Problem Set 4

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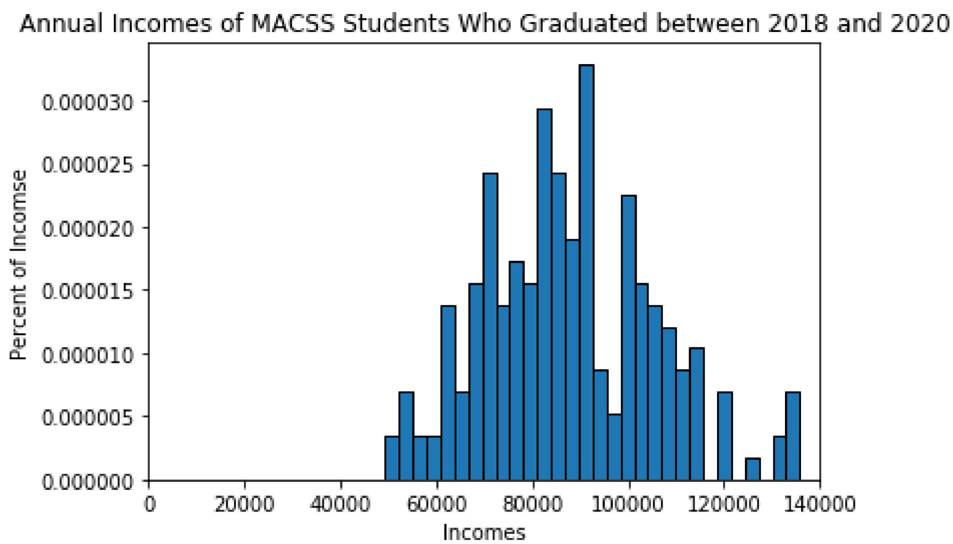
In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.stats as sts
from scipy.integrate import quad
import scipy.optimize as opt
import warnings
warnings.filterwarnings("ignore")
```

Problem 1(a)

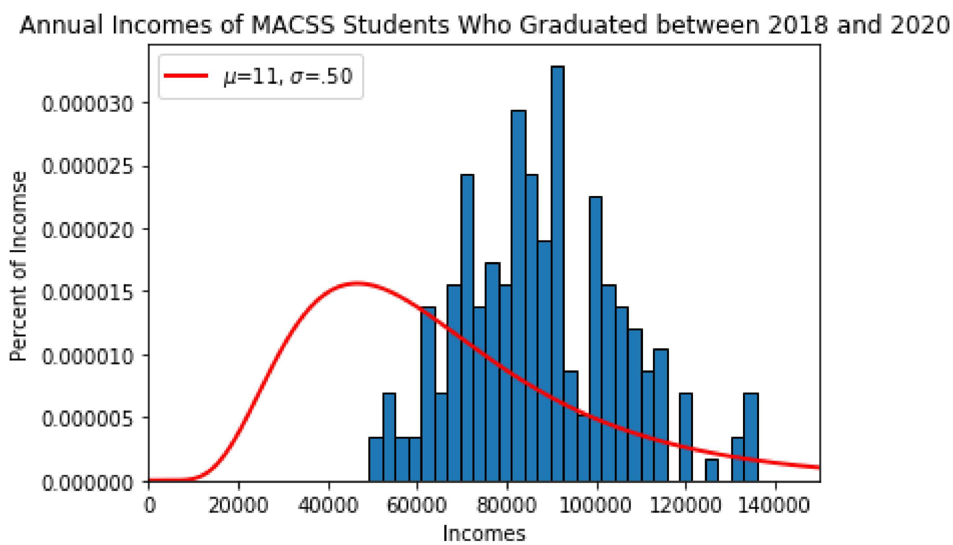
In [2]:
```
pts = np.loadtxt('data/incomes.txt')

num_bins = 30
count, bins, ignored = plt.hist(pts, num_bins, density=True, edgecolor='k')
plt.title('Annual Incomes of MACSS Students Who Graduated between 2018 and 202
0')
plt.xlabel(r'Incomes')
plt.ylabel(r'Percent of Incomse')
plt.xlim([0, 140000])
plt.show()
```



Problem 1(b)

In [3]:
```
def trunc_lognorm_pdf(xvals, mu, sigma, cut_lb, cut_ub):
    if cut_ub == 'None' and cut_lb == 'None':
        prob_notcut = 1.0
    elif cut_ub == 'None' and cut_lb != 'None':
        prob_notcut = 1.0 - sts.lognorm.cdf(cut_lb, scale=np.exp(mu), s=sigma)
    elif cut_ub != 'None' and cut_lb == 'None':
        prob_notcut = sts.lognorm.cdf(cut_ub, scale=np.exp(mu), s=sigma)
    elif cut_ub != 'None' and cut_lb != 'None':
        prob_notcut = (sts.lognorm.cdf(cut_ub, scale=np.exp(mu), s=sigma) -
                       sts.lognorm.cdf(cut_lb, scale=np.exp(mu), s=sigma))

    pdf_vals    = ((1/(xvals*sigma * np.sqrt(2 * np.pi)) *
                    np.exp( - (np.log(xvals) - mu)**2 / (2 * sigma**2))) /
                    prob_notcut)

