Module 2 : History of "big ideas" of causal inference

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Design of Experiments - Stat 140

Spring 2026

History of "big ideas" in causal inference (1/2)

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1925 : Fisher argues for the necessity of physical randomization for credibly assessing causal effects.

1952 : Kempthorne uses potential outcomes in general randomized experiments.

History of "big ideas" in causal inference (2/2)

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1996 : Angrist, Imbens, and Rubin describes causal estimands in randomized experiments with non-compliance.

Potential outcomes framework (Rubin, 1974)

- Potential outcomes :
  - $Y_i(W_i = 0)$ denotes the outcome under the control treatment
  - $Y_i(W_i = 1)$ denotes the outcome under the active treatment

- Observed and missing outcomes :
  - $Y_i^{obs} = Y_i(W_i) = \begin{cases} Y_i(W_i = 0) & \text{if } W_i = 0 \\ Y_i(W_i = 1) & \text{if } W_i = 1 \end{cases}$
  - $Y_i^{mis} = Y_i(1 - W_i) = \begin{cases} Y_i(W_i = 1) & \text{if } W_i = 0 \\ Y_i(W_i = 0) & \text{if } W_i = 1 \end{cases}$

- One-to-one correspondence :
  - $Y_i(W_i = 0) = \begin{cases} Y_i^{obs} & \text{if } W_i = 0 \\ Y_i^{mis} & \text{if } W_i = 1 \end{cases}$
  - $Y_i(W_i = 1) = \begin{cases} Y_i^{mis} & \text{if } W_i = 0 \\ Y_i^{obs} & \text{if } W_i = 1 \end{cases}$

Causal inference as a missing data problem (1976)

- Unit-level causal effect (τ_i)
  - $\tau_i = Y_i(W_i = 1) - Y_i(W_i = 0)$
  - $\tau_i = \frac{Y_i(W_i=1)}{Y_i(W_i=0)}$
- With a single unit, we can at most observe a single potential outcome.
- Average causal effect (τ)
  - $\tau = \frac{1}{N} \sum_{i=1}^{N} [Y_i(W_i = 1) - Y_i(W_i = 0)]$
  - $\tau = \frac{1}{N} \sum_{i=1}^{N} \frac{Y_i(W_i=1)}{Y_i(W_i=0)}$

    where $\mathrm{N}=\mathrm{N}_T+\mathrm{N}_C$, $\mathrm{N}_T = \sum_{i=1}^{N} W_i$, and $\mathrm{N}_C = \sum_{i=1}^{N}(1 - W_i)$

Learning about causal effects

Although the definition of causal effects does not require more than one unit, learning about causal effects typically requires multiple units.

In other words, because with a single unit we can at most observe a single potential outcome, we must rely on multiple units to make causal inferences.

More specifically, we must observe multiple units, some exposed to the active treatment, some exposed to the alternative (control) treatment.

SUTVA

- Stable Unit Treatment Value Assumption (Rubin, 1980) : the potential outcomes are a function of the unit and treatment only.
- No interference between units (Cox, 1958)
  - The potential outcomes for any unit do not vary with the treatments assigned to other units.
- No hidden variations of treatments
  - For each unit, there are no different forms or versions of each treatment level, which lead to different potential outcomes.