

# MAT1372, Quiz8, Fall2025

ID: \_\_\_\_\_

Name: \_\_\_\_\_

- This quiz consists of 1 question for a total of 10 points. You have 15 minutes to complete the quiz.
- Show all work and justify your answers.
- Wishing you success.
- Useful formulas:

Bernoulli:  $P(X = 1) = p, P(X = 0) = 1 - p; \quad \mu = p; \quad \sigma = \sqrt{p(1 - p)}$

Binomial:  $P(\text{exactly } k \text{ successes out of } n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}; \quad \mu = np; \quad \sigma = \sqrt{np(1-p)}$

1. Six **fair** coins are flipped. If the outcomes are independent, determine

- (a) the probability that there are a total of  $k$  heads, for  $k = 0, 1, 2, 3, 4, 5, 6$ . (Hint: Let  $X$  be a random variable which represents the number of heads when flipping 6 fair coins. Find  $P(X = 0), P(X = 1), P(X = 2), P(X = 3), P(X = 4), P(X = 5)$ , and  $P(X = 6)$ )
- (b) the expected value of the number of the heads.
- (c) the standard deviation of the number of the heads. (please keep the square root form as an answer)

(a)  $P(X=0) = \frac{6!}{0!6!} \cdot \left(\frac{1}{2}\right)^0 \left(1-\frac{1}{2}\right)^6 = 1 \cdot 1 \cdot \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

$n=6, p=\frac{1}{2}, k=0, 0!=1$

$k=1 \quad P(X=1) = \frac{6!}{1!5!} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{6-1} = \frac{6}{1} \left(\frac{1}{2}\right)^6 = \frac{6}{64}$

$k=2 \quad P(X=2) = \frac{6!}{2!4!} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2} = \frac{6 \cdot 5}{2!} \left(\frac{1}{2}\right)^6 = 15 \frac{1}{64}$

$k=3 \quad P(X=3) = \frac{6!}{3!3!} \left(\frac{1}{2}\right)^3 \left(1-\frac{1}{2}\right)^{6-3} = \frac{6 \cdot 5 \cdot 4}{6} \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^3 = 20 \frac{1}{64}$

$k=4 \quad P(X=4) = \frac{6!}{4!2!} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{6-4} = \frac{6 \cdot 5}{2!} \left(\frac{1}{2}\right)^6 = 15 \frac{1}{64}$

$k=5 \quad P(X=5) = \frac{6!}{5!1!} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{6-5} = \frac{6}{1!} \left(\frac{1}{2}\right)^6 = 6 \frac{1}{64}$

$k=6 \quad P(X=6) = \frac{6!}{6!0!} \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{6-6} = 1 \cdot \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

*Note: The probabilities for k=1, 2, 3, 4, 5 are symmetric around k=3, with k=1 and k=5 having the same probability, k=2 and k=4 having the same probability, and k=3 being the center. The word "same" is written next to the k=3 calculation with arrows pointing to the symmetric pairs.*

(b)  $E(X) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2) + 3 \cdot P(X=3) + 4 \cdot P(X=4) + 5 \cdot P(X=5) + 6 \cdot P(X=6) = 6 \cdot \frac{1}{2} = 3$

$n=6, p=\frac{1}{2}$

(c)  $SD(X) = \sqrt{6 \cdot \frac{1}{2} \cdot \frac{1}{2}} = \sqrt{\frac{3}{2}}$