    return pdf_vals
```

In [4]:
```
plt.hist(pts, num_bins, density=True, edgecolor='k')
plt.title('Annual Incomes of MACSS Students Who Graduated between 2018 and 202
0')
plt.xlabel(r'Incomes')
plt.ylabel(r'Percent of Incomse')
plt.xlim([0, 150000])

dist_pts = np.linspace(0.001, 150000, 1000)
mu_1 = 11
sig_1 = 0.5

plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_1, sig_1, 1, 150000),
         linewidth=2, color='r', label='$\mu$=11, $\sigma$=.50')
plt.legend(loc='upper left')

plt.show()
```



In [5]:
```
def log_lik_truncnorm(xvals, mu, sigma, cut_lb, cut_ub):
    pdf_vals = trunc_lognorm_pdf(xvals, mu, sigma, cut_lb, cut_ub)
    ln_pdf_vals = np.log(pdf_vals)
    log_lik_val = ln_pdf_vals.sum()

    return log_lik_val

print('Log-likelihood :', log_lik_truncnorm(pts, mu_1, sig_1, 1, 150000))
```

```
Log-likelihood : -2379.120591931827
```

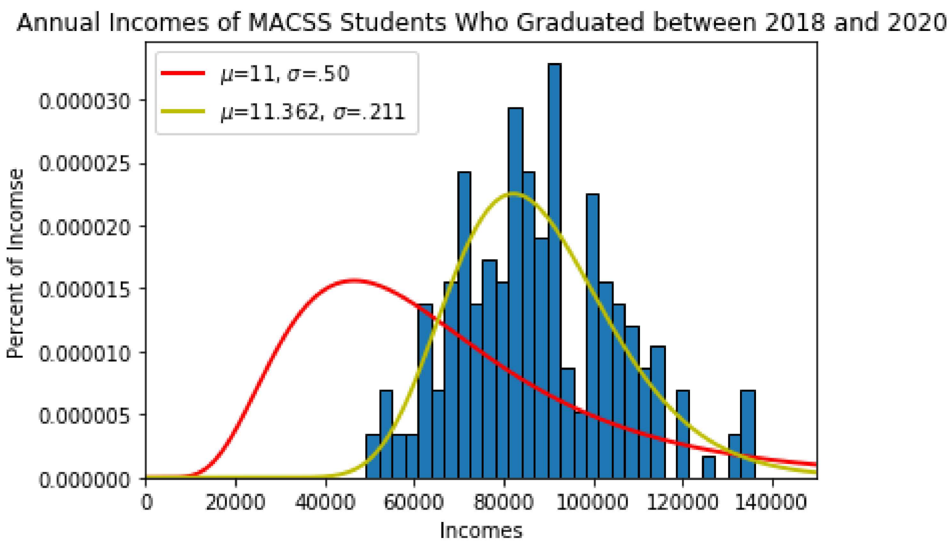
Problem 1(c)

In [6]:
```
def crit(params, *args):
    mu, sigma = params
    xvals, cut_lb, cut_ub = args
    log_lik_val = log_lik_truncnorm(xvals, mu, abs(sigma), cut_lb, cut_ub)
    neg_log_lik_val = -log_lik_val

    return neg_log_lik_val
```

In [7]:
```
mu_init = 11
sig_init = 0.5
params_init = np.array([mu_init, sig_init])
mle_args = (pts, 0, 150000)
results_uncstr = opt.minimize(crit, params_init, args=(mle_args))
mu_MLE, sig_MLE = results_uncstr.x
MLE = log_lik_truncnorm(pts, mu_MLE, sig_MLE, 0, 150000)
print('mu_MLE=', mu_MLE, ' sig_MLE=', sig_MLE)
print('value of likelihood function is', MLE)
print('The Hessian matrix is', results_uncstr.hess_inv)
```

```
mu_MLE= 11.361699976140056  sig_MLE= 0.21174326472241192
value of likelihood function is -2240.934337511636
The Hessian matrix is [[0.00032821 0.00066662]
 [0.00066662 0.00147221]]
```

In [8]:
```
plt.hist(pts, num_bins, density=True, edgecolor='k')
plt.title('Annual Incomes of MACSS Students Who Graduated between 2018 and 202
0')
plt.xlabel(r'Incomes')
plt.ylabel(r'Percent of Incomse')
plt.xlim([0, 150000])

dist_pts = np.linspace(0.001, 150000, 1000)
mu_1 = 11
sig_1 = 0.5

plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_1, sig_1, 1, 150000),
         linewidth=2, color='r', label='$\mu$=11, $\sigma$=.50')
plt.legend(loc='upper left')

plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_MLE, sig_MLE, 1, 150000),
         linewidth=2, color='y', label='$\mu$=11.362, $\sigma$=.211')
plt.legend(loc='upper left')

plt.show()
```



Problem 1(d)

In [9]:
```
mu_new, sig_new = np.array([11, 0.5])
print(mu_new, sig_new)
print(mu_MLE, sig_MLE)
log_lik_h0 = log_lik_truncnorm(pts, mu_new, sig_new, 0, 150000)
print('hypothesis value log likelihood', log_lik_h0)
log_lik_mle = log_lik_truncnorm(pts, mu_MLE, sig_MLE, 0, 150000)
print('MLE log likelihood', log_lik_mle)
LR_val = 2 * (log_lik_mle - log_lik_h0)
print('likelihood ratio value', LR_val)
pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 2)
print('chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
11.0 0.5
11.361699976140056 0.21174326472241192
hypothesis value log likelihood -2379.120591931827
MLE log likelihood -2240.934337511636
likelihood ratio value 276.3725088403826
chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

Since the p-value is extremely small, we reject the null hypothesis and conclude that the data has a different distribution from 1(b).

