PS2 Yuming Liu

In [1]:
```
import numpy as np
import scipy as sp
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import norm
from scipy.integrate import quad
from scipy.optimize import root
import math
from IPython.display import Image
from scipy.special.orthogonal import p_roots
```

Problem 2.1

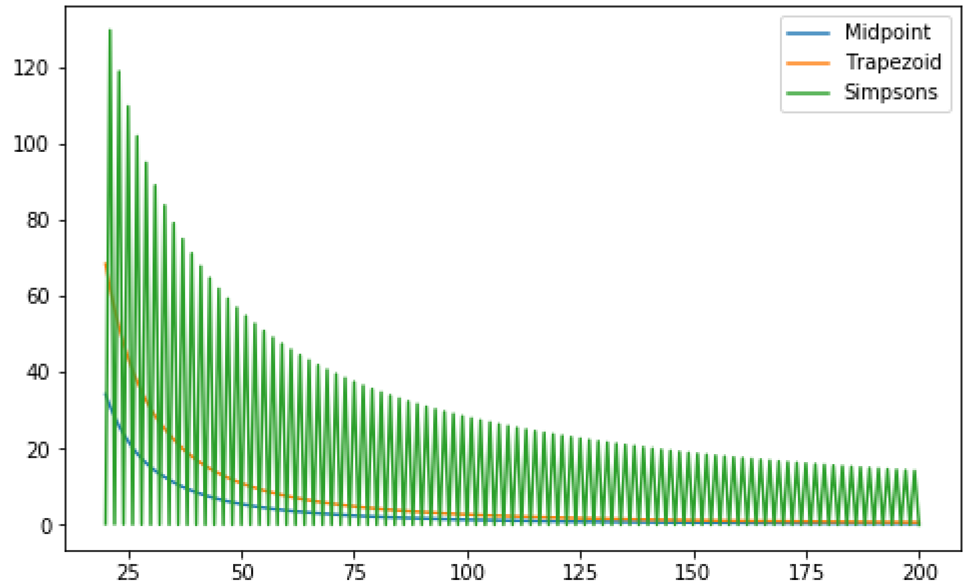
In [2]:
```
def g(x):
    return 0.1*x**4 -1.5*x**3 + 0.53*x**2 + 2*x + 1
```

In [3]:
```
def error(f, a, b, n, method, intgvalue):
    if method not in {'midpoint','trapezoid', 'Simpsons'}:
        raise ValueError
    else:
        intg = 0
        if method == 'midpoint':
            for i in range(n):
                intg += ((b-a)/n)*f(a+((2*i+1)*(b-a))/(2*n))
        elif method == 'trapezoid':
            for i in range(1,n):
                intg+=((b-a)/(2*n))*(2*f(a+i*(b-a)/n))
            intg+=((b-a)/(2*n))*(f(a)+f(b))
        else:
            for i in range(1,n):
                if i%2==1:
                    intg+=((b-a)/(3*n))*(4*f((a+((i*(b-a)))/n)))
                else:
                    intg+=((b-a)/(3*n))*(2*f((a+((i*(b-a))/n))))
            intg+=((b-a)/(3*n))*(f(a)+f(b))
    return np.abs(intgvalue - intg)
```

In [4]:

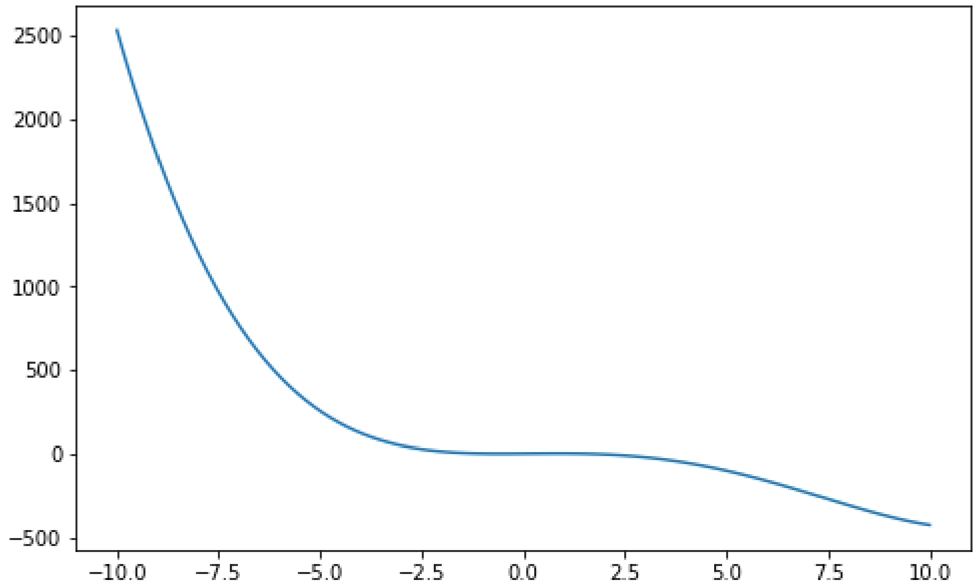
```
rr = np.arange(20, 201, 1)
plt.figure(figsize = (8,5))

def listerr(r, method):
    l = []
    for i in r:
        l.append(error(g, -10, 10, i, method, 4373+1/3))
    return l
plt.plot(rr, listerr(rr, 'midpoint'), label='Midpoint')
plt.plot(rr, listerr(rr, 'trapezoid'), label='Trapezoid')
plt.plot(rr, listerr(rr, 'Simpsons'), label='Simpsons')
plt.legend()
plt.show()
```



