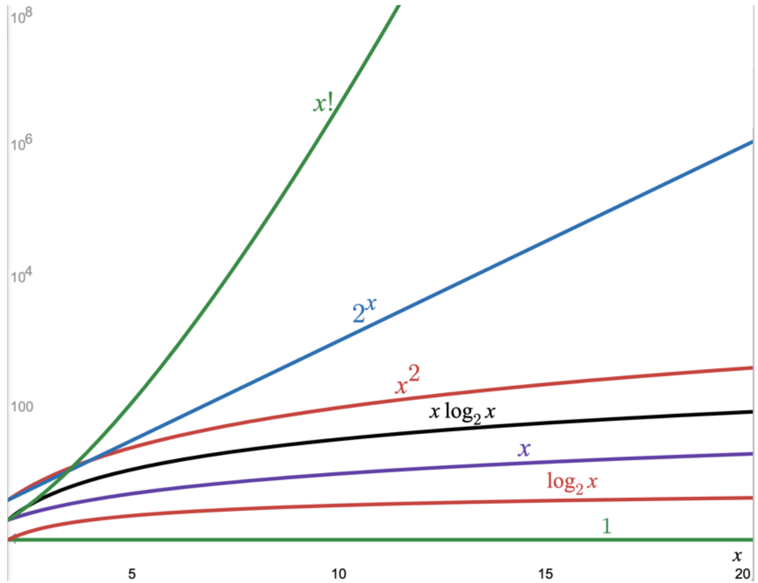
4. Other growth functions.

$$x^p \underline{<} b^x \underline{<} x! \underline{<} x^x \underline{<} x^{x^x}$$



5. The often-used list of the big-*O* relationship.

$$1 \underline{\ <\ } \log(x) \underline{\ <\ } x \underline{\ <\ } x\log(x) \underline{\ <\ } x^2 \underline{\ <\ } 2^x \underline{\ <\ } x!$$



6. Arrange the functions $8\sqrt{n}$, $(\log(n))^2$, $2n \cdot \log(n)$, $n!$, $(1.1)^n$, and n^2 in a list so that each function is the big-*O* of the next function.

$(\log(n))^2$, $8\sqrt{n}$, $2n \cdot \log(n)$, n^2, $(1.1)^n$, $n!$

7. Find the least integer n such that $f(x) = \frac{x^5+2x^3}{2x^2+1}$ is $O(x^n)$.

8. Give a big-*O* estimate for $f(x) = (x+1) \cdot \log(x^2+1) + 3x^2$.