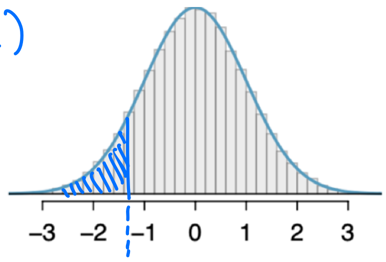
Mat 1372 HW9

4.1 Area under the curve, Part I. What percent of a standard normal distribution $N(\mu = 0, \sigma = 1)$ is found in each region? Be sure to draw a graph.

(a) $Z < -1.35$ (b) $Z > 1.48$ (c) $-0.4 < Z < 1.5$ (d) $|Z| > 2$

(a)

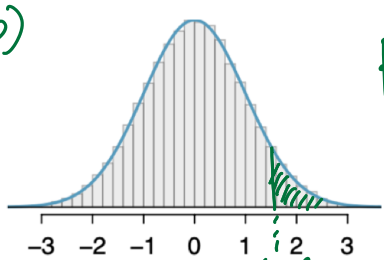


-1.35

$P(Z < -1.35) = 0.0885$

by the table

(b)

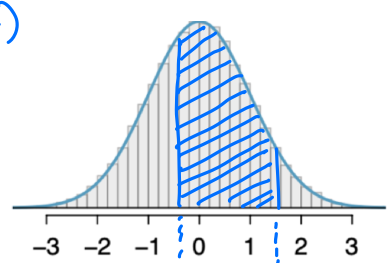


1.48

$P(Z > 1.48) = 1 - P(Z < 1.48)$

$= 1 - 0.9306 = 0.0694$

(c)



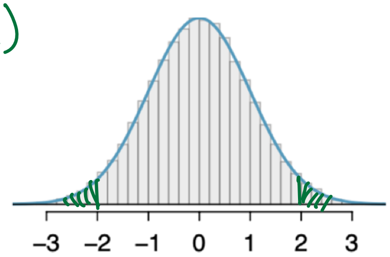
-0.4 1.5

$P(-0.4 < Z < 1.5) = P(Z < 1.5) - P(Z < -0.4)$

$= 0.9332 - 0.3446 = 0.5886$

"$|Z| > 2$" means $Z > 2$ or $Z < -2$

(d)



$P(|Z| > 2) = P(Z > 2) + P(Z < -2)$

$= 1 - P(Z < 2) + P(Z < -2)$

$= 1 - 0.9772 + 0.0228$

$= 0.0228 + 0.0228 = 0.0456$

4.3 GRE scores, Part I. Sophia who took the Graduate Record Examination (GRE) scored 160 on the Verbal Reasoning section and 157 on the Quantitative Reasoning section. The mean score for Verbal Reasoning section for all test takers was 151 with a standard deviation of 7, and the mean score for the Quantitative Reasoning was 153 with a standard deviation of 7.67. Suppose that both distributions are nearly normal.

(a) Write down the short-hand for these two normal distributions. N (mean, SD)

(b) What is Sophia's Z-score on the Verbal Reasoning section? On the Quantitative Reasoning section? Draw a standard normal distribution curve and mark these two Z-scores. $Z = \frac{X-\mu}{SD}$

(c) What do these Z-scores tell you?

(d) Relative to others, which section did she do better on?

(e) Find her percentile scores for the two exams.

(f) What percent of the test takers did better than her on the Verbal Reasoning section? On the Quantitative Reasoning section?

(g) Explain why simply comparing raw scores from the two sections could lead to an incorrect conclusion as to which section a student did better on.

(h) If the distributions of the scores on these exams are not nearly normal, would your answers to parts (b) - (f) change? Explain your reasoning.

