Cheatsheet - Equivalence Relations & Classes

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NOTE You should read the cheatsheet on *Relations*, too.

1. Definitions

1.1. Equivalence Relation

Let R be a relation of elements on set S. R is an **equivalence relation** if and only if R is **reflexive**, **symmetric** and **transitive**.

1.2. Equivalence Classes

Let R be an **equivalence relation** on a set S. Then, the **equivalence class** of $a \in S$ is the **subset** of S containing all the **elements related** to a through R:

$$[a] = \{x : x \in S \quad \text{and} \quad xRa\}$$

For example:

$$S = \{1, 2, 3, 4, 5\}$$
$$R = \left\{(a, b) \in S^2 \mid a - b \quad \text{is an even number}\right\}$$

The set R has two equivalence classes:

$$[1] = [3] = [5] = \{1, 3, 5\}$$
$$[2] = [4] = \{2, 4\}$$

2. Partial & Total Order

Let R be a relation on elements in a set S. R is a **partial order** if and only if R is **reflexive**, **anti-symmetric** and **transitive**.

Additionally, R is a **total** order if and only if:

- R is a **partial order**.
- $\forall (a, b) \in S$ we have **either** aRb **or** bRa.

For example, the following relation is a total order:

$$R = \left\{(a, b) \in \mathbb{Z}^2 \mid a \leq b\right\}$$

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