Cheatsheet - Postulates of Boolean Algebra

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1. Intro

Boolean algebra describes the field of study where the values of variables are true or false. The following operators are used:

- AND: represented as $x. y, x \cap y$ or $x \wedge y$
- ullet OR: represented as $x+y, x\cup y$ or $x\vee y$
- NOT: represented as x', \bar{x} or $\neg x$

Order of precedence: NOT > AND > OR

2. Axioms

The following axioms must be satisfied by any boolean algebra:

- **closure**: any result of logical operations belongs to the set $\{0, 1\}$ (*true* or *false*).
- identity: elements have an identity for the applied operator:
- $\circ x + 0 = x$
- $\circ x.1 = x$
- commutativity: the order of the applied operators does not matter.
 - $\circ \ x + y = y + x$
 - $\circ x. y = y. x$
- distributivity:
 - x(y+z) = (x,y) + (x,z)
 - x + (y. z) = (x + y). (x + z)
- complements: exist for all the elements
 - $\circ x + x' = 1$
 - $\circ x.x'=0$
- distinct elements:
 - $0 \neq 1$

2.1. Basic Theorems

Based on those axioms, we can establish basic theorems:

Theorem 1: Idempotent Laws

$$x + x = x$$

$$x. x = x$$

Theorem 2: Tautology and Contradiction

$$x+1=1$$

$$x.0 = 0$$

Theorem 3: Involution

$$(x')' = x$$

Theorem 4: Associative Laws

$$(x+y)+z=x+(y+x)$$

$$(x. y). z = x. (y. z)$$

Theorem 5: Absorption Laws

$$x + (x.y) = x$$

$$x.\left(x+y\right) =x$$

Theorem 6: Uniqueness of Complement

if
$$y + x = 1$$
 and $y \cdot x = 0$, then $x = y'$

$$0' = 1, 1' = 0$$

2.2. De Morgans' Theorems

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\overline{x + y} = \overline{x} \cdot \overline{y}$$

2.3. Principles of Duality

Starting with a boolean relation, we can build another equivalent boolean relation by:

- changing each OR(+) to an AND(.)
- changing each AND(.) to an OR(+)
- changing each 0 to 1 and each 1 to 0.

For example:

$$(a. 1). (0 + \overline{a}) = 0$$
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 $(a + 0) + (1. \overline{a}) = 1$

3. Boolean Functions

3.1. Standardized Forms of a Functions

The two most common standardized forms are sum-of-products and product-of-sums.

The sum-of-products form: f(x, y, z) = xy + xz + yz The product-of-sums form: f(x, y, z) = (x + y)(x + z)(y + z)

For example:

$$f(x,y) = x'y + xy' + xy$$

3.2. Useful Functions

The **exclusive-or** function:

$$x \oplus y = x'y + xy'$$

The **implies** function:

$$x \rightarrow y = x' + y$$

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