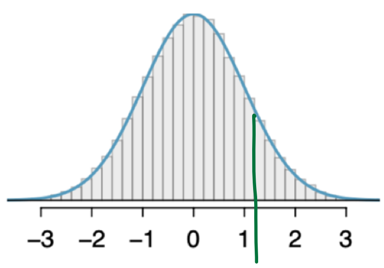
Sol

(a) Verbal : $N(151, 7)$

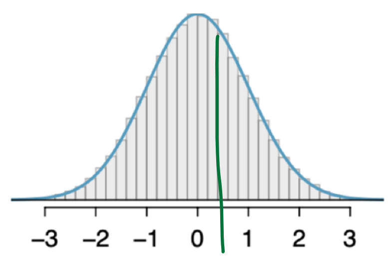
Quantitative : $N(153, 7.67)$

(b) Let Z_V be z-score on verbal section and Z_Q be z-score on quantitative section. We have

$$Z_V = \frac{160-151}{7} = \frac{9}{7} = 1.29 \qquad Z_Q = \frac{157-153}{7.67} = \frac{4}{7.67} = 0.52$$



1.29



0.52

(c) Z_V is 1.29 stand deviation above the mean, and Z_Q is 0.52 stand deviation above the mean

(d) Since $Z_V > Z_Q$, thus she did verbal better than quatitative.

(e) $P(Z > Z_V) = P(Z > 1.29) = 0.9015 = 90.15\%$

$P(Z > Z_Q) = P(Z > 0.52) = 0.6985 = 69.85\%$

(f) On verbal section, $1 - 0.9015 = 0.0985$ of people is better than her.

On quantitative section, $1 - 0.6985 = 0.3015$ of people is better than her.

(g) Since the means and SDs are different across these 2 sections.

then we can't compare them via the raw scores.

(h) Answer to part (b) would not change as Z scores can be calculated for distributions that are not normal. However, we can't answer part (c) ~ (f) since we don't have the Z table if it is not a normal distribution.

4.5 GRE scores, Part II. In Exercise 4.3 we saw two distributions for GRE scores: $N(\mu = 151, \sigma = 7)$ for the verbal part of the exam and $N(\mu = 153, \sigma = 7.67)$ for the quantitative part. Use this information to compute each of the following:

(a) The score of a student who scored in the 80^{th} percentile on the Quantitative Reasoning section.

(b) The score of a student who scored worse than 70% of the test takers in the Verbal Reasoning section.

Sol: (a) Let the quantitative score be x which is in the 80^{th} percentile

Then its $Z_Q = \frac{x-153}{7.67}$ and $P(Z < Z_Q) = 0.80$.

Based on the Z table, $0.7995 < \underset{=0.8}{P(Z < Z_Q)} < 0.8023$

$\Rightarrow 0.84 < Z_Q < 0.85$, Let's say $Z_Q \approx 0.84$

Then $\frac{x-153}{7.67} = 0.84 \Rightarrow x = 7.67 \times 0.84 + 153 = 159.44$

(b) Let the verbal score be y which is worse than 70%

Then its $Z_V = \frac{y-151}{7}$ and $P(Z > Z_V) = 70\%$

which implies $P(Z < Z_V) = 1 - 0.7 = 0.3$.

Based on Z table, $0.2981 < P(Z < Z_V) < 0.3015$

$\Rightarrow -0.53 < Z_V < -0.52$ Let's say $Z_V = -0.52$

Then $\frac{y-151}{7} = -0.52$, $y = 151 - 0.52 \times 7$
$= 147.36$

4.7 LA weather, Part I. The average daily high temperature in June in LA is 77°F with a standard deviation of 5°F. Suppose that the temperatures in June closely follow a normal distribution.

(a) What is the probability of observing an 83°F temperature or higher in LA during a randomly chosen day in June?

(b) How cool are the coldest 10% of the days (days with lowest high temperature) during June in LA?

Sol

(a) The temperature in June T_J follows $N(\mu=77, \sigma=5)$

$$P(T_J > 83) = P(Z > 1.2) = 1 - P(Z < 1.2)$$

$$\boxed{Z = \frac{83-77}{5} = \frac{6}{5} = 1.2}$$

$$= 1 - 0.8849$$

$$= 0.1151$$

(b) Let t be the coldest 10% of the days in June.

