MAT2440, Classwork22, Spring2025
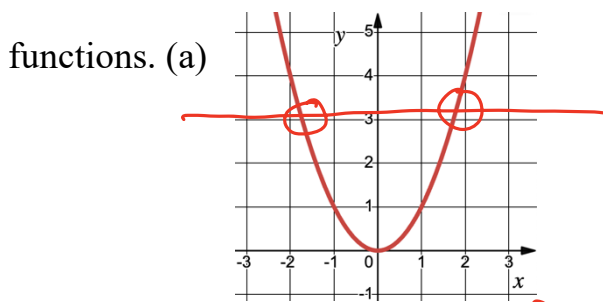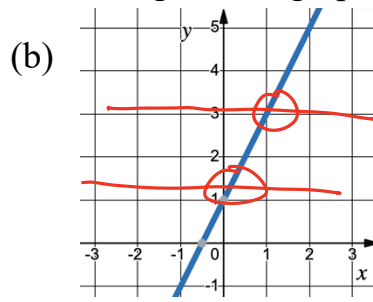
ID:_____   Name:_____

1. Use **Horizontal Line Test** to determine which of the following are the graphs of one-to-one functions. (a)   (b)

NOT a one-to-one function ↑
(more than one intersection per horizontal line)

Yes, it is one-to-one
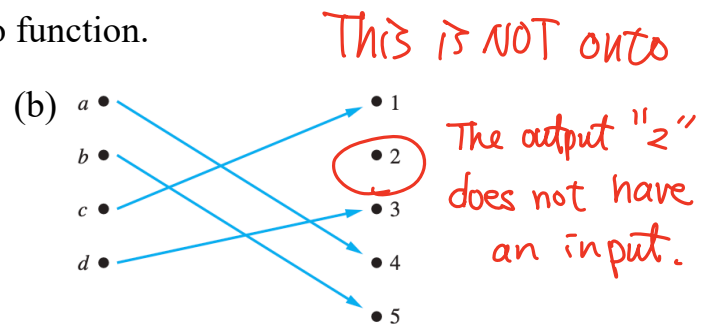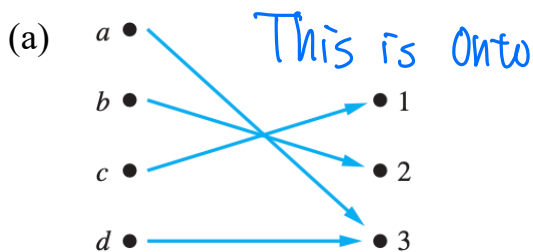(at most one intersection point per horizontal line)

2. The definition of an **Onto** Function:

A function f from A to B is called **Onto**, or a **subjection** _____, if and only if for every element

$b \in B$ there is an element $a \in A$ with $f(a) = b$ and we say f is **subjective**. Hence,

$$\forall b \in B \exists a \in A(\underline{f(a) = b}).$$

3. Check if the given function (or mapping) an onto function.

(a)   **This is Onto**

(b)   **This is NOT onto**

The output "2" does not have an input.

4. Check if the given function an onto function. (a) $f(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$.

(b) $g(x) = 2x + 1$ from $\mathbb{R}$ to $\mathbb{R}$.   (c) $h(x) = 2x + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$.

To prove f is NOT Onto: provide a counterexample to show that there exists an output without input.

(a) NO, here is an counterexample: Let $b = -1 \in \mathbb{R}$, we have

$x^2 = -1$ which has no solution in $\mathbb{R}$.

$\Rightarrow$ NOT all output has an input $\Rightarrow f(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$ is NOT onto.

(c) NO, here is an counterexample, Let $b = 2 \in \mathbb{Z}$, we have

$2x + 1 = 2 \Rightarrow x = \frac{1}{2} \notin \mathbb{Z}$

$\Rightarrow$ NOT all output has an input $\Rightarrow h(x) = 2x + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$ is NOT onto.

To prove f is onto: need to show that $\forall y \in B$, we can solve for $x \in A$, such that $f(x) = y$.

(b) To show "$g(x) = 2x+1$ from $\mathbb{R}$ to $\mathbb{R}$". is onto. we have

Given $y \in \mathbb{R}$, $2x+1 = y$

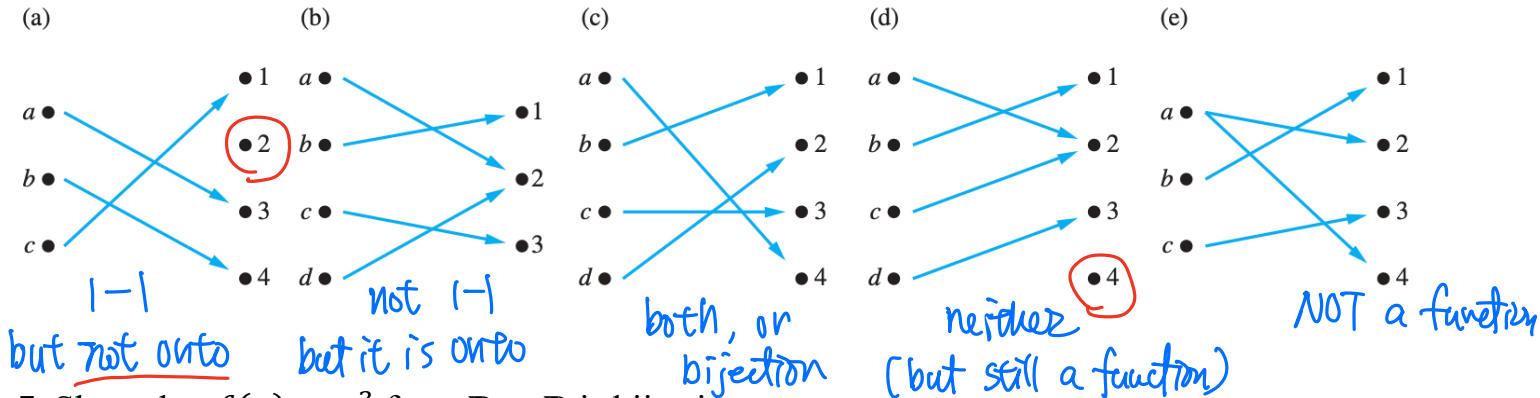$$\Rightarrow 2x = y-1$$

$$\Rightarrow x = \frac{y-1}{2}$$

$\forall y \in \mathbb{R}$, we can find $x = \frac{y-1}{2}$ which is also a real number.

Therefore, $g(x) = 2x+1$ from $\mathbb{R}$ to $\mathbb{R}$ is onto.

5. The definition of a **Bijective** Function:

If a function f is both __one-to-one__ and __onto__ , then f is bijective.

6. Check if the given function (or mapping) a bijective function.

(a)   (b)   (c)   (d)   (e)

1-1
but not onto

not 1-1
but it is onto

both, or
bijection

neither
(but still a function)

NOT a function

7. Show that $f(x) = x^3$ from $\mathbb{R}$ to $\mathbb{R}$ is bijective.

8. The definition of an **Inverse** Function:

Let a function f be one-to-one from a set A to a set B. Then f is _____ and the

_____ function of f, denoted _____, assigns $f^{-1}(\_\_) = \_\_$ if $f(x) = y$.