In [5]:
```
rr = np.linspace(-10, 10, 100)
plt.figure(figsize = (8,5))
plt.plot(rr, g(rr))
```

Out[5]: `[<matplotlib.lines.Line2D at 0x271ba462198>]`



In [6]:
```
print('Using midpoint method, the difference with true value is: ', erro
r(g, -10, 10, 200, 'midpoint', 4373+1/3))
print('Using trapezoid method, the difference with true value is: ', err
or(g, -10, 10, 200, 'trapezoid', 4373+1/3))
print('Using Simpsons method, the difference with true value is: ', erro
r(g, -10, 10, 200, 'Simpsons', 4373+1/3))
```

```
Using midpoint method, the difference with true value is:  0.3421608333
3649294
Using trapezoid method, the difference with true value is:  0.684326666
6628551
Using Simpsons method, the difference with true value is:  2.6666670237
318613e-05
```

Problem 2.2

In [7]:
```
def disc(N, mu, sigma, k):
    Z = np.linspace(mu - k * sigma, mu + k * sigma, N)
    weight = np.zeros(N)
    weight[0] = norm.cdf((Z[0]+Z[1])/2, loc=mu, scale=sigma)
    for i in range(1,N-1):
        f = lambda x : norm.pdf(x, loc=mu, scale=sigma)
        weight[i] = quad(f, (Z[i - 1] + Z[ i ]) / 2,(Z[i + 1] + Z[i])/2)
[0]
    weight[-1] = 1 - norm.cdf((Z[-2] + Z[-1]) / 2, loc=mu, scale=sigma)
    return Z, weight
Z, weight = disc(11, 5, 1.5, 3)
df = pd.DataFrame({'Z': Z, 'Weight': weight})
df
```

Out[7]:

	Z	Weight
0	0.5	0.003467
1	1.4	0.014397
2	2.3	0.048943
3	3.2	0.117253
4	4.1	0.198028
5	5.0	0.235823
6	5.9	0.198028
7	6.8	0.117253
8	7.7	0.048943
9	8.6	0.014397
10	9.5	0.003467

Problem 2.3

In [8]:
```
def log_disc(N, mu, sigma, k):
    Z, weight = disc(N, mu, sigma, k)
    A = np.exp(Z)
    apx = sum(A * weight)
    return A, weight, apx
A, weight, apx = log_disc(11, 5, 1.5, 3)
log_df = pd.DataFrame({'A': A, 'Weight': weight})
log_df
```

Out[8]:

	A	Weight
0	1.648721	0.003467
1	4.055200	0.014397
2	9.974182	0.048943
3	24.532530	0.117253
4	60.340288	0.198028
5	148.413159	0.235823
6	365.037468	0.198028
7	897.847292	0.117253
8	2208.347992	0.048943
9	5431.659591	0.014397
10	13359.726830	0.003467

In [9]:
```
print(apx)
```

```
460.5426522031043
```

Problem 2.4

In [10]:
```
A, weight, apxn = log_disc(11, 10.5, 0.8, 3)
income= np.e**(10.5+0.5*(0.8**2))
diff = apxn - income
print(income, apxn, diff)
```

```
50011.08700852173 50352.456192765894 341.36918424416217
```

Problem 3.1

```
In [11]: def gauss(f,n,a,b):
             [x,w] = p_roots(n+1)
             G = 0.5*(b-a)*sum(w*f(0.5*(b-a)*x+0.5*(b+a)))
             return G

         Gaussian = gauss(g, 3, -10, 10)
         Sim = error(g, -10, 10, 200, 'Simpsons', 4373+1/3)
         Mid = error(g, -10, 10, 200, 'midpoint', 4373+1/3)
         Tra = error(g, -10, 10, 200, 'trapezoid', 4373+1/3)
         print(Sim, Mid, Tra, np.abs(Gaussian - (4373+1/3)))
```

```
2.6666670237318613e-05 0.34216083333649294 0.6843266666628551 9.0949470
17729282e-13
```

Gaussian is more accurate than the Newton methods.

