Math 1372 HW 11

4.4 Geometric Distribution

Use the following information to answer the next six exercises: The Higher Education Research Institute at UCLA collected data from 203,967 incoming first-time, full-time first-year students from 270 four-year colleges and universities in the U.S. 71.3% of those students replied that, yes, they believe that same-sex couples should have the right to legal marital status. Suppose that you randomly select first-year students from the study until you find one who replies "yes." You are interested in the number of first-year students you must ask.

45. In words, define the random variable *X*. X is the number of students you must ask until one says yes.

46. $X \sim$ (,) $X \sim G(p=0.713)$

47. What values does the random variable *X* take on? the number of the students you asked and p.

48. Construct the probability distribution function (PDF). Stop at *x* = 6.

$P(X=1) = (1-p)^{1-1} p^1 = (0.287)^0 \cdot 0.713 = 0.713$

$P(X=2) = (1-p)^{2-1} \cdot p^1 = 0.2046$

$P(X=3) = (1-p)^{3-1} \cdot p^1 = (0.287)^2 \cdot 0.713 = 0.058$

$P(X=4) = (1-p)^{4-1} \cdot p = (0.287)^3 \cdot 0.713 = 0.01685$

$P(X=5) = (1-p)^{5-1} \cdot p = (0.287)^4 \cdot 0.713 = 0.00483$

$P(X=6) = (1-p)^{6-1} \cdot p = (0.287)^5 \cdot 0.713 = 0.00138$

x	*P*(*x*)
1	
2	
3	
4	
5	
6	

Table 4.30

49. On average (μ), how many first-year students would you expect to have to ask until you found one who replies "yes?" $\mu = \frac{1}{p} = \frac{1}{0.713} = 1.4$

50. What is the probability that you will need to ask fewer than three first-year students?

$P(\text{fewer than } 3) = P(X=1) + P(X=2)$
$= 0.713 + 0.2046 = 0.9176.$

4.5 Hypergeometric Distribution

Use the following information to answer the next five exercises: Suppose that a group of statistics students is divided into two groups: business majors and non-business majors. There are 16 business majors in the group and seven non-business majors in the group. A random sample of nine students is taken. We are interested in the number of business majors in the sample.

51. In words, define the random variable *X*. X the number of business majors out of 9 students

52. $X \sim$ ____(____,____) $X \sim H(r=16, b=7, n=9)$

53. What values does *X* take on? r: number of business majors, b: number of non-business majors

54. Find the standard deviation. $N = r+b = 23$, $p = \frac{r}{r+b} = \frac{16}{23}$, $\sigma = \sqrt{n \cdot p(1-p) \cdot \frac{N-n}{N-1}}$
$= \sqrt{10 \cdot \frac{16}{23} \cdot \frac{7}{23} \cdot \frac{14}{22}} = 0.3654$

55. On average (μ), how many would you expect to be business majors?

$\mu = n \cdot \frac{r}{r+b} = 9 \cdot \frac{16}{23} = 6.26 \cdots$

4.31 Customers at a coffee shop. A coffee shop serves an average of 75 customers per hour during the morning rush.

(a) Which distribution have we studied that is most appropriate for calculating the probability of a given number of customers arriving within one hour during this time of day?

(b) What are the mean and the standard deviation of the number of customers this coffee shop serves in one hour during this time of day?

(c) Would it be considered unusually low if only 60 customers showed up to this coffee shop in one hour during this time of day?

(d) Calculate the probability that this coffee shop serves 70 customers in one hour during this time of day.

Sol (a) Poisson distribution.

(b) The time unit is "hour".

mean $\mu = \lambda = 75$

standard deviation $\sigma = \sqrt{\lambda} = \sqrt{75} = 5\sqrt{3} = 8.66$

(c) The z-score of 60 is $z = \frac{60-75}{8.66} = -1.73$

which means it is 1.73 SD below 75 and it is not unusual.

(d) $P(\text{exactly } 70 \text{ customers}) = \frac{75^{70} \cdot e^{-75}}{70!} = 0.0402$

$k = 70$

4.32 Stenographer's typos. A very skilled court stenographer makes one typographical error (typo) per hour on average.

(a) What probability distribution is most appropriate for calculating the probability of a given number of typos this stenographer makes in an hour?

