Calculus I Question

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Question:

Your answer is INCORRECT.

Given that $f(x) = \dfrac{\sqrt{x+1}-3}{x-8}$, define the function $f(x)$ at 8 so that it becomes continuous at 8.

a) Not possible because there is an infinite discontinuity at the given point.

b) $f(8) = 1$

c) $f(8) = \dfrac{1}{6}$

d) $f(8) = 6$

e) $f(8) = 0$

Solution:
Given that $f(x) = \frac{\sqrt{x+1}-3}{x-8}$. Define the function $f(x)$ at 8 so that it becomes continuous at 8.
Like the definition of the continuous I stated in Lab, we need to check the following:

$$\lim_{x\to 8^+} \frac{\sqrt{x+1}-3}{x-8} = \lim_{x\to 8^-} \frac{\sqrt{x+1}-3}{x-8} = f(8). \tag{1}$$

First, we check the limit from the left and right. Then we got the undetermined from "$\frac{0}{0}$" which means it is a removable discontinuity at $x = 8$ and $f(8)$ does not exist. If we can find the limit from the left and right, check they are the same value, and define the value of $f(8)$ to be this value we found, then we are done.

And $\frac{0}{0}$ also means there is a same factor on the top and buttom.

To simplify this fraction, we times $\sqrt{x+1}+3$ on the top and buttom, we get

$$\frac{\sqrt{x+1}-3}{x-8} = \frac{(\sqrt{x+1}-3)(\sqrt{x+1}+3)}{(x-8)(\sqrt{x+1}+3)} = \frac{(\sqrt{x+1})^2-3^2}{(x-8)(\sqrt{x+1}+3)} = \frac{x+1-9}{(x-8)(\sqrt{x+1}+3)} = \frac{1}{\sqrt{x+1}+3}$$

and we have

$$\lim_{x\to 8^+} \frac{\sqrt{x+1}-3}{x-8} = \lim_{x\to 8^-} \frac{\sqrt{x+1}-3}{x-8} = \frac{1}{6}.$$

Thus, if we redefine $f(8) = \frac{1}{6}$ which is satified the definition (1) then we can say this new f is continuous at $x = 8$.