Problem 3.2

```
In [12]: Gaussian = sp.integrate.quad(g, -10, 10)
         print(Gaussian)
```

```
(4373.333333333334, 8.109531705284936e-11)
```

The python Gaussian result has a greater absolute error than the error we had for problem 3.1.

Problem 4.1

```
In [13]: def f(x,y):
             if x**2+y**2 <= 1:
                 return 1
             else:
                 return 0

         def Monte(f, Omega, N):
             x = np.random.uniform(Omega[0][0], Omega[0][1], N)
             y = np.random.uniform(Omega[1][0], Omega[1][1], N)

             t = 0
             for i in range(N):
                 t += f(x[i],y[i])
             figure = (Omega[0][1]-Omega[0][0])*(Omega[1][1]-Omega[1][0])
             result = figure*t/N
             return result
```

In [14]:
```
np.random.seed(25)
N = 1
O = np.array([[-1,1],[-1,1]])
while round(Monte(f, O, N),4) != 3.1415:
    N += 1
print(N)
```

```
615
```

Problem 4.2

In [15]:
```
def isPrime(n):
    for i in range(2, int(np.sqrt(n) + 1)):
        if n % i == 0:
            return False

    return True

def primes_ascend(N, min_val=2):
    primes_vec = np.zeros(N, dtype=int)
    MinIsEven = 1 - min_val % 2
    MinIsGrtrThn2 = min_val > 2
    curr_prime_ind = 0
    if not MinIsGrtrThn2:
        i = 2
        curr_prime_ind += 1
        primes_vec[0] = i
    i = min(3, min_val + (MinIsEven * 1))
    while curr_prime_ind < N:
        if isPrime(i):
            curr_prime_ind += 1
            primes_vec[curr_prime_ind - 1] = i
        i += 2

    return primes_vec
```

In [16]:
```
def element_seq(Name, n, d):
    primev = primes_ascend(d)
    if Name == 'Weyl':
        return [math.modf(i)[0] for i in n*np.sqrt(primev)]
        # Reference: https://www.geeksforgeeks.org/python-modf-function/
    elif Name == 'Haber':
        return [math.modf(i)[0] for i in (n * (n+1) / 2)*np.sqrt(primev
)]
    elif Name == 'Niederreiter':
        exp = [i / (n+1) for i in range(1,d+1)]
        return [math.modf(i)[0] for i in n * np.power(2,exp)]
    elif Name == 'Baker':
        rational=[1 / i for i in range(1,d+1)]
        return [math.modf(i)[0] for i in n * np.exp(rational)]
```

In [17]:
```
element_seq('Weyl', 1073, 2)
```
Out[17]:
```
[0.45115242633119124, 0.49051652140519764]
```

In [18]:
```
element_seq('Haber', 1073, 2)
```
Out[18]:
```
[0.2688529398292303, 0.40737199457362294]
```

In [19]:
```
element_seq('Niederreiter', 1073, 2)
```
Out[19]:
```
[0.6927253065300647, 0.38589783426255053]
```

In [20]:
```
element_seq('Baker', 1073, 2)
```
Out[20]:
```
[0.716401936555485, 0.07792346123756033]
```

Problem 4.3

In [21]:
```
def quasi_monte(f, Omega, Name, d, N):
    var = []
    for i in range(N):
        var.append(tuple(d * element_seq(Name, i ,d)[j] - 1 for j in ran
ge(d)))
    t = 0
    for i in range(N):
        t += f(var[i][0], var[i][1])
    area = (Omega[0][0] - Omega[0][1]) * (Omega[1][0] - Omega[1][1])
    return area * t / N
```

In [22]:
```
np.random.seed(25)
N = 1
O = np.array([[-1,1],[-1,1]])
while round(quasi_monte(f, O, 'Weyl', 2, N),4) != 3.1415:
    N += 1
print(N)
```
```
1230
```

In [23]:
```
np.random.seed(25)
N = 1
O = np.array([[-1,1],[-1,1]])
while round(quasi_monte(f, O, 'Haber', 2, N),4) != 3.1415:
    N += 1
print(N)
```
```
2064
```

In [24]:
```
'''
np.random.seed(25)
N = 1
O = np.array([[-1,1],[-1,1]])
while round(quasi_monte(f, O, 'Niederreiter', 2, N),4) != 3.1415:
    N += 1
print(N)
'''
```

Out[24]:
```
"\nnp.random.seed(25)\nN = 1\nO = np.array([[-1,1],[-1,1]])\nwhile roun
d(quasi_monte(f, O, 'Niederreiter', 2, N),4) != 3.1415:\n    N += 1\npr
int(N)    \n"
```

In [25]:
```
np.random.seed(25)
N = 1
O = np.array([[-1,1],[-1,1]])
while round(quasi_monte(f, O, 'Baker', 2, N),4) != 3.1415:
    N += 1
print(N)
```

```
205
```

The Baker method has the smallest number of draws that is 205 times, and the Niederreiter method has the largest number.