(b) What are the mean and the standard deviation of the number of typos this stenographer makes?

(c) Would it be considered unusual if this stenographer made 4 typos in a given hour?

(d) Calculate the probability that this stenographer makes at most 2 typos in a given hour.

Sol: (a) Poisson with $\lambda = 1$

(b) Mean : $\lambda = 1$, Standard deviation $= \sqrt{\lambda} = \sqrt{1} = 1$

(c) The z-score of 4 typos is $z = \frac{4-1}{1} = 3$ and

4 is 3 SD above mean which is unusual.

(d) $P(\text{at most } 2 \text{ typos}) = P(0 \text{ typo}) + P(1 \text{ typo}) + P(2 \text{ typos})$

$$= \frac{1^0 e^{-1}}{0!} + \frac{1^1 e^{-1}}{1!} + \frac{1^2 e^{-2}}{2!}$$

$$= 0.3679 + 0.3679 + 0.1839 = 0.9197.$$

4.33 How many cars show up? For Monday through Thursday when there isn't a holiday, the average number of vehicles that visit a particular retailer between 2pm and 3pm each afternoon is 6.5, and the number of cars that show up on any given day follows a Poisson distribution.

(a) What is the probability that exactly 5 cars will show up next Monday?

(b) What is the probability that 0, 1, or 2 cars will show up next Monday between 2pm and 3pm?

(c) There is an average of 11.7 people who visit during those same hours from vehicles. Is it likely that the number of people visiting by car during this hour is also Poisson? Explain.

Sol: Let X be the number of cars per hour which follow a Poisson distr.

Then $\mu(X) = \lambda = 6.5$.

(a) $P(\text{exactly 5 cars}) = \dfrac{\lambda^5 \cdot e^{-\lambda}}{5!} = \dfrac{(6.5)^5 \cdot e^{-6.5}}{5!} = 0.1454$

(b) $P(0 \text{ car}) = \dfrac{(6.5)^0 \cdot e^{-6.5}}{0!} = 0.0015$

$P(1 \text{ car}) = \dfrac{(6.5)^1 \cdot e^{-6.5}}{1!} = 0.0098$

$P(2 \text{ cars}) = \dfrac{(6.5)^2 \cdot e^{-6.5}}{2!} = 0.0318$

$P(0, 1, \text{or } 2 \text{ cars}) = 0.0015 + 0.0098 + 0.0318 = 0.0431$

(c) We can find the avarge people per car in an hour is

$\dfrac{11.7}{6.5} = 1.8$ which means that people are coming in together.

It implies that one person to another coming is not independent, and then it can't be described by Poisson distribution.

4.34 Lost baggage. Occasionally an airline will lose a bag. Suppose a small airline has found it can reasonably model the number of bags lost each weekday using a Poisson model with a mean of 2.2 bags.

(a) What is the probability that the airline will lose no bags next Monday?

(b) What is the probability that the airline will lose 0, 1, or 2 bags on next Monday?

(c) Suppose the airline expands over the course of the next 3 years, doubling the number of flights it makes, and the CEO asks you if it's reasonable for them to continue using the Poisson model with a mean of 2.2. What is an appropriate recommendation? Explain.

Sol: Assume X be the number of lose bag and $\mu(X) = \lambda = 2.2$ (per day)

(a) $P(0 \text{ loss bag}) = \dfrac{\lambda^0 \cdot e^{-\lambda}}{0!} \underset{\lambda = 2.2}{=} \dfrac{(2.2)^0 \cdot e^{-2.2}}{0!} = \dfrac{1}{e^{2.2}} = 0.1108$

(b) $P(0, 1, \text{ or } 2 \text{ loss bags}) = P(0 \text{ loss bag}) + P(1 \text{ loss bag}) + P(2 \text{ loss})$

$= \dfrac{(2.2)^0 \cdot e^{-2.2}}{0!} + \dfrac{(2.2)^1 \cdot e^{-2.2}}{1!} + \dfrac{(2.2)^2 \cdot e^{-2.2}}{2!}$

$= 0.1108 + 0.2438 + 0.2681 = 0.6227$

(c) NO. It is not reasonable because the mean may have changed.