Then its Z-score $Z_t = \frac{t-77}{5}$ and $P(Z < Z_t) = 0.1$

Based on Z table, we have $0.0985 < P(Z < Z_t) < 0.1003$

$\Rightarrow -1.29 < Z_t < -1.28$, Let's say $Z_t = -1.28$, then

$$\frac{t-77}{5} = -1.28 \Rightarrow t = 77 - 1.28 \times 5 = 70.6°F$$

4.9 LA weather, Part II. Exercise 4.7 states that average daily high temperature in June in LA is 77°F with a standard deviation of 5°F, and it can be assumed that they to follow a normal distribution. We use the following equation to convert °F (Fahrenheit) to °C (Celsius):

$$C = (F - 32) \times \frac{5}{9}.$$

(a) Write the probability model for the distribution of temperature in °C in June in LA.

(b) What is the probability of observing a 28°C (which roughly corresponds to 83°F) temperature or higher in June in LA? Calculate using the °C model from part (a).

(c) Did you get the same answer or different answers in part (b) of this question and part (a) of Exercise 4.7? Are you surprised? Explain.

(d) Estimate the IQR of the temperatures (in °C) in June in LA.

Sol (a) Given $\mu_F = 77$ and $\sigma_F = 5$. By the formula converting °F to °C, we have

$$\mu_C = (77-32) \times \frac{5}{9} = 25 \quad \text{(linear Combination)}$$

$$\sigma_C = 5 \cdot \frac{5}{9} = 2.78 \quad \text{(linear Combination)}$$

Thus, $T_C \sim N(\mu_C = 25, \sigma_C = 2.78)$

the temperature in °C

4.9 (b) The Z-score of 28°C is $Z_c = \frac{28-25}{2.78} = 1.08$.

Then the probability that the day temperature is higher than 28°C is $P(Z > Z_c) = P(Z > 1.08)$

$= 1 - P(Z < 1.08) = 1 - 0.8599 = 0.1401$

(c) It is a little bit different since 28°C = 82.4°F.

But if we check the Z-score of 82.4°F, it will be exactly the same as the Z-score of 28°C

(d) $IQR = Q_3 - Q_1 = t_3 - t_1$ where $Z_3 = \frac{t_3 - 25}{2.78}$.

$Z_1 = \frac{t_1 - 25}{2.78}$

$P(Z < Z_3) = 0.75$ & $P(Z < Z_1) = 0.25$.

Then we have

$0.7486 < P(Z < Z_3) < 0.7517$ and $0.2483 < P(Z < Z_1) < 0.2514$

$\Rightarrow \quad 0.67 < Z_3 < 0.68$

$Z_3 = 0.67$

$\Rightarrow \quad \frac{t_3 - 25}{2.78} = 0.67$

$t_3 = 26.8626$

$-0.68 < Z_1 < -0.67$

$Z_1 = -0.67$

$\frac{t_1 - 25}{2.78} = -0.67$

$t_1 = 23.1372$

$\Rightarrow IQR = t_3 - t_1 = 26.86 - 23.13 = 3.73$.

4.10 Find the SD. Cholesterol levels for women aged 20 to 34 follow an approximately normal distribution with mean 185 milligrams per deciliter (mg/dl). Women with cholesterol levels above 220 mg/dl are considered to have high cholesterol and about 18.5% of women fall into this category. What is the standard deviation of the distribution of cholesterol levels for women aged 20 to 34?

Sol. Let Cholesterol levels for women aged 20 to 34 C_W follows $N(\mu = 185, \sigma)$ where σ is standard deviation.

If the probability $P(C_W > 220) = 18.5\%$, we have

$P(C_W < 220) = 1 - 18.5\% = 0.8150$

The z-score of 220 is $\frac{220-185}{\sigma} = \frac{35}{\sigma}$, and $P(Z < \frac{35}{\sigma}) = 0.8150$.

Based on the z table, we have

$0.8133 < P(Z < \frac{35}{\sigma}) < 0.8159$

$\Rightarrow \quad 0.89 < \frac{35}{\sigma} < 0.90$

$\Rightarrow \quad \frac{1}{0.9} < \frac{\sigma}{35} < \frac{1}{0.89} \Rightarrow \frac{35}{0.9} < \sigma < \frac{35}{0.89}$

$\Rightarrow \quad 38.88 < \sigma < 39.32 \Rightarrow \sigma \approx 38.88$