Problem 1(e)

In [10]:
```
m100k = 1-sts.lognorm.cdf(100000, scale=np.exp(mu_MLE), s=sig_MLE)
l75k = sts.lognorm.cdf(75000, scale=np.exp(mu_MLE), s=sig_MLE)
print('The probability of earning more than 100k is ', m100k)
print('The probability of earning less than 75k is ', l75k)
```

```
The probability of earning more than 100k is  0.23755402258976566
The probability of earning less than 75k is  0.2596439222572218
```

Problem 2(a)

In [11]:
```
data = pd.read_csv('data/sick.txt')
```

In [12]:
```
data.head()
```

Out[12]:

	sick	age	children	avgtemp_winter
0	1.67	57.47	3.04	54.10
1	0.71	26.77	1.20	36.54
2	1.39	41.85	2.31	32.38
3	1.37	51.27	2.46	52.94
4	1.45	44.22	2.72	45.90

In [13]:
```
def norm_pdf(xvals, sigma):
    sigma = abs(sigma)
    pdf_vals = (1/(sigma * np.sqrt(2 * np.pi)) *
                np.exp( - (xvals)**2 / (2 * sigma**2)))
    return pdf_vals

def log_lik(y, x1, x2, x3, beta_0,
            beta_1, beta_2, beta_3, sigma):
    epsilon = y - beta_0 - beta_1 * x1 - beta_2 * x2 - beta_3 * x3
    pdf_vals = norm_pdf(epsilon, sigma)
    log_lik_func = np.log(pdf_vals).sum()

    return log_lik_func

def crit_lr(params, *args):
    beta_0, beta_1, beta_2, beta_3, sigma = params
    y, x1, x2, x3 = args
    neg_log_lik = -log_lik(y, x1, x2, x3, beta_0,
                           beta_1, beta_2, beta_3, sigma)
    return neg_log_lik
```

In [14]:
```
params_init = np.array([1, 0, 0, 0, (0.01 ** 0.5)])
mle_args = data
y = data['sick']
x1, x2, x3 = data['age'], data['children'], data['avgtemp_winter']

results = opt.minimize(crit_lr, params_init, args = (y, x1, x2, x3))
b0, b1, b2, b3, sigma = results.x
print('beta_0 = ', b0)
print('beta_1 = ', b1)
print('beta2 = ', b2)
print('beta3 = ', b3)
print('sigma = ', sigma)
print('The value of the log likelihood function is: ', abs(results.fun))
print('The estimated variance covariance matrix of the estimates is: ', result
s.hess_inv)
```

```
beta_0 =  0.25164638358979413
beta_1 =  0.01293335004243094
beta2 =  0.40050204832986624
beta3 =  -0.009991673034136558
sigma =  0.0030176821762459183
The value of the log likelihood function is:  876.8650462887076
The estimated variance covariance matrix of the estimates is:  [[ 9.28952977e
-07 -2.59333953e-09 -6.03075470e-08 -1.51766333e-08
  -5.81786380e-09]
 [-2.59333953e-09  2.62444545e-09 -2.14991777e-08 -1.55480976e-09
  -1.64135303e-10]
 [-6.03075470e-08 -2.14991777e-08  2.19103324e-07  1.29390796e-08
   3.28714537e-09]
 [-1.51766333e-08 -1.55480976e-09  1.29390796e-08  1.28927814e-09
   1.58273297e-10]
 [-5.81786380e-09 -1.64135303e-10  3.28714537e-09  1.58273297e-10
   2.21354184e-08]]
```

Problem 2(b)

In [15]:
```
b00, b01, b02, b03, sigma0 = np.array([1, 0, 0, 0, 0.1])
log_lik_h0 = log_lik(y, x1, x2, x3, b00, b01, b02, b03, sigma0)
print('hypothesis value log likelihood', log_lik_h0)
log_lik_mle = log_lik(y, x1, x2, x3, b0, b1, b2, b3, sigma)
print('MLE log likelihood', log_lik_mle)
LR_val = 2 * (log_lik_mle - log_lik_h0)
print('likelihood ratio value', LR_val)
pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 2)
print('chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2253.700688042125
MLE log likelihood 876.8650462887076
likelihood ratio value 6261.131468661665
chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

Therefore, since the p-vale is extremely small, we reject H0 and conclude that age, number of children, and average winter temperature have effect on the number of sick days.