MAT2440, Classwork15, Spring2025

ID:______________________ Name:______________________

1. The Second Method: A Proof by **Contradiction**

(a) To prove a statement $\boldsymbol{p}$ is **true**, we first find a contradition q such that $\neg p \to q$ is true. Since $\boldsymbol{q}$ **is false** and $\boldsymbol{\neg p \to q}$ **is true**, it concludes that $\neg p$ is true which implies p is true.

(b) To prove a statement $\boldsymbol{p \to q}$ is **true**, we first ***assume*** p and $\neg q$ are true. Then using $\neg q$ shows $\neg p$ is true. Because p and $\neg p$ are both true, we have a contradition. It implies the ***assumption*** "$\neg q$ is true" **is wrong** which means q is true.

2. Give a contradiction proof of the theorem "If n^2 is an odd integer, then n is odd."

P(n) Q(n)

Assume n^2 is odd and n is even $(\neg Q(n))$

Then $n = 2k$ which implies $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$ and it is even

Here we get a $\boxed{\text{contradition}}$ since n^2 cannot both even and odd.

Therefore, n is odd.

3. Rational and Irrational numbers:

The real number r is rational if there exist integers a and b with $b \neq 0$ such that

$$r = \frac{a}{b}.$$

A real number that is not rational is called irrational.

4. Prove that a product of a non-zero rational number and an irrational number is irrational.

Assume "the product of a rational number and an irrational is rational"

$$\frac{a}{b} \cdot i = \frac{c}{d}$$ (a, b, c, d are non-zero integers)
i = non-zero irrational

Then $i = \frac{c}{d} \cdot \frac{b}{a} = \frac{cb}{da} \Rightarrow$ "i" is a rational number

Here is $\boxed{\text{a contradition}}$ that "i" is both rational and irrational which implies the assumption is wrong, and a product of a non-zero rational number and an irrational one is irrational.

5. The Third Method: A Proof by **Contraposition**

Proofs by Contraposition make use of the fact that the conditional statement $p \to q$ is **equivalent** to its contrapositive $\neg q \to \neg p$. This means that $p \to q$ can be proved by showing $\neg q \to \neg p$ is true.

6. Give a proof by Contraposition of the theorem "If n^2 is an odd integer, then n is odd."

In this theorm, p is " n^2 is odd" and q is " n is odd"

Assume $\neg q$: n is Not odd $\Rightarrow$ n is even

Let $n = 2k$, then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$

which implies n^2 is an even number and this is the $\neg p$ proposition.

We proved that $\neg q \to \neg p$. implies $p \to q$

7. Mistakes in Proofs: An Example

What is wrong with this famous supposed "proof" that $1 = 2$?

Proof: We use these steps, where a and b are two equal positive integers.

Step	**Reason**
(1). $a = b$	Given
(2). $a^2 = ab$	Multiply both sides of (1) by a
(3). $a^2 - b^2 = ab - b^2$	Subtract b^2 from both sides of (2)
(4). $(a-b)(a+b) = b(a-b)$	Factor both sides of (3)
(5). $a + b = b$	Divide both sides of (4) by $a - b$
(6). $2b = b$	Replace a by b in (5) since $a = b$
(7). $2 = 1$	Divide both sides of (6) by b

since $a=b \Rightarrow \boxed{a-b=0}$, then we can not cancel $(a-b)$ on both sides in (4)