

# Cheatsheet - Set Theory

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## 1. Definition

A set refers to an **unordered** collection of any kind of (abstract) **unique** objects, so called "elements", such as numbers, figures, things, etc.

### 1.1. Basic Notation

We have the set  $E$  containing four elements:

$$E = \{1, 2, 3, 4\}$$

or set  $V$ :

$$V = \{a, b, c, d, e, f\}$$

We can see that the element 2 is in  $E$ , respectively:

$$2 \in E$$

but *not* in  $V$

$$2 \notin V$$

## 2. Cardinality

Given the set  $S$ , the **cardinality** of  $S$  is the number of elements contained in  $S$ . We write the cardinality of  $S$  as  $|S|$ .

$$S = \{a, b, c\}$$

$$|S| = 3$$

and the following **empty** set:

$$S = \emptyset = \{\}$$

$$|S| = 0$$

## 3. Subset and Superset

We have set  $E$  and  $\{1, 2\}$  is a **subset** ( $\subseteq$ ) of  $E$ . We can also say that  $E$  is a **superset** ( $\supseteq$ ) of  $\{1, 2\}$ . The set  $\{a, b\}$  is **not** a subset ( $\not\subseteq$ ) of  $E$ , however.

$$E = \{1, 2, 3, 4\}$$

$$E \subseteq E$$

$$E \supseteq E$$

$$\{1, 2\} \subseteq E$$

$$E \supseteq \{1, 2\}$$

$$\{a, b\} \not\subseteq E$$

$$E \not\subseteq \{a, b\}$$

$$E \not\supseteq \{1, 2, 3, 4, 5\}$$

Importantly, we distinguish between a subset ( $\subseteq$ ) and a **proper subset** ( $\subset$ ):

$$\{1, 2, 3\} \subset E$$

$$\{1, 2, 3, 4\} \not\subset E$$

$$\{1, 2, 3, 4\} \subseteq E$$

Do note that any set also contains an empty subset:

$$(\{\} = \emptyset) \subseteq E$$

## 4. Special Sets

Commonly defined and used sets in mathematics:

$$\mathbb{N} = \{1, 2, 3, 4, \dots\} \quad \text{natural numbers}$$

$$\mathbb{W} = \{0, 1, 2, 3, \dots\} \quad \text{whole numbers}$$

$$\mathbb{Z} = \{\dots, -1, 0, 1, 2, \dots\} \quad \text{integers (no fractions)}$$

$$\mathbb{Q} = \left\{ \dots, -1, -\left(\frac{1}{2}\right), 0, \frac{2}{3}, 1, \dots \right\} \quad \text{rational numbers}$$

$$\mathbb{R} = \quad \text{real numbers (non complex-numbers)}$$

Note that infinite numbers are *irrational numbers* and are not part of  $\mathbb{Q}$ , such as  $\pi \notin \mathbb{Q}$ .

Additionally:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

## 5. Set Builder

We define:

$$E = \{1, 2, 3, 4\}$$

and use the set builder to create a new set, for example:

$$V = \{2n \mid n \in E\}$$

$$V = \{2, 4, 6, 8\}$$

Read as "each element set  $V$  is two times  $n$  for each  $n$  in set  $E$ ".

We now define the set  $J$ :

$$J = \{2n \mid n \in E \text{ and } n < 3\}$$

$$J = \{2, 4\}$$

## 6. Powerset

Given a set  $S$ , the powerset of that set is denoted as  $P(S)$  and contains all subsets of  $S$ :

$$S = \{a, b\}$$

$$P(S) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

since

$$\emptyset \subseteq S$$

$$\{a\} \subseteq S$$

$$\{b\} \subseteq S$$

$$\{a, b\} \subseteq S$$

### 6.1. Cardinality of a powerset

We define:

$$|P(S)| = 2^{|S|}$$

For example:

$$S = \{1, 2, 3\}$$

$$|S| = 3$$

$$P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

$$|P(S)| = 2^3 = 8$$

## 7. Set Operations

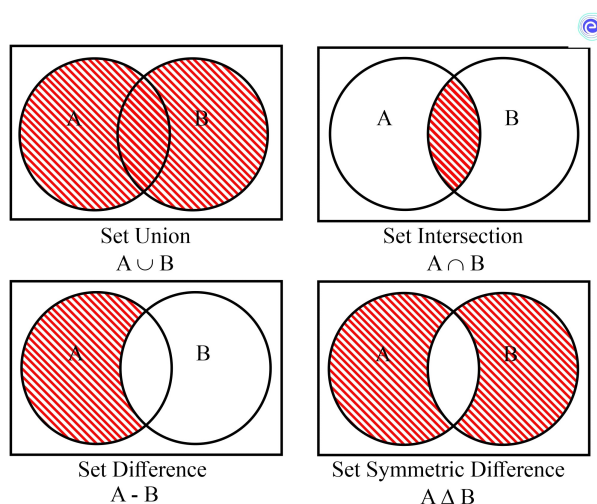


Figure 1. Venn Diagram, source: <https://www.embibe.com/exams/set-theoretic-approach/>

7.1. Union

Given two sets  $A$  and  $B$ , the union of  $A$  and  $B$ ,  $A \cup B$ , contains all the element in **either**  $A$  and  $B$ .

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

For example:

$$A = \{1, 2, 3\}$$

$$B = \{4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

and

$$A = \{1, 2, 3\}$$

$$C = \{2, 3, 4\}$$

$$A \cup C = \{1, 2, 3, 4\}$$

7.2. Intersection

Given two sets  $A$  and  $B$ , the intersection of  $A$  and  $B$ ,  $A \cap B$ , contains all the elements in both  $A$  **and**  $B$ .

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

For example:

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4\}$$

$$A \cap B = \{2, 3\}$$

7.3. Set Difference

Given two sets  $A$  and  $B$ , the set difference,  $A - B$ , contains the elements that are in  $A$  but not in  $B$ .

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

For example:

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4\}$$

$$A - B = \{1\}$$

NOTE

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$A - B = A \setminus B$

7.4. Symmetric Difference

Given two sets  $A$  and  $B$ , the symmetric difference,  $A \oplus B$ , contains the elements that are in  $A$  or in  $B$  but **not in both**.

$$A \oplus B = \{x \mid (x \in A \text{ or } x \in B) \text{ and } x \notin A \cap B\}$$

For example

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4\}$$

$$A \oplus B = \{1, 4\}$$

NOTE

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$A \oplus B = A \Delta B$