Cheatsheet - Graphs: Isomorphism

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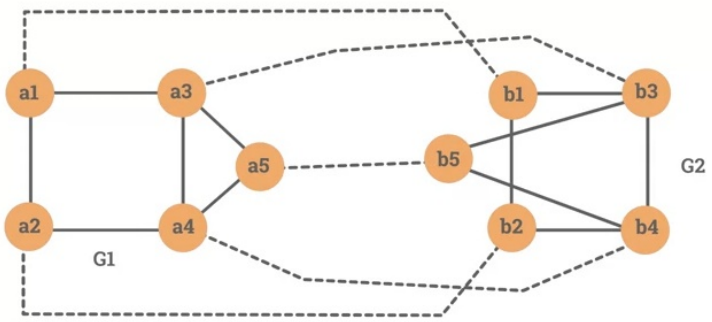
1. Definition

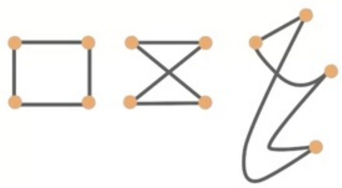
Two graphs G_1 and G_2 are isomorphic if there is a bijection (invertible function) $f: G_1 \to G_2$ that preserves adjacency and non-adjacency. Given two vertices u and v, if $u \times v$ is in $E(G_1)$ then $f(u) \times f(v)$ is in $E(G_2)$.



$f: G_1 \to G_2$

G_1	a_1	a_2	a_3	a_4	a_5
$f(G_1)=G_2$	b_1	b_2	b_3	b_4	b_5

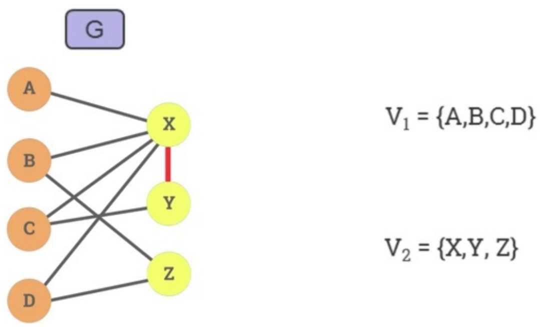
Two graphs with different degree sequences **cannot be isomorphic**. Two graphs with the same degree sequence **are not necessarily isomorphic**.



Isomorphic graphs

2. Bipartite Graph

A graph $G(V, E)$ is called a bi-partite graph if the set of vertices V can be partitioned in two non-empty disjoint sets V_1 and V_2 in such a way that each edge e in G has one endpoint in V_1 and another endpoint in V_2.



The graph is 2-colourable

No odd-length cycles

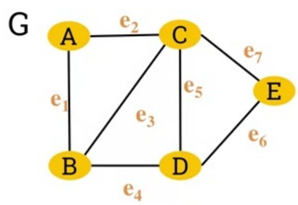
2.1. Matching

A **matching** is a set of pairwise non-adjacent edges, none of which are loops. That is, no two edges share a common endpoint. A vertex is matched (or saturated) if it is an endpoint of one of the edges in the matching. Otherwise the vertex is unmatched.

A **maximum** matching is a matching of maximum size such that if any edge is added, it is no longer a matching. The **Hopcroft-Karp algorithm** is commonly used for solving the maximum matching problem in a bipartite graph (*the algorithm is not specified in this cheatsheet*).

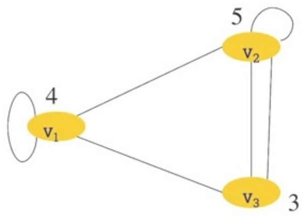
3. Adjacency Matrix of Graph

The adjacency list of a graph G is a list of all the vertices in G and their corresponding individual adjacent vertices.



a = b, c
b = a, c, d
c = a, b, d, e
d = b, c, e
e = c, d

A graph can also be represented by its **adjacency matrix**. The number of edges in an **undirected** graph is equal to half the sum of all the elements (m_{ij}) of it's corresponding adjacency matrix.



$$M(G) = . \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$\sum m_{ij} = 1+1+1+1+2+2+2+2 = 5+4+3 = 12$$

$$\text{Number of edges in G} = (\sum m_{ij})/2 = 12/2 = 6$$

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