Graphing Functions Review

Polynomial Functions:

If the degree of the polynomial is **even**:

And the leading coefficient is **positive**; the graph will open **up** on **both ends.**

And the leading coefficient is **negative**; the graph will open **down** on **both ends.**

If the degree of the polynomial is **odd**:

And the leading coefficient is **positive**; the graph **rises** from **left to right**.

And the leading coefficient is **negative;** the graph **falls** from **left to right**.

Relative Extrema of a polynomial function:

As graphs are drawn from left to right, there may be highs (maxima) and lows (minima) across the graph.

These highs and lows are called relative extrema or relative maxima or relative minima. They are called relative, because they are only an extrema relative to a restricted part of the graph, not over the whole thing.

The number of relative extrema can be calculated by taking one less than the degree of the polynomial.

Example 1: $P(x) = -2(x-1)(x+2)(x+1)^2(x-2)^2$ has a degree of 6. Therefore it can have 5 relative extrema. However, it doesn't have to have that many. It could have 5 or 3 or 1 extrema.

Example 2: $P(x) = 3(x+2)^3(x+1)^4$ has a degree of 7. Therefore, it can have 6 relative extrema. However, it doesn't have to have that many. It could have 6 or 4 or 2 or 0 extrema.

Domain – The domain of any function is the set of all possible x values for that function. The domain of a polynomial function is the set of real numbers.

To find the x-intercepts: Factor the function and set it equal to zero to determine the values of x.

Behavior through (at) the x-intercepts: The powers of each factor are used to determine the behavior of the graph through the intercept. If the power is even, the graph is tangent to the axis. If the power is one, the graph goes straight through the intercept. If the power is odd (other than one), the graph has a concavity change at the intercept.

To find the y-intercept: Set x = 0 and determine the value of the function.

Rational Functions:

A <u>rational function</u> is a function of the form $f(x) = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomials in x.

Domain – The domain of any function is the set of all possible x values for that function. The only x values that are not possible in a rational function are the ones that make the denominator equal 0. To find the domain of a rational function, set the denominator equal to zero. The domain will always be all real numbers except those values.

Vertical Asymptotes – A vertical asymptote is a line that a graph will approach but never cross. If there are no common factors, you will find a vertical asymptote everywhere the denominator equals zero. Remember that asymptotes are lines. So when you label a vertical asymptote, you must write the equation of a vertical line. Just make x equal everything it couldn't be in the domain.

Horizontal asymptotes:

A rational function may also have a horizontal asymptote. A **rational** function will have at most one horizontal asymptote. (Other functions will have, at most two horizontal asymptotes.) Horizontal asymptotes may be crossed.

- If the degree of the numerator (highest power of x) is less than the degree of the denominator, then the line y = 0 (x-axis) is the horizontal asymptote.
- If the degree of the numerator is equal to the degree of the denominator, then the line $\mathbf{y} = \frac{\mathbf{a}}{\mathbf{b}}$ is the horizontal asymptote, where a is the leading coefficient of the numerator, and b is the leading coefficient of the denominator.
- If the degree of the numerator is greater than the degree of the denominator, then the graph has no horizontal asymptote.
- If the degree of the numerator is exactly one greater than the degree of the denominator, it has a slant (oblique) asymptote, which is determined by long division.

To find the x-intercepts: Use the factors of the numerator set equal to zero.

Behavior through the x-intercepts: The powers of each factor of the numerator are used to determine the behavior of the graph through the intercept. If the power is even, the graph is tangent to the axis. If the power is one, the graph goes straight through the intercept. If the power is odd (other than one), the graph has a concavity change at the intercept.

To find the y-intercept: Set x = 0 and determine the value of the function.

Behavior at the vertical asymptotes: Use the factors of the denominator to determine the behavior of the graph at the vertical asymptotes. If the factor has an even power, as the graph approaches the asymptote from both sides, the ends of the graph will go in the same direction (both up to positive infinity, or both down to negative infinity). If the factor has an odd power, as the graph approaches the asymptote from both sides, the ends of the graph will go in opposite directions (one up, one down).