Cheatsheet - Turing Machines

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NOTE Make sure to check the *Automata Theory* cheatsheet, too.

1. Intro

A **turing machine** is a finite automation with **unbounded random access memory**. The **finite state automaton** (FSA) provides instructions on an **infinite tape**, where the input is given and can also be the working space. Every cell contains one character, but some cells are empty. A **tape head** reads and writes according to the instructions given by the FSA.

2. Formal Definition

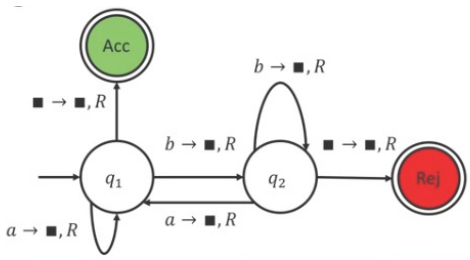
A turing machine ™ consists of:

$$(Q, \Sigma, \Gamma, \delta, q_1, q_{\text{Acc}}, q_{\text{Rej}})$$

where

- Γ is the tape alphabet that included the blank symbol.
- $\Sigma \subseteq \Gamma$ is the input alphabet.
- $\delta : Q \times \Gamma \to (Q \times \Gamma \times \{L, R\})$ is the transition function.
- $q_1 \in Q$ is the start state.
- q_{Acc} is the accepting state.
- q_{Rej} is the rejecting state.

The **transition function** takes one state and one letter from Γ and returns a **state** of the automaton, a **letter to be written** on the current cell of the tape and the **direction** instructing the tape head where to go, L for left, R for right.



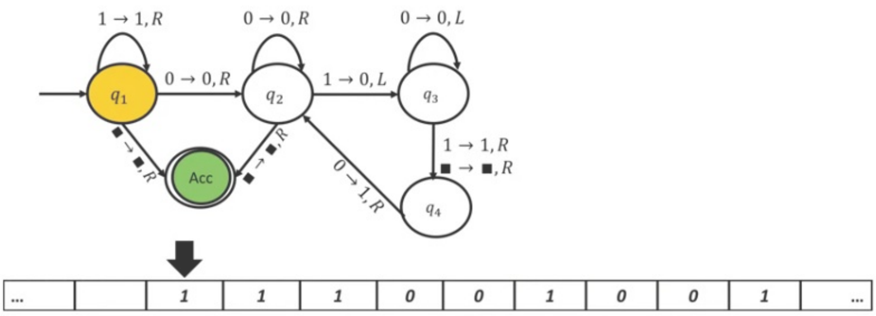
δ	a	b	$\blacksquare$
q_1	q_1, ■, R	q_2, ■, R	Accept
q_2	q_1, ■, R	q_2, ■, R	Reject

Figure 1. Note that the black box means "blank".

NOTE Seeing an interactive example makes the behavior of the direction $\{L, R\}$ clearer. See Week 13, "7.201 Turing machines: examples" and "7.202 Designing Turing Machines" in the FCS course.

2.1. Example

Given $w \in (1 \cup 0)^*$, make $w \in 1^*0^*$. For example, given 111001001, make 111110000.



NOTE For the interactive process, see Week 14, "7.301 The power of Turing machines".

3. DFA vs TM

The difference between a Deterministic Finite Automata (DFA) and a Turing Machine (TM) is that a TM **may not terminate** when the input is completely processed and may process the input **several times**. A TM **always terminate** at the accepting or rejecting state, while in DFA the process can pass through those states and continue. Additionally, a TM may **manipulate** the input, may enter **an infinite loop** and is **deterministic**.

4. The Language of Turing Machines

The language of a TM is:

$$\mathscr{L}(M) = \{w \in \Sigma^* \mid M \quad \text{accepts} \quad w\}$$

If $w \in \mathscr{L}(M)$, M reached accept state. If $w \notin \mathscr{L}(M)$, M **does not** reach accept state (either it reaches reject state or enters an infinite loop). A language is **recognizable** if it is accepted by a TM, where the TM is called the **recognizer** of $\mathscr{L}(M)$.

A TM that does not enter an infinite loop is called a **decider**. The language is decidable if it is **accepted** by the decider.

4.1. Halting Problem

RE ("recursively enumerable") is a class of **all** recognizable languages R is a class of **all** decidable languages.

Additionally:

$$R \subseteq RE$$

In other words, every decider is a recognizer, but not the other way around. The **halting problem** states that we cannot determine whether an arbitrary TM and an input will eventually halt or run forever.

4.2. Language Hierarchy

$$\text{RL} \subseteq \text{CFL} \subseteq R \subseteq RE \subseteq \text{all languages}$$

where "RL" refers to Regular Languages and "CFL" refers to Context-Free Languages.

4.2.1. Chomsky Hierarchy

Grammar	Languages	Automaton	Example
Type-0	Recursively enumerable	Turing machine	
Type-1	Context-sensitive	Turing machines with bounded tape	$a^n b^n c^n$
Type-2	Context-free	Push-down	$a^n b^n$
Type-3	Regular	Finite state	$a^* b^*$

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