Cheatsheet - Recursion

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1. Intro

When a function calls itself, we refer to this process as "recursion". A recursive formula can be useful to solve certain kinds of problems.

1.1. Example Exercise

Example 1. Disclaimer: The following exercise was solved by ChatGPT and includes direct quotes from its output.

> *There are 10 steps and you can take either one step at the time or two steps at the time. How many ways are there to walk all 10 steps?*

Let $S(n)$ be the number of ways to walk n steps by taking either one or two steps at a time. Then:

$$S(n) = S(n-1) + S(n-2)$$

The intuition behind this formula is that to walk n steps, we can either take a single step and then walk the remaining $n-1$ steps, or take two steps and then walk the remaining $n-2$ steps. Therefore, the total number of ways to walk n steps is the sum of the number of ways to walk $n-1$ steps and the number of ways to walk $n-2$ steps.

We can use this recursive formula to calculate $S(10)$:

$$\begin{aligned} S(1) &= 1 \\ S(2) &= 2 \\ S(3) &= S(2) + S(1) = 3 \\ S(4) &= S(3) + S(2) = 5 \\ S(5) &= S(4) + S(3) = 8 \\ &\dots \\ S(10) &= 89 \end{aligned}$$

NOTE

We define $S(1) = 1$ (only one way to walk one step) and $S(2) = 2$ (two ways to walk two steps, either one step at the time or two steps at once). Therefore, we don't use the recursive formula directly on $S(1)$ and $S(2)$.

Therefore, there are 89 ways to walk all 10 steps by taking either one or two steps at a time.

2. Euclid's Algorithm

Consider the following: there are two non-zero integers a and b, now lets find the greatest integer that divides a and b without leaving a remainder.

With Euclid's algorithm, lets first assume that $a \geq b$, then lets divide a by b and the remainder is r.

Importantly:

$$\gcd(a, b) = \gcd(b, r) = \gcd(a, r)$$

By using division once, we have $b \leq a$, $r < b$ since the arguments of gcd have decreased. We continue with this until $r = 0$, then b is the final gcd.

More formally:

$$\gcd(a, b) = \begin{cases} \gcd(b, |a \bmod b|) & \text{if } \ b > 0 \\ a & \text{if } \ b = 0 \end{cases}$$

For example, if we want to solve $\gcd(27, 36)$:

$$\begin{aligned} \gcd(27, 36) &= \gcd(36, 27) \\ \gcd(36, 27) &= \gcd(27, 9) \\ \gcd(27, 9) &= \gcd(9, 0) \\ \gcd(9, 0) &= 9 \end{aligned}$$

Or, if the numbers were reversed:

$$\begin{aligned} \gcd(36, 27) &= \gcd(27, 9) \\ \gcd(27, 9) &= \gcd(9, 0) \\ \gcd(9, 0) &= 9 \end{aligned}$$

Hence, $\gcd(27, 36) = \gcd(36, 27) = 9$.

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